Homework 4

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1. We can just prove this by proving every case. $1^2 \geq 2^1$, $2^2 \geq 2^2$, $3^2 \geq 2^3$, $4^2 \geq 2^4$

6. We can assume that x is the odd one here. Since y is even, and we know that an odd number and an even number make an odd number, $((2x+1)+2x = 4x+1$, if we let $k = 2x$, then we have $2k+1$, which is odd.) $5(x+y)$ (distributive property) is just 5 times an odd number. We know that odd*odd=odd. $((2r+1)*(2s+1) = 2(2rs+r+s)+1$, which has to be odd.)

8. Prove that there is a positive integer that equals the sum of the positive integers not exceeding it. Is your proof constructive or nonconstructive?

We can just find a witness that makes the existence clause in the statement true, $(3 = 1+2)$ which makes it a constructive proof.

14. Prove or disprove that if a and b are rational numbers, then a^b is also rational.

We can just prove this by example. $1^2 = 1$, so both sides of the statement are true.

18. Show that if r is an irrational number, there is a unique integer n such that the distance between r and n is less than $\frac{1}{2}$.

No matter what n is, we can subtract r from r, n and the irrational number, and this proof becomes, without any loss of generalization, "show that any irrational number r between 1 and 0 is less than $\frac{1}{2}$ away from one of the two. Since there's only one value between 0 and 1 where this is false, ($\frac{1}{2}$ is exactly $\frac{1}{2}$ away from both 0 and 1) and that number is rational, all irrational numbers have to satisfy the condition.

19. Show that if n is an odd integer, then there is a unique integer k such that n is the sum of $k-2$ and $k+3$.

$k-2+k+3 = 2k+1$. This is literally the definition of an odd integer.

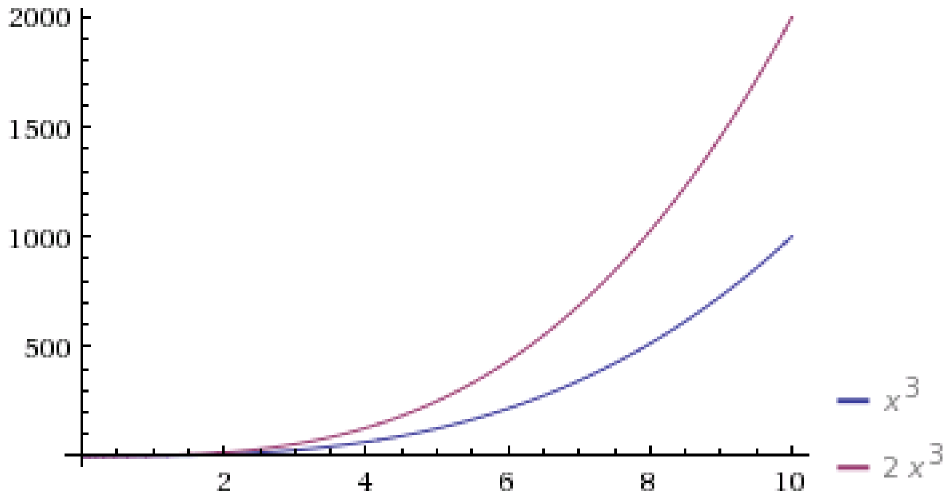
23. The **harmonic mean** of two real numbers x and y equals $\frac{2xy}{x+y}$. By computing the harmonic and geometric means of different pairs of positive real numbers, formulate a conjecture about their relative sizes and prove your conjecture.

#1	#2	HM	GM
1	2	$\frac{4}{3}$	$\sqrt{2}$
10	20	$\frac{40}{3}$	$10\sqrt{2}$
80	90	$\frac{1440}{17}$	$60\sqrt{2}$
100	120	$\frac{1200}{11}$	$20\sqrt{30}$

Without loss of generality, we already have all 3 means in terms of x and y. If we move some terms around, we can get $\frac{h}{2} = \frac{xy}{x+y}$, $2a = x + y$ and $g^2 = xy$. Do some substitutions and we get $\frac{h}{2} = \frac{g^2}{2a}$. Multiply by 2 and you get $h = \frac{g^2}{a}$.

34. Prove that $\sqrt[3]{2}$ is irrational.

We prove this by contradiction. We assume that $\sqrt[3]{2}$ is rational, and therefore the quotient of 2 rational numbers, a and$b$. Cube both sides and we get $2 = \frac{a^3}{b^3}$. $2b^3 = a^3$. This is impossible, as seen below.



The lines will never intersect.

40. Verify the $3x + 1$ conjecture for these integers:

16, 8, 4, 2, 1

11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1

113, 340, 170, 85, 256, 128, 64, 32, 16, 8, 4, 2, 1