Group # 2
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Sectional Written Homework #2: (**75 points**):

1. **(10 points)** Given the observed data below,

Name	Give Birth	Can Fly	Live in Water	Have Legs	Class
human	yes	no	no	yes	mammals
python	no	no	no	no	non-mammals
salmon	no	no	yes	no	non-mammals
whale	yes	no	yes	no	mammals
frog	no	no	sometimes	yes	non-mammals
komodo	no	no	no	yes	non-mammals
bat	yes	yes	no	yes	mammals
pigeon	no	yes	no	yes	non-mammals
cat	yes	no	no	yes	mammals
leopard shark	yes	no	yes	no	non-mammals
turtle	no	no	sometimes	yes	non-mammals
penguin	no	no	sometimes	yes	non-mammals
porcupine	yes	no	no	yes	mammals
eel	no	no	yes	no	non-mammals
salamander	no	no	sometimes	yes	non-mammals
gila monster	no	no	no	yes	non-mammals
platypus	no	no	no	yes	mammals
owl	no	yes	no	yes	non-mammals
dolphin	yes	no	yes	no	mammals
eagle	no	yes	no	yes	non-mammals

Show your stepwise calculation for assigning the class label for a new animal with the following attribute values, using Naïve Bayes.

Give Birth	Can fly	Live in water	Have Legs	Class (Mammal or non-mammal)
No	yes	yes	no	?

(**No score** will be given if you only answer “mammal” or “non-mammal”)

Your answer:

Important -> Naïve Bayes assumes independence among attributes x_j when given a class.

Mathematically,

p(x1, x2, … , xn | Ci) = p(x1 | Ci) p(x2 | Ci) … p(xn | Ci)

Here, X: attributes -> Give Birth, Can fly, Live in water, Have Legs.

Two classes -> M = Mammal, N = Non-Mammal

P(M) = 7/20 -(1)
P(N) = 13/20 –(2)

Two cases:

a.) P (Class = Mammal|X) α p(x | Class = mammal) p(Class = mammal)
b.) P (Class = Non-Mammal|X) α p(x | Class = Non-Mammal) p(Class = Non-Mammal)

Case 1 -> P (Class = Mammal|X) α p(x | Class = mammal) p(Class = mammal)

*p(x | Class = mammal) = p(GiveBirth = No | Class = mammal) * p(CanFly= Yes | Class = mammal) *p(LiveinWater=Yes| Class = mammal) * p(HaveLegs = No| Class = mammal)* - (3)

From (1)

n(Mammals) = 7

p(X|M) = 1/7 * 1/7 * 2/7 * 2/7 = 0.0016 -(4)

p(X|M) p(M) = 0.0016 * 7/20 = $5.83 * 10^{-4}$ - (5)

Case 2 -> P (Class = Non-Mammal|X) α p(x | Class = Non-Mammal) p(Class = Non-Mammal)

*p(x | Class = Non-mammal) = p(GiveBirth = No | Class = Non-mammal) * p(CanFly= Yes | Class = Non-mammal) * p(LiveinWater=Yes| Class = Non-mammal) * p(HaveLegs = No| Class = Non-mammal)* - (6)

From (2)

n(Non-Mammals) = 13

p(X|N) = 12/13 * 3/13 * 3/13 * 4/13 = 0.0151 - (7)

p(X|N) p(N) = 0.0151 * 13/20 = 0.0098 - (8)

From (5) and (8) ->

P(Class = M | X) < P(Class = N | X) => This case is a Non-Mammal.

2. **(10 points)** Given the observed data and the reference table below,

Tid	Refund	Marital Status	Taxable Income	Evade
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

naive Bayes classifier:

- (Refund = Yes | No) = 3/7
- (Refund = No | No) = 4/7
- (Refund = Yes | Yes) = 0
- (Refund = No | Yes) = 1
- (Marital Status = Single | No) = 2/7
- (Marital Status = Divorced | No) = 1/7
- (Marital Status = Married | No) = 4/7
- (Marital Status = Single | Yes) = 2/7
- (Marital Status = Divorced | Yes) = 1/7
- (Marital Status = Married | Yes) = 0

For Taxable Income:
If Class = No: sample mean = 110
sample variance = 2975
If Class = Yes: sample mean = 90
sample variance = 25

Show your stepwise calculation for assigning the class label for a new customer with the following attribute values, using Naïve Bayes.

Refund	Marital Status	Taxable Income	Evade Class (No or Yes)
Yes	Single	200K	?

(No score will be given if you only answer "Yes" or "No")
Hint: For Taxable income, it follows the normal distribution.

$$P(x_j \mid C_i) = \frac{1}{\sqrt{2\pi s^2_{ji}}} e^{-\frac{(x_j - \mu_{ji})^2}{2s^2_{ji}}}$$

Your answer:

Assumption -> Income follows a Gaussian/Normal Distribution.

New case, x = (Refund = Yes, Status = Single, Taxable Income = 200K)

P(Yes) = 0.3 -(1)

P(No) = 0.7 -(2)

*p(x | Class = No) = p(Refund = Yes | Class = No) * p(Single | Class = No) * p(Income = 200K | Class = No)*

⇨ p(x | Class = No) = 3/7 * 2/7 * p(Income = 200K | Class = No) - (3)
⇨ If class = No, then μ_{ji} = 110; Sample variance = 2975
⇨ P(Income = 200K | Class = No) = $1/(\sqrt{2\pi} * 2975) * e^{(200-110/(2 * 2975))}$ = 0.0085 -(4)
⇨ Substituting (4) in (3) -> p(x | Class = No) = 0.001 -(5)

*p(x | Class = Yes) = p(Refund = Yes | Class = Yes) * p(Single | Class = Yes) * p(Income = 200K | Class = Yes)*

⇨ P(x | Class = Yes) = 0 * 2/7 * p(Income = 200K | Class = Yes) – (6)
⇨ If class = Yes, then μ_{ji} = 90; Sample variance = 25
⇨ P(Income = 200K | Class = No) = $1/(\sqrt{2\pi} * 25) * e^{(200-90/(2 * 25))}$ = 0.482 -(7)
⇨ Substituting (7) in (6) -> p(x | Class = Yes) = 0 -(8)

From (1), (2), (5), and (8) ->

p(x | Class = No) * P(No) > p(x | Class = Yes) * P(Yes) = 0.3

Hence, P(No | x) > P(Yes | x), and this case will be classified as No Evade.

3. **(15 points; 5 points *3)**
   1) Is the total variance of a dataset equal to the variance explained by components identified in PCA?

Your answer:

Yes. The total variance of the dataset is explained by the principal components identified. To understand the strength of each component, we compute the Proportion of Variance (PVE) explained by each component.

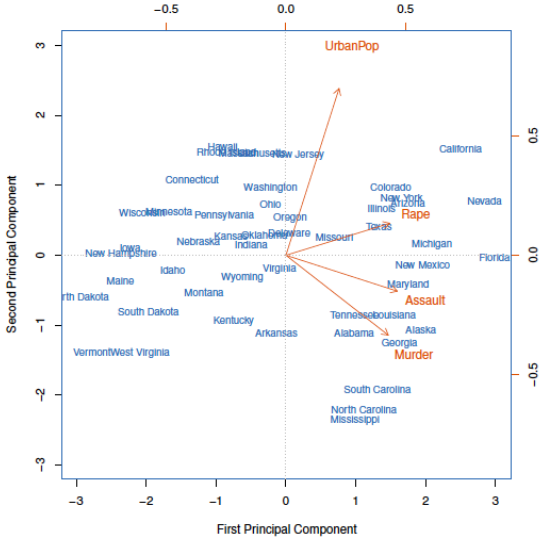
   2) Based on the loading matrix from the USarrests data, which variables will be counted into PC1 and which one will be calculated into PC2?

	PC1	PC2
Murder	0.5358995	-0.4181809
Assault	0.5831836	-0.1879856
UrbanPop	0.2781909	0.8728062
Rape	0.5434321	0.1673186

Your answer:

Based on the loading matrix, the variables Murder, Assault, and Rape will be counted in PC1 and thus indicate the measure of overall rates of serious crimes. A high correlation between the variables is observed. The variable, UrbanPop, will be counted in PC2 and thus measure the level of urbanization in the state.

   3) What are the principal components scores shown on this bi-plot fusing USarrest data? What do the arrows indicate?



Your answer:

The arrows in the biplot indicate the first two principal components' loading vectors. The PC loading scores for each variable is -> Murder(0.52, -0.41), Assault(0.58, -0.18), UrbanPop(0.27,0.87), Rape(0.54, 0.16). The negative sign indicates the direction.

4. **(10 points; 2 points *5)**

1) How to deal with random initialization issues in K-means?

Your answer: Trying multiple initializations and choosing the best results. We can use other robust algorithms like K-means ++.

2) What algorithm can deal with outliers if k-means is sensitive to outliers?

Your answer: Use the K-medians algorithm.

3) What are the assumptions for K-means?

Your answer: Following are the assumptions of K – Means:

a.) Clusters are spherical, i.e., all data points in a cluster are centered around that cluster.

b.) The spread/variance of the clusters is similar, i.e., each data point belongs to the closest cluster

4) What algorithm can we use to prevent local minima resulting from K-means?

Your answer: K-Means ++

5) How to choose the optimal number of K clusters?

Your answer: Choose the elbow point from the scree-plot.

5. **(10 points) Write the K-means pseudo-code for choosing 2-clusters for a sample of 100 cases with two attributes.**

Your answer: <u>Pseudocode</u> ->

Select 2 points as the initial centroids
Repeat:
Form 2 clusters by assigning all 100 points to the closest centroid
Recompute the centroid of each cluster
Compare each individual's distance to its updated cluster mean and to that of the other cluster
Until:
The centroids don't change

6. **(10 points) Write the pseudo code for agglomerative hierarchical clustering.**

Your answer: <u>Pseudocode</u> ->

Begin with 'n' observations and a measure of all the (n * (n – 1))/2 pairwise dissimilarities. Treat each observation as its cluster.

For (i = n, n-1, …., 2) :
Examine all pairwise inter-cluster dissimilarities among the clusters and identify the pair of clusters that are least dissimilar. Fuse the two clusters.

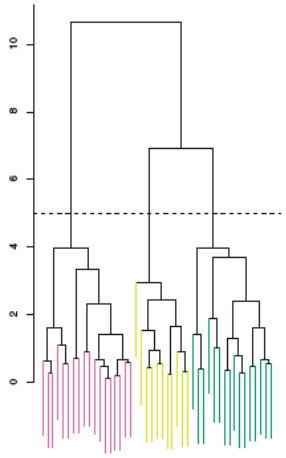
Compute the new pairwise inter-cluster dissimilarities among the remaining clusters.

7. **(5 points) What are the three dissimilarity measures in hierarchical clustering?**

   Your answer: The three dissimilarity measures in hierarchical clustering are ->

   a.) Min- link -> The minimum distance between the data points of each cluster.
   b.) Max – link -> The maximum distance between the data points of each cluster
   c.) Average link -> The mean distance between the data points of each cluster

8. (**3 points)** How many clusters do we have if we cut at the height of 5 in this Figure?



Your answer: Three (3)

9. (2 Points) Gap statistics and silhouette plots can be used to select the optimal number of clusters in hierarchical clustering? True or False

Your answer: True