Star Problems

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§ 1.8, #6

Prove using the notion, without loss of generality, that $5x + 5y$ is an odd integer when x and y are integers of opposite parity.

For the sake of argument, let x be odd and y be even. This means that the equation can be represented as $5(2a + 1) + 5(2b)$, for 2 integers a and b. We can rewrite this as $10a + 10b + 5$, or $2(5a + 5b) + 5$. Since $5a + 5b$ is an integer, this equation can be written as $2k + 5$, or $2k + 1 + 4$ for some integer k. Since 4 is even, we can further write this as $2k + 2j + 1$, where $j = 2$. Simplify this to $2(k + j) + 1$, and since $k + j$ is an integer, we can further simplify it to $2h + 1$, the definition of an odd integer.

§ 1.8, #20

Prove that given a real number x, there exist unique numbers n and a such that $x = n + a$, n is an integer, and $0 \leq a < 1$.

If we choose an integer b to be the greatest integer so that $b \leq x$ and let $a = x - b$, we know through the priciples of subtraction that $a \geq 0$. If $e \geq 1$, (which would violate uniqueness), then we can use subtraction to show $(a - 1) + 1 = x - n$, and $n + 1 = x - (a - 1)$. Problem is that this implies $n + 1 \leq x$, which we know isn't true. Therefore, a has to be somewhere between 0 and 1. We know they're unique by assuming 2 integers, c and d add up to x. We now know that $c + d = n + a$, and therefore that $c - n = a - d$. If $d \geq 0$ and $n < 1$, then $|a - d| < 1$. Since a and d are both integers, that means they'd have to be equal. Since $c - n = a - d = 0$, $c = n$ and $a = d$, which proves n and a are unique.