Quantum Information
Week 2

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1 Review

$|u\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$
density matrix $\rho = |\phi\rangle\langle\phi|$
$\rho^\dagger = \rho$, $\mathrm{Tr}(\rho) = 1$.
$\forall\langle\psi|\rho|\psi\rangle \geq 0, \rho^2 = \rho \Rightarrow \rho(\rho-1) = 0$. Eigenvalue 0 or 1.
$\langle\psi|M|\psi\rangle = \mathrm{Tr}(\langle\psi|M|\psi\rangle) = \mathrm{Tr}(M|\psi\rangle\langle\psi|) = \mathrm{Tr}(M\rho)$. M is a measurement (projection).
Unitary operation. $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$. $H^\dagger H = \mathbf{1}$.
Tensor product $|\phi\rangle|0\rangle$.

2 Two-Qubit System

2.1 Notation

Tensor Product.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \otimes \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} & a_{12}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \\ a_{21}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} & a_{22}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \end{bmatrix}$$

Example. $|0\rangle_A|0\rangle_B \qquad \uparrow, |1\rangle_A|1\rangle_B \qquad \downarrow$. $|\psi\rangle_{AB} = a|0\rangle_A|0\rangle_B + b|1\rangle_A|1\rangle_B$. Then $|\psi\rangle_A = a|0\rangle + b|1\rangle$. Let $a = b = 1/\sqrt{2}$, then $|\psi\rangle_A = |+\rangle$.
$|+\rangle_A|+\rangle_B \qquad \rightarrow\rightarrow, |-\rangle_A|-\rangle_B \qquad \leftarrow\leftarrow$.

$|\psi'\rangle_{AB} = (|+\rangle|+\rangle + |-\rangle|-\rangle)/\sqrt{2}$. $|\psi'\rangle = (|+\rangle + |-\rangle)/\sqrt{2} = |0\rangle$. $|\psi\rangle_{AB} = |\psi'\rangle_{AB}$, but $|\psi\rangle_A = |\rangle'_A\rangle$.

EPR paradox. If Bob measure by σ_z and get $|1\rangle$ then Alice learned that with some probability, so some information transmits instantly (faster than the light speed). Actually, Alice can not rule out any case. no-signaling(stronger, must non-local), non-locality.

(partial trace)

$$\begin{aligned} \rho_A &= \mathrm{Tr}_B(\rho_{AB}) = \mathrm{Tr}_B((|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)(\langle 0|_A\langle 0|_B + \langle 1|_A\langle 1|_B)) \\ &= |0\rangle_A\langle 0|_A + |1\rangle_A\langle 1|_A = \frac{1}{2}\mathbf{1}. \end{aligned}$$

. $\rho_A^2 = \frac{1}{4}\mathbf{1} \neq \rho_A$. Mixed state. ($\rho^2 = \rho$ for pure state.)

Property 1. $\rho_A^\dagger = \rho_A$

2. $\forall |\psi\rangle, \langle\psi|\rho_A|\psi\rangle \geq 0$.

3. $\mathrm{Tr}(\rho_A) = \mathrm{Tr}_A(\mathrm{Tr}_B(\rho_{AB})) = \mathrm{Tr}(\rho_{AB}) = 1$

$\rho_A = a_0\sigma_1 + \ldots$ $\mathrm{Tr}(\rho_A) = 2a_0 = 1 \Rightarrow a_0 = \frac{1}{2}$, so $\rho_A = \frac{1}{2}(\sigma_1 + \vec{P}\cdot\vec{\sigma})$.