Homework 8: Partial Fractions II

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29. Give the appropriate PFD:

$$\frac{2x^2+3}{(x^2-8x+16)(x^2+3x+4)} = \frac{2x^2+3}{(x+4)(x+4)(x^2+3x+4)} = \boxed{\frac{A}{(x+4)} + \frac{B}{(x+4)^2} + \frac{Cx+D}{x^2+3x+4} + C}$$

32.

$$\int \frac{x+1}{x(x^2+4)}dx = \int \left(\frac{A}{x} + \frac{Bx+C}{x^2+4}\right)dx \qquad x+1 = Ax^2+4A+Bx^2+Cx \qquad A=\frac{1}{4}, B=-\frac{1}{4}, C=1$$

$$\int \left(\frac{1}{4x} + \frac{-\frac{x}{4}+1}{x^2+4}\right)dx = \boxed{\frac{x^2 - \ln\left|x^2+4\right| + 4\tan^{-1}\frac{x}{2}}{8} + C}$$

34. How do you decompose this fraction? It's completley irreducible already.

$$\int \frac{2x+1}{x^2+4}dx = \int \left(\frac{Ax+B}{x^2+4}\right)dx \qquad A=2, B=1$$

36*.

$$\int \frac{1}{(x^2+1)(x^2+2)}dx = \int \left(\frac{Ax+B}{x^2+2} + \frac{Cx+D}{x^2+1}\right)dx \qquad \boxed{\tan^{-1}x - \frac{\tan^{-1}\frac{x}{\sqrt{2}}}{\sqrt{2}} + C}$$

37. Determine if the following statements are true or false:
To evaluate $\int \frac{4x^6}{x^4+3x^2}dx$, the first step is to find the PFD of the integral.
Nope. The numerator has a higher degree than the denominator, so some means of division is necessary.
The easiest way to evaluate $\int \frac{6x+1}{3x^2+x}dx$ is with a PFD of the integrand.
Yes. The denominator breaks down very nicely to $x(3x+1)$, which lets us break down the fraction.
The rational function $\frac{1}{x^2-13x+42}$ has an irreducible quadratic denominator.
Nuh-uh! The denominator breaks down very nicely into $(x-6)(x-7)$.
The rational function $\frac{1}{x^2-13x+43}$ has an irreducible quadratic denominator.
Yep! There's no way to account for the irregular constant in the denominator.
39*. Find the area of the reigon bounded by $y = \frac{10}{x^2-2x-24}$, the x axis, and the bounds -2 and 2.

$$\int \frac{10}{x^2-2x-24}dx = 10\int\left(\frac{A}{x+4}+\frac{B}{x-6}\right) \qquad A=-\frac{1}{10}, B=\frac{1}{10} \qquad \int\left(-\frac{1}{x+4}+\frac{1}{x-6}\right) = \ln|6-x| - \ln|x+4| + C = f(x)$$

$$\left| f(x)|_{-2}^{2} \right| = |-\ln|6|| = \boxed{\ln 6}$$

50. Use polynomial long division.

$$\begin{array}{r|lllll} & x & & & & \\ \hline x^3+9 & x^4 & +0x^3 & +0x^2 & +0x & +1 \\ & x^4 & & & +9x & \\ & & & & -9x & +1 \\ & & & & & \end{array}$$

$$\int \frac{x^4+1}{x^3+9}dx = \frac{x^2}{2} + \frac{-9x+1}{x^3+9}$$

Again, I'm not really sure what can be done here. The fraction is completley irreducible.

51.

$$\begin{array}{r|lll} & 3 & & \\ \hline x^2-3x+2 & 3x^2 & +4x & -6 \\ & 3x^2 & -6x & +6 \\ & & 10x & -12 \\ & & & \end{array}$$

$$\int \frac{3x^2+4x-6}{x^2-3x+2}dx = 3x + \int \frac{10x-12}{(x-1)(x-2)}dx \qquad A=8, B=2 \qquad 3x+\int\left(\frac{8}{x-1}+\frac{2}{x-2}\right)dx = \boxed{3x - \ln|1-x| + 14\ln|2-x| + C}$$