Homework 7: Partial Fractions I

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6.

$$\boxed{x - 9 = A\,(x - 3) + B\,(x + 6)}$$

8.

???

9.

$$\int \frac{dx}{(x-1)(x-2)} = \int \left(\frac{A}{x-1} + \frac{B}{x-2}\right) \qquad A(x-2) + B(x-1) \quad A = -1, B = 1 \qquad \int \left(\frac{-1}{x-1} + \frac{1}{x-2}\right) =$$

$$\boxed{-\ln|x-1| + \ln|x-2| + C}$$

11.

$$\int \frac{3dx}{x^2-1} = 3\int \left(\frac{A}{x+1} + \frac{B}{x-1}\right) \qquad A = -\frac{1}{2}, B = \frac{1}{2} \qquad 3\left(-\frac{1}{2}\int \frac{1}{x+1} + \frac{1}{2}\int \frac{1}{x-1}\right) = \boxed{\ln\left|\frac{x-1}{x+1}\right|^{\frac{3}{2}} + C}$$

13.

???

14.

$$\int \frac{3dx}{x^3 - x^2 - 12x} = 3\int \left(\frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-4}\right) \qquad A = -\frac{1}{12}, B = \frac{1}{21}, C = \frac{1}{28} \qquad 3\left(-\frac{1}{12}\int \frac{1}{x}dx + \frac{1}{21}\int \frac{1}{x+3}dx + \frac{1}{28}\int \frac{1}{x-4}dx\right)$$

$$\boxed{3\left(-\frac{\ln|x|}{12} + \frac{\ln|x+3|}{21} + \frac{\ln|x-4|}{28}\right) + C}$$

19.

$$\int \frac{3dx}{x^3 - 9x^2} = 3\int \left(\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-9}\right) = \boxed{3\left(\frac{x\left(\ln|9-x| - \ln|x| + 9\right)}{81x}\right) + C}$$

23.

$$\int \frac{2dx}{x^3 + x^2} = \boxed{2\left(-\frac{1}{x} - \ln|x| + \ln|x+1|\right) + C}$$

24.

$$\int \frac{2dt}{t^3(t+1)} = 2\left(\int \frac{1}{x^3} - \int \frac{1}{x^2} + \int \frac{1}{x} - \int \frac{1}{x+1}\right) = \boxed{2\left(\frac{1}{2x^2} - \frac{1}{x} + \ln|x| - \ln|x+1|\right) + C}$$

25.

$$\int \frac{x-5}{x^2(x+1)}dx \qquad x - 5 = Ax^2(x+1) + Bx^2 + Bx + Cx^3 \qquad \boxed{\frac{5}{x} + \ln\left|\frac{x}{x+1}\right|^6 + C}$$