

*Crypto &security

The least you should know about Cryptography and Security

Knowing how a camera works does not make you a great photographer. Knowing what cryptographic designs are and how they work does not gives you proficiency in using cryptography.

Think as a Cryptographer does

Agenda

- History
- Introduction and Principles
- Algorithms
 - Symmetric Key
 - Asymmetric Key
 - Hashing
 - Prime Numbers & Prime Factorization
- Real World Applications
- Think as a Cryptographer does
 - Professional Paranoia
 - The Weakest Link

Introduction and Principles

Cryptography is an extremely varied field, including Computer Security, higher algebra, economics, quantum physics, civil and criminal law, statistics, chip design, extreme software optimization, politics, user interfaces and everything between.

It has to be part of a much greater system in order to be useful.

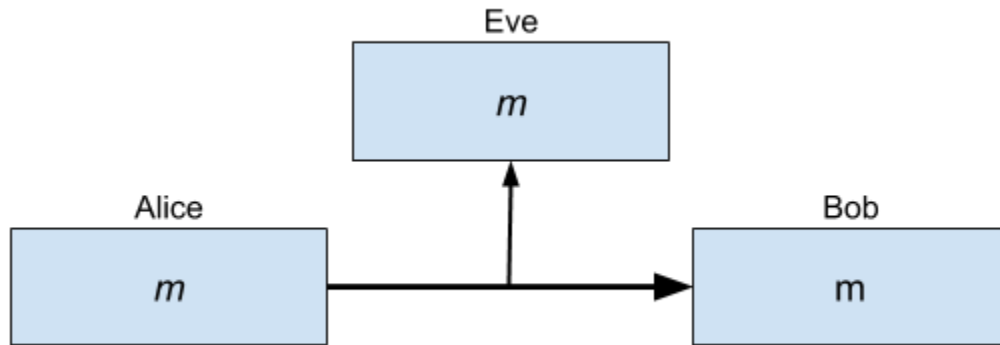
Definition

Cryptography is the art and science of encryption, which allows a sender to *disguise* data for secure communication so that an intruder can't gain information from the intercepted data.

It has to be...

- Reliable
- Secure

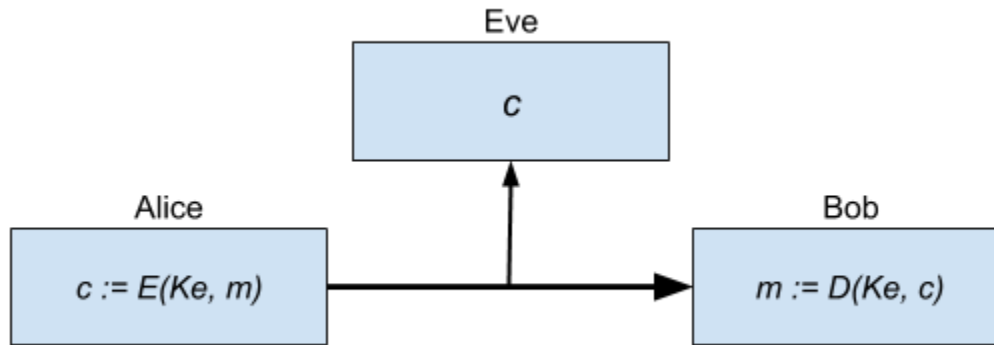
Not to be confused with Cryptology, which is a broader subject consisting of two branches: *Cryptography* and *Cryptoanalysis*



Alice and Bob want to communicate with each other. However, communication channels are assumed not to be secure.

Eve is eavesdropping on the channel. Any *message* m that Alice sends to Bob is also received by Eve.

How can **Alice** and **Bob** communicate securely?



To prevent Eve from understanding the conversation that Alice and Bob are having, they use encryption.

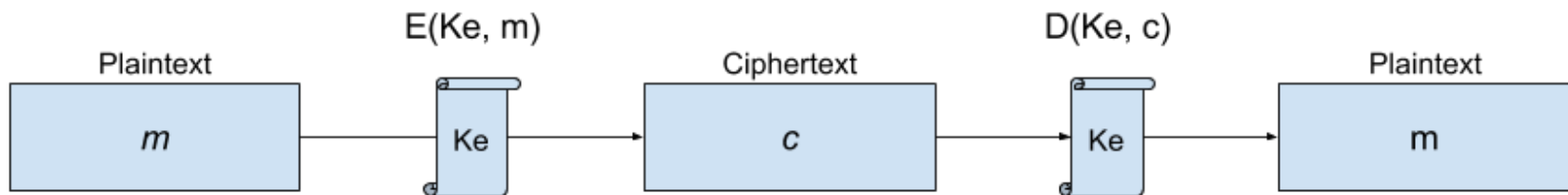
Alice and Bob first agree on a secret key K_e and an encryption and decryption algorithm $E()$, $D()$.

Instead of sending a *plaintext* message m , they are communicating using the *ciphertext* c .

What encryption methods or algorithms do you know?

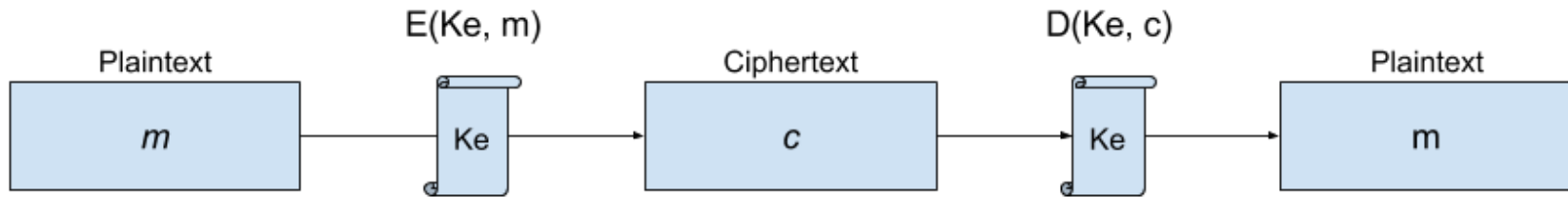
Let's design one!

Symmetric Key



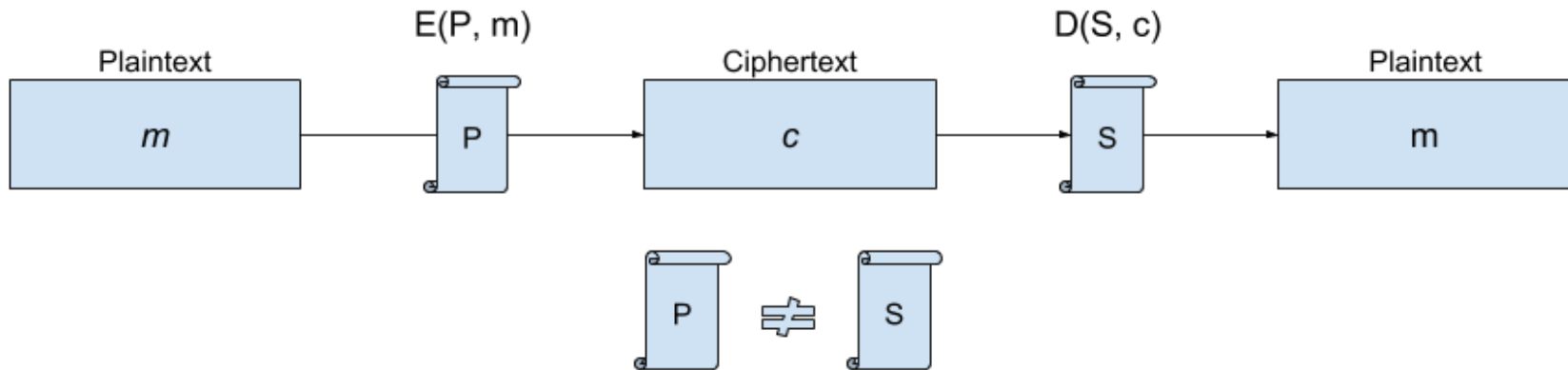
The *encryption key* K_e is used to Encrypt and Decrypt the message.

Symmetric Key



- Fairly simple
- You should be extremely careful with to whom you are sharing it with (and how it will be stored)
- There is no way to allow unknown third-parties to send encrypted messages to you without exposing the key.

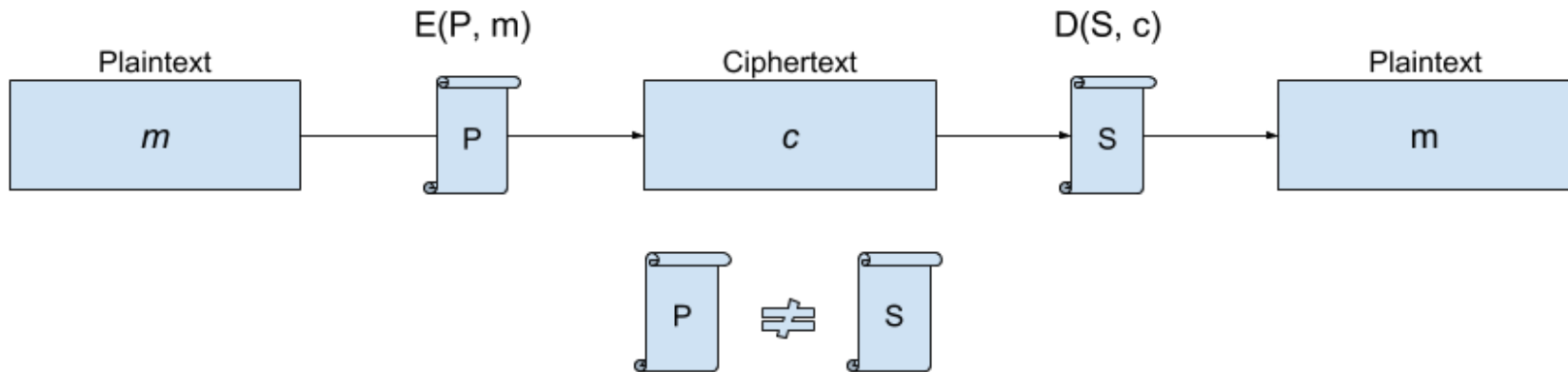
Asymmetric Key



The encryption key consist of two components. Secret Key S and the Public Key P .

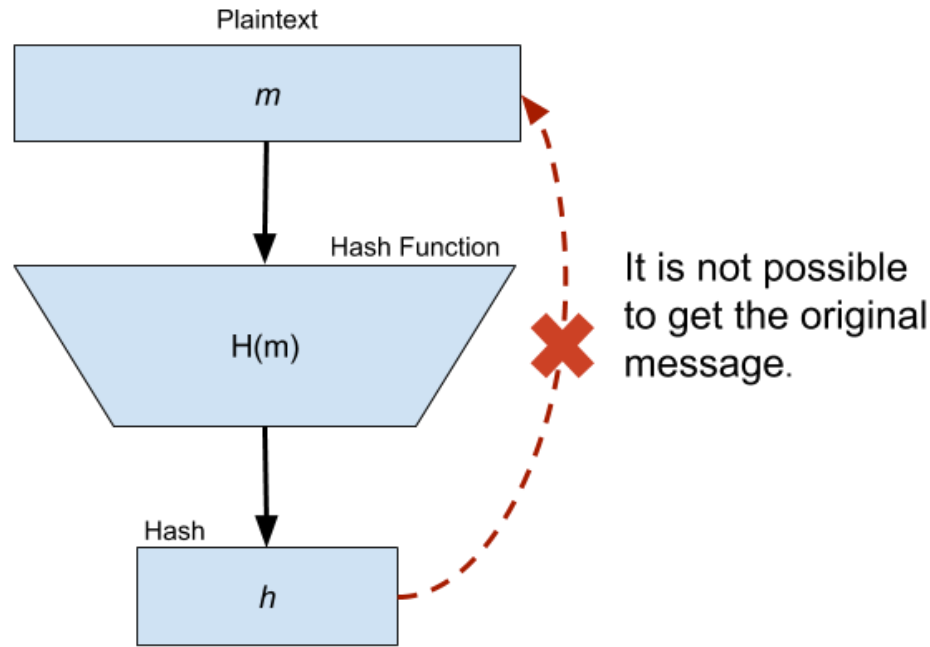
Even though the Private Key S is derived from the Public Key P , they are not the same.

Asymmetric Key



- Broadly used in the modern technology stack
- Third-parties are allowed to send encrypted messages to you without exposing the key.
- Everything depends on how secure the Secret Key is stored

Hashing



A hash function $H()$ is a function that takes as input an *arbitrarily long string of bits* m and produces a fixed-size result h .

It's important to know that there is no way back from the hash result h to the original message m .

Why anyone would use a Hash Function if the result cannot be decrypted back?

Yes, I'm asking You...

**THE REASON ANYONE WOULD DO THIS IF THEY COULD, WHICH
THEY CAN'T; WOULD BE BECAUSE THEY COULD, WHICH THEY CAN'T**

14
DLSV



PICKLE RICK

Hashing

They are actually useful:

- Having a fixed-size result facilitates *signing* and comparisons
- Allows you to save *the message* without actually knowing what it is or what does it says. (*hint: passwords*)
- Works as a glue for several cryptography systems

Prime Numbers

A **prime number** is a natural number greater than 1 that has no positive divisors other than **1** and **itself**.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 ...

Prime Numbers

Even though there are infinitely many `Primes`, verifying them becomes more complex and *costly* as the number grows.

The simplest way to verifying the `primality` of a given number `n` is known as trial division. It consists of testing whether `n` is a multiple of any integer between 2 and `sqrt(n)`.

As of January 2016, the largest known prime number has 22,338,618 decimal digits.

Prime Factorization

Finding which prime numbers multiply together to make the original number.

Calculate the prime factorization of 12.

Prime Numbers: 1, 2, 3, 5, 7, 11, 13, 17, 23, ...

$$12 \div 2 = 6$$

$$6 \div 2 = 3$$

$$3 \div 3 = 1$$

$$12 = 2 \times 2 \times 3$$

! There is one and only one prime factorization for each number.

Prime Factorization

Calculate the prime factorization of 147.

Prime Numbers: 1, 2, 3, 5, 7, 11, 13, 23, ...

$$147 \div 2 = 73.5$$

The result should be an integer, 2 is discarded.

$$147 \div 3 = 49$$

$$49 \div 7 = 7$$

$$7 \div 7 = 1$$

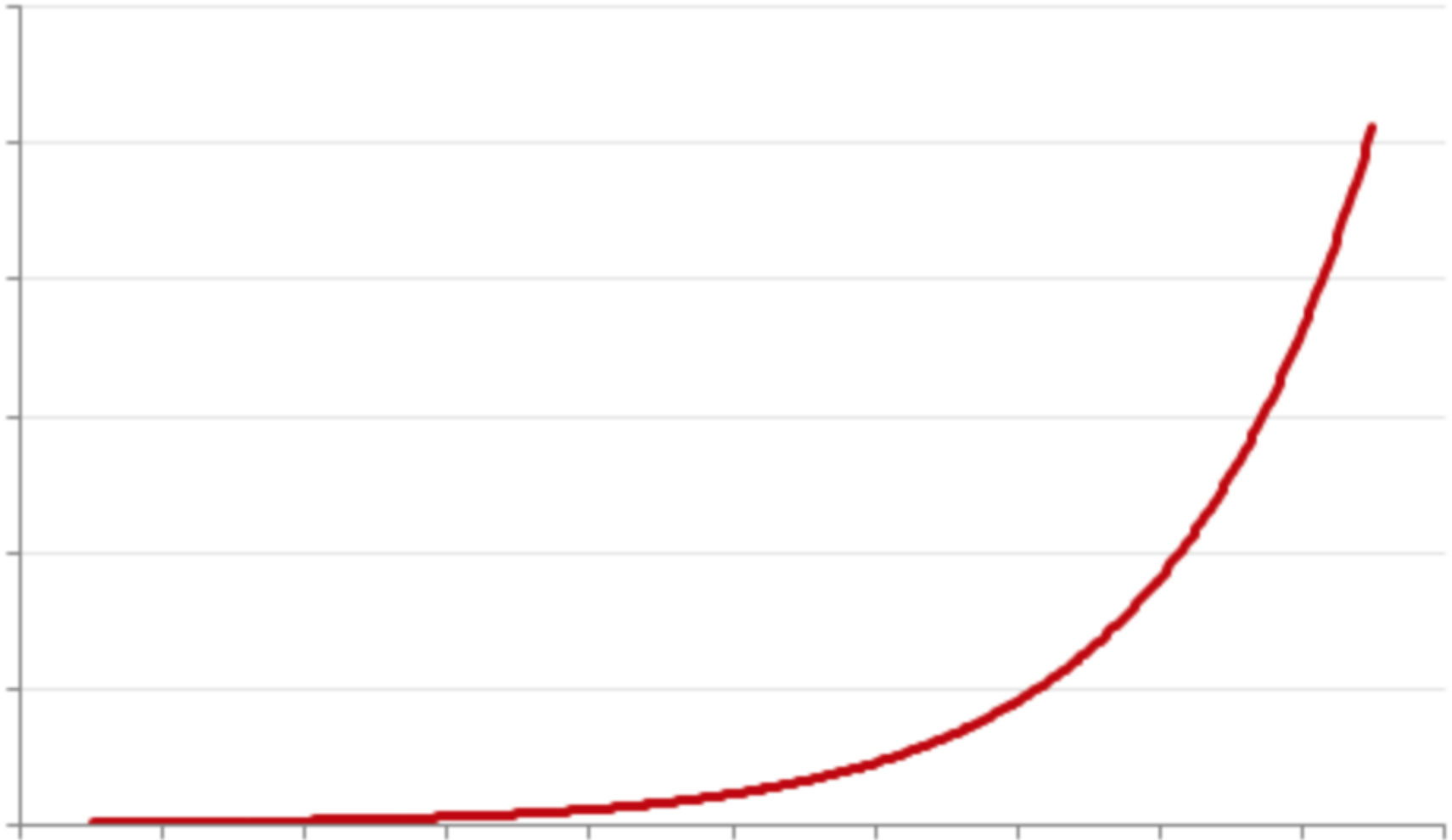
$$147 = 3 \times 7 \times 7$$

Prime Factorization

What about 721261014729547490954452378504349240996938
21481867654600825000853935195565259214555887054230207
51421525921455588705423020752592145558870542302072222?

hmmm....

Prime Factorization



Computational cost grows exponentially as the number goes bigger

Prime Factorization

Lets cheat!

Chose two random, equal *bit size* primes

```
p1 = 1500450271  
p2 = 3267000013
```

Multiply them:

```
n = p1*p2 = 4901971054862853523
```

Now we know the *Prime Factorization* of n and it's unique.

Euler's totient function (Phi Function)

Counts the positive integers up to a given integer n that are relatively prime to n .

$$\Phi(8) = 4$$

1, 2, 3, 4, 5, 6, 7, 8 \rightarrow 1, 3, 5, 7 \rightarrow 4

$$\Phi(7) = 7 - 1 = 6$$

1, 2, 3, 4, 5, 6, 7 \rightarrow 1, 2, 3, 4, 5, 6 \rightarrow 6

Phi of any Prime is equal to Prime - 1.

$$\Phi(21377) = 21376$$

Euler's totient function (Phi Function)

Phi function is *multiplicative*, which means...

$$\Phi(AB) = \Phi(A)\Phi(B)$$

Remember our last Prime Factorization?

$$n = p_1 p_2$$

$$\Phi(n) = (p_1 - 1)(p_2 - 1)$$

$$\begin{aligned}\Phi(4901971054862853523) &= 1500450270 \times 3267000012 \\ &= 4901971050095403240\end{aligned}$$

Now, Magic... or Sorcery?

$$m^{\Phi(n)} \equiv 1 \text{ mod } n$$

$$m = 5, n = 8, \Phi(8) = 4$$

$$5^{\Phi(8)} \equiv 1 \text{ mod } 8$$

$$625 \text{ mod } 8 = 1$$

$$625 \div 8 = 78 \frac{1}{8}$$

Remember... it will always be **1**

$$1^k = 1$$

$$m^{k\Phi(n)} \equiv 1 \pmod{n}$$

$$k = 2$$

$$5^{2\Phi(8)} \equiv 1 \pmod{8}$$

$$390625 \pmod{8} = 1$$

$$390625 \div 8 = 48828 \frac{1}{8}$$

Wait!, there is more...

$$1m = m$$

$$mm^{k\Phi(n)} \equiv (1 \times m) \bmod n$$

$$mm^{k\Phi(n)} \equiv m \bmod n$$

$$m^{k\Phi(n)+1} \equiv m \bmod n$$

$$m^{ed} \equiv m \bmod n$$

$$ed = k\Phi(n) + 1$$

$$d = \frac{k\Phi(n) + 1}{e}$$

`d` is the Private Key `S`

`e` and `n` are the Public Key `P`

Generating Keys

Given

$$p_1 = 53, p_2 = 59, n = 3127, k = 2$$

$$\Phi(n) = 3016$$

Chose an encryption key e

$$e = 3$$

$$d = \frac{2(3016) + 1}{3} = 2011$$

d is now our Secret encryption key.

e and n form together our *Public Key* and we can send it to Alice

Encrypting 89

Given

$$m = 89, n = 3127, e = 3$$

$$c = 89^3 \bmod 3127 = 1394$$

The encrypted number will be 1394

Decrypting **1394**

Using the Private Key **d**

$$d = 2011$$

$$1394^{2011} \equiv 89 \text{ mod } 3127$$

$$1394^{2011} = \textit{REALLYBIGNUMBER}$$

$$\textit{REALLYBIGNUMBER} \text{ mod } 3127 = 89$$

REALLYBIGNUMBER is also a **REALLYBIGPROBLEM** ...

$$3^{200}$$

has more than 99 ~~problems~~ digits...

$$490197137^{1500450270} = OH\ BOY..$$

In practice, **d** can be a number with more than **100** digits.

Solving the ***REALLYBIGNUMBER*** problem.

$$x = 1394^{2011} \bmod 3127$$

Modular Exponentiation + Fermat's Little Theorem

$$1394^2 \equiv 1369 \bmod 3127$$

$$1394^4 \equiv (1394^2)^2 \equiv 1088 \bmod 3127$$

$$1394^8 \equiv (1394^4)^2 \equiv 1738 \text{ mod } 3127$$

$$1394^{16} \equiv (1394^8)^2 \equiv 3089 \text{ mod } 3127$$

$$1394^{32} \equiv (1394^{16})^2 \equiv 1444 \text{ mod } 3127$$

$$1394^{64} \equiv (1394^{32})^2 \equiv 2554 \text{ mod } 3127$$

$$1394^{128} \equiv (1394^{64})^2 \equiv 3121 \text{ mod } 3127$$

$$1394^{256} \equiv (1394^{128})^2 \equiv 36 \text{ mod } 3127$$

$$1394^{512} \equiv (1394^{256})^2 \equiv 1296 \text{ mod } 3127$$

$$1394^{1024} \equiv (1394^{512})^2 \equiv 417 \text{ mod } 3127$$

Exponents

$$1024 + 512 + 256 + 128 + 64 + 16 + 8 + 2 + 1 = 2011$$

Product of Exponentiations Remainders

$$\begin{aligned} 1394^{2011} \equiv & 1394^{1024} \times 1394^{512} \times 1394^{256} \times 1394^{128} \\ & \times 1394^{64} \times 1394^{16} \times 1394^8 \times 1394^2 \times 1394^1 \end{aligned}$$

Replacing Values

$$417 \times 1296 \times 36 \times 3121 \times 2554 \times 3089 \times 1738 \times 1369 \times 1394$$

<http://bit.ly/2fugVwF>

Real World Applications

Enough of ur fancy maths ya' clever homeless man...

Real Stuff

RSA in Golang, using a key length of **2048** bits

```
package main

import (
    "crypto/rsa"
    "crypto/rand"
    "fmt"
)

func main() {
    priv, _ := rsa.GenerateKey(rand.Reader, 2048)

    fmt.Println(priv.D)
    fmt.Println(priv.E)
    fmt.Println(priv.N)
    fmt.Println(priv.Primes)
}
```

SSL TLS

Transport Layer Security is a cryptographic protocol that provides communications security (privacy and data integrity) over a computer network.

[Secure] https://....

- Asymmetric Crypto (Authentication A.K.A *handshake* w/ AES)
- Symmetric Crypto (Data Transmission w/ RSA)
- Hashing (Integrity Check w/ HMAC)
- Relies on a set of trusted third-party certificate authorities

Protocols can vary depending TLS version.

Password Hashing

Who's system is storing Passwords as plain text?

Why is not a good idea to **encrypt** them?

Password Hashing

You don't need to know the original value to verify if two given password strings are identical.

You only need to know that they are identical.

```
const stored = hash('123456');  
// stored === 'A7FB9...041C' <-- This is stored in DB  
  
// Next time you have to validate if passwords match  
stored === hash(userPassword);
```


Password Hashing

- Slow is a must.

Prevent brute force attacks from working.

- The algorithm must be secure

Prevent collisions, enough randomness

- Use a Salt and Iterations

Those parameters will make your hashed string unique from the rest of the world using the same algorithm

- - bcrypt
 - PBKDF2

SAT - Hacienda

Secretaria de Hacienda y Credito Publico (SHCP) and it's Servicio de Administracion Tributaria (SAT) relies on Cryptography systems.

- Invoices uses Digital Sign
- e.firma as a Asymmetric Key w/ Cert.

JSON Web Tokens

Tokens are `signed` using HMAC256.

The `Header` and the `Payload` are encoded to Base64URL, then concatenated, and then, along with a `secret` are passed to the `HMAC256` Hash Function to generate a `Digital Signature` that will authenticate the token.

The signature is the last component of the JWT
(header.payload.signature)

- Cryptocurrencies & Blockchains
- PGP
- Token
- ◦ Stored Payment Data
- ◦ Cookies and session data
- Tor & Dark Web

How does it look in Node.js?

An small set of code examples

HMAC256

```
const crypto = require('crypto');

const secret = 'abcdefg';
const hash = crypto.createHmac('sha256', secret)
    .update('WHAT WE WANT TO HASH')
    .digest('hex');

console.log(hash);
// Prints:
// c0fa1bc00531bd78ef38...a959d6658e
```

AES192 Encrypt

```
const crypto = require('crypto');
const cipher = crypto.createCipher('aes192',
                                    'a password');

let encrypted = cipher.update('plain text',
                              'utf8',
                              'hex');

encrypted += cipher.final('hex');
console.log(encrypted);
// Prints: ca981be48e90867...ed84d815504
```

AES192 Decrypt

```
const crypto = require('crypto');
const decipher = crypto.createDecipher('aes192',
                                        'a password');

const encrypted =
    'ca981be48e90867604588...b10616ed84d815504';
let decrypted = decipher.update(encrypted, 'hex', 'utf8')
decrypted += decipher.final('utf8');
console.log(decrypted);
// Prints: plain text
```


Professional Paranoia

You have to think like a malicious attacker to find weaknesses in your own work.

Once you start thinking about how to attack your systems, you apply that to everything around you.

Professional Paranoia

You suddenly see how you could cheat the people around you, and how they could cheat you.

Developing this mindset will help you observe things about systems and your environment that most other people don't notice.

The Weakest Link

A security system is only as strong as its weakest link.

Print the sentence above in a very large font and paste it along the top of your monitor.

The Weakest Link

Every security system consists of a large number of parts. We must assume that our opponent is smart and that he is going to attack the system at the weakest part.

It doesn't matter how strong the other parts are.

Think about your users writing down their passwords in a sticky note...

... What if the Super Admin does that as well?

The Weakest Link

Adding cryptography to your systems without previous analysis will lead to failure.

Imagine a reinforced titanium chain that locks your house door and an open window in the backyard at the same time.

Ruben Rivera ([@nullrocks](#))

Based on the book:

Cryptography Engineering

Design Principles and Practical Applications

Niels Ferguson, Bruce Schneier, Tadayoshi Kohno

Plus Several Sources

<http://www.math.utah.edu/mathcircle/crypto3.pdf>

https://math.dartmouth.edu/archive/m19w03/public_html/Section2-3

<https://a-nikolaev.github.io/ds/docs/lec16.pdf>