

Quantum Information

Week 2

Instructed by *Xiongfeng Ma*

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Wu Yijie 吴艺杰 JK30 2013011314

1 Review

$$|u\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle$$

$$\text{density matrix } \rho = |\phi\rangle\langle\phi|$$

$$\rho^\dagger = \rho, \text{Tr}(\rho) = 1.$$

$$\forall \langle\psi|\rho|\psi\rangle \geq 0, \rho^2 = \rho \Rightarrow \rho(\rho - I) = 0. \text{ Eigenvalue } 0 \text{ or } 1.$$

$$\langle\psi|M|\psi\rangle = \text{Tr}(\langle\psi|M|\psi\rangle) = \text{Tr}(M|\psi\rangle\langle\psi|) = \text{Tr}(M\rho). \text{ } M \text{ is a measurement (projection).}$$

$$\text{Unitary operation. } H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \quad H^\dagger H = \mathbf{1}.$$

$$\text{Tensor product } |\phi\rangle|0\rangle.$$

2 Two-Qubit System

2.1 Notation

Tensor Product.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \otimes \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} & a_{12} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \\ a_{21} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} & a_{22} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \end{bmatrix}$$

Example. $|0\rangle_A|0\rangle_B \quad \uparrow, |1\rangle_A|1\rangle_B \quad \downarrow. \quad |\psi\rangle_{AB} = a|0\rangle_A|0\rangle_B + b|1\rangle_A|1\rangle_B.$ Then $|\psi\rangle_A = a|0\rangle + b|1\rangle.$ Let $a = b = 1/\sqrt{2},$ then $|\psi\rangle_A = |+\rangle.$

$$|+\rangle_A|+\rangle_B \quad \rightarrow\rightarrow, |-\rangle_A|-\rangle_B \quad \leftarrow\leftarrow.$$

$$|\psi'\rangle_{AB} = (|+\rangle|+\rangle + |-\rangle|-\rangle)/\sqrt{2}. \quad |\psi'\rangle = (|+\rangle + |-\rangle)/\sqrt{2} = |0\rangle. \quad |\psi\rangle_{AB} = |\psi'\rangle_{AB}, \text{ but } |\psi\rangle_A = |\psi'\rangle_A.$$

EPR paradox. If Bob measure by σ_z and get $|1\rangle$ then Alice learned that with some probability, so some information transmits instantly (faster than the light speed). Actually, Alice can not rule out any case. no-signaling(stronger, must non-local), non-locality.

(partial trace)

$$\begin{aligned} \rho_A &= \text{Tr}_B(\rho_{AB}) = \text{Tr}_B(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)(\langle 0|_A\langle 0|_B + \langle 1|_A\langle 1|_B) \\ &= |0\rangle_A\langle 0|_A + |1\rangle_A\langle 1|_A = \frac{1}{2}\mathbf{1}. \end{aligned}$$

. $\rho_A^2 = \frac{1}{4}\mathbf{1} \neq \rho_A$. Mixed state. ($\rho^2 = \rho$ for pure state.)

Property 1. $\rho_A^\dagger = \rho_A$

2. $\forall |\psi\rangle, \langle\psi|\rho_A|\psi\rangle \geq 0$.

3. $\text{Tr}(\rho_A) = \text{Tr}_A(\text{Tr}_B(\rho_{AB})) = \text{Tr}(\rho_{AB}) = 1$

$\rho_A = a_0\sigma_1 + \dots$ $\text{Tr}(\rho_A) = 2a_0 = 1 \Rightarrow a_0 = \frac{1}{2}$, so $\rho_A = \frac{1}{2}(\sigma_1 + \vec{P} \cdot \vec{\sigma})$.