Ordinary Differential Equations.

Differential Equations - An equation which contains derivatives of various order and the variables is called a differential equation.

Differential Equations (DE)

Ordinary Differential
Equation (ODE)

Partial Differential Equation (PDE)

ODF - A differential equation with only one independent variable, is called ordinary Differential Equation.

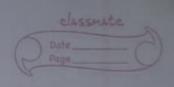
PDF - A differential equation with more than one independent variables and partial derivatives of dependent variable with respect to them, is called Partial Differential Equation

fx: i) $\frac{d^2y}{dt} + \left(\frac{dy}{dt}\right)^3 - y^4 = \sin t \longrightarrow ODE$

(i) $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 4 \longrightarrow PDE$

f(0) do . of f(0) da

bbo & (a) } 0 = (v)



Linear Ordinary Differential Equation:

An ODE is called linear, if the dependent variable and its derivatives occur only in first degree, and no products of the dependent variable and its derivative occur.

Non-Linear ODE: - Otherwise the DE is known as non-Linear ODE.

ii)
$$y'' + y \cdot y' + xy = 0 \rightarrow Non-Linear$$

iii)
$$x^2(y')^4 + y = \sin x \rightarrow Non-Linear$$

iv)
$$\chi^3 y''' + \chi^2 y'' + (\chi - 1) y' = \sin \chi \longrightarrow \text{Linear}$$

v)
$$y' = \sqrt{x^2 + y} \leftrightarrow (y')^2 = x^2 + y \rightarrow Non-Linear$$

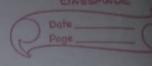
* Order of a Differential Equation.

The order of a differential equation is the order of the highest order derivative involved in the equation.

Degree of a Differential Equation.

of the highest order derivative involved in the equation after the equation is made free from radicals and fractions in its derivatives

Ex: i)
$$y''' + (y'')^3 + xy'^2 + y^5 = e^{3x}$$
 [Order-3, Degree-1, NL]
ii) $(y'')^{3/2} + y' + y^2 = \sin x \Rightarrow (y'')^{3/2} = \sin x - y' - y^2$
 $\Rightarrow (y'')^3 = (\sin x - y' - y^2)^2$ [Order-2, Degree-3, NL]



Formulation of differential equations:

- · Differential equations are formed by eliminating arbitary constants from a relation in the variables and constants
- . If we have a relation containing n arbitrary constants, then we have to differenciate the relation n-times, which will produce nequations.
- · Eliminating n arbitrary constants from n equations, we will get a differential equation.

Ex: Y = Acosx + Bsinx, AlB are arbitrary constant

dy = - Asina + Bcox

 $\frac{d^2xy}{dx^2} = -A\cos x - B\sin x = -\left(A\cos x + B\sin x\right) = -y$

 $\Rightarrow \frac{d^2y}{dz^2} + y = 0$

It is a second order, first degree, linear differential equation. formulation of ODE.

Solo
$$y' = -Am \sin m\alpha + Bm \cos m\alpha$$

 $y'' = -Am^2 \cos m\alpha - Bm^2 \sin m\alpha$

$$y'' = y + e^{\alpha} (-Asinx + Bcosx) + e^{\alpha} (-Asinx + Bcosx)$$

$$y'' = 2(y'-y)$$
or, $y'' - 2y' + 2y = 0 \rightarrow 2^{nd}$ Order
Degree 1
Linear

Question Find the DE corresponding to the relation - 2y = Aex + Be-x

24 = Ae2 + Be-x Solo

or, y + xy' = Aex - Be-x

or, y'+ 2y" + y' = Aex + Be-x

or, y'+ xy" + y' = xy

or, $2y'+\alpha y''-\alpha y=0$ \rightarrow 2^{nd} Order Degree 1 Linear

How i) $y = A \sec x + B \tan x$ ans: $y'' - \tan x y' - y \sec^2 x = 0$ ii) $ax^2 + by^2 = 1$ ans: $xyy'' + xy'^2 - yy' = 0$

Solution of a DE:

A function y = f(x) is called solution of a DE if y is a continuous function and differentiable up to required order and if we substitute the value of y and les derivative in the given equation, then the equation is reduced to an identity.

- i) General solution or complete solution.
- ii) Particular solution
 iii) Singular solution

Ex: i) y"+ y = 0 + 10 A

General solution: - y = A cosx + B sin x

if A=1, B=2: Particular solution: y = cosx + 2 sin x

ii) (y')2 + xy' = y General solution: y = Ax + A2

if A=1; Particular solution :- y=x+1

Singular Solution: $4y + \chi^2 = 0$ $\Rightarrow y = -\chi^2$

H/W

1) y = A sec x + B tan x

y'= Asecxtanx + Bsecx)

y'= seex (Atanx + Bsecx)

y" = sec x tan x (Atan x + Bsec x) + sec x (Asec 2x + Bsec x tan x)

= y'tana + sec2 a (A sec x + B tan x)

= y'tann + y sec2x

or, y"-y"tan x - y sec2x = 0

ii) ax2 + by2 = 1. - differenciating wrt x

or, ax + byy'=0 - (i)

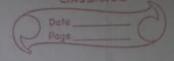
more, y'= - an - (ii)

differenciating (i);

=> a + b (yy" + y'y') = 0

or, $yy'' + (y')^2 = -\frac{a}{b} - (iii)$

Plugging eq. (iii) in eq. (ii); we get,



or,
$$\alpha yy'' + \alpha (y')^2 - yy' = 0$$
.

Initial Value Problem (IVP) - A differential equation with the condition for an initial value of independent variable is called initial value problem.

for x=01, y=0+1) + 10 11 (3+1)

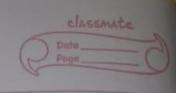
Boundary value Problem (BVP) - A differential equation is called boundary value problem if the conditions are imposed on the dependant variables at the boundary points of the domain of independent variable.

* Variable Seperation method for solving first order first degree differential equation.

Represented as: dy = f(x,y)

Say, M(x,y) dx + N(x,y) dy = 0.

we can apply the variable seperation method of Mis a function of x only M=f(x) and N is a function of y only N=f(y).



Then, we can easily integrate the terms indivisually and get the solution at the problem.

General look: M(x)dx + N(y)dy = 0

Ex: (1+x2) dy-(1+y2) dx =0

 $\Rightarrow \int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$

or, tan'y = tan'x + C, where C & an arbitrary constant

Question (1+ex)y dy - (1+y)exdx = 0

or, (1+ea)y dy = (1+y)eada

or, $\frac{y}{(1+y)} dy = \frac{e^{\alpha}}{(1+e^{\alpha})} d\alpha$

- integrating both sides

or, $\int \frac{dy}{(1+y)} dy = \int \frac{e^{x}}{(1+e^{x})} dx$ $\int \frac{e^{x}}{(1+e^{x})} dx$

or, $\int \left(\frac{1+y}{1+y} - \frac{1}{1+y}\right) dy = \int \left(\frac{1+e^{\alpha}}{1+e^{\alpha}} - \frac{1}{1+e^{\alpha}}\right) dx + c = \ln t$

or, I dy - I dy = I dx - I dx + c I da - x-la 1+ex (11ex)

or, y-ln(1+y) = ln(1+ex)+c

or,
$$\alpha \sqrt{1-y^2} d\alpha = -y \sqrt{1-\alpha^2} dy$$

or,
$$\frac{\chi}{\sqrt{1-\chi^2}} d\chi = -\frac{y}{\sqrt{1-y^2}} dy$$

or,
$$\int \frac{\chi}{\sqrt{1-\chi^2}} dx = -\int \frac{y}{\sqrt{1-y^2}} dy$$

or,
$$\int -t dt$$
, $-\int -Z dz$

or,
$$\frac{1+2}{\sqrt{1-x^2}} + \sqrt{1-y^2} = C$$

Let,
$$t = \sqrt{1-x^2}$$

or, $t^2 = \pm 1-x^2$
or, $2tdx = 2x$

or,
$$t dt = -x dx$$

HIW

HID is tona sing da + costa coty dy =0

=> tona sin2y dx = - cos2x coty dy

or, tan a da = cot y dy

or, - sin x dx = cos my dy

 $\frac{1}{1}$ or, $\frac{1}{1}$ $\frac{1}{1}$

Let, $u = \cos x$ $\omega = \sin y$ $d\omega = \cos y dy$

 $\int \frac{du}{u^3} = \int \frac{d\omega}{\omega^3}$

or, [u-3 du = [w-3 dw]

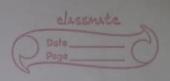
or, $u^{-2} - w^{-2} + e$

or, $\frac{1}{u^2} = \frac{1}{\omega^2} + (-2)C$

or, $\frac{1}{\cos^2 x}$ $\frac{1}{\sin^2 y}$ + K

Let, K = -2C.

or, sec2n - cosec2y = K.



ii) $3e^{x} + \tan y \, dx + (1-e^{x}) \sec^{2} y \, dy = 0$ $\Rightarrow \frac{3e^{x}}{(e^{x}-1)} \, dx = \frac{\sec^{2} y}{\tan y} \, dy$ or, $\int \frac{3e^{x}}{(e^{x}-1)} \, dx = \int \frac{\sec^{2} y}{\tan y} \, dy$

Let, $e^{x}-1=Z$ tany = k or, $e^{x}dx=dz$ tany = k sec²y dy = dk

on, $\int \frac{3}{Z} \frac{dz}{dz} = \int \frac{dk}{k}$

or, 3 log Z = log K + log C

or, log z3 = log Ke

or, z3= Kc (you)

or, $(e^{x}-1)^3 = C.4any$.

strayed - a not a homogeneous diffe of

iii) dy = e^{2a-y} + x³e^{-y}

 $\frac{dy}{dx} = \frac{e^{2\alpha}}{e^{2\beta}} + \frac{\alpha^3}{e^{2\beta}} \qquad (1-e^{2\beta})$

or, et dy = (e2x + x3) dx

or, [ey dy: (e2x + 23) dx

or, $e^{i\frac{\pi}{2}} = \frac{e^{2x} + x^4 + c}{2}$ or, $4e^{i\frac{\pi}{2}} = 2e^{2x} + x^4 + 4c$

or, 4ed-2e2x-x4= K.

2 los x + log C