Homework 21: Alternating Series

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Determine whether the following series converge.

11. Nuke the alternator of $\sum \frac{(-1)^{k+1}}{k^3}$ and we get $\frac{1}{k^3}$, which decreases to 0. Therefore this sum converges.

12. Exactly the same as #11. Get rid of the alternator $\sum \frac{(-1)^k}{k^2+10}$ and take the limit of $\frac{1}{k^2}$ once the constant goes away and the series converges.

13. The alternator of $\sum(-1)^{k+1}\frac{k^2}{k^3+1}$ is already separated for us. When we remove the constant and take the limit of $\frac{k^2}{k^3}$, we see it decreases to 0 and the series converges.

14. Same here. $\sum(-1)^k \frac{\ln k}{k^2}$ becomes $\frac{\ln k}{k^2}$, which increases to 0 as the denominator overpowers the numerator, so the series converges.

15. Killing the alternator, getting rid of the constants and taking the limit of $\frac{k^2}{k^2}$ gets us 1, not 0, which means the series diverges.

16. Getting rid of the alternator here gives us the limit of $\frac{1}{5}^k$, which decreases to 0, and the series converges.

17. Another easy alternator removal. Take the limit of $1+\frac{1}{k}$, limit of a sum is the sum of the limits, and we converge to 1, so the series doesn't converge.

18. We can't just take the alternator out, but we can see that the numerator endlessly alternates, so we can just get rid of it like it's a standard alternator. $\frac{1}{k^2}$ goes to 0, so the series converges.

20. Knock the alternator out of $\sum \frac{(-1)^k}{k \ln^2 k}$ and we get $\frac{1}{k \ln^2 k}$, which approaches 0, so the series converges.

23. Knock out the alternator again and take the limit of $\frac{1}{\sqrt{x^2+4}}$, which goes to 0, so the series converges.