CIS522 Advanced Algorithm Design and Complexity

Name:__________________________ Learning Activity Last 3-Digit ID: □□□

Homework #8. Identify Problem Complexities and Problem-Solving.

For each problem below:

- Build a model of the problem if such a model is not provided.
- Decide which class it belongs to: P, NP, NP-Complete, PSPACE.
- If possible to solve this problem (optimally or approximately) with a **polynomial-time** algorithm:
    - Describe the overall idea of the algorithm in one paragraph using English language,
    - Present the algorithm details using pseudo code with the similar format and detail level as in textbook. Be clear of the meaning of each variable, use comments on important steps to explain its purpose.
    - Must walk through the algorithm step by step with a small problem instance (create a problem instance yourself if the problem does not provide one) to show how the algorithm works. (Required, unless you have implemented the algorithm and show the output instead).
    - Analyze time complexity of algorithm and present results in big-O notation.
    - If this algorithm is an approximation algorithm provide the approximation guarantee.
- If you model a problem as a network-flow problem or its extension:
    - Describe what the nodes, edges, capacities, source, sink represent and any necessary information.
    - Describe how the solution to the (extended) network-flow problem may be interpreted as a solution to the original problem.
    - If you use any standard algorithm (such as max-flow, augmenting path) presented in Chapter 7, you do NOT need to describe the details of the algorithm, but you need analyze the time complexity of this algorithm in the context of this specific problem you are solving here, such as what n means and what m means.
- If no polynomial-time algorithm (neither optimal nor approximate) can be found for this problem, show the complexity of this problem is at least NP-Complete following the general three-step strategy on page 473.
    - To prove a problem X is NP is to show there is an efficient certifier for X, which means, there is a polynomial time algorithm to check if s is a solution to X or not. (Page 464 – 465)
    - To show problem Y (that is known to be NP-Complete) is polynomial-time reducible to X, following the outline described in the middle of Page 473.
    - Besides general description of the transformation, construct a concrete small problem instance of X and show its relationship to an arbitrary problem instance of Y satisfies the properties (a) and (b) described in the middle of Page 473.

1. Given k network routes from one source to one destination, and n files to be transfer from the source to the destination; each file fi has length li, the transfer speed along all k paths are identical. Find out how to assign files to each of the k routes so that to finish the transfer of all n files in the shortest time.
   For example, given 10 files, their lengths are: 5, 6, 7, 3, 2, 10, 9, 8, 6, 4, and 3 paths, the optimal solution is to transfer them in 20/speed time unit.

Exercise Type: Preparation In Class Practice P41
Grade Type: Just for fun Boolean Numeric
Submission time: ______________________ Graded By:__________________ Grade: ________

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2. Consider a Multiple Interval Scheduling problem. You have a processor that is available to run jobs over some period of time (e.g., 9 A.M. to 5 P.M). People submit jobs to run on the processor; the processor can only work on one job at any single point in time. Each job requires **a set of** intervals of time during which it needs to use the processor. Thus, for example, a single job could require the processor from 10 A.M. to 11 A.M., **and** again from 2 P.M. to 3 P.M. If you accept this job, it ties up your processor during those two hours, but you could still accept jobs that need any other time periods (including the hours from 11 A.M. to 2 A.M.). Now you're given a set of n jobs, each specified by a set of time intervals, and you want to answer the following question: For a given number k, is it possible to accept at least k of the jobs so that no two of the accepted jobs have any overlap in time?

3. There are n airport sites and m direct flight scheduled among these sites. To select a subset of airport sites to build service facility so that each direct flight can be served at either it's start airport or its destination airport. The cost to build such service facility at each site is different, specified as Ci for airport site Ai. The goal is to select a subset of airport sites with minimized cost and also ensure all flights are severed.

4. Telecommunication company needs to base stations in various location to provide mobile phone service. The distance to a based station affects the communication quality. Given houses location in n different locations, select k locations to build k base stations so that the maximum distance from any house to its closet station is minimized.

5. We have a collection of n job requests to select from, each job i pays p_i amount of reward. However, certain pairs of jobs cannot be selected together due to their time conflict. Gladly discovered that there are only exactly (n-1) pairs of conflicts but unfortunately, there is no job that is free of conflict with other jobs. The goal is to select the set of jobs that have no conflicts among themselves and maximized the total amount of reward.

6. In a city with m roads and n crossroad point, the mayor is planning to install security cameras at some selected crossroad points to monitor all roads in the city. Limited budget only allows 10 security cameras to be installed. Find out if it is possible to place 10 or less security cameras at the crossroads in a way that will let you see the whole city roads.

Exercise Type:	Preparation	In Class	Practice	P42
Grade Type:	Just for fun	Boolean	Numeric	

Submission time: _________________________ Graded By:____________________ Grade: _________