A brief history of dark matter

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1. Introduction

The title not withstanding, this is not a history of dark matter. Until we know what the dark matter is, we cannot know its history. Instead, this is a brief history of how astronomers converged to the view that most of the matter in the universe is dark. This paper deals principally with the early studies which helped to answer the questions "Are rotation curves flat? If so, why?" It also includes some early history in deciphering the signature of clusters of galaxies as gravitational lenses, which seems to have been little investigated. This account covers the years up to 1980; achievements since 1980 are science, not history. Several excellent, informative brief histories exist, and interested readers should see Trimble (1987, 1995) and van den Bergh (1999). We can all thank Sidney van den Bergh for correctly translating Zwicky's "dunkle (kalte) materie" as "dark (cold) matter" and finally putting to rest the myth that Zwicky called it "missing matter."

The notion that there are stars that are dark was a common one in the 18th and 19th Century. Walt Whitman's (1855) lines in *Leaves of Grass*, "The bright suns I see and the dark suns I cannot see are in their place" and Bessel's "Foundation of an Astronomy of the Invisible" (Clerke 1885 and reference therein) are early manifestations of this belief. Based on his decade-long observations of Sirius and Procyon, Bessel announced in 1844 that each was a binary star system, whose irregular motion on the sky was due to the presence of its invisible companion. When Maria Mitchell taught her students at the new (1865) Vassar Female College, one of the classes featured Dark Stars.

Even earlier, the Reverend Mr. Mitchell (1784) had imagined a star, 500 times the solar radius but of equal density, and realized that "all light emitted from such a body would be made to return toward it, by its own proper gravity." Moreover, "if there really should exist in nature [such] bodies... since their light could not arrive at us, we could have no information from light. Yet if any other luminous bodies should happen to revolve about them, we might still perhaps from the motions of these revolving bodies infer the existence of the central ones." An impressive intuition, almost 200 years before we knew of black holes.

And of course, Vincent van Gogh's "Starry Night" (1889) is surely every optical astronomer's nightmare of what the night sky would look like if the dark matter were not dark.

2. The early 20th century

Kapteyn (1922), in his efforts to study the arrangement and motion of the sidereal system, estimated "the amount of dark matter" in a universe with the sun at the center of similar ellipsoids of revolution. Using star counts and physics, he concluded that "it appears at once that this (dark) mass cannot be excessive." Kapteyn references Jeans (1919, p. 239) for a dark star determination. However, in my 1919 edition of Jeans' book, there is no mention of dark stars on the cited page 239. Instead, the only estimate I have located in a quick perusal is on page 222, where Jeans writes "these estimates evaluate the density of matter in the bright stars only; the dark stars, of which it is impossible even to guess at the number, will increase the density to a quite unknown extent, so that the estimates only provide lower limits to the true density." But in a few years Jeans had changed his mind. With later work, Jeans (1922) counts "about three dark stars in the universe for every bright star." Trimble (1995) has pointed out that this range of dark matter density matches closely the range of dark matter density currently discussed. A decade later, Oort's (1932) study of the mass density in the Galactic plane also left its mark with the name "Oort limit."

Zwicky's (1933; also 1937c) analysis of the velocity dispersion for galaxies in the Coma cluster marks the beginning of the contemporary study of dark matter in the universe, albeit a slow beginning. His study, plus that of Smith (1936) for the Virgo cluster, noted that the large relative motions for individual galaxies would disrupt the clusters, unless each galaxy has a mass about 100 times the accepted mass. Zwicky also cited good evidence that clusters are not dissolving.

The discrepancy between the high galaxy masses calculated from the viral mass of the clusters, and the low masses calculated from the very inner rotation curves for five galaxies, troubled Hubble (1936). "The discrepancy seems to be real and is important," he wrote. It is not surprising that these early absorption line rotation curves extended only over the brightest nuclear regions, and were poor indicators of galaxy mass. Several decades would pass before the cluster dark matter would be associated with the flat rotation curves derived for individual galaxies.

Observations of M31's rotation by Babcock (1939) and Mayall (1951) extended major axis rotation velocities to almost $120'$ from the nucleus, but exposure times were tens of hours, and spectrographs had stability problems. Interestingly, both Babcock's velocities for M31 and Humason's unpublished velocities for NGC 3115 showed the last measured point to have a rotation velocity of over 400 km s^{-1} (almost two times actual), but consequently raised questions of mass distribution.

At a symposium celebrating the dedication of the McDonald Observatory in 1939, Oort (1940) noted that "... the distribution of mass [in NGC 3115] appears to bear almost no relation to that of the light." His conclusion, "The strongly condensed luminous system appears embedded in a large more or less homogeneous mass of great density," was a clear statement of the puzzle that would grip astronomers again in the 1970s. However, it seems to have impressed few in 1940 and in the decades following.

3. Instrumentation starts catching up: Mid-century

Early radio observations of neutral hydrogen in external galaxies showed a slowly falling rotation curve for M31 (van de Hulst el al. 1957) and a flat rotation curve for M33 (Volders 1959). Because the flatness could be attributed to the side lobes of the beam, it was consequently ignored. Louise Volders must also have realized that a flat rotation curve conflicted with the value of the Oort constants for our Galaxy, which implied a falling rotation curve at the position of the sun. Jan Oort was one of her thesis professors.

With eight rotation curves available by 1959, de Vaucouleurs (1959) concluded "In all cases the rotation curve consists of a straight inner region... beyond which the rotational velocity decreases with increasing distance to the center and tends asymptotically toward Kepler's third law." From a reanalysis of the same scattered velocities for M31, Schwarzschild (1954) reached the opposite conclusion. He stated that the approximately flat rotation curve was "not discordant with the assumption of equal mass and light distribution." With the 20/20 vision of hindsight, plots of the data reveal only a scatter of points, from which no certain conclusions can be drawn.

A paper that was more in the spirit of what was to come than what had taken place was Kahn and Woltjer's (1959) investigation of the dynamical stability of the Local Group. They concluded that the Local Group must contain an appreciable amount of invisible matter. In a sense, this was a contemporary formulation of Zwicky's virial cluster problem.

Although Zwicky's cluster results were not forgotten, they only came to the forefront of astronomy in discussions of the stability of clusters. At two symposia in Santa Barbara preceding the Berkeley 1961 General Assembly, Ambartsumian (1961) had significant support for his view that clusters were explosively disintegrating (but see van den Bergh 1961; 1999). Much of the discussion centered on galaxy radial velocities which were beginning to be obtained in fairly large numbers. My notes from the symposia mention dark matter once, in connection with the disks of early type spirals.

In an effort to learn how our Galaxy "ended," my graduate students at Georgetown and I made a study of the three dimensional velocities of almost 1000 O and B stars beyond the solar circle (Rubin et al. 1962). Our 1962 conclusion, "the stellar (rotation) curve is flat, and does not decrease as is expected for Keplerian orbits," apparently influenced very few, and was not emphasized by the senior author when she returned to the problem of galaxy rotation a decade later. In my earliest Kitt Peak observing, I attempted to obtain a rotation curve for the Galaxy beyond the solar circle by observing O and B stars near the anticenter direction (Rubin 1965). It was clear that while many studies of the center of the Galaxy were underway, there was little attention being paid to the outer limits of galaxies.

The many galaxies studied by Margaret and Geoffrey Burbidge (e.g. Burbidge, Burbidge, & Prendergast 1962) generally showed an inner velocity rise; velocities were then expected to fall. For at least one galaxy, NGC 7331 (Rubin, Burbidge, & Burbidge 1965), we showed three possible density laws which extended the velocity curve beyond the turnover; one predicted rotation curve is rising slightly, one is slightly falling. This was our way of saying that we did not know what happens beyond the final measured velocities.

4. The decade of seeing is believing: The seventies

Science often advances when ideas, formerly very disparate, are united. In retrospect, it took a long time for astronomers to relate Zwicky's dark matter to the flat rotation curves for some galaxies that were beginning to attract attention. If I were to choose a date when astronomers decided that dark matter must "really" exist, I would pick 1978. In 1977, many astronomers hoped that dark matter might be avoided; by 1979 the Annual Review article by Gallagher and Faber convinced most of the remaining skeptics. Of the two questions to be answered, "Are rotation curves flat? If so, why?" we would arrive at an answer to the first.

Freeman (1970) discussed 21-cm velocity maps of a number of nearby galaxies. For NGC 300 and M33 he concluded "if [these data] are correct, there must be in these galaxies additional matter that is not detected, either optically or at 21 cm. Its mass must be as large as the mass of the detected galaxy, and its distribution must be quite different from the exponential distribution which holds for the optical galaxy." His remarks are reminiscent of Oort's (1940) concerning NGC 3115, but technology had now made it possible to measure velocities beyond the optical galaxy.

In the same year, Kent Ford and I completed our study of the velocities of emission regions in M31 (Rubin & Ford 1970), and produced a rotation curve which extended to 120′, the extent of the optical disk. The curve was flat over the last 30% of the galaxy.

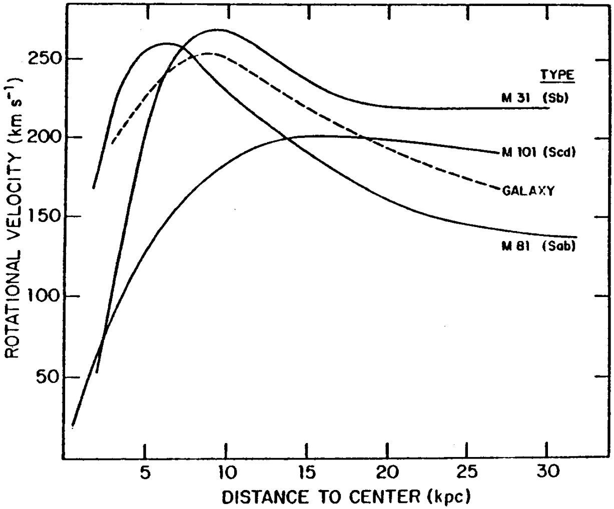

FIGURE 1. Rotation curves for the galaxies M31, M101, M81 and the rotation curve of our Galaxy, from Roberts and Rots (1973).

Perhaps that is why we chose not to employ the typical mass models that assumed a Keplerian fall beyond the last observed region, but instead wrote that "extrapolation (of the mass) beyond that distance is clearly a matter of taste." In an era when rotation curves were routinely extended in a Keplerian fashion beyond the final observed point, we chose not to do so.

More flat 21-cm rotation curves followed. Rogstad and Shostak (1972) and later Krumm and Salpeter (1976) obtained flat 21-cm rotation curves for more galaxies, but rumors of sidelobe problems continued to plague such studies. Mort Roberts, whose very extended rotation curve of M31 (1995) would lead to the acceptance of flat rotation curves, delayed that acceptance by publishing (Roberts & Rots 1973) a plot (Fig. 1) of superposed rotation curves for M31, M101, our Galaxy, and M81. For our Galaxy and M81 the outer velocity decreases were so substantial that the eye of the beholder remembered mostly the falling parts.

However, the theorists had their eyes wide open. Ostriker, Peebles, and Yahill (1974) introduced their paper on galaxy masses with the stunning sentence "There are reasons, increasing in number and quality, to believe that the masses of ordinary galaxies may have been underestimated by a factor of 10 or more." This work, plus that of Einasto, Kaasik, and Saar (1974) "The mass of galactic coronas exceeds that mass of populations of known stars by one order of magnitude, as do the effective dimensions," made use of arguments both observational and theoretical. They emphasized the evidence showing that masses of nearby giant spirals increase linearly with radius from 20 to 100 kpc and to as much as 500 kpc. These papers, coupled with an earlier paper (Ostriker & Peebles 1973) which demonstrated (with 150 to 500 mass points) that disk galaxies were "found

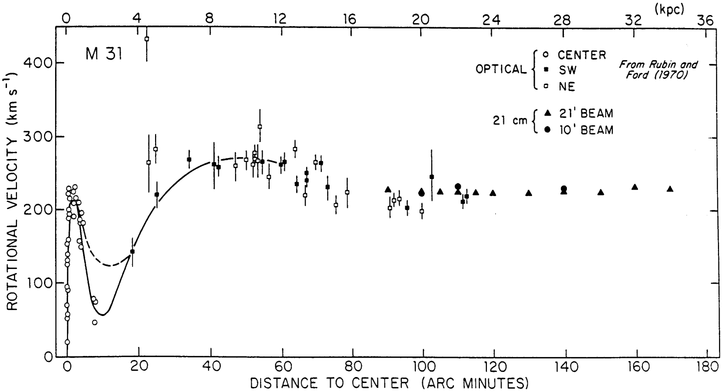

FIGURE 2. Rotation velocities in M31 as a function of distance from the nucleus. Optical Hα velocities come from Rubin and Ford (1970); 21-cm velocities come from Roberts and Whitehurst (1975).

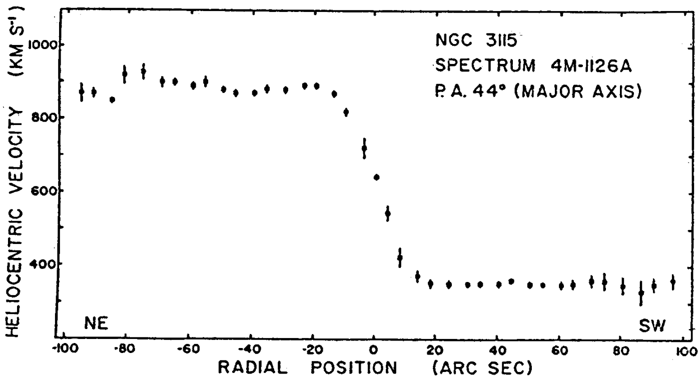

FIGURE 3. Rotation velocities for NGC 3115 from stellar absorption lines from Rubin, Peterson, and Ford (1976; @American Astronomical Society).

to be rapidly and grossly unstable to barlike modes," attracted considerable attention. It is surprising that it did not send scores of observers scurrying to their telescopes.

By 1975, the 21-cm studies by Mort Roberts and his collaborators produced (Roberts & Whitehurst 1975) a rotation curve for M31 (Fig. 2) that extended to 35 kpc (175′), almost 50% farther than the optical curve (Rubin & Ford 1970). At least for M31, there was no longer room for argument: the rotation curve was mathematically flat, over more than 50% of the detectable disk, much of it beyond the optical galaxy. For this best observed disk galaxy, astronomers knew that mass rose linearly with radius, and did not asymptotically approach a limit.

Flat rotation curves were not restricted to spirals. Rubin, Peterson, and Ford (1976) submitted an AAS abstract that consisted of only a plot and a title: The Rotation Curve of the E7/S0 Galaxy NGC 3115 (Fig. 3). It was a textbook plot: lat, steep, flat.

For a few astronomers' views of rotation curves in 1977, we have an impeccable source: a discussion at the Yale meeting, *The Evolution of Galaxies and Stellar Populations* (Tinsley & Larson 1977). I reproduce some of the discussion which followed the talk by Freeman.

> FREEMAN (to Krumm and Salpeter): Rumor has it that your flat rotation curves may be affected by sidelobe problems. Could you comment please?
>
> SALPETER: Sidelobe problems affect only the last point on a rotation curve. Even if the last point were removed, a large enough range of radial distances remains to demonstrate flat rotation curves.
>
> FREEMAN: Is anyone prepared to offer any alternative way of explaining the flat rotation curves aside from having a massive halo?
>
> M. ROBERTS: Why do you need a massive halo? Why not an unmassive disk? The mass goes up with the radius so it only need to be doubled. All you need is the most common type of star in the solar neighborhood, M dwarfs, to account for the M/L.
>
> The problem with the flat rotation curves is not peculiar to only one telescope, but is common to all the large telescopes that have been used. Any given case may be arguable, but overall there do seem to be flat rotation curves—although not ALL rotation curves are flat.
>
> L. SMITH: If it were true that 50% of stars formed were below 0.1 $M_\odot$ and if it were true that only MASSIVE star formation stops at $\approx$ 13 kpc galactic radius, would that solve the problem.
>
> M. ROBERTS: Yes.
>
> KING: Although I'm not particularly fond of massive halos, I can think of one argument that would favor an extended mass being in a halo rather than in a disk. This is based on the Westerbork observations of the high incidence of disks with twisted edges. If a twisted disk has much mass in its outer parts, it is very hard for it to maintain a clean twist without turning into a washboard. But it is much easier to maintain the twisted shape if the mass is in a halo instead.
>
> OSTRIKER: From observations by Hy Spinrad and myself, it does seem difficult to account for the mass with ordinary late M dwarfs, for they would give too much light. Of course, stars of even lower mass are possible. My other point is that if all the mass is in a flat, cold disk it is very likely to be unstable. But since the mass is invisible, it could be in a flat, hot disk.

As these exchanges indicate, astronomers were willing, in 1977, to accept that some rotation curves were flat. But at the same meeting, Rees (1977) focused his talk, Galaxy Formation, on the "implications of massive halos and 'missing mass' (which, if the participants in this conference are an unbiased sample, are seriously believed to exist...)." Yet not a single author referenced Zwicky's studies of dark matter in clusters of galaxies.

The next year, Bosma (1978) completed his thesis, observing and compiling 21-cm rotation curves for 25 galaxies (Fig. 4). All but a few had flat or almost flat curves. Only M81, M51, and M101 showed significant outer falling velocities; explanations in terms of tidal interactions would ultimately arise. Also in 1978, Kent Ford and I (Rubin & Ford 1978) published data for eight rotation curves of high luminosity spirals, and photos (Fig. 5) which showed their emission line spectra all strikingly flat to the eye. I think that Bosma's plots, plus the visual spectra, convinced many astronomers that rotation curves are flat. Not flat was the exception. But there were still non-believers. One eminent astronomer said to me "When you observe low luminosity galaxies, you'll find Keplerian falling rotation curves." Not so, of course. We know now that the lower

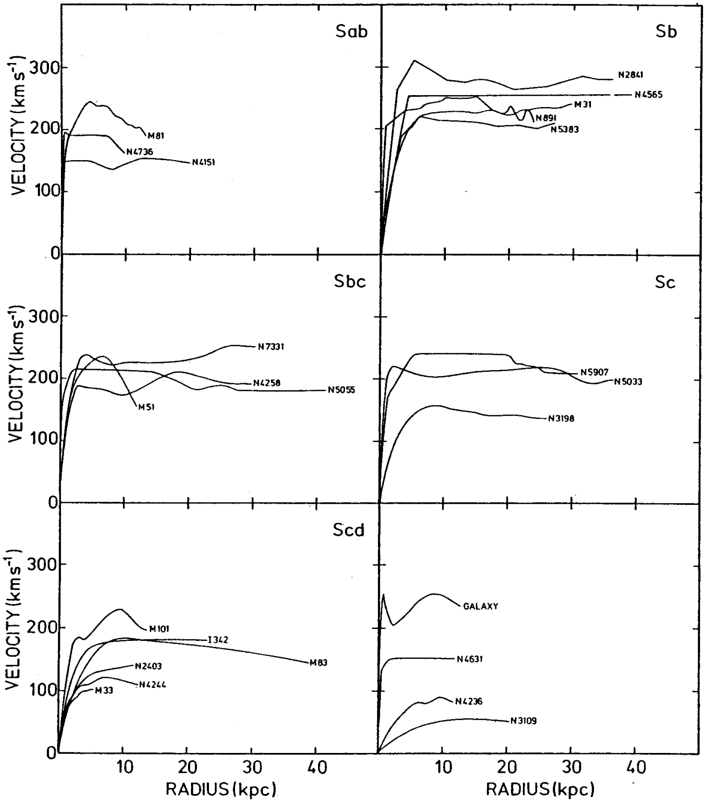

FIGURE 4. 21-cm rotation curves for 25 galaxies, from Bosma's (1978) thesis.

the luminosity, the fractionally more dark matter required. Kalnajs' (1983) insistence that dark matter is not required, at least for a few galaxies with spatially limited data, convinced a few astronomers that dark matter could be avoided. In retrospect, I think it is fair to say that many astronomers hoped that Kalnajs was right; dark matter was to be avoided, if at all possible.

In their review "The Kinematics of Spiral and Irregular Galaxies," van der Kruit and Allen (1978) waffled in their conclusion. "It is certainly true that more mass is waiting to be found beyond the last measured HI points on many rotation curves... However, the great increase (factors of 10 to 100) in masses favored by Einasto et al. (1974) and by Ostriker et al. (1974) involve estimates at much greater distances, from 200–500 kpc. ...There is no evidence in favor of such massive halos *within* the visible disks of galaxies...

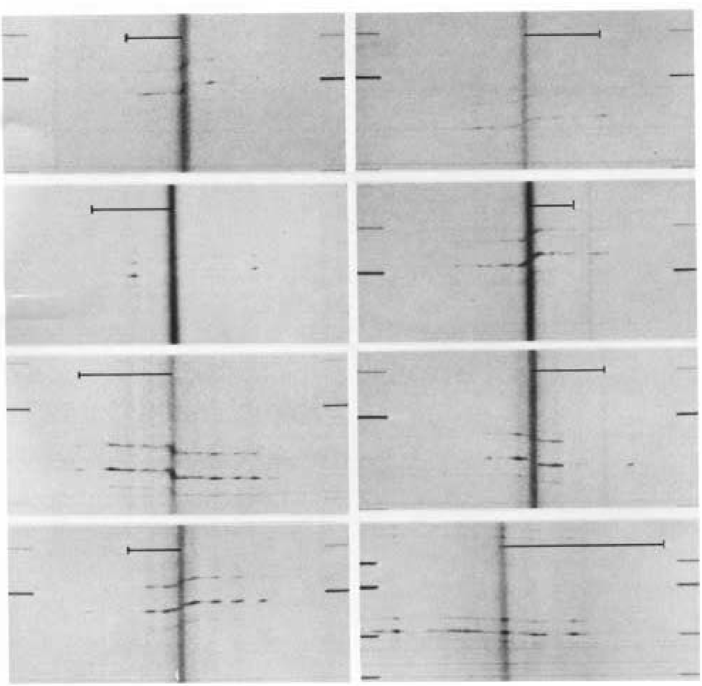

FIGURE 5. Spectra showing emission lines (dark) of Hα and [NII] for galaxies of different Hubble types, taken with the *Kitt Peak* 4-m spectrograph plus Carnegie image tube. Exposures range from 120 to 200 minutes (Rubin and Ford 1978).

We must conclude that the results from rotation curves are not inconsistent with the existence of extensive, massive halos around galaxies, although the prime evidence for them comes from studies of binary galaxies and outlying globular clusters (e.g. Turner and Ostriker 1977, Sargent 1977)."

Only one year later, a comprehensive review by Faber and Gallagher (1979) concluded more emphatically "After reviewing all the evidence, it is our opinion that the case for invisible mass in the Universe is very strong and getting stronger." Finally, a paper (since Hubble 1936) had united in print Zwicky's dark matter with flat rotation curves of galaxies.

Ostriker (1999) chose Zwicky's (1937c) *Astrophysical Journal* paper "On the Masses of Nebulae and Clusters of Nebulae" for reprinting in the *ApJ Centennial Issue* (Abt 1999). In a glowing discussion of Zwicky's paper, Ostriker wrote "Thus, we (Ostriker et al. 1994) took comfort in the fact that our estimates for the total mass were consistent with those reached by Zwicky decades earlier." Yet even this ground-breaking paper of

Ostriker et al. (1994) failed to reference Zwicky. However, these authors were in good company. A decidedly incomplete survey of papers in the 1960s and 1970s (but which does include most of the relevant papers referenced in the present paper) turned up not a single paper pre-Faber and Gallagher (1979) that referenced Zwicky.

Following the influential Faber and Gallagher (1979) review, it was "general belief" that rotation curves were flat. As more rotation curves accrued, dark matter became the accepted cause. But in as much as we have not yet succeeded in identifying the composition of dark matter, attributing flat rotation curves to 'dark matter' seems at times only a semantic construct.

5. Another approach: Gravitational lenses

As usual, Zwicky (1937a, 1937b, 1937c) was right. Gravitational lenses do offer a method for inferring the existence of dark matter in clusters of galaxies.

Early work concentrated on single galaxies as lenses. Just a few years after detecting the deflection of light by the sun during the 1919 total solar eclipse, Frost (1923, mentioned in Zwicky 1937b) suggested searching for the gravitational deflection of background sources by stars. A decade later, Zwicky (1937a, 1937b, 1937c) realized that the more massive galaxies would be far more important as lenses. Refsdal (1964) cites the literature through 1964, and Barnothy (1989) traces the detailed history preceding his suggestion (Barnothy 1965) that QSOs were lensed Seyfert galaxies.

Clusters of galaxies as gravitational lenses have a convoluted history. During tests of a SIT Vidicon, Jim Westphal (1973) observed the cluster Abell 370 with the 200-inch telescope. Images comparing the cluster photographed with the SIT Vidicon and an unaided photographic plate were published by *Science* magazine. In retrospect, both images are very noisy, but do show a gravitational arc, not mentioned. Hoag (1981) discussed a filament-like feature (the arc) in A370 at an AAS meeting; the source of the image is a *KPNO* 4-m prime focus plate.

A decade after the SIT Vidicon images, Butcher, Oemler, and Wells (1983) published a beautiful *Kitt Peak* 4-m prime focus image of A370 (Fig. 6), taken for their detailed study of galaxies in clusters. The very sharp and prominent arc is not referred to in the paper. When I sent an email request to Butcher and Oemler to reproduce the image in this paper, I gently warned them that is was included in papers which had been published, but had not noted the arc. Oemler replied (almost instantly) "Of course you may use it, and of course you may mention that we stared at that damn arc for endless hours without recognizing it—I always do."

Several years later, Lynds and Petrosian (1986) presented a paper "On the Giant Luminous Arcs in Clusters" at the AAS meeting. They noted their properties: spatially coherent, narrow arc-like shape of enormous lengths, whose radii of curvature point toward the central cD galaxies and the centers of gravity of the clusters. As in the previous work, a gravitational lens was not mentioned. A discussion with Lynds (private communication, 2000) suggests that he and Petrosian were considering various explanations, but were waiting to get a good spectrum of the arc.

Soucail and colleagues solved the puzzle, following a false start. Initially they (Soucail et al. 1987a) attributed the arcs "in" clusters to a characteristic of the cluster, perhaps a cooling flow." Katz (1987) and Milgrom (1987) thought them to be light echos from a beaming cluster source. Later, from initial spectra of poor quality, Soucail et al. (1987b) identified the arc in A370 as a gravitational lens. The next year, Soucail et al. (1988), in a paper received by *A&A* on November 17, 1987, showed a beautiful spectrum taken with an arc-shaped slit, which confirmed that the arc is the image of a galaxy at $z = 0.724$,

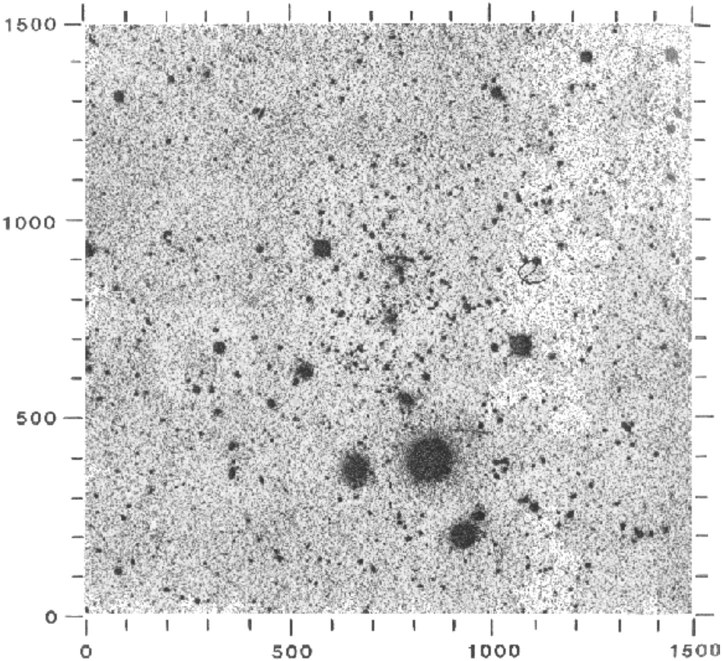

Figure 6. *Kitt Peak* 4-m prime focus image of Abell cluster A370, from Butcher, Oemler, and Well (1983). Note the arc, which was not mentioned in the paper.

gravitationally lensed by A370 at $z = 0.374$. It is true, as textbooks instructed, that lensing is optimal for a mass midway between the observer and the lensed object.

At about the same time in late 1987, Lynds and Petrosian (1988) submitted a late paper for the AAS meeting in Austin, Texas, January 1988. They too had obtained spectra and redshifts and announced the arc as the lensed image of a background galaxy. Their detailed paper (Lynds and Petrosian 1989) gives a slight history leading up to this conclusion.

6. Conclusion

By the end of the decade of the 70s, astronomers generally agreed that rotation velocities did not show a Keplerian decline toward the edge of the optical disk. An extended spheroidal distribution of matter, roughly centered on the galaxy center, more extended than the optical galaxy, and more massive that the luminous galaxy, was the generally adopted solution. Today, although we think we know the quantity of luminous matter in the universe, and can put a limit on the amount of dark matter in the universe, we have made little progress in deducing the composition of the dark matter.

For a few scientists (Finzi 1963; Bekenstein & Milgrom 1984, Milgrom 1983; Sanders 1990, McGaugh and de Blok 1998), flat rotation curves have been a sufficient reason to investigate alternatives to Newtonian gravitational theory. Rather than discuss these alternatives which are generally as valid as dark matter in fitting the observations (although gravitational lensing might stress the theories), I will close with a few historical comments relating to scales in nature.

At the close of the 19th century, physicists were discovering phenomena that did not fit into the science they had grown up with. Late in life, in discussing the early years of quantum mechanics, Heisenberg (1961) wrote "Although the previous laws of nature, e.g., Newtonian mechanics, contained so-called constants, the constants referred to the properties of objects... On the other hand, Planck's action quantum, which is the characteristic constant in his law of radiation, does not represent a property of objects, but a property of nature. It establishes a scale in nature."

Heisenberg continues "While the laws of former physics, e.g., Newtonian mechanics, should basically be valid for all orders of magnitude (the movement of the moon around the earth should obey the same laws as the fall of the apple from the tree or the deviation of an alpha particle that grazes the nucleus of an atom), Planck's law of radiation shows for the first time that there are scales in nature, that phenomena in different ranges of magnitude are not necessarily of the same type." It is interesting that Heisenberg's large scale example does not extend beyond the solar system.

Feynman, in "The Character of Physical Law" (1965) starts with the solar system, and the problem with the apparent motion of Mercury which "was shown by Einstein that Newton's Laws were slightly off and then they had to be modified. The question is, how far does this law extend? Does it extend outside the solar system?" After discussing a binary star system and a globular cluster, he shows a "typical galaxy, and it is clear once again that this thing is held together by some force, and the only candidate that is reasonable is gravitation. When we get to this size we have no way of checking the inverse square law, but there seem to be no doubt that in these great agglomerations of stars... gravity is extending even over these distances."

He ends by naming a characteristic that gravity shares with other physical laws. "It is mathematical, ...it is not exact; Einstein had to modify it, and we know that it is not quite right yet, because we have still to put the quantum theory in. That is the same with all our other laws—they are not exact. There is always an edge of mystery, a place where we have some fiddling around to do yet. This may or may not be a property of Nature, but it is certainly common to all the laws as we know them today. It may be only a lack of knowledge."

Only the future will tell us what the dark matter is, or whether our lack of knowledge of gravitation on the largest scales has fooled us. It will be exciting to follow the path that leads us from this edge of mystery to answer the question, "What is dark matter?"

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