Problems

1. Based on 3-nearest neighbor, the data point, $x = 3.2$, has $(3, 5)$, $(3, 8)$, and $(2, 11)$ neighbors. By averaging the y value we can get,

$$g(x) = \frac{1}{3}\sum_{i=1}^{3} y_{[i]}(x) = 8$$

2. The examples map from $[x_1, x_2]$ to $[x_1, x_1x_2]$ coordinates as follows:

   $[-1, -1]$ (negative) maps to $[-1, +1]$

   $[-1, +1]$ (positive) maps to $[-1, -1]$

   $[+1, -1]$ (positive) maps to $[+1, -1]$

   $[+1, +1]$ (negative) maps to $[+1, +1]$

   In the parenthesis are XOR function. The positive examples have $x_1x_2 = -1$, the negative examples have $x_1x_2 = +1$. The maximum margin separator is the line x_1x_20, with the margin of 1.

   Transforming from x_1, x_1x_2 space back to x_1, x_2 the separator becomes either $x_1 = 0$ or $x_2 = 0$

3.

$$\begin{aligned} K(x_i, x_j) =& \Phi(x_i)\Phi(x_j) \\ =& (\Phi(x_i)\ - Phi(x_j))^2 \\ =& (\Phi(x_i))^2 - 2\Phi(x_i)\Phi(x_j) + (\Phi(x_j))^2 \\ =& K(x_i, x_i) + 2K(x_i, x_j) + K(x_j, x_j) \end{aligned} \tag{1}$$

   Based on the equation above, we conclude that a kernel function, which is used to compute squared Euclidean distance in the projected space, can be simplified to compute dot product of two transformed dimensions.

Problem

4.

$$XOR(AND(X_1, X_2), X_3)$$

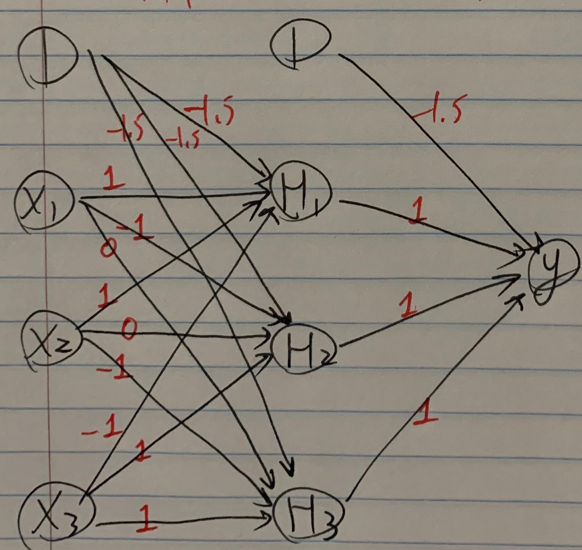
$$= X_1 \cap X_2 \oplus X_3$$

$$\Rightarrow p \oplus q = (p \wedge \neg q) \vee (\neg p \wedge q)$$

$$\Rightarrow (X_1 \cap X_2 \cap \neg X_3) \cup (\neg(X_1 \cap X_2) \wedge X_3)$$

$$= (X_1 \cap X_2 \cap \neg X_3) \cup ((\neg X_1 \cup \neg X_2) \wedge X_3)$$

$$= \underbrace{(X_1 \cap X_2 \cap \neg X_3)}_{H_1} \cup \underbrace{(\neg X_1 \wedge X_3)}_{H_2} \cup \underbrace{(\neg X_2 \wedge X_3)}_{H_3}$$



Collaboration Statement

I didn't collaborate with anyone for this assignment.