Homework 25: Power Series-Interval and Radius of Convergence

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6. What is the radius of convergence of the power series $\sum c_k \left(\frac{x}{2}\right)^k$if the radius of convergence of $\sum c_k x^k$ is R?

It would also be R, but it would occur at a different point, because c_k remains the same.

Determine the radius and interval of convergence for the following.

10. $\sum(-1)^k \frac{x^k}{5^k}$

The only time we get a real result here is between 5 and -5, so that's the radius of convergence. The series diverges if $|x| > 5$.

12. $\sum(-1)^k \frac{k(x-4)^k}{2^k}$

In this case $\left|2 - \frac{x}{2}\right| < 1$, or the series won't converge, which means it only converges from 2 to 6, and diverges otherwise.

13. $\sum \frac{k^2 x^{2k}}{k!}$

$k!$ will always be greater than the numerator, which means that we'llalways go to 0, which means an infinite radius of convergence and an interval of convergence of all real numbers.

15. $\sum \frac{x^{2k+1}}{3^{k-1}}$

We know that the series converges between $-\sqrt{3}$ and $\sqrt{3}$, thanks to the ratio test. Test it on either end, and find divergence, so the radius of convergence is $\sqrt{3}$, and the series only converges if $|x| < \sqrt{3}$.

18. $\sum \frac{(-2)^k (x+3)^k}{3^{k+1}}$

$2\,|x+3| < 3$ if we want the series to converge, so that means the radius of convergence is 2. Test the endpoints, and we find that the interval of convergence goes from -5 to -1.

20. $\sum(-1)^k \frac{x^{3k}}{27^k}$

$|x| < 3$ if we want the series to converge, so that means the radius of convergence is 3. Test the endpoints, and we find that the interval of convergence goes from -3 to 3.