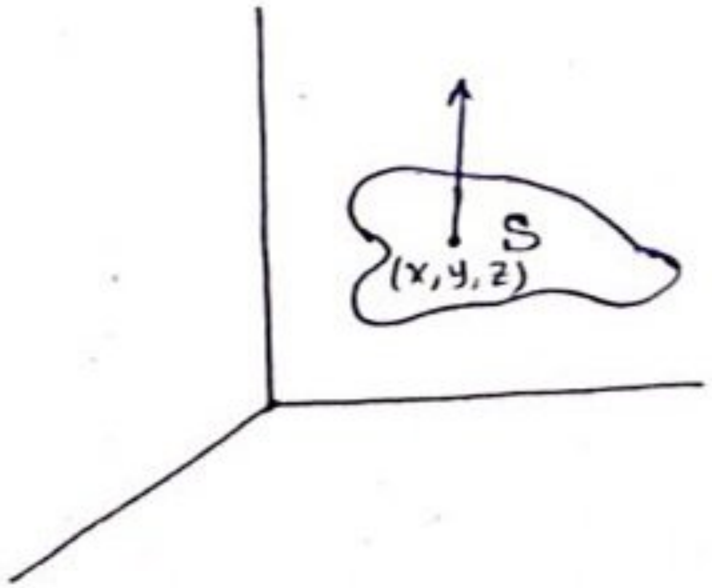
Differential Operators

(a) **Gradient:** Gradient of a scalar point function f is denoted as ∇ (nabla) and defined as

$$\vec{\nabla} f = \text{grad} f = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \right) f$$

$$= \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$



Normal to the tangent plane at (x, y, z)

Gradient descent Method

(i) If $u = x^3 + 3yz^2$, find $\vec{\nabla} u$.

$$\vec{\nabla} u = \frac{\partial u}{\partial x}\hat{i} + \frac{\partial u}{\partial y}\hat{j} + \frac{\partial u}{\partial z}\hat{k}$$

$$= 3x^2\,\hat{i} + 3z^2\,\hat{j} + 6yz\,\hat{k}$$

(b) Divergence: Divergence of a vector point function $\vec{f}$ is denoted as div. $(\vec{\nabla})$ and defined as

$$\text{div.}(\vec{f}) = \vec{\nabla}\cdot\vec{f} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\vec{f}$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(a(t)\hat{i} + b(t)\hat{j} + c(t)\hat{k}\right).$$

$$= \frac{\partial a}{\partial x} + \frac{\partial b}{\partial y} + \frac{\partial c}{\partial z}.$$

~~Qs. If $u = x^3 +$~~ # if $\text{div}\,\vec{F} = 0$, then the vector find is called solenoidal.

(c) Curl:

$$\vec{\nabla}\times\vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix} \qquad \vec{f} = f_1\hat{i} + f_2\hat{j} + f_3\hat{k}$$

If Curl $\vec{F} = 0$, then the vector field $\vec{F}$ is called Irrotational.