Exact Differential Equation.

Let us consider a first order linear diff. equation as M(x,y) dx + N((x,y) dy = 0

i) we will check the exactness of this type of equations

ii) If it is exact, then how to find the solution of the given D.E

The DF & D is said to be exact diff. equation if and only if it satisfies the following conditions:

2M 30N

Ex: (2ay - 3a2) da + (x2-2y) dy = 0

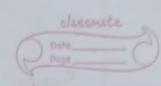
 \rightarrow Here, $M = 2\alpha y - 3\alpha^2$ $N = \alpha^2 - 2y$

: 2 dy = 2 (2 ay - 3 x2) = 2 x

 $\frac{\partial N}{\partial x} = \frac{\partial}{\partial x} (x^2 - 2y) = 2a$

: 3M = 3N 3y 32

Gren, eg. is exact.



Solution	of	Exact	DE.
	_		

Step: 1 IMda, treating y as a constant

Step: 2 INdy, treating a as a constant

Step:3 Compute, fMdx + fNdy=C, tout common term are only counted once.

Ex: yda + ady =0

→ Hore, M=y

Mdx: Jydx: ay

Judy = Jady = my

: Solution + my = C

Ex: Cosacosy dx - Sinasiny dy = 0

Mode = cosy sina findy = - sina (-cosy)

= cosy sina = cosy sina

:- Solution => stracosy = C

Question Solve the following IVP:

ex (cosydz - siny dy) =0

, $y(0) = 0 \rightarrow Initial condition$ for x=0, y=0

classmate X

Sol- Comparing the given equation with

Mdx + Ndy = 0

whe have, $M = e^{\alpha} \cos y$ $N = -e^{\alpha} \sin y$ $\cos y = \cos y$ $\cos y = \cos y$

now, am = -exsing

DN 1 - 12x siny (x+4) 6 - MG

: We observe that day , day 16

Hence, the given equation is an exact DF.

let, f(x,y)=C be the solution of given Df, then-

f(a,y): Mdx (partial integration with x)

excosy dx = cosy fexdx = excosy

again, f(x,y): [N dy (partial integration wrty)

 $= -e^{\alpha} \sin y \, dy = -e^{\alpha} \int \sin y \, dy = e^{\alpha} \cos y$

: f(x,y) = excosy = C is the general solution of the equation

now, for x=0, y=0, e°cos(0)=0 => 0=1. : ezery: 1 is the solution of the IVP. Find the value of a l b so that the following equation becomes an exact D.f. Hence solve the equation.

(y+x3) dz + (ax + by3) dy > 0 Section Comparing to Mox + Ndy = 0 an M=y+a3 N=ax+by3 ay = ay (y+x3) = 1+0-1. 16 DN = da (ax + by3) - a now, for exact Df, and i. a=1., beR now, let of (x,y) of be the solution of given DE. f(2,y) = [y+x3d2 = xy+ 2" again, f (2,y) = [N dy = [(1,2+by3) dy = 2y+by4 : f(x,y)= xy + xy + b yy = xy+ 1/(x4.1 by") where ber : f(a,y) = xy + 1/4(x4 by4) = C is the general solution of the Df.