24. Consider a single serve queue where customers arrive according to a Poisson process with rate 2 per minute and the service times are exponentially distributed with mean 1 minute. Let T_i denote the amount of time that customer i spends in the system. We are interested in using simulation to estimate $\theta = E[T_1 + \cdots + T_{10}]$.

(a) Do a simulation to estimate the variance of the raw simulation estimator. That is, estimate $\mathrm{Var}(T_1 + \cdots + T_{10})$.

(b) Do a simulation to determine the improvement over the raw estimator obtained by using antithetic variables.

(c) Do a simulation to determine the improvement over the raw estimator obtained by using $\sum_{i=1}^{10} S_i$ as a control variate, where S_i is the ith service time.

(d) Do a simulation to determine the improvement over the raw estimator obtained by using $\sum_{i=1}^{10} S_i - \sum_{i=1}^{9} I_i$ as a control variate, where I_i is the time between the ith and $(i+1)$st arrival.

(e) Do a simulation to determine the improvement over the raw estimator obtained by using the estimator $\sum_{i=1}^{10} E[T_i|N_i]$, where N_i is the number in the system when customer i arrives (and so $N_1 = 0$).

(a)

Step1 : Generate U ~ U(0,1) , t = t – log(U)/2,

Step2 : S[i] = t , repeat step 1~2 10 times

Step3 : Generate U ~ U(0,1), G = -log(U)

Step4 : D[1] = S[1]+G, if S[j] > D[j-1] , D[j] = S[j]+G ; else D[j] = D[j-1]+G ,
for j = 2~10

Step5 : theta = sum(D[1],…,D[10])

Step6 : Repeat step 1~5 n times , get theta(1),…,theta(n)

Step7 : Variance = Var(theta(i))

```
theta hat 35.73279441986678
variance 359.4205742922175
```

(b)

Step1 : Generate U ~ U(0,1) , t = t – log(U)/2,

Step2 : S[i] = t , repeat step 1~2 10 times

Step3 : Generate U ~ U(0,1), G = -log(U)

Step4 : D[1] = S[1]+G, if S[j] > D[j-1] , D[j] = S[j]+G ; else D[j] = D[j-1]+G ,
for j = 2~10

Step5 : theta1 = sum(D[1],…,D[10])

Step6 : Repeat step 1~5 by changing U to 1-U , get another theta2 & theta(i)=
(theta1+ theta2)/2

Step7 : Repeat step 1~6 n times , get theta(1),…,theta(n)

Step8 : Variance = Var(theta(i))

```
theta hat 35.797677153050145
variance 82.175129640269
```

(c)

Step1 : Generate U ~ U(0,1) , t = t – log(U)/2,

Step2 : S[i] = t , repeat step 1~2 10 times

Step3 : Generate U ~ U(0,1), G = -log(U)

Step4 : D[1] = S[1]+G, if S[j] > D[j-1] , D[j] = S[j]+G ; else D[j] = D[j-1]+G , for j = 2~10

Step5 : Y = 10*E(G)

Step6 : theta = sum(D[1],…,D[10])

Step7 : Repeat step 1~6 n times , get theta(1),…,theta(n) , Y1,…,Yn

Step8 : Variance = Var(theta(i)) – cov(theta(i),Yi)^2/Var(Yi)

```
theta hat 35.61693359343921
variance 99.02987546133897
```

(d)

Step1 : Generate U ~ U(0,1) , t = t – log(U)/2,

Step2 : S[i] = t , repeat step 1~2 10 times , I[i] = S[i+1]-S[i] for i = 1,…,9

Step3 : Generate U ~ U(0,1), G = -log(U)

Step4 : D[1] = S[1]+G, if S[j] > D[j-1] , D[j] = S[j]+G ; else D[j] = D[j-1]+G , for j = 2~10

Step5 : Y = 10*E(G) – 9*E(I])

Step6 : theta = sum(D[1],…,D[10])

Step7 : Repeat step 1~6 n times , get theta(1),…,theta(n) , Y1,…,Yn

Step8 : Variance = Var(theta(i)) – cov(theta(i),Yi)^2/Var(Yi)

```
theta hat 35.67969807305372
variance 80.63995444695621
```

(e)

Step1 : Generate U ~ U(0,1) , t = t – log(U)/2,

Step2 : S[i] = t , repeat step 1~2 10 times

Step3 : Generate U ~ U(0,1), G = -log(U)

Step4 : D[1] = S[1]+G, if S[j] > D[j-1] , D[j] = S[j]+G ; else D[j] = D[j-1]+G , for j = 2~10

Step5 : If S[j] < D[i] for j<i , N[i] +=1 for i = 2,…,10, N[1]=0

Step6 : theta = sum(N[i]+1)*1

Step7 : Repeat step 1~6 n times , get theta(1),…,theta(n)

Step8 : Variance = Var(theta(i))

```
theta hat 35.4683
variance 103.45879511000001
```