Homework 18: The Integral Test and P-Series

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2. Is it true as the terms of a positive series approach 0 that the series converges? Give an example.

Yes. As the sequence approaches 0, there's less and less to add, and the function levels off. $\left(0.6 + 0.06 + 0.006 + \ldots + 0.\overline{06} = \frac{2}{3}\right)$

3. Can the integral Test be used to determine is a series diverges?

Yep! If the integral diverges, so does the series.

4. For what values of p does the series $\sum_{k=1}^{\infty} \frac{1}{k^p}$ converge? For what values does it diverge?

It converges when p is positive. When p is negative, the thing osccilates and diverges.

23. Use the Integral Test to test the convergence of $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$.

$\int_2^{\infty} \frac{1}{x \ln x} dx$ doesn't converge to anything, which means the series diverges.

25. Use the Integral Test to test the convergence of $\sum_{k=1}^{\infty} k e^{-2k^3}$.

The integral of the summed term will eventually converge to something as the base is overpowered by the exponent, so this series converges.

28. Use the Integral Test to test the convergence of $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$.

This integral of the summed term gradually gets bigger and doesn't approach anything, so this series diverges.

30.

Use the Integral Test to test the convergence of $\sum_{k=2}^{\infty} \frac{1}{k \ln k \ln(\ln k)}$.

This integral of the summed term also gets bigger and doesn't approach anything, so this series diverges as well.

31. Determine the convergence or divergence of $\sum_{k=1}^{\infty} \frac{1}{k^{10}}$.

The exponent is bigger than 1. The series converges.

33. Determine the convergence or divergence of $\sum_{k=3}^{\infty} \frac{1}{(k-2)^4}$.

Doesn't matter what's in the denominator, if the exponent is bigger than 1, we converge.

43A. True or false? If a summed series starting at 1 converges, that same series starting at 10 converges.

True. Convergence has to do with the end of a sequence, not the beginning.

43B. True or false? If a summed series starting at 1 diverges, that same series starting at 10 diverges.

True. Dinvergence also has to do with the end of a sequence, not the beginning.

43C. If the sum of a_n converges, will the sum of $a_n + .000001$?

No. As you keep adding this new constant, the sequence gradually converges to a linear function, not a constant.

43D. If the sum of p^k diverges, will the sum of $(a_n + .000001)^k$?

Yes. The constant won't affect the behavior of the exponent.

43E. If the sum of p^k diverges, will the sum of $p^{k+.000001}$?

No. Eventually, the exponent will cross 1, and the series will converge.