Direct Collocation for Quantum Optimal Control

Aaron Trowbridge, Aditya Bhardwaj, Kevin He, David I. Schuster, & Zachary Manchester

Robotic Exploration Lab

schuster.lab

Quantum Optimal Control Review

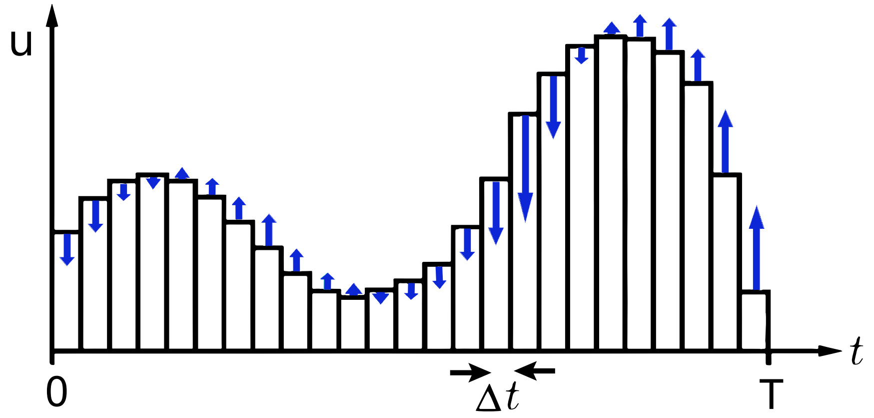
Know the dynamics of my system $$i\frac{\partial|\psi\rangle}{\partial t} = H(\mathbf{u}(t))|\psi\rangle$$

Achieve $$|\psi_0\rangle \rightarrow |\psi_{\text{target}}\rangle$$

Minimize $$J = 1 - |\langle\psi_{\text{final}}|\psi_{\text{target}}\rangle|^2$$

Procedure

1. Rollout
2. Calculate gradient of cost function w.r.t controls
3. Update controls with gradient
4. Repeat



GRAPE and Other Indirect Methods

Question: What if you care about things besides the fidelity?

$$J = 1 - |\langle \psi_{\text{final}} | \psi_{\text{target}} \rangle|^2 + \sum_i w_i \text{cost}_i$$

Answer: Cost function shaping

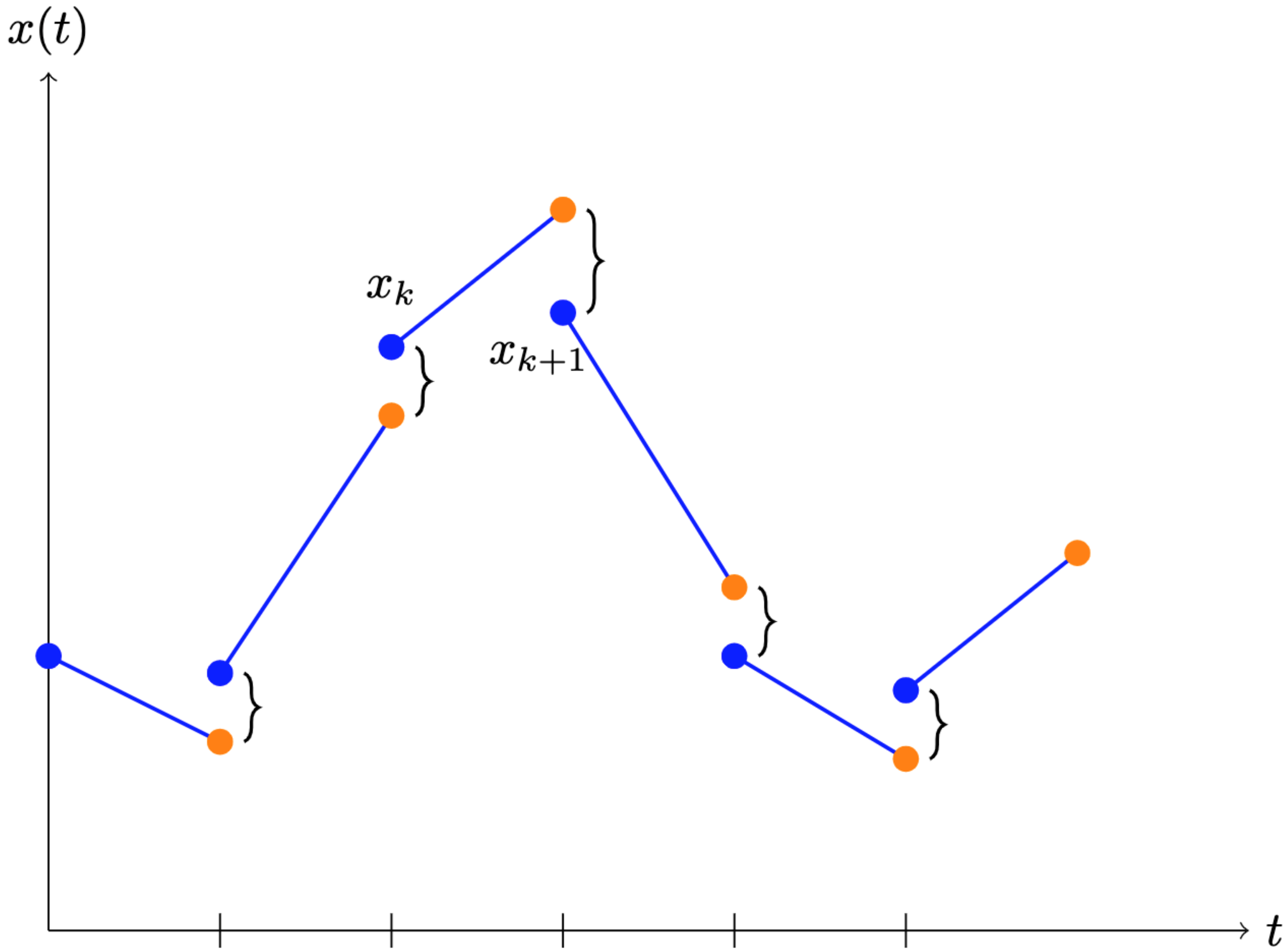
Direct Methods

Treat states and controls as decision variables

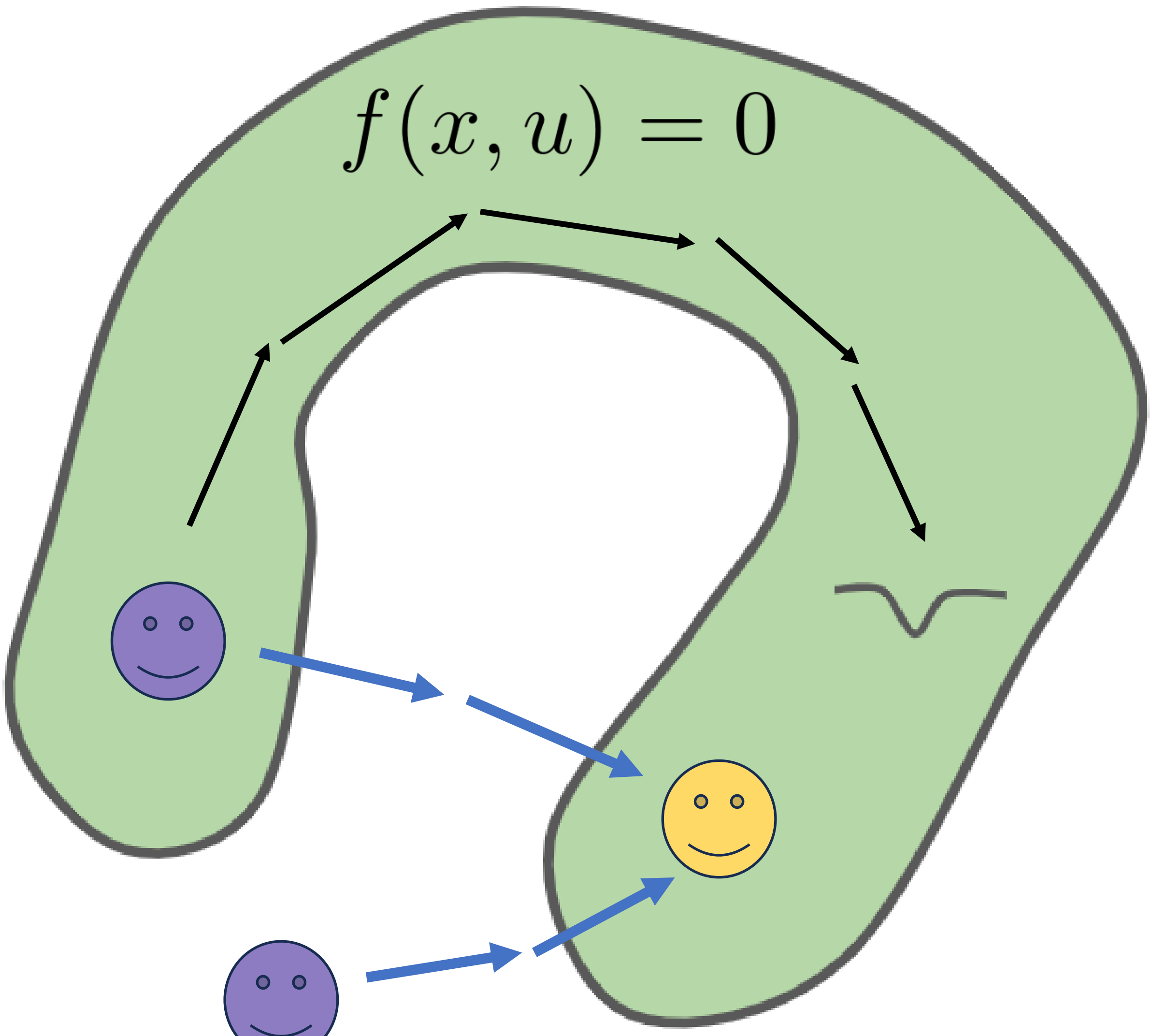
$$
\begin{aligned}
\underset{x,u}{\text{minimize}} \quad & J(x) = 1 - \mathcal{F}(x_N, x_{\text{goal}}) \\
\text{subject to} \quad & f(x_{k+1}, x_k, u_k) = 0 \\
& c(x, u) \geq 0 \\
& d(x, u) = 0
\end{aligned}
$$

Solve using nonlinear program solver (IPOPT)

Direct Collocation



Direct Collocation



Padé Approximants

Taylor Series

$$\exp(A) \approx T_4(A) = 1 + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \frac{A^4}{4!}$$

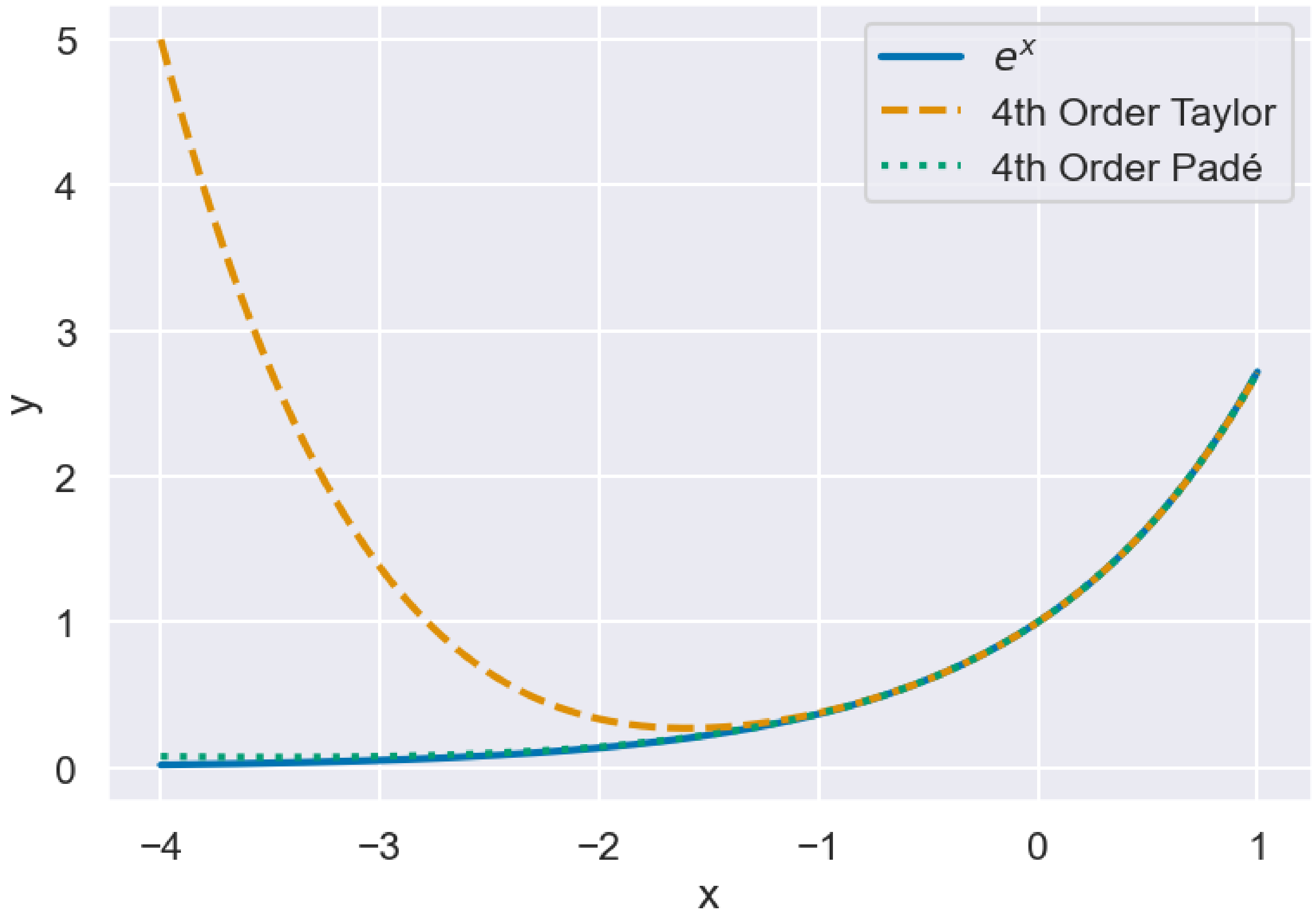
Padé Approximant

$$\exp(A) \approx \frac{F_2(A)}{B_2(A)} = \frac{b_0 + b_1 A + b_2 A^2}{1 + c_1 A + c_2 A^2}$$

Demand

$$\frac{F_2(A)}{B_2(A)} = T_4(A) \xrightarrow[\text{coefficients}]{\text{match}} \frac{F_2(A)}{B_2(A)} = \frac{1 + \frac{1}{2}A + \frac{1}{12}A^2}{1 - \frac{1}{2}A + \frac{1}{12}A^2}$$

Padé Approximants



Padé Integrator Collocation (PICO)

$$
\begin{aligned}
0 &= f(x_{k+1}, x_k, u_k) \\
&= x_{k+1} - \exp(-i\Delta t H(u_k))x_k \\
&\approx x_{k+1} - B(u_k)^{-1}F(u_k)x_k \\
&= B(u_k)x_{k+1} - F(u_k)x_k
\end{aligned}
$$

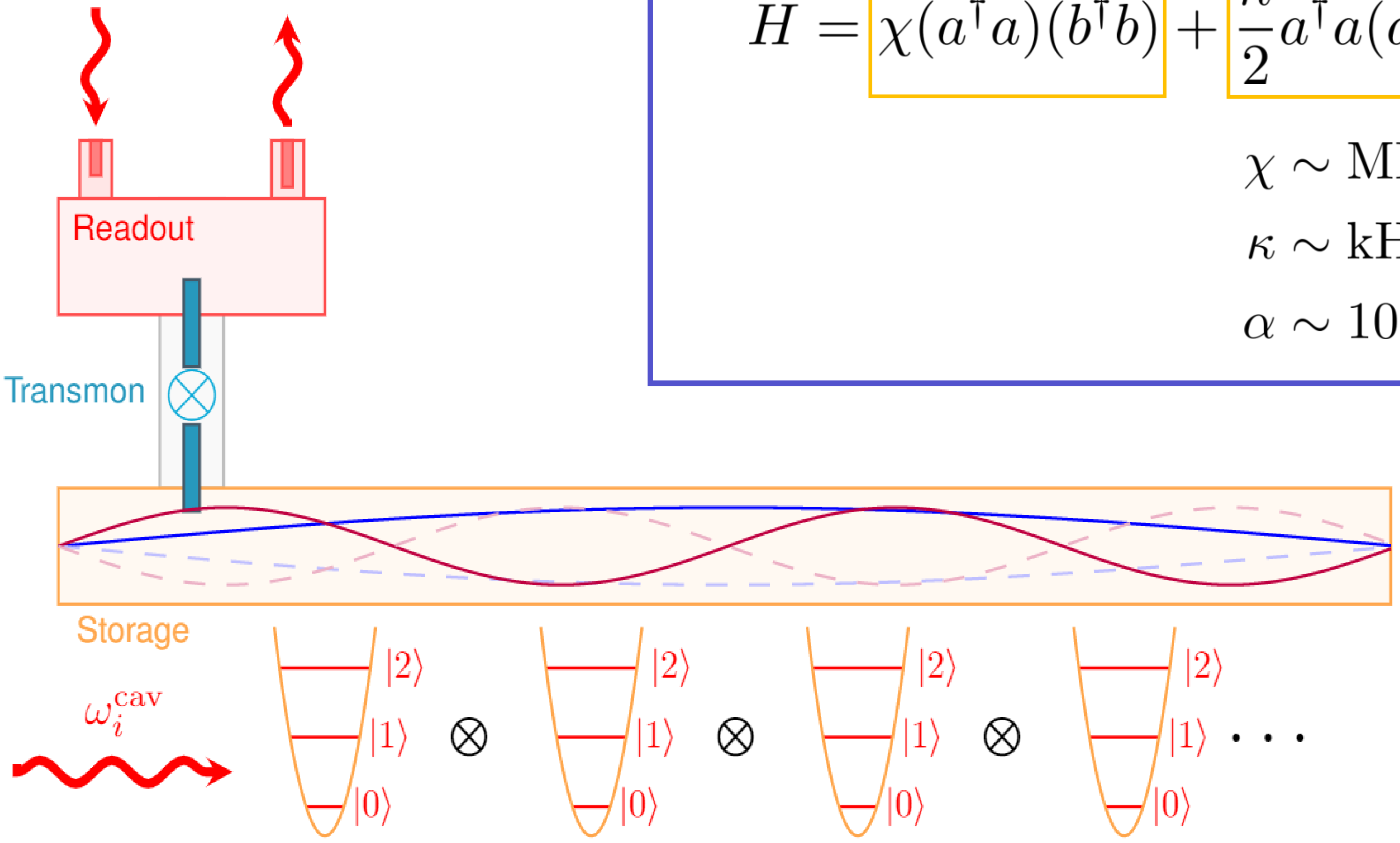
Hardware System

$$H = \boxed{\chi(a^\dagger a)(b^\dagger b)} + \boxed{\frac{\kappa}{2} a^\dagger a(a^\dagger a - I)} - \boxed{\frac{\alpha}{2} b^\dagger b(b^\dagger b - I)}$$

$\chi \sim \mathrm{MHz}$

$\kappa \sim \mathrm{kHZ}$

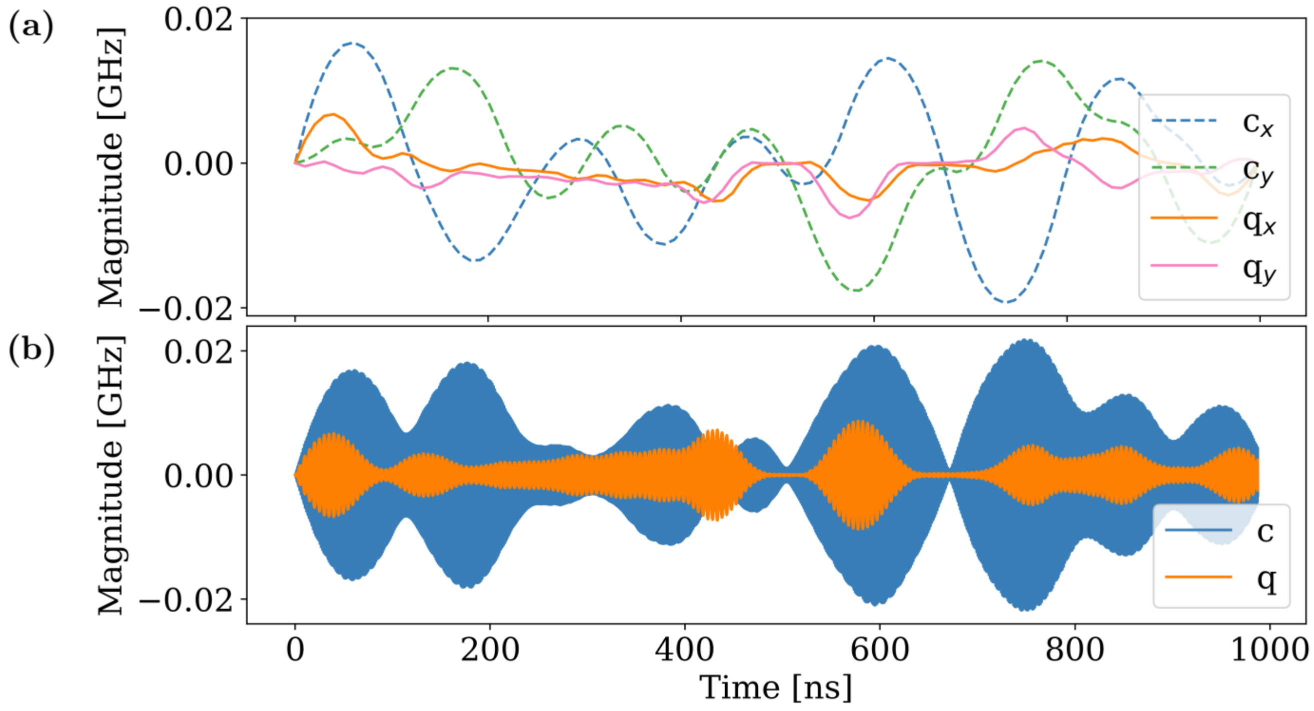
$\alpha \sim 100\ \mathrm{MHz}$

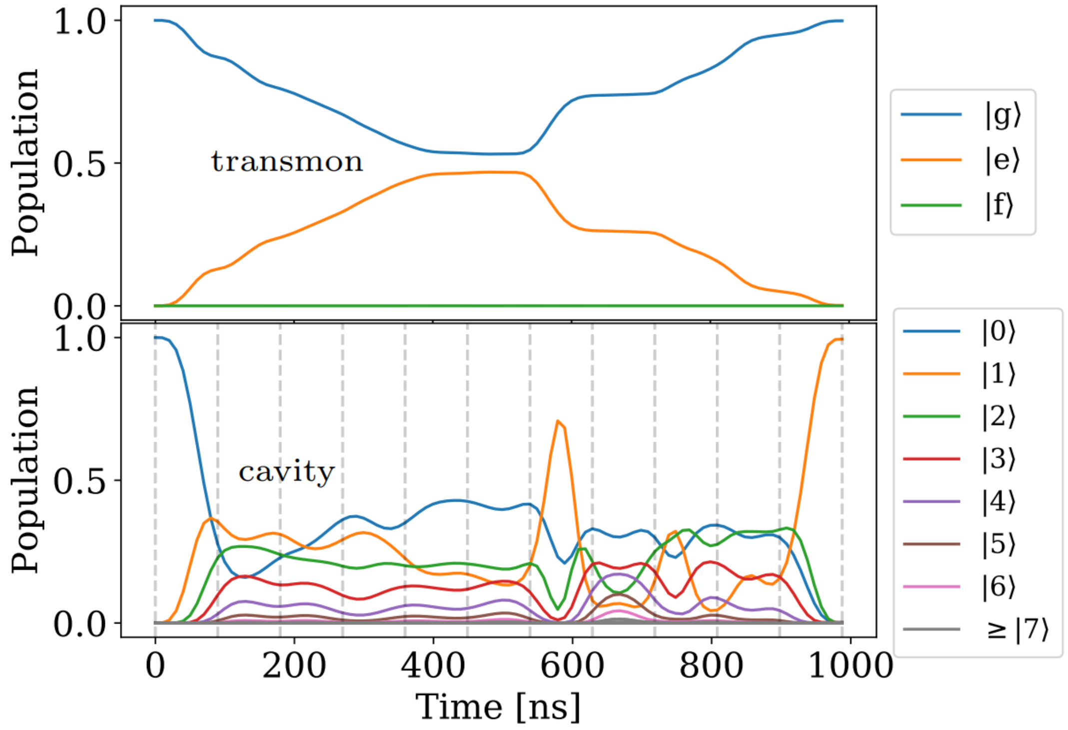


$$|g0\rangle \rightarrow |g1\rangle$$

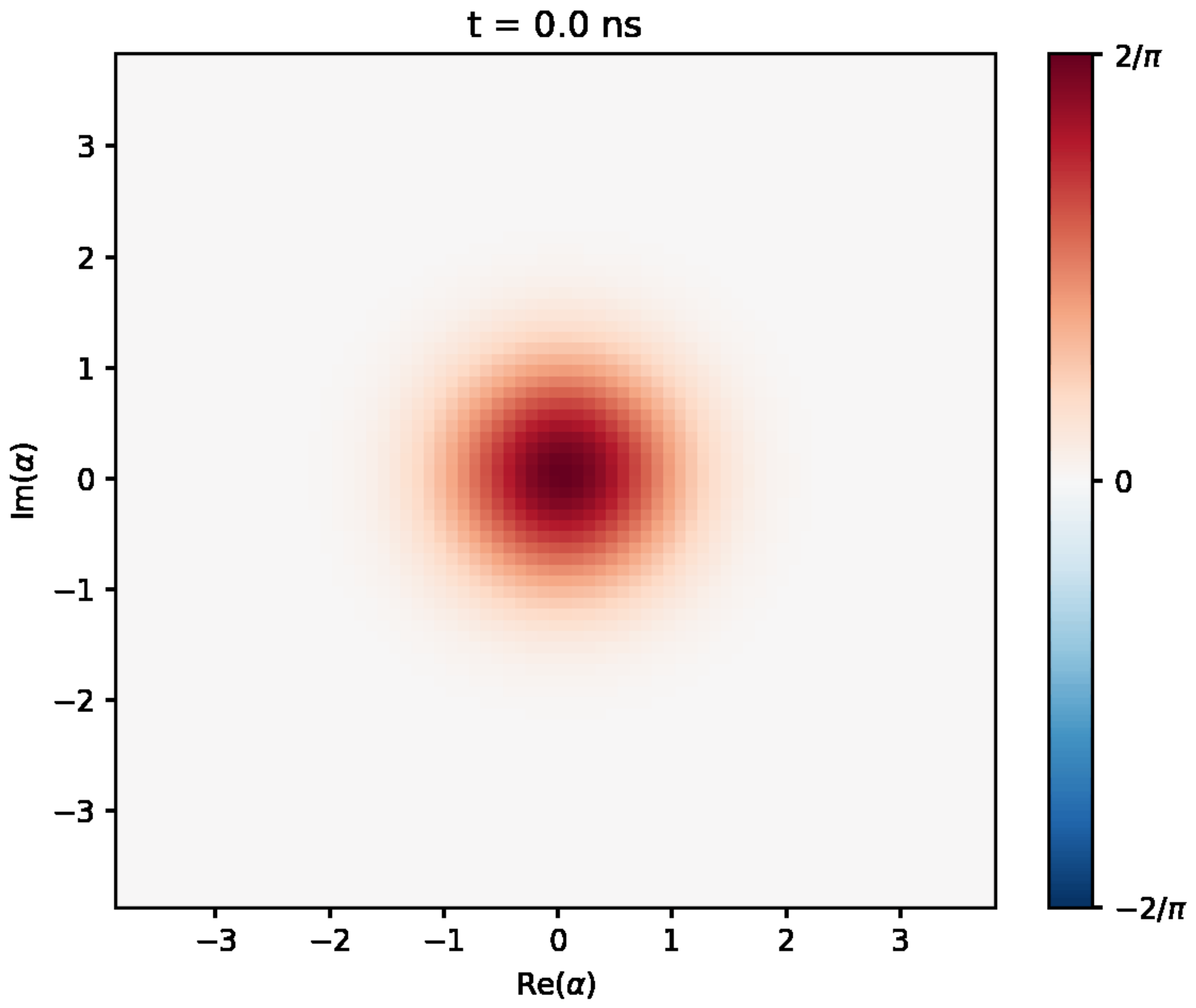
Hardware Result

$$|g0\rangle \rightarrow |g1\rangle$$





$$\text{Fidelity} = 0.988 \pm 0.011$$


t = 0.0 ns
Im(α)
Re(α)
3
2
1
0
−1
−2
−3
2/π
0
−2/π

Direct vs. Indirect Methods

Direct	Indirect

Direct vs. Indirect Methods

	Direct	Indirect
Dynamics	Implicit and parallelizable	Explicit and requires rollouts

Direct vs. Indirect Methods

	Direct	Indirect
Dynamics	Implicit and parallelizable	Explicit and requires rollouts
State constraints	Easily implemented	More difficult

Direct vs. Indirect Methods

	Direct	Indirect
Dynamics	Implicit and parallelizable	Explicit and requires rollouts
State constraints	Easily implemented	More difficult
Initial guess	More freedom	Dynamically constrained

Direct vs. Indirect Methods

	Direct	Indirect
Dynamics	Implicit and parallelizable	Explicit and requires rollouts
State constraints	Easily implemented	More difficult
Initial guess	More freedom	Dynamically constrained
Convergence	Great tail convergence	Poor tail convergence

Overclocking Quantum Collocation

Free time problem

Free time problem

$$\underset{x,u,\Delta t}{\text{minimize}} \quad \ell(x_N)$$

Free time problem

$$\underset{x,u,\boxed{\Delta t}}{\text{minimize}} \quad \ell(x_N)$$

Free time problem

$$\underset{x,u,\Delta t}{\text{minimize}} \quad \ell(x_N)$$

Free time problem

$$\begin{aligned} \underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \end{aligned}$$

Free time problem

$$\underset{x,u,\Delta t}{\text{minimize}} \quad \ell(x_N)$$

$$\text{subject to} \quad \boxed{f(x_{k+1}, x_k, u_k, \Delta t_k) = 0}$$

Free time problem

$$\begin{aligned} \underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \end{aligned}$$

Minimum time problem

$$
\begin{aligned}
\underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) \\
\text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0
\end{aligned}
$$

Minimum time problem

$$\begin{aligned} \underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) + \sum_k \Delta t_k \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \end{aligned}$$

Minimum time problem

$$\begin{aligned} \underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) + \boxed{\sum_k \Delta t_k} \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \end{aligned}$$

Minimum time problem

$$\begin{aligned}\underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) + \sum_k \Delta t_k \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0\end{aligned}$$

Minimum time problem

$$
\begin{aligned}
\underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) + \sum_k \Delta t_k \\
\text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \\
& \mathcal{F}(x_N) \geq \bar{\mathcal{F}}
\end{aligned}
$$

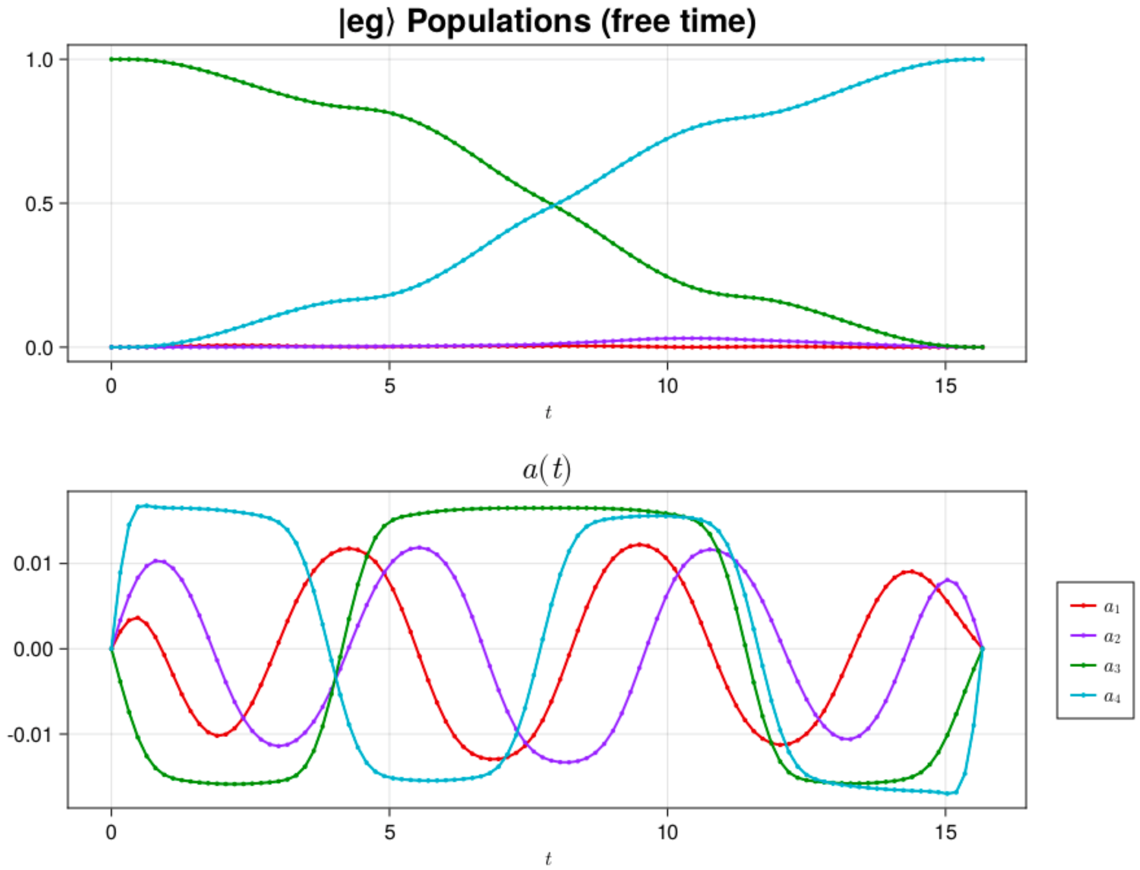
Minimum time problem

$$\begin{aligned} \underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) + \sum_k \Delta t_k \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \\ & \boxed{\mathcal{F}(x_N) \geq \bar{\mathcal{F}}} \end{aligned}$$

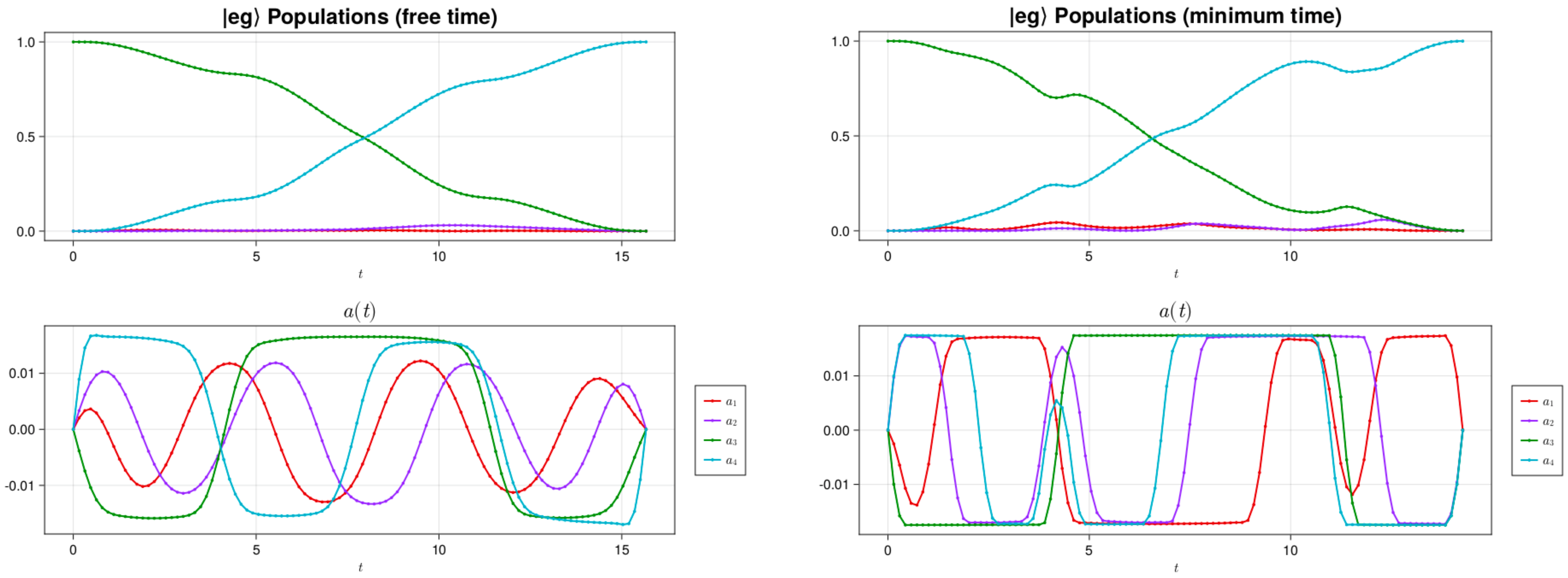
Minimum time problem

$$
\begin{aligned}
\underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) + \sum_k \Delta t_k \\
\text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \\
& \mathcal{F}(x_N) \geq \bar{\mathcal{F}}
\end{aligned}
$$

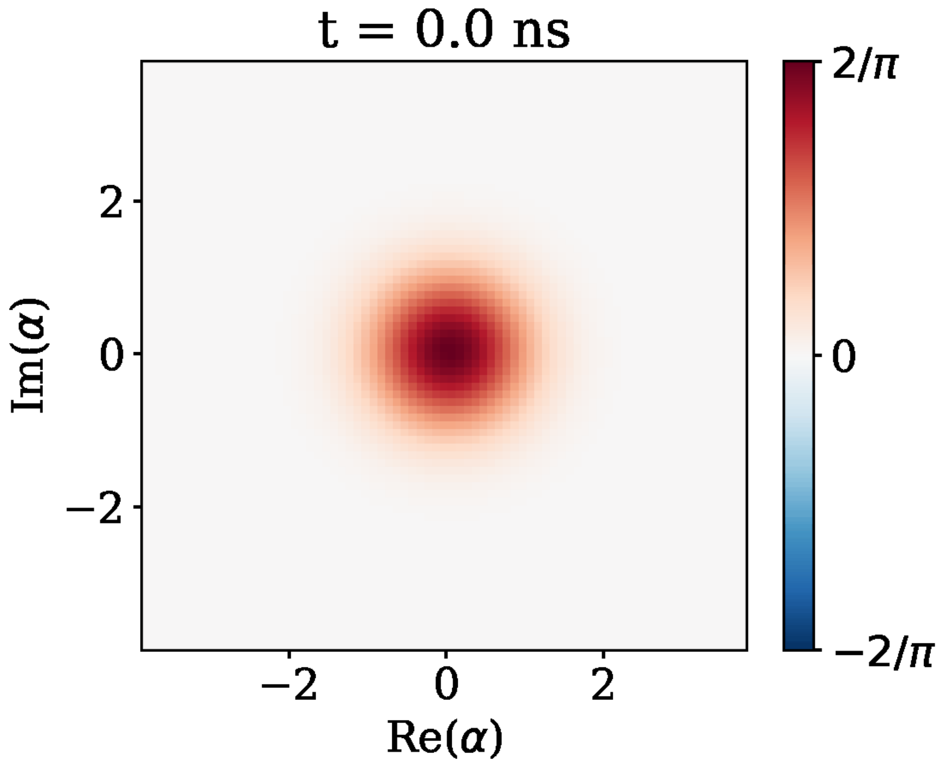
Minimum time problem



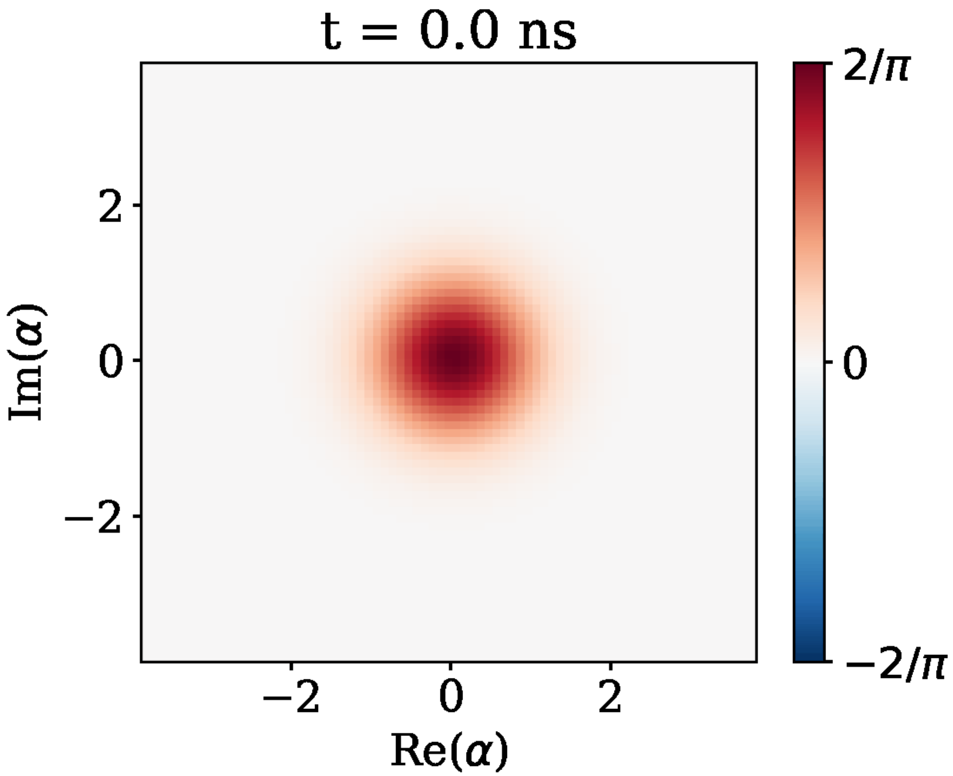
Minimum time problem



Minimum time problem



PICO minimum time pulse



GRAPE pulse

Leakage Suppression

$$\begin{aligned} \underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \end{aligned}$$

Leakage Suppression

$$\begin{aligned} \underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \\ & \|x_{\text{guard}}\| < \epsilon \end{aligned}$$

Leakage Suppression

$$\begin{aligned} \underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \\ & \boxed{\|x_{\text{guard}}\| < \epsilon} \end{aligned}$$

Leakage Suppression

$$\begin{aligned} \underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \\ & \|x_{\text{guard}}\| < \epsilon \end{aligned}$$

Leakage Suppression

$$\begin{aligned} \underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \end{aligned}$$

Leakage Suppression

$$\begin{aligned} \underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) + \|x_{\text{guard}}\|_1 \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \end{aligned}$$

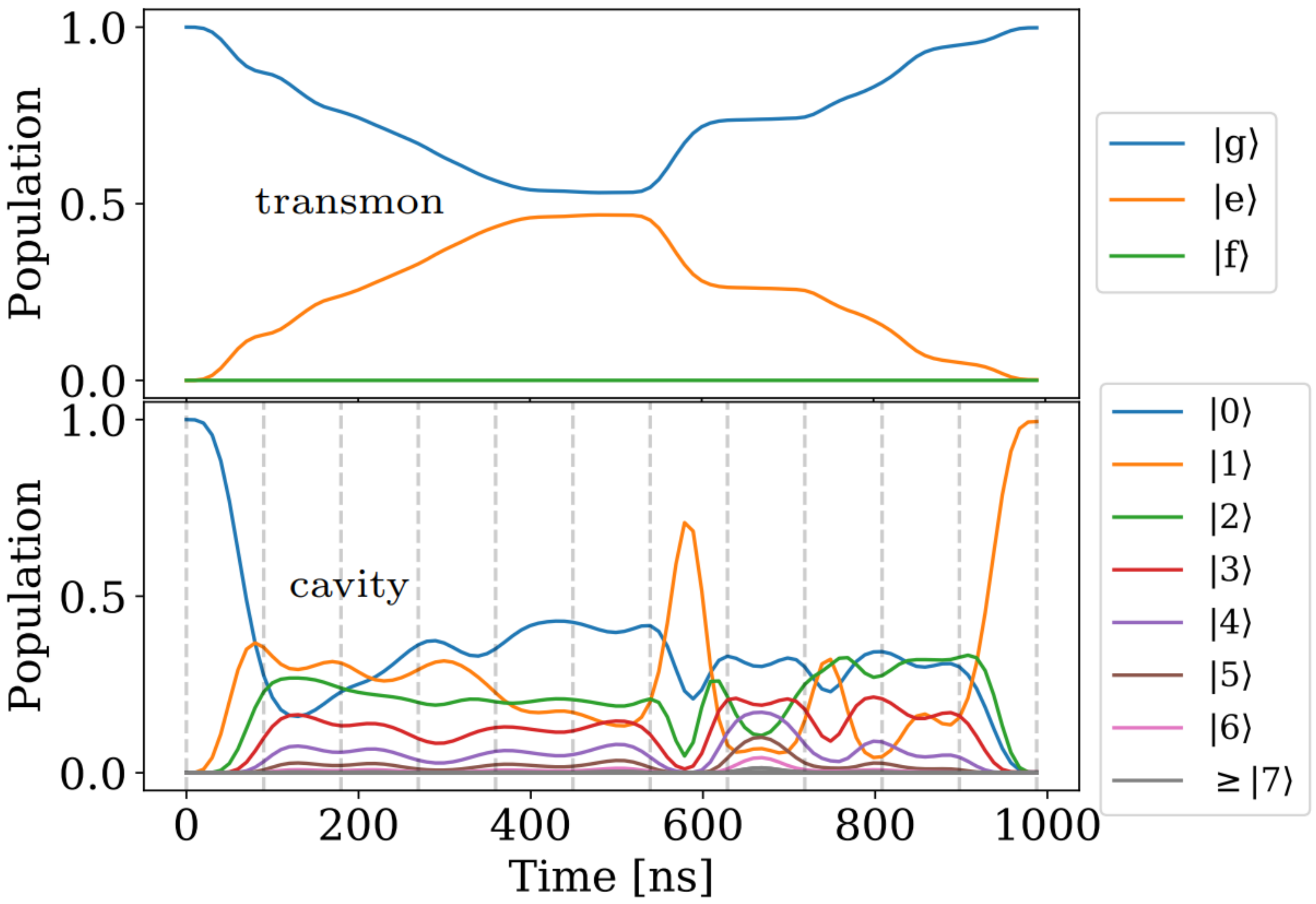
Leakage Suppression

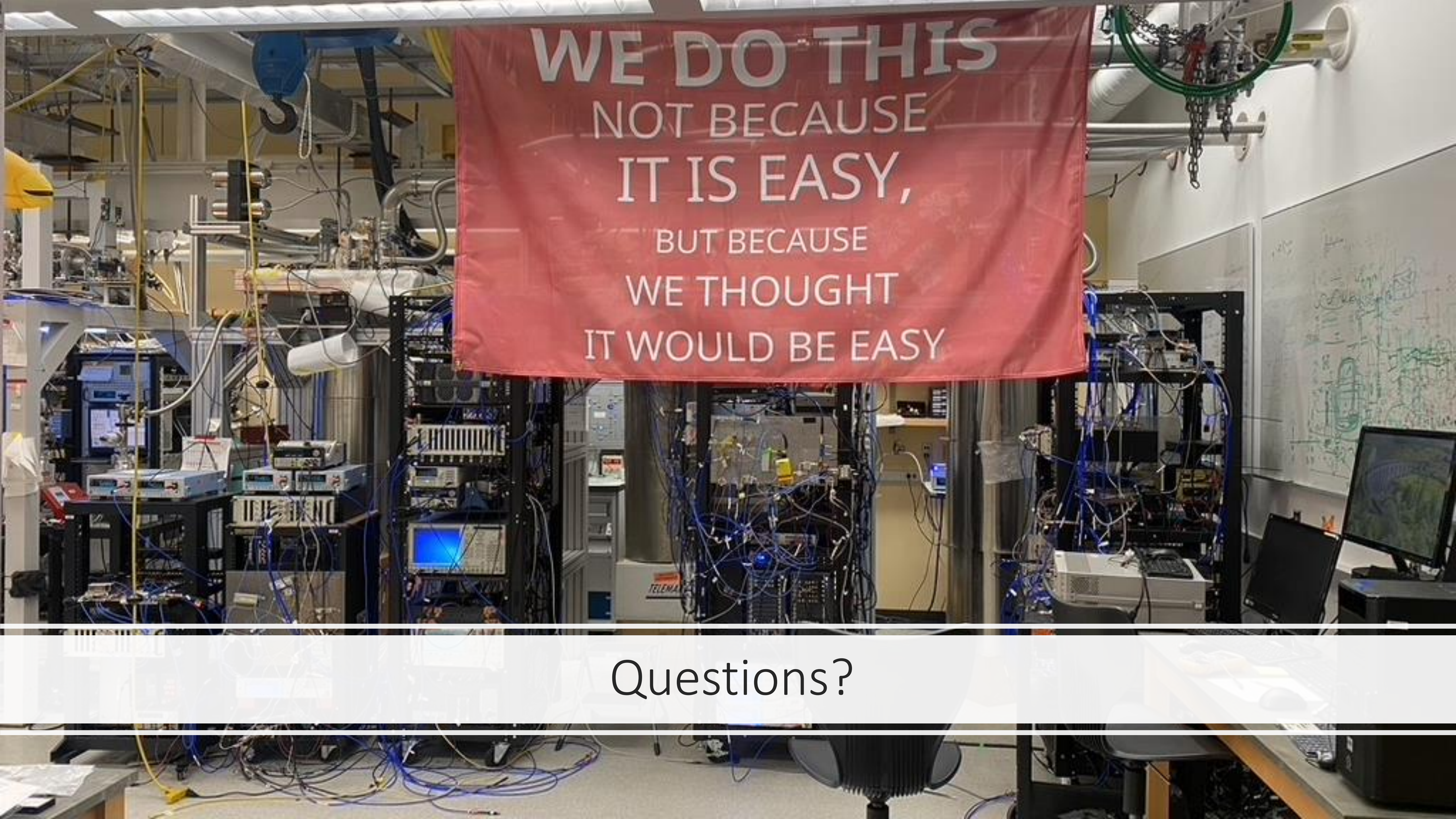
$$\begin{aligned} \underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) + \boxed{\left\| x_{\text{guard}} \right\|_1} \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \end{aligned}$$

Leakage Suppression

$$\begin{aligned} \underset{x,u,\Delta t}{\text{minimize}} \quad & \ell(x_N) + \|x_{\text{guard}}\|_1 \\ \text{subject to} \quad & f(x_{k+1}, x_k, u_k, \Delta t_k) = 0 \end{aligned}$$

Leakage Suppression




WE DO THIS
NOT BECAUSE
IT IS EASY,
BUT BECAUSE
WE THOUGHT
IT WOULD BE EASY

Questions?