Homework 2

Alexander Gould

January 23, 2015

1. A and B are true. C is false.
3. Only A is true. (Although D should be...)
5A. There exists a student who spends more than five hours every weekday in class.
5B. All students spend more than five hours every weekday in class.
5C. There exists a student who doesn't spend more than five hours every weekday in class.
5D. No students spend more than five hours every weekday in class.
9A. $\exists x\,(P\,(x) \wedge Q\,(x))$
9B. $\exists x\,(P\,(x) \wedge \neg Q\,(x))$
9C. $\forall x\,(P\,(x) \vee Q\,(x))$
9D. $\forall x\,(\neg P\,(x) \wedge \neg Q\,(x))$
13. A is true because of the laws of addition, and 1 and 0 make B and C true. D is false because the statement doesn't hold for negative integers, yet the expression calls for all integers to fulfill it.
15. A is true, but B is false because $\sqrt{2}$ isn't an integer. C is true, but D is false. (All squares of integers are positive.)
17A. $P\,(0) \vee P\,(1) \vee P\,(2) \vee P\,(3) \vee P\,(4)$
17C. $\neg P\,(0) \vee \neg P\,(1) \vee \neg P\,(2) \vee \neg P\,(3) \vee \neg P\,(4)$
17E. $\neg P\,(0) \wedge \neg P\,(1) \wedge \neg P\,(2) \wedge \neg P\,(3) \wedge \neg P\,(4)$
3A. There exists a sender and a recipient.
3B. There is someone who has emailed everybody.
3C. Everyone has sent an email to someone.
3D. (At least) One person has gotten an email from everyone.
3E. Everyone has recieved an email from someone.
3F. Everyone has emailed everyone.
9A. $\forall x\,(L\,(x, Jerry))$
9B. $\forall x \exists y\,(L\,(x, y))$
9C. $\exists y \forall x\,(L\,(x, y))$
9D. $\neg\,(\exists x \forall y\,(L\,(x, y)))$
9E. $\exists y\,(\neg\,(L\,(Lydia, y)))$
29A. $P\,(1,1) \wedge P\,(1,2) \wedge P\,(1,3) \wedge P\,(2,1) \wedge P\,(2,2) \wedge P\,(2,3) \wedge P\,(3,1) \wedge P\,(3,2) \wedge P\,(3,3)$
29B. $P\,(1,1) \vee P\,(1,2) \vee P\,(1,3) \vee P\,(2,1) \vee P\,(2,2) \vee P\,(2,3) \vee P\,(3,1) \vee P\,(3,2) \vee P\,(3,3)$

29C. $(P(1,1) \wedge P(1,2) \wedge P(1,3)) \vee (P(2,1) \wedge P(2,2) \wedge P(2,3)) \vee (P(3,1) \wedge P(3,2) \wedge P(3,3))$

29D. $(P(1,1) \vee P(1,2) \vee P(1,3)) \wedge (P(2,1) \vee P(2,2) \vee P(2,3)) \wedge (P(3,1) \vee P(3,2) \vee P(3,3))$

30A. $\forall y \neg P(x,y)$

30B. $\exists x \forall y \neg (P(x,y))$

30C. $\neg \exists y (Q(y) \wedge \forall x \neg R(x,y)) \equiv \forall y \neg (Q(y) \wedge \forall x \neg R(x,y)) \equiv \forall y (\neg Q(y) \vee \neg \forall x \neg R(x,y)) \equiv \boxed{\forall y (\neg Q(y) \vee \exists x R(x,y))}$

30D. $\neg \exists y (\exists x R(x,y) \vee \forall x S(x,y)) \equiv \forall y \neg (\exists x R(x,y) \vee \forall x S(x,y)) \equiv \forall y (\neg \exists x R(x,y) \wedge \neg \forall x S(x,y)) \equiv \boxed{\forall y (\forall x \neg R(x,y) \wedge \exists x \neg S(x,y))}$

30E. $\neg \exists y (\forall x \exists z T(x,y,z) \vee \exists x \forall z U(x,y,z)) \equiv \forall y (\neg \forall x \exists z T(x,y,z) \wedge \neg \exists x \forall z U(x,y,z)) \equiv \boxed{\forall y (\exists x \forall z \neg T(x,y,z) \wedge \forall x \exists z \neg U(x,y,z))}$

33A. $\neg \forall x \forall y P(x,y) \equiv \boxed{\exists x \exists y \neg P(x,y)}$

33B. $\neg \forall y \exists x P(x,y) \equiv \boxed{\exists y \forall x \neg P(x,y)}$

33C. $\neg \forall y \forall x (P(x,y) \vee Q(x,y)) \equiv \exists y \exists x \neg (P(x,y) \vee Q(x,y)) \equiv \boxed{\exists y \exists x (\neg P(x,y) \wedge \neg Q(x,y))}$

33D. $\neg (\exists x \exists y \neg P(x,y) \wedge \forall x \forall y Q(x,y)) \equiv (\neg \exists x \exists y \neg P(x,y) \vee \neg \forall x \forall y Q(x,y)) \equiv \boxed{(\forall x \forall y P(x,y) \vee \exists x \exists y \neg Q(x,y))}$

33E. $\neg \forall x (\exists y \forall z P(x,y,z) \wedge \exists z \forall y P(x,y,z)) \equiv \exists x (\neg \exists y \forall z P(x,y,z) \vee \neg \exists z \forall y P(x,y,z)) \equiv \boxed{\exists x (\forall y \exists z \neg P(x,y,z) \vee \forall z \exists \neg y P(x,y,z))}$

34. $x = \{4\}\, y = \{5\}\, z = \{4\}$would make the statement true, $x = \{4\}\, y = \{5\}\, z = \{6\}$would make it false.

35. $x = \{1,2,3\}\, y = \{1,2,3\}\, z = \{1,2,3\}\, w = \{4\}$would make the statement true, $x = \{4\}\, y = \{4\}\, z = \{4\}\, w = \{4\}$ would make it false.