Homework 17: Infinite Series

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47. Find the formula for the nth term of $\sum_{k=-1}^{\infty}\left(\frac{1}{k+1}-\frac{1}{k+2}\right)$. Then find its limit.

The nth term is $1-\frac{1}{n+2}$. As $x \to \infty$, we approach 1.

51. Find the formula for the nth term of $\sum_{k=-1}^{\infty}\left(\ln\frac{k+1}{k}\right)$. Then find its limit.

The nth term is $\ln((n+1)!)-\ln(n!)$, which means that the series dierges.

59A. Is $\sum_{i=1}^{\infty}\left(\frac{\pi}{e}\right)^{-k}$ a convergent geometric series? Explain why or why not.

We can tell that the values of the summed term approach 0, so we can conclude that the sum approaches a constant.

59B. If the sum of a series in terms of k starts at $k=12$ and converges, will it still converge if the sum starts at $k=1$?

Yes. Convergence has to do with the end of the series, not the beginning.

59C. If $\sum_{i=1}^{\infty}a^k$ converges, will $\sum_{i=1}^{\infty}b^k$ converge if $|a|<|b|$?

Yes. Bases don't matter, exponents do.

9.

$$\sum_{k=0}^{\infty}\left[3\left(\frac{2}{5}\right)^k-2\left(\frac{5}{7}\right)^k\right]$$

The limit of the summed term as $k\to\infty$ is 0 thanks to the continuity of k at infinity making the term go to $\infty-\infty=0$.