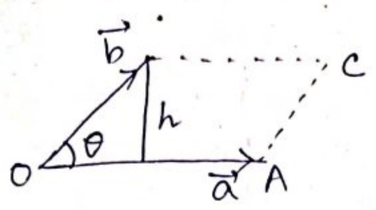
EM-II

$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos\theta$

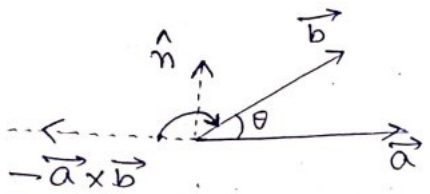
SCALAR PRODUCT



$|\vec{OA}| = |\vec{a}|$, $|\vec{OB}| = |\vec{b}|$, $h = |\vec{b}| \cos\theta$

$\square OACB = |\vec{OA}| \cdot h = |\vec{a}| \cdot |\vec{b}| \cos\theta$

VECTOR PRODUCT



$\vec{a} \times \vec{b} = |\vec{a}| \cdot |\vec{b}| \sin\theta \cdot \hat{n}$

- Vector Function

1. $f(t) = a(t)\hat{i} + b(t)\hat{j} + c(t)\hat{k}$

e.g. $f(t) = e^t \hat{i} + \sin t \hat{j} + (t^2+2)\hat{k}$

$\frac{df}{dt} = e^t \hat{i} + \cos t \hat{j} + 2t \hat{k}$

$$\int_0^1 f(t)\,dt = \left(\int_0^1 e^t dt\right)\hat{i} + \left(\int_0^1 \cos t\, dt\right)\hat{j} + \left(\int_0^1 2t\, dt\right)\hat{k}$$

$$= [e^t]_0^1 \hat{i} + [\sin t]_0^1 \hat{j} + [t^2]_0^1 \hat{k}$$

$$= (e-1)\hat{i} + \sin 1 \hat{j} + \hat{k}$$

2. If $\vec{f}(t) = t\hat{i} + (t^2-2t)\hat{j} + (3t^2+3t^3)\hat{k}$, find $\int_0^1 \vec{f}(t)\,dt$.

$$\int_0^1 \vec{f}(t)\,dt = \left(\int_0^1 t\,dt\right)\hat{i} + \left(\int_0^1 (t^2-2t)\,dt\right)\hat{j} + \left(\int_0^1 (3t^2+3t^3)\,dt\right)\hat{k}$$

$$= \left[\frac{t^2}{2}\right]_0^1 \hat{i} + \left[\frac{t^3}{3}\right]_0^1 \hat{j} - [t^2]_0^1 \hat{j} + [t^2]_0^1 \hat{k} + \frac{3}{4}[t^4]_0^1 \hat{k}$$

$$= \frac{1}{2}\hat{i} + \frac{1}{3}\hat{j} - 1\hat{j} + 1\hat{k} + \frac{3}{4}\hat{k}$$

$$= \frac{1}{2}\hat{i} - \frac{2}{3}\hat{j} + \frac{7}{4}\hat{k}.$$

3. $\vec{A}(t) = (3t^2-2t)\hat{i} + (6t-4)\hat{j} + 4t\hat{k}$, find $\int_2^3 \vec{A}(t)dt$.

$$\int_2^3 \vec{A}(t)dt = \int_2^3 (3t^2-2t)\,dt\,\hat{i} + \int_2^3 (6t-4)\,dt\,\hat{j} + \int_2^3 4t\,dt\,\hat{k}$$

$$= [t^3]_2^3\hat{i} \cancel{- [t^2]_2^3\hat{i} + [3t^2]_2^3\hat{j} - [4t]_2^3\hat{j} + [2t^2]_2^3\hat{k}}$$

$$= \cancel{10\hat{i}}\ 14\hat{i} + 11\hat{j} - 10\hat{k}$$

4. If $\vec{p} = t\hat{i} - t^2\hat{j} + (t-1)\hat{k}$ and $\vec{s} = 2t^2\hat{i} + 6t\hat{k}$, then find (i) $\int_0^2 \vec{p}\cdot\vec{s}\,dt$ (ii) $\int_0^2 \vec{p}\times\vec{s}\,dt$

$\rightarrow$

$$\vec{p}\cdot\vec{s}\ \cancel{= |\vec{p}||\vec{s}|\cos\theta} = 2t^3 + 6t^2 - 6t$$

$$\int_0^2 \vec{p}\cdot\vec{s}\,dt = \int_0^2 (2t^3+6t^2-6t)\,dt$$

$$= \frac{2}{4}[t^4]_0^2 + \frac{6}{3}[t^3]_0^2 - \frac{6}{2}[t^2]_0^2$$

$$= \frac{1}{2}\times 16 + \frac{6}{3}\times 6 - \frac{6}{2}\times 4 = 8 + 16 - 12 = 12$$

$$\vec{p}\times\vec{s} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t & -t^2 & t-1 \\ 2t^2 & 0 & 6t \end{vmatrix}$$

$$= \hat{i}(-6t^3 - 0) - \hat{j}\,(6t^2 - 2t^3 + 2t^2) + \hat{k}\,(0 + 2t^4)$$

$$= (-6t^3)\hat{i} - (8t^2 - 2t^3)\hat{j} + (2t^4)\hat{k}$$

$$\int_0^2 \vec{p}\times\vec{s}\,dt = -\frac{6}{4}\times[t^4]_0^2\hat{i} - \frac{8}{3}[t^3]_0^2\hat{j} + \frac{2}{4}[t^4]_0^2\hat{j} + \frac{2}{5}[t^5]_0^2\hat{k}$$

$$= -\frac{6}{4}\times 16\hat{i} - \frac{8}{3}\times 8\hat{j} + \frac{2}{4}\times 16\hat{j} + \frac{2}{5}\times 32\,\hat{k}$$

$$= -24\hat{i} - \frac{64}{3}\hat{j} + 8\hat{j} + \frac{64}{5}\hat{k}$$

$$= -24\hat{i} + \frac{(-64+24)\hat{j}}{3} + \frac{64}{5}\hat{k}$$

$$= -24\hat{i} - \frac{40}{3}\hat{j} + \frac{64}{5}\hat{k}.$$

Let, $\vec{r}(t)$ denotes the position of a moving particle on a straight line at time t.

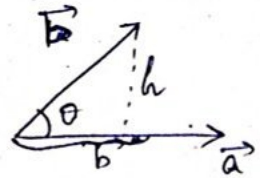
$$\underset{}{\vdash}\!\!\longrightarrow \quad \overset{\vec{r}(t)}{\underset{A}{\cdot}}$$

Velocity $= \dfrac{d\vec{r}}{dt}$

Acceleration $= \dfrac{d^2\vec{r}}{dt^2}$.

Scalar Dot Product

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta \quad \text{—— (i)}$$



$h = |\vec{b}|\sin\theta$, $\quad b = |\vec{b}|\cos\theta$

$$\text{(i)} \Rightarrow \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = |\vec{b}|\cos\theta = b = \text{projection of } \vec{b} \text{ on } \vec{a}$$

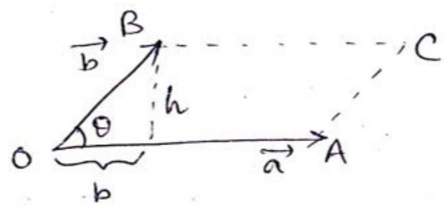
Qs. Find projection of $\hat{i} - 2\hat{j} + \hat{k}$ on $2\hat{i} + 3\hat{k}$.

$\vec{a} = 2\hat{i} + 3\hat{k} \qquad \vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{2+3}{\sqrt{2^2+3^2}} = \frac{5}{\sqrt{13}}$$

Vector Cross Product

$$\vec{a}\times\vec{b} = |\vec{a}|\,|\vec{b}|\sin\theta\,\hat{n} \quad \text{—— (i)}$$

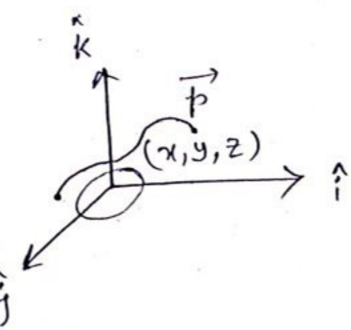


$h = |\vec{b}|\sin\theta$

Area of $\square$ OACB = base × height $= |\vec{a}|\cdot h$

$= |\vec{a}|\,|\vec{b}|\sin\theta$

- Position vector



$\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$.

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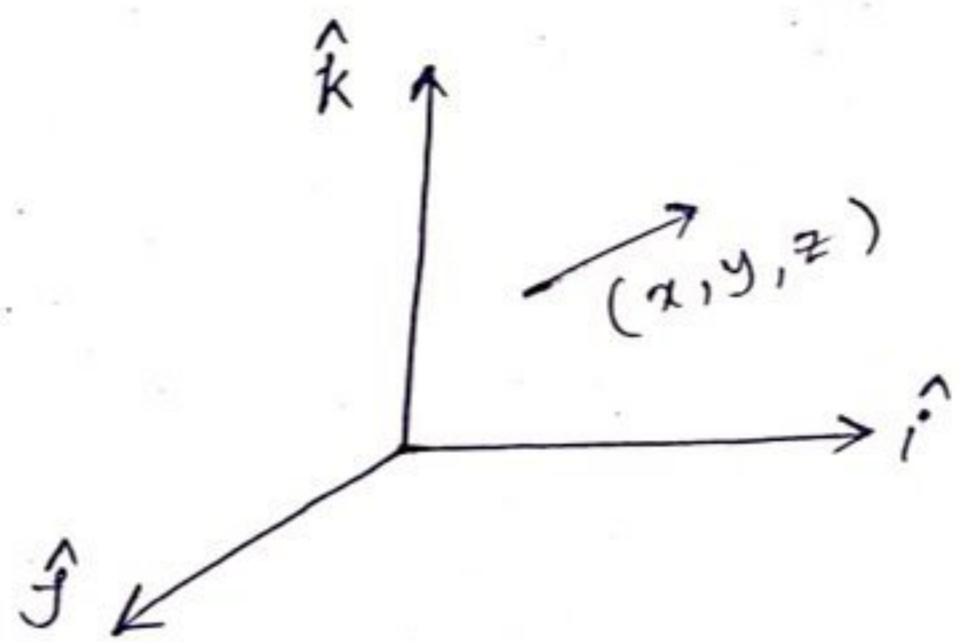
• Point Function

A function defined on space or $\mathbb{R}^3$ is called a point function.

$$f : \mathbb{R}^3 \to \mathbb{R}$$

Scalar: If the point function is a scalar function, i.e., at each point it defines a magnitude only, then it is called a scalar point function. Further, $\mathbb{R}^3$ with the scalar function is called a scalar field.

e.g., $f : \mathbb{R}^3 \to \mathbb{R}$ defined as $f(x, y, z) = x + y + z$.



Vector Point Function: A point function defines vectors at each point of the space is called a vector Function.

e.g., (1) $\vec{f} : \mathbb{R}^3 \to \mathbb{R}$ defined as $f(x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}$.