Homework 19: Ratio and Root Tests

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10.

$$\sum_{k=1}^{\infty}\left[2\left(\frac{3}{5}\right)^k+3\left(\frac{4}{9}\right)^k\right]=\sum_{k=1}^{\infty}2\left(\frac{3}{5}\right)^k+\sum_{k=1}^{\infty}3\left(\frac{4}{9}\right)^k=\frac{\frac{6}{5}}{1-\frac{3}{5}}+\frac{12}{9}=3+\frac{108}{45}=\frac{27}{5}$$

11.

$$\sum_{k=1}^{\infty}\left[\frac{1}{3}\left(\frac{5}{6}\right)^k+\frac{3}{5}\left(\frac{7}{9}\right)^k\right]=\sum_{k=1}^{\infty}\frac{1}{3}\left(\frac{5}{6}\right)^k+\sum_{k=1}^{\infty}\frac{3}{5}\left(\frac{7}{9}\right)^k=\frac{\frac{5}{18}}{1-\frac{5}{6}}+\frac{\frac{21}{45}}{1-\frac{7}{9}}=\frac{30}{18}+\frac{42}{50}=\frac{188}{75}$$

15. Does $\sum_{k=0}^{\infty}\frac{k}{2k+1}$ converge according to the Divergence Test?

No. The limit of the summed term is $\frac{1}{2}$,not 0.

18. How about $\sum_{k=0}^{\infty}\frac{k^2}{2^k}$?

Yep! Applying L'Hopital's Rule multiple times gives us $\frac{\lim_{x\to\infty}2^{1-x}}{\ln^2 2}$. This goes to 0.

19. How about $\sum_{k=0}^{\infty}\frac{1}{1000+k}$?

Should work. The summed term goes to 0.

20. How about How about $\sum_{k=0}^{\infty}\frac{k^3}{k^3+1}$.

Nope. The 1 in the denominator goes away and the limit is 1, not 0.

24. Does $\sum_{k=1}^{\infty}\frac{k}{\sqrt{k^2+4}}$ converge according to the Integral Test?

Some u-substitution shows us that the integral of the summed term grows with k. Therefore this series doesn't converge.

25. How about $\sum_{k=0}^{\infty}ke^{-2k^2}$?

This is another u-substitution problem. The e cancels out nicely and we're left with a definite integral of $\frac{1}{4}$, so that works!

26. How about $\sum_{k=1}^{\infty}\frac{1}{\sqrt[3]{k+10}}$?

We can expand it, but the integral grows with k, meaning this series doesn't converge.