Homework 11: Differental Equations

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9. These first two just needed basic integration.

$f'(t) = 3x^2 - 4x + 10 \quad f(0) = 20 \qquad x^3 - 2x^2 + 10x + C \quad f(x) = \boxed{x^3 - 2x^2 + 10x + 20}$

10.

$$f'(x) = 8e^{-4x} + 1 \quad f(0) = 5 \qquad \boxed{2e^{-4x} + x + 5}$$

15.

$f'(x) = -2f - 4 \qquad \int \frac{df}{-2f-4} = \int 1dx \quad -\frac{\ln|-2f-4|}{2} = x + C \quad \boxed{f(x) = -\frac{1}{2e^{2x+C}} - 2}$

17.

$f'(x) = 3f - 6 \quad y(0) = 9 \qquad \int \frac{df}{3y-6} = \int 1dx \quad \frac{\ln|3y-6|}{3} = x + C \quad y = \frac{e^{3(x+C)} + 6}{3} \quad C = \frac{\ln 21}{3} \quad \boxed{y = 7e^3}$

23.

$$\frac{dy}{dt} = \frac{3t^2}{y} \quad \int y dy = \int 3t^2 dt \quad \frac{y^2}{2} = x^3 + C \quad \boxed{y = \pm\sqrt{2}\sqrt{x^3 + C}}$$

24.

$$\frac{dy}{dx} = y\left(x^2 + 1\right) \quad \int \frac{dy}{y} = \int \left(x^2 + 1\right) dx \quad \ln|y| = \frac{x^3}{3} + x + C \quad \boxed{y = C^{\frac{x^3}{3} + x}}$$

25.

$f'(x) = e^{\frac{f}{2}} \sin x \quad \int \frac{df}{e^{\frac{f}{2}}} = \int \sin x dx \quad -\frac{2}{e^{\frac{f}{2}}} = -\cos x + C \quad \boxed{y = -2\ln|\cos x + C| + \ln|4|}$

28.

$$f'(x) = f(4x^3+1) \quad f(0) = 4 \quad \int \frac{df}{f} = \int \left(4x^3+1\right) dx \quad \ln|y| = x^4+x+C \quad y = Ce^{x^4+x} \quad \boxed{y = 4x^{x^4+x}}$$

30.

$$f'(x) = \frac{x^3}{\sec x} \quad f(0) = 3 \quad \int df = \int x^3 \cos x dx \quad y = \boxed{3\cos x\left(x^2-2\right) + x\left(x^2-6\right)\sin x + 9}$$

31.

$$\frac{dy}{dx} = e^x e^{-y} \quad y(0) = \ln 3 \quad \int e^y \frac{dy}{dx} = \int e^x \quad y = \ln|e^x + C| \quad \boxed{y = \ln\left(e^x+2\right)}$$