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Power Series:

A series of the form $a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$ is called power series.

It is also denoted as $\sum_{n=0}^{\infty} a_n x^n$, where an are coefficients.

Note: If x = 0, then the power series is always convergent, and if the series is convergent only for x = 0, then it is called 'nowhere' convergent.

Note: If the series is convergent for all values of x, then it is called 'everywhere' convergent.

1. $(1-x)^{-1}$ $= 1 + x + x^{2} + x^{3} + \dots$ 2. $(1+x)^{-1}$ $= 1 - x + x^{2} - x^{3} + \dots$

move express + 1 + $2x + 3x^2 + 4x^3 + \dots$ = $(1-2)^{-2}$

Note: If the power series converges for some values of x, then the values of x for which the series converges is called 'region of convergence'.

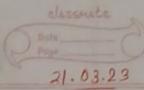
Note: If for the power series $\sum_{n=0}^{\infty} a_n x^n$, there exists a positive number R. such that the series converges for |x| < R and divergence for |x| < R, then R is called radius of convergence.

Radius of Convergence:

The radius of convergence R of a power series

Zanxn is defined to be equal to: $R = \frac{1}{\lim_{n \to \infty} |a_n|^{\frac{1}{n}}}$, where $\lim_{n \to \infty} |a_n|^{\frac{1}{n}} > 0$, when $\lim_{n\to\infty} |a_n|^{\frac{1}{n}} = 0$ R= do R = 0, when $\lim_{n \to \infty} |a_n|^{\frac{1}{n}} = \infty$ The radius of convergence for a power series $\sum a_n x^n$ is also given by: $R = \lim_{n \to \infty} \left| \frac{a_n}{a_{n+1}} \right|$

Radius of Convergence. 21.03.23



guestion: Find the Radius of convergence of the series:

- Hore, an = n+1

now, lim | an | n = lim (n+1) /n = A (say)

:. log A = lim 1 log (n+1)

 $=\lim_{n\to\infty}\frac{\log(n+1)}{n}\left[\frac{s}{s}\text{ form}\right]$

= $\lim_{n\to\infty} \frac{1}{n} = \lim_{n\to\infty} \frac{1}{n+1} = 0$.

now, log A = 0 . A = e° = 1.

2. R = 1 = 1 = 1.

ii) 1+x+2! x2+3! x3+...

-> Here, an = n!

mow lim an mos (n!) In

now, ann = (n+1)!

: R = lim | an | = lin n! = lim n!

noo | ann | no (n+1)! = noo (n+1)n!

2 Um 1 = 0

in R=0. -t: The series is not convergent for (no region of convergence) any non-zero, of a.

iii) of + x + x3 + :. on = (n+)1 (ii) $n + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ $a_2 = \frac{1}{2!}$: $a_n = \frac{1}{n!}$ an+1 = (n+1)! = (n+1)n! R = lim | an | - m lim not mod : for all Series is convergent when x \$0 + x iv) x + 22x2 + 33x3 + v) 1- x + 22 1- x3+ 24 - x4+...