Deep Learning - Assignment 3

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1

1.1

The result of this convolution is as follows (see theory.ipynb):

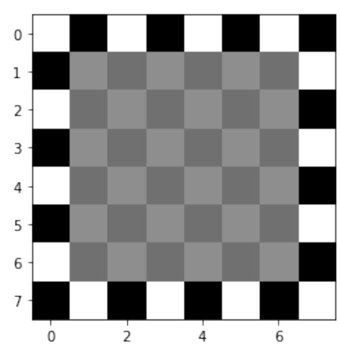

Figure 1: Result of convolution.

where the edge pixels are preserved as mentioned.

1.2

This filter act similar to a average pooling filter with $kernel_size = (3, 3)$, where each pixel is replaced by the average of its 8 neighbors and itself.

2

Dimension of input of the size $H \times W \times C$ after a Conv$k - N(S, P)$ layer is $H' \times W' \times N$, where:

$$H' = \left\lfloor \frac{H - k + 2P}{S} \right\rfloor + 1 \tag{2.1}$$

$$W' = \left\lfloor \frac{W - k + 2P}{S} \right\rfloor + 1 \tag{2.2}$$

Layer	Output Dimension	Parameters
Input	$32 \times 32 \times 3$	0
CONV3-10	$32 \times 32 \times 10$	$3 \times 3 \times 3 \times 10 + 10$
ReLU	$32 \times 32 \times 10$	0
POOL-2	$16 \times 16 \times 10$	0
CONV3-20(3,2)	$6 \times 6 \times 20$	$3 \times 3 \times 10 \times 20 + 20$
ReLU	$6 \times 6 \times 20$	0
POOL-2	$3 \times 3 \times 20$	0
FLATTEN	180	0
FC-10	10	$180 \times 10 + 10$

3

3.1

k_1, k_2, k_3, b, w_1, w_2 and a are the parameters of the network.

3.2

$$\hat{y} = w_1 v_1 + w_2 v_2 + b \tag{3.1}$$

$$L = \frac{1}{2}(y - \hat{y})^2 \tag{3.2}$$

$$\frac{\partial L}{\partial \hat{y}} = \hat{y} - y \tag{3.3}$$

$$\frac{\partial L}{\partial a} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial a} \tag{3.4}$$

$$= \hat{y} - y \tag{3.5}$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w_1} \tag{3.6}$$

$$= (\hat{y} - y) v_1 \tag{3.7}$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w_2} \tag{3.8}$$

$$= (\hat{y} - y) v_2 \tag{3.9}$$

3.3

$$\frac{\partial L}{\partial v_1} = \delta_1 \tag{3.10}$$

$$\frac{\partial L}{\partial v_2} = \delta_2 \tag{3.11}$$

$$\frac{\partial L}{\partial z_1} = \begin{cases} \delta_1 & \text{if } z_1 > 0 \text{ and } z_1 \geq z_2 \\ 0 & \text{otherwise} \end{cases} \tag{3.12}$$

$$\frac{\partial L}{\partial z_2} = \begin{cases} \delta_1 & \text{if } z_2 > 0 \text{ and } z_2 > z_1 \text{ and } z_2 \leq z_3 \\ \delta_2 & \text{if } z_2 > 0 \text{ and } z_2 \leq z_1 \text{ and } z_2 > z_3 \\ \delta_1 + \delta_2 & \text{if } z_2 > 0 \text{ and } z_2 > z_1 \text{ and } z_2 > z_3 \\ 0 & \text{otherwise} \end{cases} \tag{3.13}$$

$$\frac{\partial L}{\partial z_3} = \begin{cases} \delta_2 & \text{if } z_3 > 0 \text{ and } z_3 \geq z_2 \\ 0 & \text{otherwise} \end{cases} \tag{3.14}$$

$$\tag{3.15}$$

(Assuming ties are broken in favor of z_1 and z_3).

3.4

$$\frac{\partial L}{\partial k_1} = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 \tag{3.16}$$

$$\frac{\partial L}{\partial k_2} = \alpha_1 x_2 + \alpha_2 x_3 + \alpha_3 x_4 \tag{3.17}$$

$$\frac{\partial L}{\partial k_3} = \alpha_1 x_3 + \alpha_2 x_4 + \alpha_3 x_5 \tag{3.18}$$

$$\frac{\partial L}{\partial b} = \alpha_1 + \alpha_2 + \alpha_3 \tag{3.19}$$

3.5

$$\frac{\partial L}{\partial k_j} = \alpha_1 x_j + \alpha_2 x_{j+1} + \cdots + \alpha_m x_{n-d+j} \tag{3.20}$$

$$\frac{\partial L}{\partial b} = \alpha_1 + \alpha_2 + \cdots + \alpha_m \tag{3.21}$$

4

4.1

Since the kernel acts similarly on width and height of the input, it is a square kernel.

$$\frac{205-k}{3}+1=66 \tag{4.1}$$

$$\implies k=205-65\times 3=10 \tag{4.2}$$

So it is a convolution with 96 kernels of size $10\times 10\times 10$ (since the input's channel size is 10) with stride 3.

4.2

$$96\times 10\times 10\times 10+96=96,096 \tag{4.3}$$

4.3

For each output value, $10\times 10\times 10$ multiplications are performed. So the number of multiplications is:

$$66\times 66\times 96\times 10\times 10\times 10=418,176,000 \tag{4.4}$$