Homework 22: Alternating Series Estimation

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Use the Root Test to see whether the following series converge.

25. $\sum_{k=1}^{\infty} \left(\frac{1}{k}\right)^k$

As we get bigger, the summed term raised to its inverse approaches 0, which means we're absolutley convergent.

26. $\sum_{k=2}^{\infty} \left(\frac{k-1}{k}\right)^k$

Taking the limit of the summed term shows that it grows along with k, showing that the sum diverges.

Use the Comparison Test or Limit Comparison test to see if these series converge.

27. $\sum_{k=1}^{\infty} \frac{1}{k^2+4}$

The summed term is always less than $\frac{1}{k^2}$, which we know decreases to 0, which means the series converges.

28. $\sum_{k=1}^{\infty} \frac{k^2+k-1}{k^4+4k^2-3}$

This summed term will also always be less than $\frac{1}{k^2}$, which we know decreases to 0. This series also converges again.

29*. $\sum_{k=1}^{\infty} \frac{k^2-1}{k^3+4}$

This term will always be less than $\frac{1}{k}$, which decreases to 0, so this series diverges due to the Harmonic Series Test.

35. $\sum_{k=1}^{\infty} \frac{1}{2k-\sqrt{k}}$

This term will always be greater than $\frac{-1}{\sqrt{k}}$, which we know diverges.

37. $\sum_{k=1}^{\infty} \frac{\sqrt[3]{k^2+1}}{\sqrt{k^3+2}}$

This sieries diverges, as the summed term simply approaches $\frac{1}{\sqrt{2}}$

38*. $\sum_{k=2}^{\infty} \frac{1}{(k \ln k)^2}$

This summed term approaches 0, so we can say the series converges.