

OCR

H240



Mathematics

This book is currently a *preliminary draft*.

It is probably full of errors, lies, paradoxes and communist propaganda.

Send corrections to <https://github.com/aDotInTheVoid/a-level-notes>.

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[HTTPS://GITHUB.COM/ADOTINTHEVOID/A-LEVEL-NOTES](https://github.com/ADOTINTHEVOID/A-LEVEL-NOTES)

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