Calculus 2

Professor Glaser

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Part I
Review

1 Basic Antiderivatives

Below are some antiderivatives you should be able to list at the drop of a hat. These are fundamental to any serious problem solving.

1.1 Exponents

$$\int x^n dx = \frac{x^{n+1}}{x+1} + C \, n \neq -1 \qquad \int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C \qquad \int a^x dx = \frac{a^x}{\ln a} + C \ (a > 0, a \neq 1)$$

1.2 Trig Functions

$$\int \cos x dx = \sin x + C \qquad \int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C \qquad \int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C \qquad \int \csc x \cot x dx = -\csc x + C$$

$$\int \tan x dx = \ln|\sec x| + C \quad (= \ln|\cos x| + C) \qquad \int \cot x dx = \ln|\sin x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C \qquad \int \csc x dx = -\ln|\csc x + \cot x| + C$$

1.3 Fractions

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{2\sqrt{x}} dx = \sqrt{x} + C \qquad \int \frac{dx}{x^2} = -\frac{1}{x} + C$$

2 U-Substitution

Again, you should know this stuff. Like, you should be able to do all of this without thinking. These problems are designed to be simple.

2.1 Basic U-Sub

$$\int 2x \left(x^2+10\right)^{98} dx$$

We assign u to the pain in the ass, so $u = x^2 + 10$ and $du = 2xdx$. Rewrite the equation so it's blantantly obvious and solve:

$$\int 2xdx \left(x^2+10\right)^{98} = \int u^{98} du = \frac{u^{99}}{99} + C$$

Then you substitute in the original like so:

$$\boxed{\frac{\left(x^2+10\right)^{99}}{99} + C}$$

Not so bad, right?

2.2 A bit harder

All right, what if the answer isn't glaringly obvious?

$$\int 2x \left(x^2+10\right)^{98} dx$$

This is also really easy. $u = x^2 + 10$ and $du = 2xdx$, so just divide until dx fits in nicely. Since $\frac{1}{2}du = xdx$, we can do the exct same thing, but with a new constant.

$$\int xdx \left(x^2+10\right)^{98} = \int \frac{1}{2} u^{98} du = \frac{1}{2} * \frac{u^{99}}{99} + C$$

Substitute again:

$$\boxed{\frac{\left(x^2+10\right)^{99}}{198}+C}$$

Again, not hard at all.

2.3 Bounded Integrals

What is there are bounds on the integral? This isn't hard, just more tedious.

$$\int_0^1 2x\left(x^2+2\right)^5 dx$$

Do the same thing with u:

$$u = x^2+2 \quad du = 2xdx$$

$$\int_?^? u^5 du$$

Great. Now that the equation is in terms of u, we have to recalculate the bounds. Since we already know the equation for u, shoudln't be that bad. Plug in 0 and 1 in this case and solve it out:

$$\int_2^3 u^5 du = \boxed{\frac{3^6}{6}-\frac{2^6}{6}}$$

Boom. Done.

2.4 Fractions

$$\int \frac{2xdx}{x^2+34}$$

If you set $u = x^2+34$ and $du = 2xdx$, this problem is actually stupidly simple. But you have to remember some log rules.

$$\int \frac{du}{u} = \ln|u| + C = \boxed{\ln\left|x^2+34\right|+C}$$

...Let's keep going.

2.5 e and Friends

$$\int xe^{x^2}dx$$

Remember, u handles the pain in the ass part. $u = x^2$and $\frac{1}{2}du = xdx$.

$$\frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \boxed{\frac{1}{2}e^{x^2}+C}$$

2.6 Trig

I'm too lazy to write commentary for this:

$$\int \frac{\cos x}{\sqrt{\sin x}}$$

$$u = \sin x \quad u = \cos x dx$$

$$\int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} = \boxed{2\sqrt{\sin x} + C}$$

3 Final Words

I swear, Alex, if you don't remember any part of this after learning it 4 times in your life, I want you to call Miguel and Carrie and Alexandra so they can yell at you. A lot.