Consider a single-server system in which potential customers arrive in accordance with a Poisson process having rate 4.0. A potential customer will only enter if there are three or fewer other customers in the system when he or she arrives. The service time of a customer is exponential with rate 4.2. No additional customers are allowed in after time $T = 8$. (All time units are per hour.) Develop a simulation study to estimate the average amount of time that an entering customer spends in the system. Using the bootstrap approach, estimate the mean square error of your estimator.

Simulation:

$S\ represent\ enter\ time, D\ represent\ exist\ time$

$Step1:\ Generate\ a\ poisson\ process\ with\ \lambda =\ 4.0,$
$and\ get\ S(1), \dots, S(I)$

$Step2:\ D(1) = S(1), D(2) = \max\big(D(1), S(2)\big) + G,$

$D(3) = max(D(2), S(3)) + G,$

$D(4) =\ max\big(D(3), S(4)\big) +\ G.\ \ Where\ G\ \sim\ Exp(\lambda = 4.2)$

$Step3: If\ \ D(i) < S(j), D(i+4) = max\big(D(i+3), S(j)\big) +\ \ G,$

$i = i + 1, j = 5, \dots, I. i = 1$

$Step4: Every\ customer\ spend\ time = D - S,$
$and\ get\ average\ spend\ time$

$Step5: Repeat\ n\ times. Take\ average =\ \hat{\theta}$

$Step6: Resampling\ with\ B\ times, and\ get\ \widehat{\theta_1}^*, \dots, \widehat{\theta_B}^*. Get\ average\ \hat{\theta}^*$

$$Step7: Bias = \hat{\theta}^* - \hat{\theta}, Var = \frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\theta}^* - \widehat{\theta_b}^*\right)^2.$$

$$MSE = Bias^2 + Var^2$$

$Via\ simulation:$

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average spend time =  0.5472781358657489
MSE =  5.2751362891470756e-05
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