Nuclear Physics B253 (1985) 375-386

COSMOLOGICAL CONSTRAINTS ON THE PROPERTIES OF WEAKLY INTERACTING MASSIVE PARTICLES

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Received 9 October 1984

Considerations of the age and density of, as well as the evolution of structure in, the universe lead to constraints on the masses and lifetimes of weakly interacting massive particles (WIMPs). The requirement that the observed large-scale structure of the universe be permitted to develop, leads to much more restrictive bounds on the properties of WIMPs than those which follow from considerations of the age and density of the universe alone.

1. Introduction

The universe provides a valuable laboratory for the study of elementary particle physics. During its early evolution particles, some of whose existence is yet to be confirmed or ruled out by experiments at terrestrial accelerators, were copiously abundant. Such particles may have survived in sufficient numbers to influence the subsequent evolution of the universe and, may have played a significant role in the development of structure in it. Data on the present age, density and large-scale structure of the universe can be marshalled to provide interesting constraints on the abundances, masses and lifetimes of such relics. The purpose of this paper is to derive constraints based on these considerations.

To date, much progress has been made in constraining the masses and abundances (related to the interaction strengths) of stable or long-lived particles through the requirement that their present mass density not exceed that observed (see, e.g., ref. [1]). Unstable relics whose decay products include photons and/or electrically charged particles, have their lifetimes, as well as their masses and abundances, constrained directly by observations of the background radiation fields (see, e.g., ref. [2]) and, indirectly by considerations of stellar structure and evolution, (see, e.g., ref. [3]) and by primordial nucleosynthesis (see, e.g., ref. [4]).

It might seem that unstable particles whose decay products interact only feebly (i.e., participate in interactions which are as weak, or weaker than, the weak interaction) might have disappeared without a trace. However, such particles and their invisible decay products may have at one time dominated the energy density of the universe and, therefore, controlled the evolution of the universe during a crucial epoch. In a natural extension of previous analyses we use the requirements that the present universe be neither too young (or, equivalently, too dense) and that the observed large-scale structure should have been able to evolve in it to derive constraints on the masses, abundances and lifetimes of unstable, weakly interacting massive particles (WIMPs) whose decay products are "invisible" (i.e. interactionless) and sufficiently light so that for all epochs of interest they are ultrarelativistic. In the recent literature there are numerous examples of models in which particles (usually neutrinos) are relatively long-lived ($\sim 10^2 - 10^{10}$ y) and decay into invisible, relativistic daughter particles, for example: $\nu \to \nu' +$ familon, $\nu \to 3\nu'$ or $\nu \to \nu' +$ Majoran (Dicus et al. [2], [5]).

In sect. 2 we consider the evolution of the density of a massive WIMP, X, and of the density of its relativistic decay products, R. When these densities are compared with those of the relic photons and neutrinos, $\gamma\nu$, and of the other stable non-relativistic particles (i.e., baryons, etc.), NR, interesting epochs during the evolution of the universe are identified. With this as background, we proceed in the following section to derive constraints on the mass, abundance and lifetime of the X. In sect. 4 we summarize our results.

2. The evolution of WIMPs and their decay products

Consider a particle X whose mass is M_X and whose abundance (relative to relic photons) is η_X. The lifetime of this particle is $t_D (\equiv \Gamma^{-1}$; $\Gamma =$ decay width). [Note, $t_D/10^9\,\text{y} = 2.08 \times 10^{-41}\,\text{GeV}/\Gamma$]. Prior to decay the energy density in X is

$$\rho_X = M_X \eta_X n_\gamma = M_X \eta_X n_{\gamma 0} R^{-3} . \tag{1a}$$

In eq. (1a), n_γ is the number density of relic photons and $R(\leq 1)$ is the cosmic scale factor normalized so that, at present, $R_0 = 1$. Throughout, the present epoch is denoted by the subscript zero. If θ is the present microwave temperature in units of 2.7 K, then $n_{\gamma 0} \approx 399\theta^3\,\text{cm}^{-3}$. For M_X measured in eV.

$$\rho_X \approx 399 (M_X \eta_X \theta^3) R^{-3}\,\text{eV cm}^{-3} . \tag{1b}$$

If X were one of the "usual" (e, μ or τ) neutrinos then, $\eta_X = \eta_\nu = \frac{3}{4}(T_\nu/T_\gamma)^3 = \frac{3}{11}$. It is useful to introduce $\tilde{M}_X \equiv M_X(\eta_X/\eta_\nu)\theta^3$, the mass of an equivalent neutrino in a universe with $T_{\gamma 0} = 2.7$ K. With this definition,

$$\rho_X \approx 109 \tilde{M}_X R^{-3}\,\text{eV cm}^{-3} . \tag{1c}$$

It is convenient to compare all densities today with the present critical (Einstein–de Sitter) density ρ_c:

$$\rho_c = \frac{3H_0^2}{8\pi G} \approx 1.05 \times 10^4 h_0^2\,\text{eV cm}^{-3} , \tag{2}$$

where the present value of the Hubble parameter is $H_0 = 100h_0(\mathrm{km\ s^{-1}\ Mpc^{-1}})$. From (1c) and (2) it follows that

$$\tilde{M}_X \approx 96.8(\Omega_X h_0^2)\ \mathrm{eV}. \tag{3}$$

At the epoch when $t = t_D = \Gamma^{-1}$ and $R = R_D$, we assume that all the X decay simultaneously. After decay, the energy density of their relativistic decay products (R) is

$$\rho_R \approx 1.05 \times 10^4 (\Omega_R h_0^2) R^{-4}\ \mathrm{eV\ cm^{-3}}, \tag{4}$$

and since $\rho_X = \rho_R$ at $R = R_D$, $\Omega_R = \Omega_X R_D$ or

$$\tilde{M}_X R_D \approx 96.8(\Omega_R h_0^2)\ \mathrm{eV}. \tag{5}$$

[Note that the errors made by assuming that all the decays occur simultaneously are 10–20% ; see ref. [6]].

In addition to the products of X-decay, there is of course also the relativistic background of relic photons and (light) neutrinos:

$$\rho_{\gamma\nu} = \rho_\gamma + \Sigma\rho_\nu \equiv A_{\gamma\nu}\rho_\gamma \approx 0.25 A_{\gamma\nu}\theta^4 R^{-4}\ \mathrm{eV\ cm^{-3}}, \tag{6a}$$

where

$$A_{\gamma\nu} = 1 + \tfrac{7}{8}(T_\nu/T_\gamma)^{4/3} N_\nu = 1.68 + 0.227(N_\nu - 3). \tag{6b}$$

In eq. (6b), N_ν is the number of relativistic 2-component neutrinos. Comparing $\rho_{\gamma\nu}$ with ρ_c we find

$$\Omega_{\gamma\nu} h_0^2 \approx 2.4 \times 10^{-5} A_{\gamma\nu}\theta^4. \tag{7}$$

Finally, there will be a background of non-relativistic (NR) particles such as baryons and, possibly, other stable massive relics (and/or those massive products of X-decay which have become NR):

$$\rho_{NR} \approx 1.05 \times 10^4 (\Omega_{NR} h_0^2) R^{-3}\ \mathrm{eV\ cm^{-3}}. \tag{8}$$

In the absence of WIMPs and their decay products, the universe evolves from early "radiation domination" to "matter domination" at the epoch $R = R_{eq}$ where

$$R_{eq} = \frac{\Omega_{\gamma\nu}}{\Omega_{NR}} \approx 2.4 \times 10^{-5} A_{\gamma\nu}\theta^4 (\Omega_{NR} h_0^2)^{-1}. \tag{9}$$

In the presence of WIMPs and their decay products there are several other epochs of interest. First, compare ρ_X with ρ_{NR} and call the ratio x

$$x \equiv \frac{\rho_X}{\rho_{NR}} \approx \frac{\tilde{M}_X}{96.8\Omega_{NR} h_0^2} = \frac{\Omega_R}{\Omega_{NR}} \frac{1}{R_D}. \tag{10}$$

If $x < 1$, neither the X nor the R ever dominate the dynamical evolution of the universe. In this case the X is "cosmologically safe" (with respect to the issues

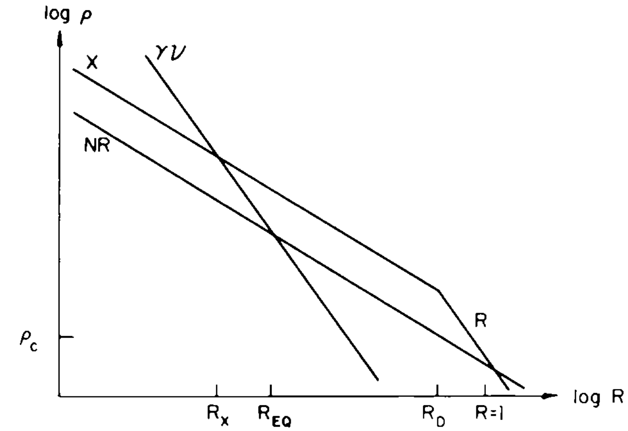

Fig. 1. The evolution of the densities of the several constituents of the Universe: X-particles (X), their relativistic decay products (R), photons and neutrinos ($\gamma\nu$), and stable, non-relativistic particles (NR). The universe becomes X-dominated at R_X; the X decay at R_D; at present ($R = 1$) the universe is R-dominated ($\Omega_R > \Omega_{NR}$).

considered in this paper), and we will not be able to place any constraints on its properties. In the more interesting case that $x \geq 1$, the universe will become X-dominated at $R = R_X$ where

$$R_X = x^{-1} R_{eq}, \tag{11}$$

provided that the X do not decay first (i.e., $R_X < R_D$); see figs 1 and 2. If, however, $R_D < R_X$, the X will decay prior to X-domination and, once again, neither the

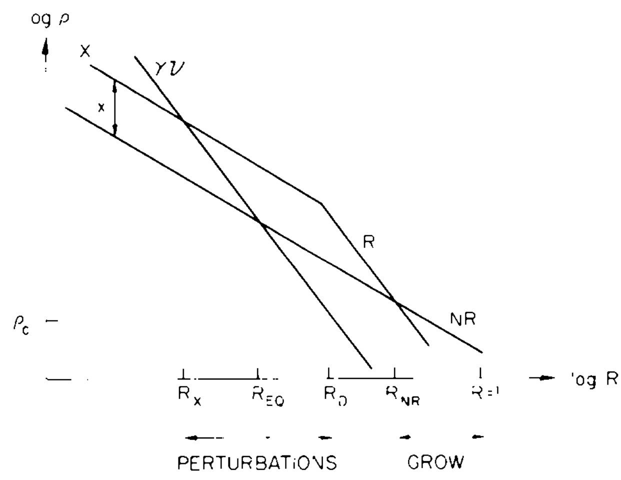

Fig. 2. The evolution of the densities in the case that the present epoch of NR domination has been preceded by epochs of R- and X-domination. The current epoch of NR domination begins at R_{NR}. Linear density perturbations can grow significantly only when the universe is matter dominated, here for $R_X < R < R_D$ and for $R > R_{NR}$.

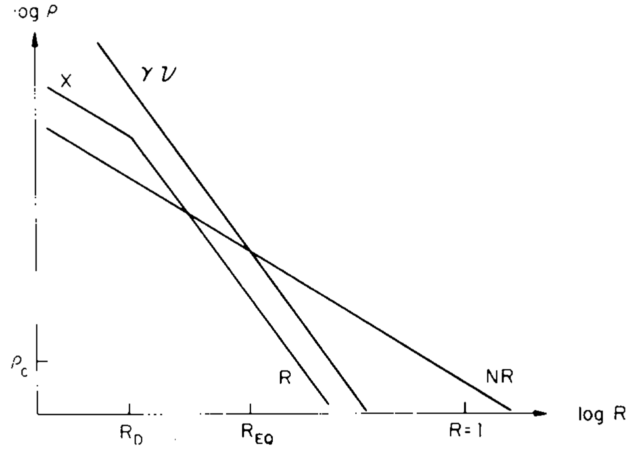

Fig. 3. The evolution of the densities in the case that the X decay before they can dominate the universe ($R_D < R_X$).

WIMPs nor their decay products are every dynamically significant; see fig. 3. This case too is "cosmologically safe". Returning to the more interesting case where $x>1$ and $R_X > R_D$, it is seen (figs. 1 and 2) that the period of X-domination lasts until the X decay at $R=R_D$. Thereafter, the universe will be R-dominated until $R=R_{NR}$ where

$$R_{NR} = xR_D = \Omega_R/\Omega_{NR}\,. \tag{12}$$

Notice that if $\Omega_R > \Omega_{NR}$, the present universe is radiation dominated(!); this is the case considered by Turner, Steigman and Krauss ([7]; hereafter TSK) in an attempt to reconcile an Einstein-de Sitter universe ($\Omega_0 = 1$) with the observational constraint that the matter associated with the visible galaxies contributes less than a third of that required. This case is illustrated in fig. 1.

For $\Omega_R < \Omega_{NR}$, the present universe is matter-dominated; see fig. 2. Although this is the "standard" assumption, it must be emphasized that the baryon density is small: $\Omega_B \leq 0.14$–0.19 (ref. [4]), so that if $\Omega_{NR} = 1$, the non-relativistic matter is dominated by something other than baryons which, in addition, must be more smoothly distributed than the luminous matter already observed.

We have, in the above, expressed the epoch of X-decay by the scale factor R_D. We are, however, interested in constraining the lifetime t_D. In relating t_D to R_D, there are three cases to be considered. If the universe is X-dominated at decay ($x>1$, $R_X < R_D$), then $6\pi G\rho_{XD}t_D^2 \approx 1$ and

$$t_D \approx \frac{6.5\times 10^9 R_D^2}{(\Omega_R h_0^2)^{1/2}}\,\mathrm{y} \approx \frac{6.5\times 10^9 R_D^{3/2}}{(\tilde{M}_X/96.8\ \mathrm{eV})^{1/2}}\,\mathrm{y}\,. \tag{13}$$

If the universe is $\gamma\nu$ dominated at decay ($x>1$, $R_\mathrm{D}<R_\mathrm{X}$), then $\frac{32}{3}\pi G\rho_{\gamma\nu\mathrm{D}}t_\mathrm{D}^2\approx 1$ and

$$t_\mathrm{D}\approx\frac{4.9\times10^9R_\mathrm{D}^2}{(\Omega_{\gamma\nu}h_0^2)^{1/2}}\,\mathrm{y}\approx\frac{10^{12}R_\mathrm{D}^2}{(A_{\gamma\nu}\theta^4)^{1/2}}\,\mathrm{y}\,. \tag{14}$$

Finally, in the less interesting case ($x<1$) where the X decay when the universe is NR-dominated, $6\pi G\rho_\mathrm{NRD}t_\mathrm{D}^2\approx 1$ and

$$t_\mathrm{D}\approx\frac{6.5\times10^9R_\mathrm{D}^{3/2}}{(\Omega_\mathrm{NR}h_0^2)^{1/2}}\,\mathrm{y}\,. \tag{15}$$

Using the relations derived in this section we now proceed, through various cosmological considerations, to find constraints on $\tilde{M}_\mathrm{X}$ and t_D.

3. Constraints on X-masses and lifetimes

Although relic WIMPs may have decayed long ago and their decay products may be "invisible", nonetheless the X and/or the R could have played a dynamically important role in the evolution of the universe. If so, data on the age, density and observed structure of the present universe can be used to constrain the masses (M_X), abundances (η_X) and lifetimes (t_D) of such particles. The results to be obtained in this section are summarized in fig. 4 which displays the t_D versus $\tilde{M}_\mathrm{X}$ plane; the regions to the left and below the various lines are not excluded by cosmological considerations.

(A) $t_\mathrm{D}\geqslant t_0$: In this case the stable or long-lived X are still present today. Such relics must not contribute excessively to the present mass density ($\Omega_\mathrm{X}\leqslant\Omega_0$). From eq. (3) this constraint leads to an *upper* limit to $\tilde{M}_\mathrm{X}$:

$$\tilde{M}_\mathrm{X}(\mathrm{A})\leqslant 96.8[\max(\Omega_0h_0^2)]\,\mathrm{eV}\,. \tag{16}$$

Current data suggests $h_0\leqslant 1$ (refs. [8-10]) and does not exclude a value as small as $h_0\approx 0.4$ (ref. [10]). The density *associated* with luminous matter is small: $\Omega\approx$ 0.1-0.3 (refs. [11-14]). However, "theoretical prejudice" (i.e., the "naturalness" of the Einstein-de Sitter model and the very attractive "inflationary" universe scenarios) strongly suggest that $\Omega_0=1$. Measurements of the deceleration parameter q_0, imply that $\Omega_0\leqslant$ few [15]. In setting an upper limit to the combination $\Omega_0h_0^2$ it is, it must be emphasized, not permissible to choose $\Omega_0\geqslant 1$ *and* $h_0=1$. The reason is that such a combination corresponds to an exceedingly youthful universe: $t_0\leqslant 6.5\times10^9$ y. The age of a matter-dominated universe is $t_0\simeq 10^{10}h_0^{-1}f(\Omega_0)$ y, where $f(\Omega)$ is monotonically decreasing, from $f(0)=1$ to $f(1)=\frac{2}{3}$. Asymptotically, $\Omega f^2(\Omega)$ approaches $(\frac{1}{2}\pi)^2$ from below, which implies that, independent of h_0 and Ω_0, $\Omega_0h_0^2\leqslant(\frac{1}{2}\pi)^2(t_0/10^{10}\,\mathrm{y})^{-2}$.

Various determinations of the age of the universe suggests a *lower bound* [16] on the age: $t_0\geqslant(10-13)\times10^9$ y, which then restricts $\Omega_0h_0^2$:

$$\Omega_0h_0^2\leqslant 1.5\text{-}2.5\,. \tag{17a}$$

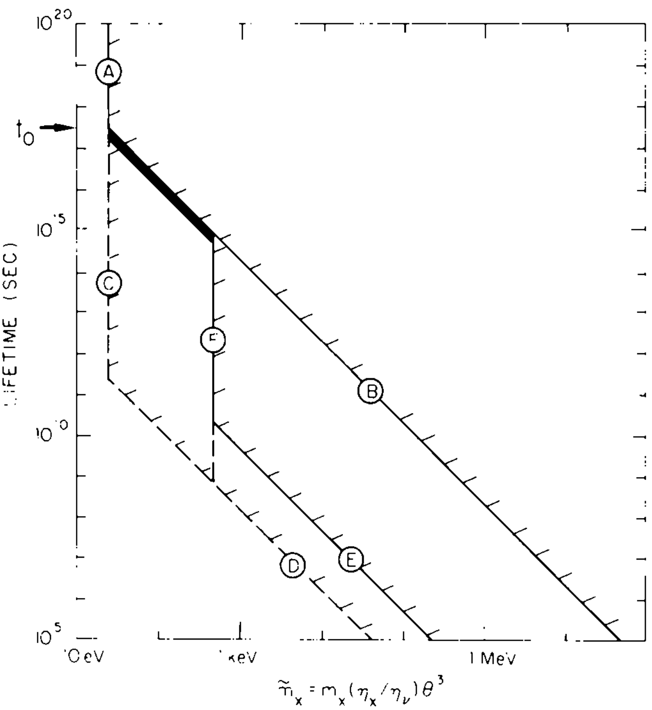

Fig. 4. The lifetime (t_D) versus mass plane; $\tilde{M}_X = M_X(\eta_X/\eta_\nu)\theta^3$ where $\eta_X = n_X/n_\gamma$ and $\eta_\nu = n_\nu/n_\gamma = \frac{3}{11}$. The areas to the left of and below the various curves (see the text) are permitted. The heavily shaded part of curve B is the $\Omega_0 = 1$, RD model proposed by TSK. The areas to the left of and below lines C and D are cosmologically safe, while the region bounded by lines C, D, E, F is, at present, permitted (more careful considerations are likely to exclude this region as well).

Restricting h_0 to be $\geq \frac{1}{2}$ and again taking $t_0 \geq (10-13) \times 10^9$ y results in the tighter constraint [17]

$$\Omega_0 h_0^2 \leq 0.25\text{-}0.75 \,. \tag{17b}$$

In preparing fig. 4 and, in subsequent numerical comparisons, we have used $\max(\Omega_0 h_0^2) = \frac{1}{4}$ so that the limit in eq. (16) becomes $\tilde{M}_X(\mathrm{A}) \leq 24$ eV.

(B) $\Omega_R > \Omega_{NR}$: This is the case, considered by TSK, in which the present universe is R-dominated. If the energy density in the R is too large, again the universe will be too young. From eq. (5) we see that

$$\tilde{M}_X(\mathrm{B}) R_D \leq 96.8[\max(\Omega_0 h_0^2)]\ \mathrm{eV} \,. \tag{18}$$

In this case, X-decay has occurred when the universe was X-dominated (see fig. 1) so that the t_D versus R_D relation is given by eq. (13). It follows then that

$$\tilde{M}_X(\mathrm{B}) \leq 78[\max(\Omega_0 h_0^2)^{3/4}](10^{10}\ \mathrm{y}/t_D)^{1/2} \,. \tag{19}$$

For the same values of Ω_0 and h_0, a radiation-dominated universe is younger than a matter-dominated universe:

$$\text{RD: } t_0 \approx \frac{0.98\times 10^{10}\,\text{y}}{h_0 + (\Omega_0 h_0^2)^{1/2}}\,. \tag{20}$$

For $t_0 \gtrsim 10\times 10^9$ y and $h_0 \gtrsim 0.4$–0.5, $\Omega_0 h_0^2 \lesssim 0.23$–$0.34$, whereas for $t_0 \gtrsim 13\times 10^9$ y and $h_0 \gtrsim 0.4$–0.5, $\Omega_0 h_0^2 \lesssim 0.06$–$0.12$. Roughly speaking then,

$$\tilde{M}_X(\text{B}) \approx \tilde{M}_X(\text{A})(t_0/t_D)^{1/2}\,, \tag{21}$$

where in fig. 4 and throughout the paper t_0 is taken to be $\approx 10^{10}$ y. This mass-lifetime constraint, for a neutrino which decays into relativistic particles, was first discussed by Dicus et al. [2].

(C) $x \lesssim 1$: If $\rho_X \lesssim \rho_{NR}$ then neither the X nor their decay products ever significantly affect the dynamical evolution of the universe (since $\rho \gg \rho_X + \rho_R$). This will be the case for X whose mass is less than

$$\tilde{M}_X(\text{C}) = 96.8[\max\,(\Omega_{NR} h_0^2)] \approx \tilde{M}_X(\text{A})\,. \tag{22}$$

This is a "cosmologically safe" case in the sense that although it *may* be possible for $\tilde{M}_X$ to exceed $\tilde{M}_X(\text{C})$ without violating any observational data, $\tilde{M}_X \lesssim \tilde{M}_X(\text{C})$ is not forbidden by any consideration in this paper.

(D) $R_D < R_X$: Even if $x > 1$ so that early on $\rho_X > \rho_{NR}$, the X may decay while the universe is $\gamma\nu$ dominated (see fig. 3). Once again, neither the X nor their decay products ever significantly influence the dynamical evolution of the universe. This, too, is a "cosmologically safe" case, defined in the $\tilde{M}_X - t_D$ plane by

$$\tilde{M}_X(\text{D}) \approx 0.023(A_{\gamma\nu}\theta^4)^{3/4}(10^{10}\,\text{y}/t_D)^{1/2} \tag{23a}$$

or

$$\tilde{M}_X(\text{D}) \approx 10^{-3}\tilde{M}_X(\text{A})(t_0/t_D)^{1/2} \approx 10^{-3}\tilde{M}_X(\text{B})\,. \tag{23b}$$

Of the cases considered to this point, (A) (C) and (D) have been "cosmologically safe" in the sense that if $\tilde{M}_X \lesssim M_X(\text{A})$ for $t_D \gtrsim t_0$ or, if $\tilde{M}_X \lesssim \tilde{M}_X(\text{C})$ for $10^{-6} \lesssim t_D/t_0 \lesssim 1$ or, if $\tilde{M}_X \lesssim \tilde{M}_X(\text{D})$ for $t_D \lesssim 10^{-6} t_0$, neither the X nor the R ever dominate the energy density of the universe. In contrast, case (B) is potentially dangerous in the sense that (for $t_D \lesssim t_0$) even for masses somewhat smaller than $\tilde{M}_X(B)$, there will be an epoch during which the universe is R-dominated (see fig. 1). It is such recent epochs during which the observed large-scale structure (galaxies, clusters, etc.) of the universe is generally believed to have formed. By requiring that such structure be able to evolve from small density perturbations in an otherwise homogeneous universe, we will find limits to $\tilde{M}_X$ which are more constraining than those found in case B above. Although these new constraints will be weaker than those of cases C and D, our analysis will permit us to deal with a region of the mass-lifetime plane which cannot easily be excluded by "cosmologically safe" considerations alone.

Since we will be using "the formation of structure in the universe" to constrain $\tilde{M}_X$ and t_D, we should first briefly outline the general picture of how structure is believed to have evolved. [For a more detailed discussion we refer the reader to ref. [18]]. The basic idea is that the structure we see today grew up from small density inhomogeneities via the Jeans or gravitational instability. The initial "spectrum" of density inhomogeneities is specified by the amplitudes of the various Fourier components of the density contrast: $\delta\rho/\rho = \sum_k \delta_k\, e^{ikx}$. A particular component is labeled by its comoving wavenumber k or comoving wavelength $\lambda(\equiv 2\pi/k)$, which are related to the "proper" (or physically measurable) wavenumber and wavelength by $\lambda_{phys} = R\lambda$ or $k_{phys} = k/R$. Note at the present epoch $\lambda_{phys} = \lambda$ and $k_{phys} = k$. The density contrast on a given scale is related to the Fourier component by $(\delta\rho/\rho)^2 \simeq k^3|\delta_k|^2$. A convenient way to specify the initial spectrum is to specify the amplitude, $(\delta\rho/\rho)_H$, on a given scale when the scale "entered the horizon", i.e., when $\lambda_{phys} \simeq ct$. For example, the scale-invariant Zeldovich spectrum predicted in inflationary models is characterized by $(\delta\rho/\rho)_H \simeq$ constant [19, 20]. Once inside the horizon, linear density perturbations grow as $\delta\rho/\rho \propto R^n (n = 1$, matter domination; $n = 0$, radiation domination) - Thus the total amount by which a linear perturbation grows is determined by the factor by which the cosmic scale factor has grown during the epoch(s) of matter domination. For reference, $\lambda \simeq 1$ Mpc corresponds to a galactic size $(\simeq 10^{12} M_\odot)$ perturbation. Studies of the galaxy-galaxy correlation function [21] indicate that the scale which is just going non-linear today is $\lambda_c \simeq 5h_0^{-1}$ Mpc. Density perturbations also result in anisotropies in the cosmic μ-wave background radiation [22-25], and thus measurements of the isotropy of the μ-wave background provide valuable and stringent constraints on the initial spectrum of density perturbations. While the precise constraints depend upon the scale dependence of the initial spectrum, for our purposes, we will use the constraint that $(\delta\rho/\rho)_H$ must be $\lesssim 0(10^{-3}\text{-}10^{-4})$ to be consistent with the measured smoothness of the μ-wave background [22-25]. We believe this to be a *very* conservative bound - in all likelihood the constraint is much more restrictive. Finally, we note that we are restricting ourselves to adiabatic (as opposed to isothermal) initial perturbations as they are strongly favored in light of the fact that Ω_0 appears to be $\gg \Omega_B$, the fact that isothermal perturbations are all but inconsistent with scenarios of baryogenesis, and the fact that the perturbations predicted in inflationary scenarios are adiabatic. Allowing for the possibility of isothermal perturbations would lessen only slightly the constraints which we will now discuss.

(E) $R_{NR} \lesssim 10^{-3}$: If $R_X < R_D$, there will be an epoch of X-domination followed at $R = R_D$ by an epoch of R-domination (see fig. 2). Then, at $R = R_{NR} = xR_D$, the universe becomes NR-dominated. If $R_{NR} \lesssim 10^{-3}$, then perturbations in the linear regime will grow by $\gtrsim 10^3$ from $R = R_{NR}$ up to the present epoch. By the standards we outlined above this may be just barely enough growth to account for the observed large-scale structure. For this case to apply, we first must have $R_X = x^{-1}R_{eq} < R_D$ so that $R_{eq} < xR_D = R_{NR} \lesssim 10^{-3}$. This condition, $R_{eq} \lesssim 10^{-3}$, requires that $\Omega_{NR} h_0^2 \gtrsim$

$0.024(A_{\gamma\nu}\theta^4)$. From eqs. (10) and (13).

$$\tilde{M}_X(E) = 78[(xR_D)(\Omega_{NR}h_0^2)]^{3/4}(10^{10}\ y/t_D)^{1/2}\,. \tag{24}$$

For $\Omega_{NR}h_0^2 \lesssim \frac{1}{4}$ and $R_{NR} = xR_D \lesssim 10^{-3}$ we have

$$\tilde{M}_X(E) \approx 6.4\times10^{-3}\tilde{M}_X(A)(t_0/t_D)^{1/2}\,, \tag{24$'$}$$

or

$$\tilde{M}_X(E) \approx 6.4\tilde{M}_X(D)\,. \tag{24$''$}$$

(F) $R_X < R_D < 1$: In this case there are two epochs during which the universe is matter dominated and during which perturbations may grow. Perturbations which enter the horizon before or at $R = R_X$ will grow by a factor R_D/R_X until the universe becomes R-dominated at $R = R_D$. From $R \approx R_D$ until $R \approx R_{NR}$ the universe is radiation dominated and perturbations grow very little. When $R = R_{NR}$ the universe becomes matter dominated again and perturbations resume their growth up to the present epoch. It is clear (see fig. 2) that the total growth factor is

$$\gamma = \frac{R_D}{R_X}\frac{1}{R_{NR}} = \frac{1}{R_{eq}} \approx \frac{4.2\times10^4\Omega_{NR}h_0^2}{(A_{\gamma\nu}\theta^4)}\,. \tag{25}$$

Notice that the growth factor γ is *independent* of R_D, in fact independent of whether or not there were relic X. Now, as TSK have emphasized, we must make sure that the "proper" scales achieve sufficient growth. Perturbations which entered the universe before X-domination have a present size $\lambda \lesssim \lambda_X$ where

$$\lambda_X \approx 53(A_{\gamma\nu}\theta^4)^{1/2}(24\ \text{eV}/\tilde{M}_X)\ \text{Mpc}\,, \tag{26}$$

[In TSK, $N_\nu = 2$, corresponding to $A_{\gamma\nu} = 1.45$ was chosen.] Larger scales ($\lambda > \lambda_X$) enter the horizon at a later epoch ($R > R_X$) and undergo less growth, by a factor $(\lambda_X/\lambda)^2$. As discussed earlier, the scale $\lambda_c \approx 5h_0^{-1}$ Mpc is just entering the non-linear regime today. If $\tilde{M}_X \gtrsim 600h_0$ eV, then $\lambda_X \lesssim \frac{1}{2}\lambda_c$ and $\gamma(\lambda_c) \lesssim \frac{1}{4}\gamma$. Therefore, to insure that initial perturbations of amplitude $\lesssim 10^{-3}$-10^{-4} (on a scale of $5h_0^{-1}$ Mpc) undergo sufficient growth, we require that $\tilde{M}_X \lesssim \tilde{M}_X(F)$ where

$$\tilde{M}_X(F) \approx 25h_0^{-1}\tilde{M}_X(A)\,. \tag{27}$$

4. Summary

The results of our considerations are illustrated in fig. 4. The regions in the t_D versus $\tilde{M}_X$ plane to the right of and above lines A and B are excluded by considerations of the present universal mass density. Here we have adopted $\Omega_0 h_0^2 \lesssim \frac{1}{4}$ so that

(i) $\tilde{M}_X < \tilde{M}_X(A) \approx 24$ eV for $t_D > t_0$, and

(ii) $\tilde{M}_X < \tilde{M}_X(B) \approx M_X(A)(t_0/t_D)^{1/2}$ for $t_D < t_0$.

Note that for $\tilde{M}_X = \tilde{M}_X(B)$ the present universe is radiation dominated and $\Omega_0 = 1$.

For $t_D \gtrsim 10^{-2} t_0$, this case is not inconsistent with the observed large-scale structure (TSK). For $t_D \lesssim 10^{-2} t_0$, the observed large-scale structure excludes case (B); consistency requires that

(iii) $\tilde{M}_X \lesssim \tilde{M}_X(F) \approx 25 h_0^{-1} M_X(A)$ for $t_D \lesssim 10^{-2} t_0$. If $\tilde{M}_X = \tilde{M}_X(F)$, then as t_D decreases, so does R_{NR}; that is, the epoch of NR domination occurs earlier. If $R_{NR} \lesssim 10^{-3}$ then perturbations *may* achieve enough growth to account for the observed large-scale structure. Therefore, for $t_D \lesssim 4 \times 10^{-7} t_0$, we only have the weaker requirement that

(iv) $\tilde{M}_X \lesssim \tilde{M}_X(E) \approx 6.4 \times 10^{-3} \tilde{M}_X(A)(t_0/t_D)^{1/2}$.

The regions below and to the left of curves E and F in fig. 4 may be permitted. However, it is quite clear that if for $10^{-6} t_0 \lesssim t_D \lesssim t_0$,

(v) $\tilde{M}_X \lesssim \tilde{M}_X(C) \approx \tilde{M}_X(A)$, or for $t_D \lesssim 10^{-6} t_0$,

(vi) $\tilde{M}_X \lesssim \tilde{M}_X(D) \approx 10^{-3} \tilde{M}_X(A)(t_0/t_D)^{1/2}$, then neither the X nor their decay products ever dominate the universal mass density. No cosmological considerations (discussed here) exclude the regions to the left and below curves C and D - these regions are cosmologically safe. The analysis in this paper allows the region bounded by curves CDEF. However, more careful consideration of the formation of structure in the universe (for example, detailed analysis of the predicted microwave fluctuations, and numerical simulations of these scenarios) are likely to exclude this region, leaving curves C and D as the relevant constraints.

[In a recent paper which primarily concerned itself with the viability of a neutrino-dominated universe, Hut and White [26] have indirectly discussed one of the mass-lifetime constraints (case E) we have derived here.]

This work was supported at Bartol by DOE grant DE-AC02-78ER-05007 and by the DOE at the University of Chicago and at Fermilab and by NASA at Fermilab. Some of this work was done while G.S. was a guest of the Theoretical Astrophysics Group at Fermilab and while both of us were at the Aspen Center for Physics.

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