Homework 6: Trigonometric Substitution

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8.

$$\int \frac{dx}{\left(9-x^2\right)^{\frac{3}{2}}} \qquad \theta = 3\sin x \quad d\theta = 3\cos x dx \qquad \int \frac{3\cos\theta}{\sqrt{\left(9-9\sin^2\theta\right)^3}} = \int \frac{3\cos\theta}{\sqrt{9\left(\cos^2\theta\right)^3}} = \frac{x}{9\sqrt{9-x^2}}+C \qquad \left.\frac{x}{9\sqrt{9-x^2}}\right|_0^{\frac{3}{2}} = \boxed{\frac{1}{9\sqrt{3}}}$$

9.

$$\int_5^{10} \sqrt{100-x^2}dx \qquad x = 10\sin u \quad dx = 10\cos u du \qquad \int \sqrt{100-\left(10\sin u\right)^2}\times 10\cos u du = 100\int \cos^2 u du = 100\int \left(\frac{1}{2}+\frac{\cos 2u}{2}du\right)$$

$$100\left(\frac{u}{2}+\frac{1}{2}\int \cos 2u du\right) \qquad w = 2u, dw = 2du \quad 100\left(\frac{u}{2}+\frac{\sin 2u}{2}\right) = 100\left(\frac{\sin^{-1}\frac{x}{10}}{2}+\frac{2\times\frac{x}{10}\sqrt{1-\left(\frac{x}{10}\right)}}{4}\right)+C = f\left(x\right)$$

$$f\left(x\right)\Big|_5^{10} = \boxed{\frac{25}{6}\left(4\pi-3\sqrt{3}\right)}$$

17.

$$\int \frac{dx}{\sqrt{36-x^2}} \qquad x = 6\sin u, dx = 6\cos u du \qquad \int \frac{6\cos u}{\sqrt{36-\left(6\sin u\right)^2}}du = \int \frac{6\cos u}{6\sqrt{\cos^2 x}} = \int du = \boxed{\sin^{-1}\frac{x}{6}+C}$$

23.

$$\int \frac{x^2}{\sqrt{16-x^2}}dx \qquad x = 4\sin u, dx = 4\cos u du \qquad \int \frac{4\cos u\left(4\sin u\right)^2}{\sqrt{16-\left(4\sin u\right)^2}}du = 16\int \sin^2 u du = 16\left(\int \frac{du}{2}-\int \frac{\cos 2u}{2}du\right)$$

$$w = 2u, dw = 2du \qquad \frac{1}{2}\int \cos w dw \qquad 16\left(\frac{u}{2}-\frac{\sin 2u}{4}\right) = \boxed{16\left(\frac{\sin^{-1}\frac{x}{4}}{2}-\frac{x\sqrt{1-\frac{x^2}{16}}}{8}\right)+C}$$

30.

$$\int \frac{x^4}{1+x^2}dx \qquad x=\tan u, dx=\sec^2 u du \qquad \int x^2 - dx + \int \frac{\sec^2 u}{\tan^2 u+1}du = \frac{x^3}{3} - x + \int du = \boxed{\frac{x^3}{3} - x + \tan^{-1} x + C}$$

33.

$$\int \frac{x^2}{\left(25+x^2\right)^2}dx = \boxed{\frac{\tan^{-1}\frac{x}{5} - \frac{5x}{25+x^2}}{10} + C} \qquad (???)$$

41.

$$\int \frac{dx}{\sqrt{x^2+16}} \qquad x=4\tan u, dx=4\sec^2 u du \qquad \int \frac{4\sec^2 u}{\sqrt{16\tan^2 u+16}}du = \int \sec u du = \ln\left(\sec u + \tan u\right) =$$

$$\boxed{\ln\left(\sqrt{1+\frac{x^2}{16}} + \frac{x}{4}\right) + C}$$

43.

$$\int \frac{dx}{\left(9x^2+1\right)^{\frac{3}{2}}} \qquad x=\frac{\tan u}{3}, dx=\frac{\sec^2 u}{3}du \qquad \int \frac{\sec^2 u}{\left(3\tan^2 u+1\right)^{\frac{3}{2}}}du = \int \frac{1}{3\sec u}du = \frac{1}{3}\int \cos u du = \frac{\sin u}{3} = \boxed{\frac{x}{\sqrt{1+9x^2}} + C}$$