Pro$\boldsymbol{\beta\alpha\beta}ili\boldsymbol{\tau}$y Th$\boldsymbol{\epsilon}$ory

Contents

1 Dynkin systems

Definition 1. *A* ***Dynkin system*** *on a set* Ω *is a subset* $\mathcal{D}$ *of the power set* $\mathcal{P}(\Omega)$*, with the following properties:*

1. $\Omega \in \mathcal{D}$;
2. $A, B \in \mathcal{D},\ A \subset B \implies B \setminus A \in \mathcal{D}$;
3. $A_n \in \mathcal{D},\ A_n \subset A_{n+1},\ n \geq 1 \implies \bigcup_{i=1}^{\infty} A_n \in \mathcal{D}$.

Definition 2. *Let* $\mathcal{A}$*be a subset of* $\mathcal{P}(\Omega)$*. The* ***Dynkin system generated*** *by* $\mathcal{A}$ *is the Dynkin system on* Ω*, denoted* $\mathcal{D}(\mathcal{A})$*, equal to the intersection of all Dynkin systems on* Ω *which contain* $\mathcal{A}$*. i.e.,*

$$\mathcal{D}(\Omega) := \ \{\mathcal{D} \text{ Dynkin systems on } \Omega : \ \mathcal{A} \subset \mathcal{D}\}.$$

Definition 3. *A* $\boldsymbol{\sigma}$***−algebra*** *on a set* Ω *is a subset* $\mathcal{F}$ *of the power set* $\mathcal{P}(\Omega)$ *with the following properties:*

1. $\Omega \in \mathcal{F}$;
2. $A \in \mathcal{F} \implies \Omega \setminus A =: A^c \in \mathcal{F}$;
3. $A_n \in \mathcal{F},\ n \geq 1 \implies \bigcup_{i=1}^{\infty} A_n \in \mathcal{F}$.

Appendix A Background

Definition 4. *A **set** is a well-defined collection of distinct mathematical objects.*

Definition 5. *Given two sets A and B, the **set difference** of A and B is defined:* $B \backslash A := \{x \text{ in } B,\ x \text{ not in } A\}$.

Definition 6. *The **power set** of a set Ω, denoted $\mathcal{P}(\Omega)$, is the set of all subsets of Ω. i.e.,*

$$\mathcal{P}(\Omega) := \{A \text{ subset of } \Omega\}.$$