Homework 5: Trigonometric Integrals

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9.

$$\int \sin^2 x dx = \frac{1}{2}\int (1-\cos 2x)\, dx = \frac{1}{4}\int (1-\cos u)\, du = \frac{1}{4}(u-\sin u) + C = \boxed{\frac{x}{2} - \frac{\sin x}{4} + C}$$

11.

$$\int \sin^3 x dx = \int \sin^2 x \sin x dx = \int \left(1-\cos^2 x\right)\sin x dx = \int -1 + u^2 du = \frac{u^3}{3} - u + C = \boxed{\frac{\cos^3 x}{3} - \cos x + C}$$

13.

$$\int \sin^2 x \cos^2 x dx = \int \frac{\sin^2 2x}{4} = \int \frac{1-\cos 4x}{8} = \boxed{\frac{x}{8} - \frac{\sin 4x}{32} + C}$$

14.

$$\int \sin^3 x \cos^5 x dx = \int \cos x \sin^5 x - \sin^7 x = \int u^5 - u^7 du = \frac{u^6}{6} - \frac{u^8}{8} + C = \boxed{\frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + C}$$

16.

$$\int \sin^{-\frac{3}{2}} \cos^3 x dx = \boxed{\sin^{\frac{5}{2}} x \left(\frac{\cos 2x}{9} + \frac{13}{45}\right) + C}$$

19.

$$\int \tan^2 x dx = \int \sec^2 x - 1 dx = \boxed{\tan x - x + C}$$

20.

$$6\int \sec^4 x dx = 6\int \sec^2 x \sec^2 x dx = 6\int \left(\tan^2 x + 1\right)\sec^2 x dx = 6\left(\int \sec^2 x \tan^2 x dx + \int \sec^2 dx\right) = \boxed{2\tan^3 x + 6\tan x + C}$$

21.

$$\int \tan^3 4x dx = \int \tan^2 4x * \tan 4x dx = \int \left(\sec^2 4x - 1\right) \tan 4x dx = \int \left(\sec^2 4x\right) \tan 4x - \tan 4x dx = \boxed{\frac{\tan^2 4x}{8} - \frac{\ln|\sec 4x|}{4} + C}$$

25.

$$\int \sec^2 x \tan^{\frac{1}{2}} x = \boxed{\frac{2}{3} \tan^{\frac{3}{2}} x + C}$$

30.

$$\int \tan^5 x \sec^4 x = \int \tan^5 x \sec^2 x \sec^2 x dx = \int \tan^5 \left(1 + \tan^2 x\right) \sec^2 x dx = \int \left(\tan^5 x + \tan^7 x\right) \sec^2 x dx = \boxed{\frac{\tan^6 x}{6} + \frac{\tan^8 x}{8} + C}$$