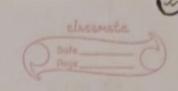
Comparision Test (Limit Form): Let Eun and Evn be two infinite series where lim un = 1 (finite), then if Eun to conveyent and then Evn is east also convergent and if Evn is divogent then Zun is also divergent. Ex: Test the convergence of: 1+2+22+23+24: am: Here: un = 2" (n-1)! : $U_n = \frac{2}{(n-1)} \cdot \frac{2}{(n-2)}$ 2 2 2 (n-1) times \\ \frac{2}{3} \frac{\rho}{3} 2 (n-3) times x 2 $=\left(\frac{2}{3}\right)^{n-3}$ more, let $v_n = 2\left(\frac{2}{3}\right)^{n-3}$ look like $\rightarrow 2\left(1+\frac{2}{3}+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3\right)$ (2) n-3 where \(\frac{1}{2}\) is a geometric series where \(\frac{2}{3}\) \land 1

since, Zun & Zun is also convergent

and is thus convergent.



Question: Test the convergence of: 6 + 8 + 10 +

any: Here-
$$u_n = \frac{2n+4}{(2n-1)(2n+1)(2n+3)}$$

$$\frac{1}{\sqrt[3]{2n}} = \frac{n^2(2n+4)}{(2n+1)(2n+3)}$$

now,
$$\lim_{n\to\infty} \frac{u_n}{u_n} = \lim_{n\to\infty} \frac{\left(2+\frac{u}{n}\right)}{\left(2-\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\left(2+\frac{3}{n}\right)} = \frac{2}{8} = \frac{1}{4}$$
 (finite)

Here, Zun = Zi/n² is a pseries where p=2>1 and is

thus convergent.

i. By comparision to test, un is also convergent

ii Vn - Vn is lim un = finite.

n->00 vn

Question Show that, the series 1 + 1 + 1 + 1 + 1 + ...

is convergent-

can a from them Sol. Un = 1

 $no\omega$, $\frac{1}{2!} = \frac{1}{2}$

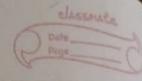
 $\frac{1}{31} < \frac{1}{22}$

41 4 23

:. let Evn = 1 + 1 + 1 + 1 + 1 + 1 +

So, Un & Vn and since Σv_n is a geometric series with $x = \frac{1}{2} \times 1$ and is thus convergent

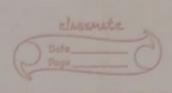
Thus, by comparision test. Zun is also convergent since, Zun < Zvn.



Cauchy's Root Test: If Ziun is a series of positive towns, Such that lim Un in = l (finite) then i) the Series is convergent if L<1. iii) if l=1, then the test fails and we need to check by some another test. E_{X} : $\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$ 80l. Here, $u_n = (1 + \frac{1}{\sqrt{n}})^{-n/2}$ i. $\lim_{n \to \infty} u_n^{-n} = \lim_{n \to \infty} (1 + \frac{1}{\sqrt{n}})^{-\sqrt{n}}$: By Cauchy's Root Test Sun is convergent

| | 21/22/22 |
|----------|--|
| | 01/03/23 |
| | 3 0 1 00 1 1 1 1 1 1 1 1 1 1 |
| | D'A lembert's ratio test: |
| | |
| | If Eun is a series of positive terms, such that |
| | lim Un+1 = l'(finite) [leR], then un |
| | n-100 Un |
| | |
| | |
| 77 | iii) there is no conteste conclusion if L=1. |
| Don | a) there is no sometime continued in |
| _0000 | 10 (deyes) + 2 la (deyes) - 2-4 - la (a |
| | The social leader to |
| Question | Test the convergence of the series $n=1$ n^n |
| | $n=1$ n^n |
| | $m! 2^m$ |
| | there, $u_n = \frac{m! 2^m}{n^m}$ |
| | : Un+1 = (n+1)! 2 n+1 (n+1) n+1 |
| | : Un+1= (n+1)! 2 |
| | |
| | $(n+1)^{1} 2^{n+1} / n! 2^{n} 2(n+1) + n$ |
| - | $\frac{U_{n+1} - (n+1)! 2^{n+1}}{U_n} / \frac{n! 2^n}{n^n} = \frac{2(n+1)}{(n+1)^{n+1}} \times n^n$ |
| | |
| | O and |
| | $= \frac{2n^n}{(n+1)^n}$ |
| | |
| | 0 0 |
| 1000 | $\frac{2}{(n+1)^n} = \frac{2}{(1+1/n)^n}$ |
| 11 - | (n) (1+ (n) |
| | |
| | $\lim_{n\to\infty} \frac{U_{nH}}{U_n} = \lim_{n\to\infty} \frac{2}{(1+\frac{1}{n})^n} = \frac{2}{\lim_{n\to\infty} (1+\frac{1}{n})^n} = \frac{2}{e}$ |
| | n->00 on (1+/n) limn->0 (1+/n)" |
| | |
| | : By D'A lembert's ratio test, $\sum_{n=1}^{\infty} \frac{n! 2^n}{n^n}$ is convergent |
| | TI-1 |

Date Page



Roabe's Test:

If Zun be a series of positive terms such that

lim n(un -1) = l (finite) [le R], then

n+a (un+1)

ii) the series is convergent if lit.

iii) the series is divergent if lit.

Question Pest the convergence of + Zun where-

un = 3.6.9...3n
7.10.13...(3(n+4)

 $\frac{1}{7.10.13} \frac{3n.(3n+3)}{(3n+7)}$

100, Un+1 = 3n+3

: lim Un+ = lim 3n+3 = lim 3+3/n = 3 = 1.

Thus, D'A lembert's test fails.

 $\frac{1}{n+\infty} \ln \left(\frac{un}{un+1} - 1 \right) = \lim_{n\to\infty} \ln \left(\frac{3n+7}{3n+3} - 1 \right) = \lim_{n\to\infty} \frac{4n}{3n+3}$

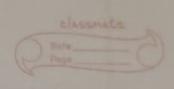
= lim 4 = 4 > 1. Thus, Raebe's Test & un is convergent.

me - 1-2.6 3 mls Question Examine the convergence of 1.2 + 2.3 + 3.4 + any Hene: $Un = \frac{n(n+1)}{2n+1}$ mow), let us consider $y_n = n$ then, $y_n = \frac{n(n+1)}{y_n} \times \frac{1}{n} = \frac{n+1}{2n+1} = \frac{1+\frac{1}{n}}{2+\frac{1}{n}}$ · lim Un = 1 which is finite. So, Un & vn convergence or diverge togethere according to convergence test (limit form)

now, I'vergent.

:. Eun is also divergent.

and The Rocket Test Silver in



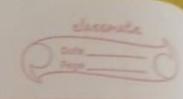
Question Pest the convergence of the series
1.2.3 + 3.4.5

Hint: Apply comparision test limit form

 $u_{n} = \frac{2n-1}{n(n+1)(n+2)} = \frac{\gamma x (2-\frac{1}{n})}{\gamma x \cdot n^{2}(1+\frac{1}{n})(1+\frac{2}{n})} \propto n (2-\frac{1}{n})$ Let, $\forall n = \frac{1}{n^2}$ $\forall n = \frac{1}{n^2}$

Now, $\lim_{n\to\infty} \frac{u_n}{\sqrt{n}} = \lim_{n\to\infty} \frac{(2-\frac{1}{n})\times n^2}{(1+\frac{1}{n})(1+\frac{2}{n})\times n^2} = 2 = finife$

Σ' Vn is a p-series and P=2>1. So Vn is convergent. .. By companision test, I'un is convengent. (limit form)



Question Test the convergence of the series-6 + 8 + 10 + 1.3.5 3.5.7 5.7.9

Hint: Apply comparision test limit form

Now, $u_n = \frac{2n+4}{(2n-1)(2n+1)(2n+3)}$ $u_n = \frac{2n+4}{(2n-1)(2n+1)(2n+1)(2n+3)}$ $u_n = \frac{2n+4}{(2n-1)(2n+1)(2n+1)(2n+3)}$ $u_n = \frac{2n+4}{(2n-1)(2n+1)(2n+3)}$ $u_n = \frac{2n+4}{(2n-1)(2n+3)}$ $u_n = \frac{2n+4}{(2n-1)(2n$

Let, $\forall n = \frac{1}{n^2}$

: $\lim_{n\to\infty} \frac{u_n}{u_n} = \frac{\left(2 + \frac{4}{n}\right)}{\left(2 - \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)\left(2 + \frac{3}{n}\right)} = \frac{2}{2 \times 2 \times 2} = \frac{1}{4} = \text{finite}$

Evn is a p-series and p=2>1 which is convengent.

So, by companision test I un is also convergent.

Question Test the convergence of the series 1 Hint: Apply comparision test limit form. an = 2(n+1)2 = 2/m+1)2 (n+1)2 (n+1) = = = (n+) 2/n+2)~ Khot = Und 113 2/nd Un a not 1+1 + 2+1 3+1 Un = (n+2) 2(n+1)2 $\frac{U_n}{V_n} = \frac{0+2}{2(n+1)} = \frac{1+\frac{2}{n}}{2(1+\frac{1}{n})}$ In un clin 1+ 2/n = 1 ifraite) = Eun il also convergent

Test the convergence of the series whose n^{th} form is: $Un = \sqrt{n^2 + 1} - n$ Here: un = / n2+1 - n ans $u_n = (\sqrt{n^2 + 1} - n) (\sqrt{n^2 + 1} + n)$ $(\sqrt{n^2 + 1} + n)$ $\sqrt{n^2+1}+n$ Let us assume, $n = \frac{1}{n}$ $\frac{Un}{\nabla n} \cdot \frac{1}{\sqrt{n^2+1}+n} \left(\frac{1}{n} \right)$ $= \frac{n}{\sqrt{n^2 + 1} + n} = \frac{1}{\sqrt{1 + \frac{1}{n^2}} + 1}$: $\lim_{n\to\infty} = \frac{1}{2}$ which is finite. So, un and un converge together according to convergence test limit form. now, Sun is a p-series of p=1 and is thus convergent : Evn is also convergent.

Question Test the convergence of the senses.

(1/3) + (2/5) + (3/7) + ... + ...

Hint: Apply Cauchy's Root Test and there: $u_n = \left(\frac{n}{2n+1}\right)^n$ Now, then $\lim_{n \to \infty} u_n \frac{1}{n} = \lim_{n \to \infty} \frac{n}{n} = \lim_{n \to \infty} \frac{1}{n} = \lim_{n \to \infty} \frac{1$: By Couchy's Root test the series is convengent Question: Test the convergence of the series: $1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \cdots$

$$1 + \frac{2^2}{2!} + \frac{3^2}{3!} + \frac{4^2}{4!} + \cdots$$

Let,
$$\sum u_n = 1 + \frac{2^2}{2!} + \frac{8^2}{3!} + \frac{4^2}{4!} + \cdots$$

:.
$$u_n = \frac{n^2}{n!}$$
, $u_{n+1} = \frac{(n+1)^2}{(n+1)!}$

now,
$$\lim_{n\to\infty} \frac{U_n}{U_{n+1}} = \lim_{n\to\infty} \frac{n^2}{n!} \times \frac{(n+)!}{(n+)^2}$$

=
$$\lim_{n\to\infty} \frac{n^2}{n!} \times \frac{(n+1)n!}{(n+1)^2}$$

=
$$\lim_{n\to\infty} \frac{n^2}{n+1} = \lim_{n\to\infty} \frac{n}{1+1/n} = \infty > 1$$

.. By D'A lembert's ratio test, Zun is convergent

Question: Test the convergence of the series:

$$\frac{1}{1+2} + \frac{2}{1+2^2} + \frac{3}{1+2^3} + \dots$$

$$\rightarrow$$
 let, $\sum_{i=1}^{n} \frac{1}{1+2} + \frac{2}{1+2^2} + \frac{3}{1+2^3}$

$$U_n = \frac{n}{1+2^n}$$
, $U_{n+1} = \frac{(n+1)}{1+2^{(n+1)}}$

Classification (C)

now, $\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \lim_{n\to\infty} \frac{n}{1+2^n} \times \frac{1+2^{(n+1)}}{(n+1)}$ $= \lim_{n\to\infty} \frac{n}{(n+1)} \times \frac{1+2^{n+1}}{1+2^n} = \lim_{n\to\infty} \frac{1}{1+\sqrt{n}} \cdot \frac{1+2^{-2^n}}{1+2^n}$

 $=\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}}\times\frac{\frac{1}{2n}+2}{\frac{1}{2n}+1}$

= 1 × 2 - 2>1

2. By D'A lembert's ratio test, Zun is convergent.

Question Pest the convergence of the series: $\frac{1+1}{2}+\frac{1+3}{2\cdot4}+\frac{1\cdot3\cdot5}{2\cdot4\cdot6}+\cdots$

Let $Zun = 1 + \frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \dots$

now, $\lim_{n\to\infty} \frac{U_n = \lim_{n\to\infty} \frac{2n+2}{2n+1} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{1+\frac{1}{2n}}$

:. D'A lambert's ratio test fails and have no conclusion

now, $\lim_{n\to\infty} n\left(\frac{Un}{U_{n+1}}-1\right) = \lim_{n\to\infty} n\left(\frac{2n+2}{2n+1}-1\right)$

 $= \lim_{n\to\infty} n\left(\frac{1}{2n+1}\right) = \lim_{n\to\infty} \frac{1}{2n+1/n} = \frac{1}{2} \cdot 1.$

.. by Raabe's Test, Zun is divorgent.

Question Test the convergence of the series: $\left(\frac{2^2}{1^2} - \frac{2}{1}\right) + \left(\frac{3}{2}^3 - \frac{3}{2}\right)^2 + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^3 + \dots$

Here, $u_n = \left[\frac{(n+1)^{n+1}}{n^{n+1}} \frac{n+1}{n}\right]^{-n}$

: $U_{n}/n = \left(\frac{n+1}{n}\right)^{n+1} - \frac{n+1}{n}$

 $\frac{n}{n+1}\left[\left(\frac{n}{n}\right)^{m}-\frac{1}{2}\right]^{-1}$

 $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right) \left[\left(1 + \frac{1}{n}\right)^n - 1 \right]^{-1}$

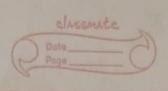
= 1[e-1]-1 < 1.

: By Couchy's Root Test & Un is convergent

Question Examine the convergence of:

$$\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3} \cdot \frac{2}{5}\right)^2 + \left(\frac{1}{3} \cdot \frac{2}{5} \cdot \frac{3}{7}\right)^2 + \dots$$

 \rightarrow Hore, $u_n = \left[\frac{1.2.3...n}{3.5.7...(2n+1)} \right]^2$



now,
$$\frac{U_{n+1}}{U_n} \cdot \left(\frac{n+1}{2n+3}\right)^2$$

$$\lim_{n\to\infty} = \left(\frac{1+\frac{1}{n}}{2+\frac{3}{n}}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \leq \frac{1}{4}$$

Thus, by D'A lemberts ratio test; Sun is convergent

Question Test the convergence of the series: 1 + 2 + 3 + 4 +

Hint: Apply D'A lemberts ratio test

Now, $u_n = \frac{n}{2^n}$: $u_{n+1} = \frac{n}{2^{n+1}}$

: lim $\frac{\ln n+1}{\ln n+1} \times \frac{2^n}{n} = \frac{\ln n+1}{\ln n} = \frac{\ln n+1}{\ln n} \times \frac{(1+\frac{1}{n})}{2}$

By D'A lembert's natio test the series is convengent.

2 1 3,5 (2n-1) (2n+1) 1 2 L E 2n (2n+2) (2n+3)

Suestion Discuse the convergence of 2 nue no

-> Here. un+1 = (n+1) 4 - (n+1) 1 - 12

 $= \left(1 + \frac{i}{n}\right)^{4} e^{-(n+i)^{2} + n^{2}}$

 $= \left(1 + \frac{1}{n}\right)^{4} e^{-2n-1}$

 $=\frac{(1+1)^{4}}{e^{2n}}$

: lim Un+1 = 0. KJ.

Thus, by D'A lemberts ratio lest, Zun is convergent

Question Test the convergence of the sories: (by Raabe's Test)

1+ 1 3 + 2 3 1 4 5 + 1 3 5 - 1 + ...

 $u_n = \frac{1.3.5...(2n-1)}{2.4.6...2n} \times \frac{1}{2n+1}$

 $u_{n+1} = \frac{1.3.5...(2n-1)(2n+1)}{2.4.6...2n(2n+2)} \times \frac{1}{(2n+3)}$

Incomplete