Suez Canal University
Faculty of Computer and Information
Computer Science Dept.

Sheet-2 **Neural Networks**

1- A two-layer neural network is to have four inputs and six outputs. The range of the outputs is to be continuous between 0 and 1. What can you tell about the network architecture? Specifically:

i. How many neurons are required in each layer?

--> num of neurons =num of output=num of diction boundary=6

ii. What are the dimensions of the first-layer and second-layer weight matrices?

--> first layer = S*R 6*4 second layer=S*S=6*6

iii. What kinds of transfer functions can be used in each layer?

--> logsig used in two layer

iv. Are biases required in either layer?

--> no answer represent this question

2- We have a classification problem with four classes of input vector. The four classes are

$$\text{class 1:}\left\{\mathbf{p}_1=\begin{bmatrix}1\\1\end{bmatrix},\mathbf{p}_2=\begin{bmatrix}1\\2\end{bmatrix}\right\},\ \text{class 2:}\left\{\mathbf{p}_3=\begin{bmatrix}2\\-1\end{bmatrix},\mathbf{p}_4=\begin{bmatrix}2\\0\end{bmatrix}\right\},$$

$$\text{class 3:}\left\{\mathbf{p}_5=\begin{bmatrix}-1\\2\end{bmatrix},\mathbf{p}_6=\begin{bmatrix}-2\\1\end{bmatrix}\right\},\ \text{class 4:}\left\{\mathbf{p}_7=\begin{bmatrix}-1\\-1\end{bmatrix},\mathbf{p}_8=\begin{bmatrix}-2\\-2\end{bmatrix}\right\}.$$

Design a perceptron network to solve this problem.

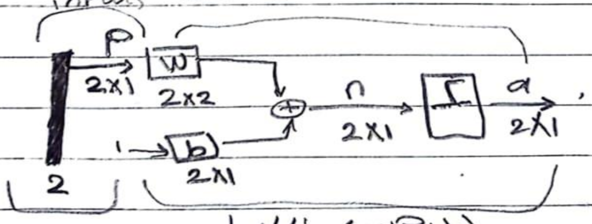
2-

Class 1: $\{P_1=\begin{bmatrix}1\\1\end{bmatrix}, P_2=\begin{bmatrix}1\\2\end{bmatrix}\}$; class 2: $\{P_3=\begin{bmatrix}2\\-1\end{bmatrix}, P_4=\begin{bmatrix}2\\0\end{bmatrix}\}$

class 3: $\{P_5=\begin{bmatrix}-1\\2\end{bmatrix}, P_6=\begin{bmatrix}-2\\1\end{bmatrix}\}$, class 4: $\{P_7=\begin{bmatrix}-1\\-1\end{bmatrix}, P_8=\begin{bmatrix}-2\\-2\end{bmatrix}\}$

no. class $= 4 = 2^S$ $\qquad R = 2$

inputs $\qquad S = 2$



$a = \text{hardlim}(WP+b)$

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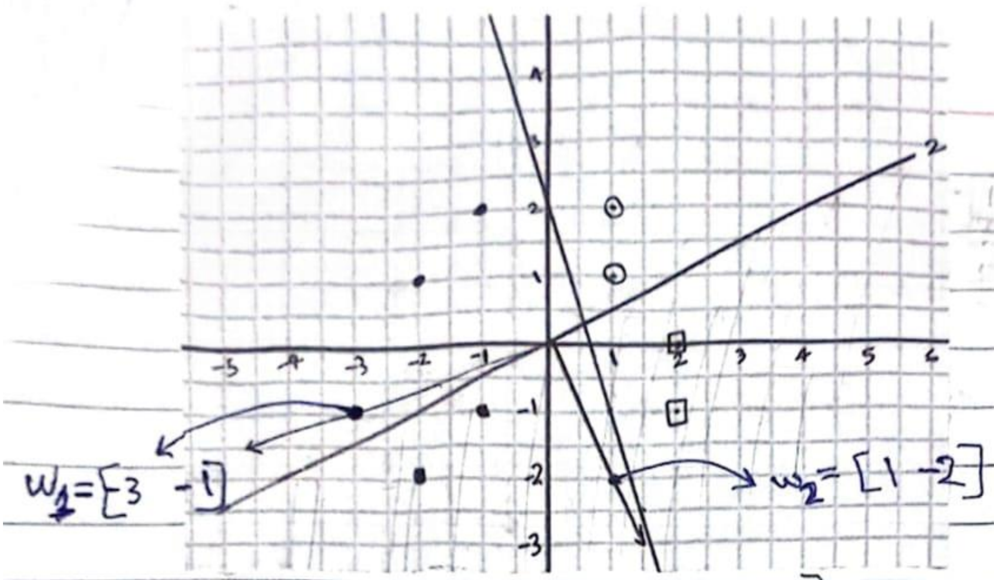


$w_1^T = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$ $\quad w_2^T = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Class 1: $\left\{ t_1 \begin{bmatrix} 0 \\ 0 \end{bmatrix}, t_2 \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ Class 2: $\left\{ t_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, t_4 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

Class 3: $\left\{ t_5 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} t_6 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$ class 4: $\left\{ t_7 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} t_8 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

$b_1 = -w_1 P = -[-3 \quad -1] \begin{bmatrix} 0 \\ 2 \end{bmatrix} = 2$

$b_2 = -w_2 P = -[1 \quad -2] \begin{bmatrix} -2 \\ -1 \end{bmatrix} = -(-2+2) = 0$

$W = \begin{bmatrix} -3 & -1 \\ 1 & -2 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

3- Solve the following classification problem with the perceptron rule. Apply each input vector in order, for as many repetitions as it takes to ensure that the problem is solved. Draw a graph of the problem only after you have found a solution.

$$\left\{ \mathbf{p}_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, t_1 = 0 \right\} \left\{ \mathbf{p}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, t_2 = 1 \right\} \left\{ \mathbf{p}_3 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}, t_3 = 0 \right\} \left\{ \mathbf{p}_4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, t_4 = 1 \right\}$$

Use the initial weights and bias:

$$\mathbf{W}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix} \qquad b(0) = 0.$$

$P_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, t_1 = 0$ $\qquad$ $P_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, t_2 = 1$

$P_3 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}, t_3 = 0$ $\qquad$ $P_4 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, t_4 = 1$

- use $W(0) = [0 \quad 0]$, $b(0) = 0$
- use transfer Function is HardLimit $\rightarrow$ $n \geq 0 \rightarrow a = 1$, $n < 0 \rightarrow a = 0$

$$\boxed{\begin{aligned} W(new) &= W(old) + eP^T \\ b(new) &= b(old) + e \\ e &= t - a \end{aligned}}$$

$a_1 = hardlim(WP_1 + b) = hardlim\left([0 \; 0]\begin{bmatrix} 2 \\ 2 \end{bmatrix} + 0\right)$

$a_1 = hardlim(0) = 1 \neq t_1$

$W(1) = W(0) + (t-a)P_1^T$

$W(1) = [0 \; 0] + (-1)[2 \quad 2] = [-2 \quad -2]$

$b(1) = b(0) + (t-a) = 0 + (-1) = -1$

$a_2 = hardlim(W(1)P_2 + b(1)) = hardlim\left([-2 \; -2]\begin{bmatrix} 1 \\ -2 \end{bmatrix} - 1\right) = hardlim(1) = 1 = t_2$

$a_3 = hardlim(W(1)P_3 + b(1)) = hardlim\left([-2 \; -2]\begin{bmatrix} -2 \\ 2 \end{bmatrix} - 1\right) = hardlim(-1) = 0 = t_3$

$a_4 = hardlim(W(1)P_4 + b(1)) = hardlim\left([-2 \; -2]\begin{bmatrix} -1 \\ 1 \end{bmatrix} - 1\right) = hardlim(-1) = 0 \neq t_4$

$W(2) = W(1) + (t-a)P_4^T$

$W(2) = [-2 \quad -2] + (1)[-1 \quad 1] = [-3 \quad -1]$

$b(2) = b(1) + (t-a) = -1 + 1 = 0$

$a_1 = hardlim(W(2)P_1 + b(2)) = hardlim\left([-3 \; -1]\begin{bmatrix} 2 \\ 2 \end{bmatrix} + 0\right) = hardlim(-8) = 0 = t_1$

$a_2 = hardlim(W(2)P_2 + b(2)) = hardlim\left([-3 \; -1]\begin{bmatrix} 1 \\ -2 \end{bmatrix} + 0\right) = hardlim(-1) = 0 \neq t_2$

$W(3) = W(2) + (t-a)P_2^T$

$W(3) = [-3 \quad -1] + (1)[1 \quad -2] = [-2 \quad -3]$

$b(3) = b(2) + (t-a) = 0 + 1 = 1$

$a_3 = hardlim(W(3)P_3 + b(3)) = hardlim\left([-2 \; -3]\begin{bmatrix} -2 \\ 2 \end{bmatrix} + 1\right) = hardlim(-1) = 0 = t_3$

$a_4 = hardlim(W(3)P_4 + b(3)) = harlim\left([-2 \; -3]\begin{bmatrix} -1 \\ 1 \end{bmatrix} + 1\right) = hardlim(0) = 1 = t_4$

$a_1 = hardlim(W(3)P_1 + b(3)) = hardlim\left([-2 \; -3]\begin{bmatrix} 2 \\ 2 \end{bmatrix} + 1\right) = hardlim(-9) = 0 = t_1$

$a_2 = hardlim(W(3)P_2 + b(3)) = hardlim\left([-2 \; -3]\begin{bmatrix} 1 \\ -2 \end{bmatrix} + 1\right) = hardlim(5) = 1 = t_2$

then $W = [-2 \quad -3]$ & $b = 1$ #

مع تحياتي بالتوفيق

4- We want to train a perceptron network with the following training set:

$$\left\{\mathbf{p}_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, t_1 = 0\right\} \left\{\mathbf{p}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, t_2 = 0\right\} \left\{\mathbf{p}_3 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, t_3 = 1\right\}.$$

The initial weight matrix and bias are

$$\mathbf{W}(0) = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; b(0) = 0.5 .$$

i. Plot the initial decision boundary, weight vector and input patterns. Which patterns are correctly classified using the initial weight and bias?

ii. Train the network with the perceptron rule. Present each input vector once, in the order shown.

iii. Plot the final decision boundary and demonstrate graphically which patterns are correctly classified.

iv. Will the perceptron rule (given enough iterations) always learn to correctly classify the patterns in this training set, no matter what initial weights we use? Explain

4 $P_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, t_1 = 0 \quad P_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, t_2 = 0 \quad P_3 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, t_3 = 1$

ii $w(0) = [1 \ \ 0]$, $b(0) = 0.5$

$\Rightarrow$ we have 2 classes $\Rightarrow 2^S = 2 \quad \therefore S = 1$

$w \Rightarrow S \times R \Rightarrow 1 \times 2$, $b \Rightarrow 1 \times 1$

$\therefore$ Target $\begin{cases} \rightarrow 0 \\ \rightarrow 1 \end{cases}$ $\therefore$ Transfer function is hardlim

$a_1 = \text{hardlim}(w(0)P_1 + b) = \text{hardlim}\left([1 \ \ 0]\begin{bmatrix} -1 \\ -1 \end{bmatrix} + 0.5\right) = \text{hardlim}(-0.5) = 0 = t_1$

$a_2 = \text{hardlim}(w(0)P_2 + b) = \text{hardlim}\left([1 \ \ 0]\begin{bmatrix} 0 \\ 0 \end{bmatrix} + 0.5\right) = \text{hardlim}(0.5) = 1 \neq t_2$

$e = T - a = 0 - 1 = -1$

$w(1) = w(0) + eP_2 = [1 \ \ 0] + [0 \ \ 0] = [1 \ \ 0]$

$b(1) = b(0) + e = 0.5 - 1 = -0.5$

$a_3 = \text{hardlim}(w(1)P_3 + b(1)) = \text{hardlim}\left([1 \ \ 0]\begin{bmatrix} -1 \\ 1 \end{bmatrix} - 0.5\right) = \text{hardlim}(-1.5) = 0 \neq t_3$

$e = t - a = 1 - 0 = 1$

$w(2) = w(1) + eP_3 = [1 \ \ 0] + [-1 \ \ 1] = [0 \ \ 1]$

$b(2) = b(1) + e = -0.5 + 1 = 0.5$

$a_1 = \text{hardlim}(w(2)P_1 + b(2)) = \text{hardlim}\left([0 \ \ 1]\begin{bmatrix} -1 \\ -1 \end{bmatrix} + 0.5\right) = \text{hardlim}(-0.5) = 0$

$a_2 = \text{hardlim}(w(2)P_2 + b(2)) = \text{hardlim}\left([0 \ \ 1]\begin{bmatrix} 0 \\ 0 \end{bmatrix} + 0.5\right) = \text{hardlim}(0.5) = 1$

$e = t - a = 0 - 1 = -1$

$w(3) = w(2) + eP_2 = [0 \ \ 1] - [0 \ \ 0] = [0 \ \ 1]$

$B(3) = b(2) + e = 0.5 - 1 = -0.5$

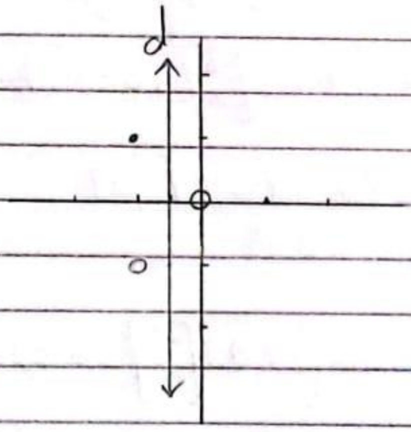
$a_3 = hardlim(w(3)P_3 + b(3)) = hardlim\left([0 \quad 1]\begin{bmatrix}-1\\1\end{bmatrix} - 0.5\right) = hardlim(0.5) = 1$

$a_1 = hardlim(w(3)P_1 + b(3)) = hardlim\left([0 \quad 1]\begin{bmatrix}-1\\-1\end{bmatrix} - 0.5\right) = hardlim(-1.5) = 0$

$a_2 = hardlim(w(3)P_2 + b(3)) = hardlim\left([0 \quad 1]\begin{bmatrix}0\\0\end{bmatrix} - 0.5\right) = hardlim(-0.5) = 0$

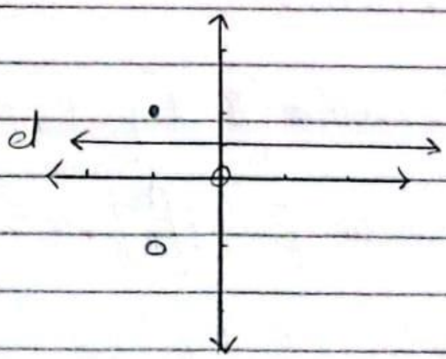
Then $w = [0 \quad 1]$ & $b = -0.5$

i-



$x = \frac{-b}{w_1} = \frac{-0.5}{1} = -0.5$

iii



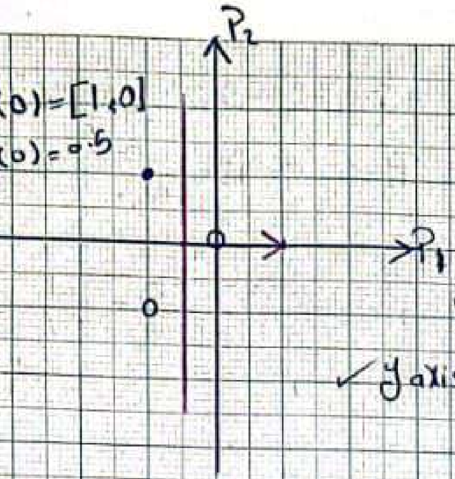
$y = \frac{-b}{w_2} = \frac{0.5}{1} = 0.5$

iv-

(i)

$w(0) = [1, 0]$

$b(0) = 0.5$



assume that $P_2 = 0$

$\therefore P_1 = \frac{-0.5}{1} = -0.5 \quad (-0.5, 0)$

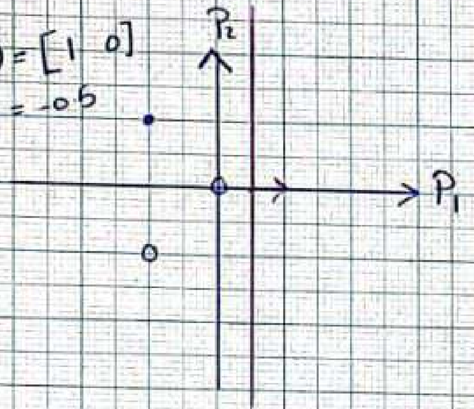
هنا مش هنقدر نفترض العكس ($P_1 = 0$)

ونجيب P_2 لأن الناتج هيكون زيرو !!! $\frac{-b}{w_{1,2}} = \frac{-b}{0}$ X

نستنتج من كدا ان الخط P_2 موازي لـ y axis ✓

$w(1) = [1 \ 0]$

$b(1) = -0.5$

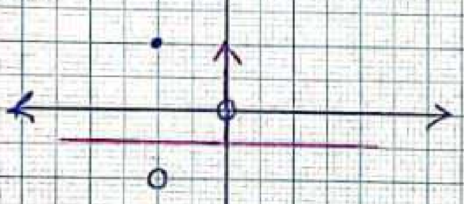


assume that $P_2 = 0$

$\therefore P_1 = \frac{0.5}{1} = 0.5 \quad (0.5, 0)$

$w(2) = [0 \ 1]$

$b(2) = 0.5$



assume that $P_1 = 0$

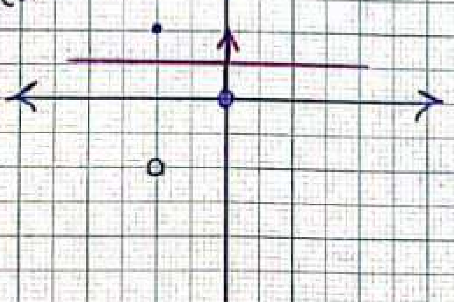
$\therefore P_2 = \frac{-0.5}{1} = -0.5$

$(0, -0.5)$

$w(3) = [0 \ 1]$

$b(3) = -0.5$

(iii)



assume that $P_1 = 0$

$\therefore P_2 = \frac{0.5}{1} = 0.5$

$(0, 0.5)$ ✓

Answer the following MCQs questions

5- What is the main function of a neural network's output layer?
a. To perform feature extraction
b. To make predictions or classifications
c. To introduce non-linearity
d. None of the above

6- In a feedforward neural network, information flows ____________.
a. Only in the forward direction
b. Only in the backward direction
c. In both forward and backward directions
d. In random directions

7- For what purpose, hamming network is suitable?
a) classification b) association c) pattern storage d) none of them

8-A perceptron is a
a) Feed-forward neural network b) Back-propagation algorithm
c) Back-tracking algorithm d) Feed Forward-backward algorithm

9-The network that involves backward links from output to the input and hidden layers is called
a) Multi layered perceptron b) Perceptron c) Recurrent neural network

10- Output of Recurrent Network layer is

a) a(t+1)=Satlin(W a(t)+b) b) a(t)=Satlin(Wa(t)+b) c)None

11- Given a two-input neuron with the following parameters: b=1.2, w= [3 2] and $p = [-5 \quad 6]^T$, calculate the neuron output for the Symmetric Hard-limit transfer function:

a) -1 b)-2 c)1

A= hardlim[3 2][-5]+1.2 = -1.8 <0 ans is 0 and if transfer function is Symmetric= -1 [6]