Homework 9: Improper Integrals I

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5.

$$\int_1^\infty \frac{1}{x^2}dx = \lim_{n\to+\infty}\int_1^n \frac{dn}{n^2} = \lim_{n\to+\infty}\left(1-\frac{1}{n}\right) = \boxed{1}$$

9.

$$\int_0^\infty e^{-2x}dx = \lim_{n\to+\infty} -\frac{e^{-2n}}{2}\Big|_0^n = \lim_{n\to+\infty}\left(\frac{e^{-2n}}{2}-\frac{1}{2}\right) = \boxed{\frac{1}{2}}$$

10.

$$\int_1^\infty \frac{1}{x\ln x}dx = \lim_{n\to+\infty} \ln\ln x|_1^n = \lim_{n\to+\infty}(\ln\ln n+\infty)\boxed{\notin\mathbb{R}}$$

13.

$$\int_0^\infty e^{-x^2}dx = ???$$

15.

$$\int_2^\infty \frac{\cos\frac{\pi}{x}}{x^2}dx = \lim_{n\to+\infty} -\frac{\sin\frac{\pi}{x}}{\pi}\Big|_2^n = \lim_{n\to+\infty}\left(0+\frac{1}{\pi}\right) = \boxed{\frac{1}{\pi}}$$

20.

$$\int_1^\infty \frac{\tan^{-1}x}{x^2+1}dx = \lim_{n\to+\infty}\frac{\tan^{-2}x}{2}\Big|_1^n = \boxed{\frac{3\pi^2}{32}}$$

22. Find the volume when the reigon $\int_1^\infty x^{-2}dx$ is revolved around the x-axis.

$$2\pi * \lim_{n\to+\infty}\frac{1}{n}\Big|_1^n = 2\pi$$

25. Same as above, but with $\int_2^\infty \frac{1}{\sqrt{x}\ln x}dx$

$$\lim_{n\to+\infty} Ei\left(\frac{\ln x}{2}\right)\Big|_2^\infty \boxed{\notin\mathbb{R}}$$

54. Use integration by parts:

$$\int_1^\infty \frac{\ln x}{x^2}dx = \lim_{n\to+\infty} -\frac{\ln(x+1)}{x}\bigg|_1^n = (0-0) = \boxed{0}$$