Infinite Series

Let {Un} be the sequence of real number, then the expression: u1 + u2 + u3 or is called infinite series and it is denoted as \(\Sum_{\text{un}} \).

To find whether the series is convergent or not, we need to construct, S,= u,

S2 = U, + U2

where {Sn} is called sequence of partial sum.

If the sequence {Sn} is convergent, then the series is also convergent.

Let us condit consider a Series:

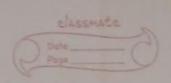
Hore, $U_n = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$

mow,
$$S_n = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \dots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

i) 30 tony da + (1-0") section of i

$$\lim_{n\to\infty} \frac{1}{n+\infty} \left(\frac{1}{n+1} \right) = \frac{1}{n+1} \left(\frac{1}{n+$$

Thus, the Series is convergent and Zun converges to 1.



Divergent Series: A series Eun is said to be divergent if $\lim_{n\to\infty} S_n = \pm \infty$

 $E_{x}: S_{\eta}=1+2+3+...+n$ $S_{\eta} = n(\eta+1)$

lim Sp = 00.

Oscillatory Series: A Series which is neither convergent nor divergent is called as cillatory series:

Ex: (+(-1) + (-1) + (-1) + (-1) + ... &

Geometric Series: The Series $1+x+x^2+x^3+\cdots \infty$ is known as geometric series. This series is convergent if -1 < x < 1 and & direvgent if x > 1.

If x = -1, then the series is oscillatory

Ex:) (+ \frac{1}{2} + \frac{1}

 $1. \alpha = \frac{1}{2} < 1 \rightarrow convergent$

i) 1+2+2+23+--3+ w

 $\therefore \alpha = 2 > 10 (3 +) \text{ divergent}$

07 1 0 1 1 K - 20

Sector + cosecty = X.

P-series.

The series of the form 1+ 1 + 1 + 1 + 1 + + 20 is called p-series. This series is convergent if P>1 and divergent if p<1.

Proporties of series:

He series I'ven is convergent and converges to 's' then the series I'ven is also convergent and converges to 'Ks'.

· If 2 series Eun and Evn converging to 's' and 't' respectively then $\Sigma(u_n + v_n)$ converges to 's+t'.

Note: If Zun is a convergent socies then its 'nth term'- un lim un = 0.

-> Let us consider a series 1+1 + 1 + 1.3.5 + ... +00

Check whether it is convergent or not

 $U_n = \frac{1}{1.3.5 - ... (2n-1)} \left(\frac{1}{1.2.2 - ... (n-1)} \right)$ times 2^{n-1}

Let, 1 = In

 $\sum_{n=1}^{\infty} y_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \cdots = \infty$

: Libr is a geometric series of x=1/2 x 1. and is thus convergent and since, un < vn
by process of comparision test;

Zun is also convergent

-> Comparision Test:

Let Eun and Evn be 2 series of positive terms and there exist an integer 'N' such that -

Unskun, tnon. then, Eun is convergent if Evn is so and Zivn is divergent if Eun is so.

> (2+v) 2v - (4×+242)

or 3 du - 1 du - - last c

on the Helps) of t to (use): - las

2 cal = (2 cola) + + (2 cola) - val / 2

[[box - la (ves) + 2 la (ve2)] . La

Roobele Test:

Alternative Series.

A series whose terms are alternatively positive and negative is called an alternative series.

Absolutely Convergent:

A series Sun is said to be absolutely convergent

if Sulun is convergent.

Ix: of alternative series: 1-d + t - 1 + 1 = 1