Chapter 1

Neural Encoding I: Firing Rates and Spike Statistics

1.1 1.1

Principle 1.1 (Conservation of angular momentum)**.** The rate of change of angular momentum of a system is equal to the net torque acting on the system, i.e.,

$$\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \boldsymbol{\tau}, \tag{1.1}$$

where $\boldsymbol{\tau}$ is the torque of all external forces on the system about any chosen axis, and $\mathrm{d}\mathbf{L}/\mathrm{d}t$ is the rate of change of angular momentum of the system about the same axis.

1.2 1.2

Remark 1.1. Many physical laws are cumbersome when written in coordinate form but become more compact and attractive looking when written in tensorial form. For example, the incompressible Navier-Stokes equations in cylindrical coordinates are

$$\begin{aligned}
\rho\left(\frac{Dv_r}{Dt} - \frac{v_\theta^2}{r}\right) &= \rho f_r - \frac{\partial p}{\partial r} + \mu\left(\Delta v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right),\\
\rho\left(\frac{Dv_\theta}{Dt} + \frac{v_r v_\theta}{r}\right) &= \rho f_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left(\Delta v_\theta + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2}\right),\\
\rho\frac{Dv_z}{Dt} &= \rho f_z - \frac{\partial p}{\partial z} + \mu\Delta v_z,
\end{aligned}$$

where

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2},$$

and

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + v_z\frac{\partial}{\partial z}.$$

1.3 1.3

Proposition 1.2. The mass of fluid in a region W at time t is

$$m(W,t) = \int_W \rho(\mathbf{x},t)\mathrm{d}V, \tag{1.2}$$

where $\mathrm{d}V$ is the area element in the plane or the volume element in space.

1.4 1.4

Assumption 1.3. From now on, assume that

$$\text{force on } S \text{ per unit area} = -p(\mathbf{x},t)\mathbf{n} + \mathbf{n}\cdot\boldsymbol{\sigma}(\mathbf{x},t), \tag{1.3}$$

where $\boldsymbol{\sigma}$ is the *(deviatoric) stress tensor* and $\mathbf{n}$ is the unit outward normal of S.

Chapter 2

Neural Encoding II: Reverse Correlation and Visual Receptive Fields

2.1 2.1

Principle 2.1 (Conservation of angular momentum)**.** The rate of change of angular momentum of a system is equal to the net torque acting on the system, i.e.,

$$\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \boldsymbol{\tau}, \tag{2.1}$$

where $\boldsymbol{\tau}$ is the torque of all external forces on the system about any chosen axis, and $\mathrm{d}\mathbf{L}/\mathrm{d}t$ is the rate of change of angular momentum of the system about the same axis.

2.2 2.2

Remark 2.1. Many physical laws are cumbersome when written in coordinate form but become more compact and attractive looking when written in tensorial form. For example, the incompressible Navier-Stokes equations in cylindrical coordinates are

$$\begin{aligned}
\rho\left(\frac{Dv_r}{Dt} - \frac{v_\theta^2}{r}\right) &= \rho f_r - \frac{\partial p}{\partial r} + \mu\left(\Delta v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right),\\
\rho\left(\frac{Dv_\theta}{Dt} + \frac{v_r v_\theta}{r}\right) &= \rho f_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left(\Delta v_\theta + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2}\right),\\
\rho\frac{Dv_z}{Dt} &= \rho f_z - \frac{\partial p}{\partial z} + \mu\Delta v_z,
\end{aligned}$$

where

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2},$$

and

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + v_z\frac{\partial}{\partial z}.$$

2.3 2.3

Proposition 2.2. The mass of fluid in a region W at time t is

$$m(W,t) = \int_W \rho(\mathbf{x},t)\mathrm{d}V, \tag{2.2}$$

where $\mathrm{d}V$ is the area element in the plane or the volume element in space.

2.4 2.4

Assumption 2.3. From now on, assume that

$$\text{force on } S \text{ per unit area} = -p(\mathbf{x},t)\mathbf{n} + \mathbf{n}\cdot\boldsymbol{\sigma}(\mathbf{x},t), \tag{2.3}$$

where $\boldsymbol{\sigma}$ is the *(deviatoric) stress tensor* and $\mathbf{n}$ is the unit outward normal of S.

Chapter 3

Neural Decoding

3.1 3.1

Principle 3.1 (Conservation of angular momentum)**.** The rate of change of angular momentum of a system is equal to the net torque acting on the system, i.e.,

$$\frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \boldsymbol{\tau}, \tag{3.1}$$

where $\boldsymbol{\tau}$ is the torque of all external forces on the system about any chosen axis, and $\mathrm{d}\mathbf{L}/\mathrm{d}t$ is the rate of change of angular momentum of the system about the same axis.

3.2 3.2

Remark 3.1. Many physical laws are cumbersome when written in coordinate form but become more compact and attractive looking when written in tensorial form. For example, the incompressible Navier-Stokes equations in cylindrical coordinates are

$$\begin{aligned}
\rho\left(\frac{Dv_r}{Dt} - \frac{v_\theta^2}{r}\right) &= \rho f_r - \frac{\partial p}{\partial r} + \mu\left(\Delta v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right), \\
\rho\left(\frac{Dv_\theta}{Dt} + \frac{v_r v_\theta}{r}\right) &= \rho f_\theta - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left(\Delta v_\theta + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2}\right), \\
\rho\frac{Dv_z}{Dt} &= \rho f_z - \frac{\partial p}{\partial z} + \mu \Delta v_z,
\end{aligned}$$

where

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2},$$

and

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + v_z\frac{\partial}{\partial z}.$$

3.3 3.3

Proposition 3.2. The mass of fluid in a region W at time t is

$$m(W,t) = \int_W \rho(\mathbf{x},t)\mathrm{d}V, \tag{3.2}$$

where $\mathrm{d}V$ is the area element in the plane or the volume element in space.

3.4 3.4

Assumption 3.3. From now on, assume that

$$\text{force on } S \text{ per unit area} = -p(\mathbf{x},t)\mathbf{n} + \mathbf{n}\cdot\boldsymbol{\sigma}(\mathbf{x},t), \tag{3.3}$$

where $\boldsymbol{\sigma}$ is the *(deviatoric) stress tensor* and $\mathbf{n}$ is the unit outward normal of S.