Pixel Geometry

Basic Relationships Between Pixels

- Neighborhood
- Adjacency
- Paths
- Connectivity
- Regions
- Boundary / Border / Contour
- Edge
- Distance Measures

Neighbors of a pixel p at (x, y)

y

x

(x-1, y-1)	(x-1, y)	(x-1, y+1)
(x, y-1)	p (x, y)	(x, y+1)
(x+1, y-1)	(x+1, y)	(x+1, y+1)

Unit Distance

Neighbors of a pixel p at (x, y)

- **4-neighbors of p**, denoted by $\mathbf{N_4(p)}$:
  (x-1, y), (x+1, y), (x,y-1), and (x, y+1).

- **4-diagonal neighbors of p**, denoted by $\mathbf{N_D(p)}$:
  (x-1, y-1), (x+1, y+1), (x+1,y-1), and (x-1, y+1).

Note: Some of the neighbors of pixel **p** lie outside the digital image if (x, y) is on the border of the image.

Neighbors of a pixel p at (x,y)

- **8 neighbors of pixel p** is the union of 4-neighbors of p and 4-diagonal neighbors of p.

- It can be denoted as:

$$N_8(p) = N_4(p) \cup N_D(p)$$

Note: Some of the points of $N_4(p)$ and $N_D(p)$ fall outside the digital image if (x, y) is on the border of the image.

Adjacency

Let V be a set of intensity (gray-level) values used to define adjacency.

- **4-adjacency**: Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.

- **8-adjacency**: Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

Adjacency

- **m-adjacency (mixed adjacency)**: Two pixels p and q with values from V are m-adjacent if

(i) q is in the set $N_4(p)$

or

(ii) q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V.

Examples: adjacency

Suppose V = {1, 2}

$$\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array}$$

8-adjacent **m-adjacent**

Mixed adjacency is a modification of 8-adjacency. It is introduced to eliminate the ambiguities that often arise when 8-adjacency is used.

Path

- A (digital) path (or curve) from pixel p with coordinates (x_0, y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

  where (x_{i-1}, y_{i-1}) and (x_i, y_i) are adjacent for $1 \le i \le n$.

- Here *n* is the *length* of the path.

Path

- If $(x_0, y_0) = (x_n, y_n)$, the path is ***closed*** path.

- We can define 4-, 8-, and m-paths based on the type of adjacency used.

Examples: path

Suppose V = {1, 2}

$$\begin{array}{ccc} 0_{1,1} & 1_{1,2} & 1_{1,3} \\ 0_{2,1} & 2_{2,2} & 0_{2,3} \\ 0_{3,1} & 0_{3,2} & 1_{3,3} \end{array} \qquad \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array}$$

8-adjacent

The 8-path from (1,3) to (3,3):

(i) (1,3), (1,2), (2,2), (3,3)

(ii)(1,3), (2,2), (3,3)

Examples: path

Suppose V = {1, 2}

$$
\begin{array}{ccc}
0_{1,1} & 1_{1,2} & 1_{1,3} \\
0_{2,1} & 2_{2,2} & 0_{2,3} \\
0_{3,1} & 0_{3,2} & 1_{3,3}
\end{array}
\qquad
\begin{array}{ccc}
0 & 1 & 1 \\
0 & 2 & 0 \\
0 & 0 & 1
\end{array}
\qquad
\begin{array}{ccc}
0 & 1 & 1 \\
0 & 2 & 0 \\
0 & 0 & 1
\end{array}
$$

8-adjacent **m-adjacent**

The m-path from (1,3) to (3,3):

(i) (1,3), (1,2), (2,2), (3,3)

Connectivity

- Let S represent a subset of pixels in an image. Two pixels p and q are said to be **connected in S** if there exists a path between them consisting entirely of pixels in **S**.

Connected Component

Let S represents a subset of pixels in an image

- For every pixel *p* in S, the set of pixels in S that are connected to *p* is called a ***connected component*** of S.

- If S has only one connected component, then S is called ***Connected Set*****.**

Region

- We call R a **region** of the image if R is a connected set.
- Two regions, R_i and R_j are said to be ***adjacent*** if their union forms a connected set.
- Regions that are not to be adjacent are said to be ***disjoint.***

Boundary or Border

- The ***boundary*** of the region R is the set of pixels in the region that have one or more neighbors that are not in R.

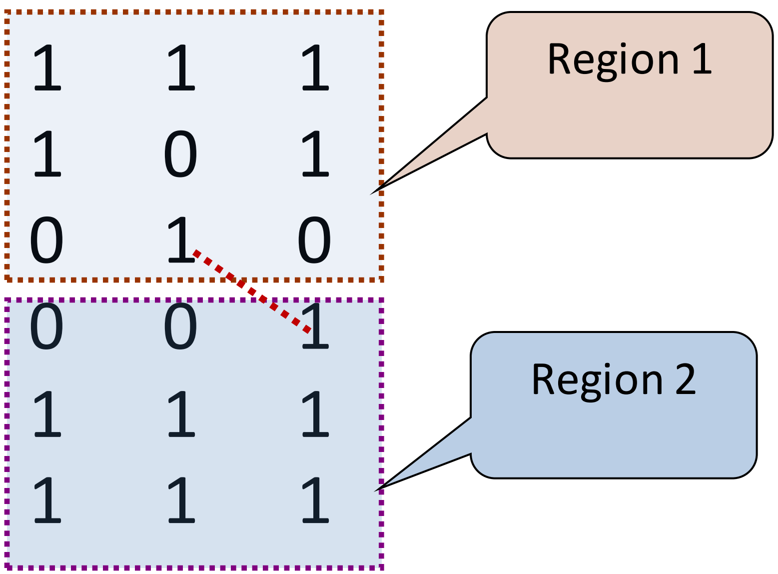
- If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

Foreground and Background

- An image contains K disjoint regions, R_l, l = 1, 2, …, K. Let R_u denotes the union of all the K regions, and let $(R_u)^c$ denote its complement.
- All the points in R_u is called **foreground.**
- All the points in $(R_u)^c$ is called **background**.

Question 1(a)

- **In the following arrangement of pixels, are the two regions (of 1s) 8-adjacent?**



Ans: YES

Question 1(b)

- **In the following arrangement of pixels, are the two regions (of 1s) 4-disjoint?**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Region 1

Region 2

Ans: NO

Question 2(a)

- **In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 4-adjacency is used, true or false?**

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Ans: NO

Question 2(b)

- **In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 8-adjacency is used, true or false?**

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Ans: YES

Distance Measures

- Given pixels *p, q* and *z* with coordinates (x, y), (s, t), (u,v) respectively, the distance function D has following properties:

a. $D(p, q) \geq 0$ [$D(p, q) = 0$, iff $p = q$]

b. $D(p, q) = D(q, p)$

c. $D(p, z) \leq D(p, q) + D(q, z)$

Distance Measures

The following are the different Distance measures:

a. Euclidean Distance:

$$D_e(p, q) = [(x-s)^2 + (y-t)^2]^{1/2}$$

b. City Block Distance:

$$D_4(p, q) = |x-s| + |y-t|$$

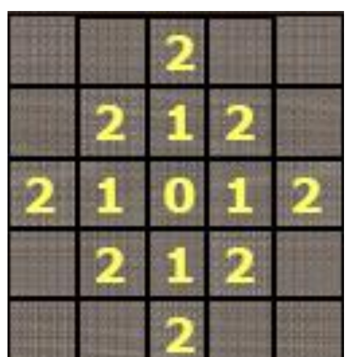
c. Chess Board Distance:

$$D_8(p, q) = \max(|x-s|, |y-t|)$$

Distance Measures

- For Euclidean distance measure, the pixels having a distance less than or equal to some value r from (x,y) are the points contained in a disk of radius r centered at (x,y).

- Pixels with D_4 distance from (x,y) form a diamond centered at (x,y).



Here r = 2

Distance Measures

- Pixels with D_8 distance from (x,y) form a square centered at (x,y).

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Here r = 2

Distance Measures

- The D_4 and D_8 distances between p and q are independent of any paths that might exist between the points because these distances involve only the coordinates of the points.

- D_m distance between two pixels depends on the values of the pixels along the path, as well as the values of their neighbors.

Distance Measures

Consider the following arrangement of pixels and assume that p, $p_{2,}$ and p_4 has value 1 and that p_1 and p_3 can have a value of 0 or 1.

	p_3	p_4
p_1	p_2	
p		

Case I: V = {1}, p_1=0, p_3=0, then $D_m(p, p_4)$ = ?   2

Distance Measures

Consider the following arrangement of pixels and assume that p, p_2, and p_4 has value 1 and that p_1 and p_3 can have a value of 0 or 1.

$$\begin{array}{ccc} & p_3 & p_4 \\ p_1 & p_2 & \\ p & & \end{array}$$

Case II: V = {1}, p_1=1, p_3=0, then $D_m(p, p_4)$ = ?   3

Distance Measures

Consider the following arrangement of pixels and assume that p, p_2, and p_4 has value 1 and that p_1 and p_3 can have a value of 0 or 1.

p_3 p_4

p_1 p_2

p

Case III: V = {1}, p_1=0, p_3=1, then $D_m(p, p_4)$ = ? 3

Distance Measures

Consider the following arrangement of pixels and assume that p, p_2, and p_4 has value 1 and that p_1 and p_3 can have a value of 0 or 1.

$$
\begin{array}{ccc}
 & p_3 & p_4 \\
p_1 & p_2 & \\
p & &
\end{array}
$$

Case IV: $V = \{1\}$, p_1=1, p_3=1, then $D_m(p, p_4)$ = ?   4

Linear and Nonlinear Operations

- Let H be an operator whose input and output are images, H is said to be a linear operator if, for any two images f and g and any two scalars a and b,

$$\boxed{H(af + bg) = aH(f) + bH(g)}$$

- The results of applying a linear operator to the sum of two images (that have been multiplied by the constants) is identical to applying the operator to the images individually, multiplying the results by the appropriate constants, and then adding those results.

Linear and Nonlinear Operations

Examples:

- An operator whose function is to compute the sum of K images.

- An operator that fails the test of the below equation is by definition nonlinear.

$$\boxed{H(af + bg) = aH(f) + bH(g)}$$

Examples:

- An operator that computes the absolute value of the difference of two images.