Homework 6

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3A. $A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$
3B. $A \cap B = \{3\}$
3C. $A - B = \{1, 2, 4, 5\}$
3D. $B - A = \{0, 6\}$
14. $A = \{1, 3, 5, 6, 7, 8, 9\}$, $B = \{2, 3, 6, 9, 10\}$

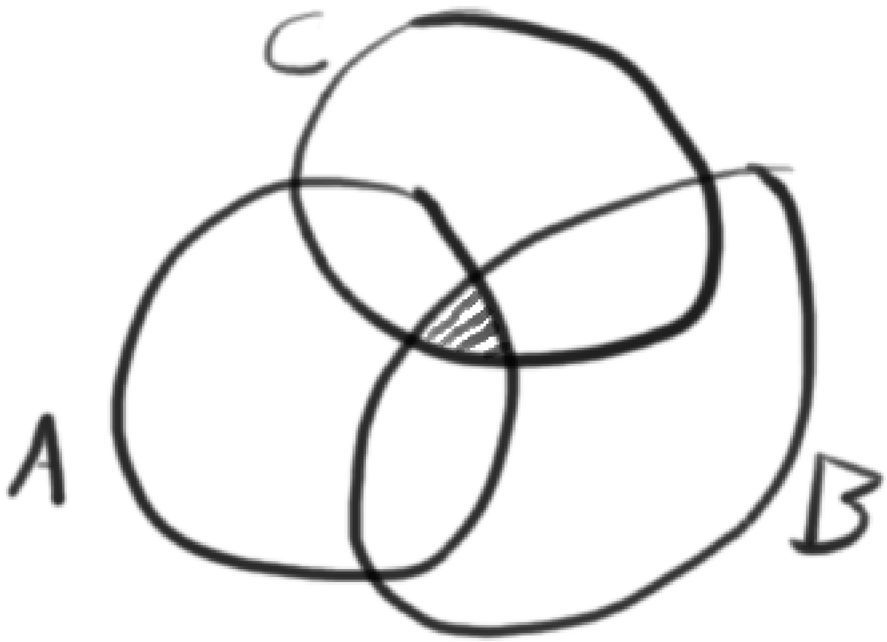
15A. Any $x \in \overline{A \cap B}$ is obviously $\notin A \cap B$, meaning $x \notin A \vee x \notin B$, or $x \in \overline{A} \vee x \in \overline{B}$. This means that $x \in \overline{A} \cup \overline{B}$, which shows that $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$. Any $x \in \overline{A} \cup \overline{B}$ is obviously $x \in \overline{A} \vee x \in \overline{B}$, meaning $x \notin A \vee x \notin B$, or $x \in \overline{A} \vee x \in \overline{B} \notin A \cap B$. This means that $x \in \overline{A \cap B}$, which shows that $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$.

18E. $x \in (B - A) \cup (C - A)$ translates to $x \in (B - A) \vee x \in (C - A)$. We can further expand this to $(x \in B \wedge x \notin A) \vee (x \in C \wedge x \notin A)$. We can then move the A outside and get $(x \in B \vee x \in C) \wedge x \notin A$. This becomes $x \in B \cup C \wedge x \notin A$. We can use the definition of set subtration to get to $x \in (B \cup C) - A$.

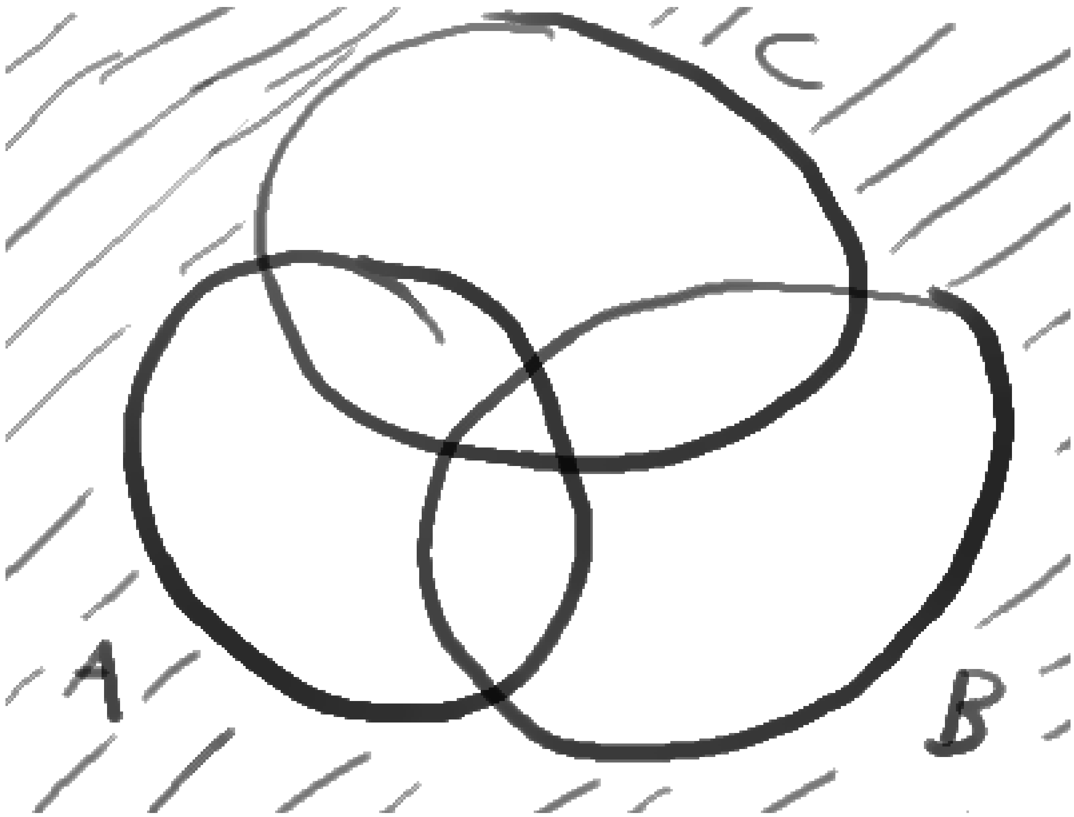
19A. $x \in (A - B)$ can become $x \in A \wedge x \notin B$. We can rewrite this in set notation as $x \in A \cap \overline{B}$. Going the other way, $x \in A \cap \overline{B}$ becomes $x \in A \wedge x \notin B$, which becomes $x \in (A - B)$.

19B. First we rewrite $x \in (A \cap B) \cup \big(A \cap \overline{B}\big)$ as $(x \in A \wedge x \in B) \vee (x \in A \wedge x \notin B)$. We can "factor out" the $x \in A$, leaving us with $x \in A \wedge (x \in B \vee x \notin B)$. The second condition will always be true, and since it's one of 2 parts of an AND, we can just eliminate it and evaluate the other statement, leaving $x \in A$.

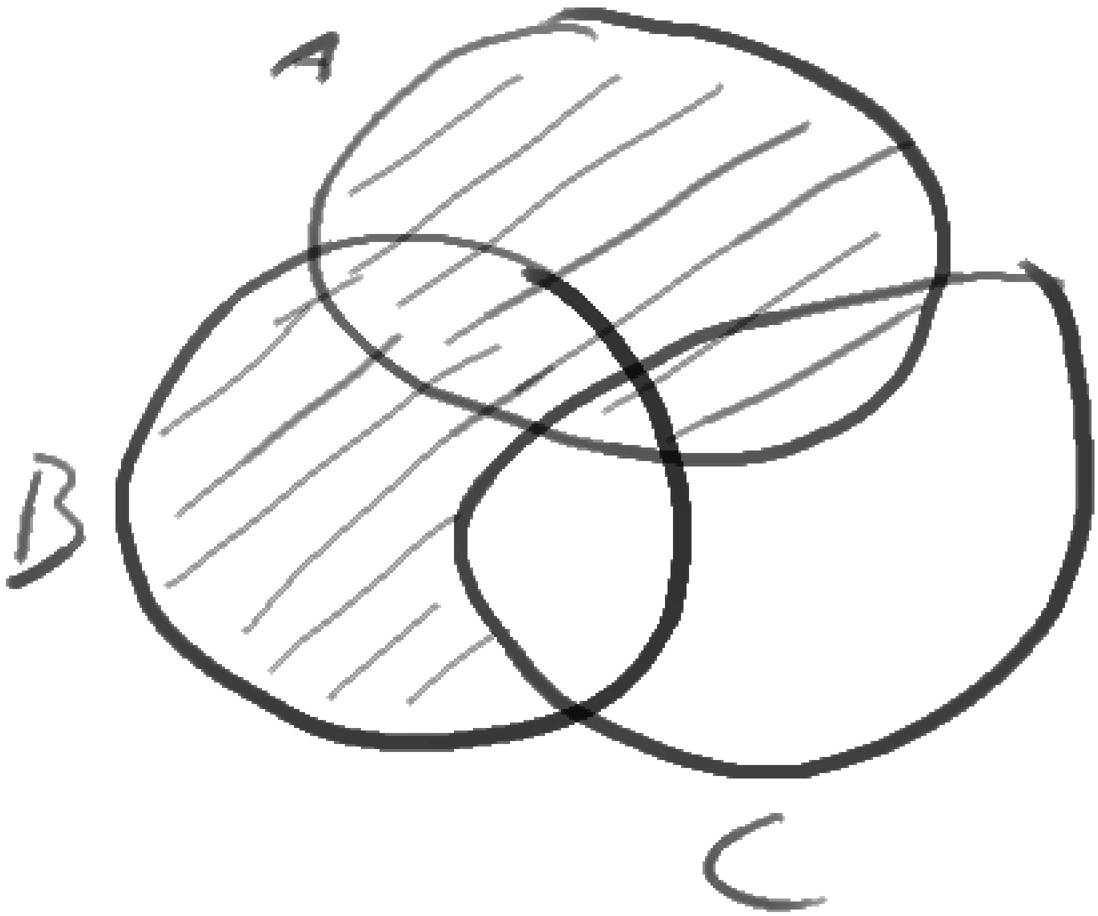
26A.



26B.



26C.



29A. If A is equivelant to the union of A and B, we know that B is empty.

29B. If the intersection of A and B is A, we know A is a subset of B.

29C. If A-B is A, we know A and B don't intersect.

29D. Knowing that the intersection of A and B and the intersection of B and A are the same doesn't tell us anything. Intersection is commutative.

29E. When $A - B = B - A$, we know that sets are equal.

32. $\{2, 5\}$

35. $x \in A \bigoplus B$ means that $(x \in A \vee x \in B) \wedge (x \notin A \vee x \notin B)$. Use DeMorgan's law to simplify this to $(x \in A \vee x \in B) \wedge \neg(x \in A \wedge x \in B)$. Rewrite this as $x \in A \cup B \wedge \neg(x \in A \cap B)$, and use the definition of set subtraction to get $x \in (A \cup B) - (A \cap B)$

41. Yes. In order for 2 set pairs to have the same symmetric difference, both their unions and intersections must be identical. That can only happen if you're comparing the same 2 sets.

50A. The union would contain every number from 1 to ∞. The intersection would be empty.

50B. The union would contain every number from 0 to ∞. The intersection would contain 0.

50C. The union would contain every number from 0 to ∞. The intersection would contain 1.

50D. The union would contain every number from 1 to ∞. The intersection would be empty/contain only ∞.