Homework 8: Partial Fractions II

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29. Give the appropriate PFD:

$$\frac{2x^2+3}{(x^2-8x+16)(x^2+3x+4)} = \frac{2x^2+3}{(x+4)(x+4)(x^2+3x+4)} = \boxed{\frac{A}{(x+4)} + \frac{B}{(x+4)^2} + \frac{Cx+D}{x^2+3x+4} + C}$$

32.

$$\int \frac{x+1}{x(x^2+4)}dx = \int \left(\frac{A}{x} + \frac{Bx+C}{x^2+4}\right)dx \qquad x+1 = Ax^2+4A+Bx^2+Cx \qquad A = \frac{1}{4}, B = -\frac{1}{4}, C = 1 \qquad \int \left(\frac{1}{4}\right.$$

34. How do you decompose this fraction? It's completley irreducible already.

$$\int \frac{2x+1}{x^2+4}dx = \int \left(\frac{Ax+B}{x^2+4}\right)dx \qquad A = 2, B = 1$$

36*.

$$\int \frac{1}{(x^2+1)(x^2+2)}dx = \int \left(\frac{Ax+B}{x^2+2} + \frac{Cx+D}{x^2+1}\right)dx \qquad \boxed{\tan^{-1}x - \frac{\tan^{-1}\frac{x}{\sqrt{2}}}{\sqrt{2}} + C}$$

37. Determine if the following statements are true or false:

To evaluate $\int \frac{4x^6}{x^4+3x^2}dx$, the first step is to find the PFD of the integral.

Nope. The numerator has a higher degree than the denominator, so some means of division is necessary.

The easiest way to evaluate $\int \frac{6x+1}{3x^2+x}dx$ is with a PFD of the integrand.

Yes. The denominator breaks down very nicely to $x(3x+1)$, which lets us break down the fraction.

The rational function $\frac{1}{x^2-13x+42}$ has an irreducible quadratic denominator.

Nuh-uh! The denominator breaks down very nicely into $(x-6)(x-7)$.

The rational function $\frac{1}{x^2-13x+43}$ has an irreducible quadratic denominator.

Yep! There's no way to account for the irregular constant in the denominator.

1

39*. Find the area of the reigon bounded by $y = \frac{10}{x^2 - 2x - 24}$, the x axis, and the bounds -2 and 2.

$$\int \frac{10}{x^2 - 2x - 24}\, dx = 10 \int \left(\frac{A}{x+4} + \frac{B}{x-6} \right) \qquad A = -\frac{1}{10}, B = \frac{1}{10} \qquad \int \left(-\frac{1}{x+4} + \frac{1}{x-6} \right) = \ln|6-x| - \ln$$

$$\left| f(x)\big|_{-2}^{2} \right| = |-\ln|6|| = \boxed{\ln 6}$$

50. Use polynomial long division.

$$
\begin{array}{r|lllll}
 & x & & & & \\
\hline
x^3 + 9 & x^4 & +0x^3 & +0x^2 & +0x & +1 \\
 & x^4 & & & +9x & \\
 & & & & -9x & +1 \\
\end{array}
$$

$$\int \frac{x^4 + 1}{x^3 + 9}\, dx = \frac{x^2}{2} + \frac{-9x + 1}{x^3 + 9}$$

Again, I'm not really sure what can be done here. The fraction is completley irreducible.

51.

$$
\begin{array}{r|lll}
 & 3 & & \\
\hline
x^2 - 3x + 2 & 3x^2 & +4x & -6 \\
 & 3x^2 & -6x & +6 \\
 & & 10x & -12 \\
\end{array}
$$

$$\int \frac{3x^2 + 4x - 6}{x^2 - 3x + 2}\, dx = 3x + \int \frac{10x - 12}{(x-1)(x-2)}\, dx \qquad A = 8, B = 2 \qquad 3x + \int \left(\frac{8}{x-1} + \frac{2}{x-2} \right) dx = \boxed{3x - \ln|1 -}$$