CIS 522: Homework 1

Anubhav Shankar(01951462)

Q1.) Chapter 1, Exercise 4, Page 23

Ans.) **Brief Description**: The basic idea of the Gale-Shapley(1962) Algorithm is to, in this case, find a way to assign each student to *at most* one hospital in such a way that all the vacant positions in the list of 'm' hospitals is filled by a student chosen from a subset of 'n' students. Concurrently, both the hospitals and the students have an ordered list of preferences.

Now, in this case, there is a slight caveat given that each student is looking to accept a position in exactly one hospital whereas each hospital is looking to fill multiple positions. Also, the question in this case assumes that the number of graduating students outnumber the total available positions across 'm' hospitals. Hence, at any point in time a particular student is either "committed" or "free". Similarly, a hospital either has available positions or it is "full".

Pseudocode:

While some hospital 'm_i' has available positions

m_i offers a position to the next student 'n_i' on it's preference list

if n_i is free then

n_i accepts the offer

else(n_j is already committed to a hospital m_k)

if n_j prefers m_k to m_i then

n_j remains committed to m_k

else n_j becomes committed to m_i

the number of available positions at m_k increases by 1

the number of available positions at m_i decreases by 1

The algorithm, in this case, will have a time complexity of O(m*n) unlike regular Gale-Shapley which has a time complexity of $O(n^2)$ owing to the possibility of a one-to-one matching in a square matrix.

Proof: Algorithm returns a stable assignment -> Extending from Lemma (1.5) wherein we prove that a set returned at termination is a perfect match, we will now use Lemma (1.6) to prove that the returned set is a stable match.

As the question stem lays out the necessary instability conditions, let us tackle them one-by-one:

1.) Let there be two students s and s`, and a hospital h.

    If h preferred s` to s, then h would have offered a position to s` before offering one to s. Consequently, s` would have a position at another hospital h` and would not be free at the termination of the algorithm. Hence, by contradiction, the first instability is disproved.

2.) Let there be a pair (h`, s`) which is an unstable pairing.

    Then it stands to argue that h` must have offered a position to s` in spite having 'x' residents whom h` prefers over s`.

    Correspondingly, at some point, s` must have rejected h` for 'h' which he/she preferred and must therefore be committed to another hospital h`` (likely different from 'h') which he/she prefers over h`. Hence, by contradiction, we see that this pairing is actually stable.

Q2.) Implementation of Gale-Shapley algorithm

Ans.)

1.) Victor to Zeus:

```
Confirmed Matches!
Victor   Amy
Xavier   Bertha
Wyatt   Clare
Zeus   Diane
Yancey   Erika

Process finished with exit code 0
```

2.) Zeus to Victor:

```
Confirmed Matches!
Victor   Amy
Xavier   Bertha
Wyatt   Clare
Zeus   Diane
Yancey   Erika

Process finished with exit code 0
```

3.) Amy to Erika:

```
Confirmed Matches!
Diane   Victor
Clare   Wyatt
Bertha   Xavier
Erika   Yancey
Amy   Zeus

Process finished with exit code 0
```