

§1.7, #18

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What is wrong with this argument? Let $S(x, y)$ be “ x is shorter than y .” Given the premise $\exists sS(s, Max)$, it follows that $S(Max, Max)$. Then by existential generalization it follows that $\exists xS(x, x)$, so that someone is shorter than himself.

There are 2 main problems here:

1. It doesn't follow at all that Max can replace s . All the \exists sign says is that there's *somebody* who's shorter than Max. That someone doesn't have to be Max. If instead of \exists , we had \forall , then it would logically follow, because the statement has to hold for everybody. But \exists means that we only need at least one person to meet the criteria, and that person doesn't have to be Max.
2. The one person in the group who meets the criteria can't be Max at all. An essential precondition for $S(x, y)$ being true is that the two arguments are different, so they can be compared to each other. If you compare an object to itself, it fails that precondition, which means $S(Max, Max)$ has to be false.