14/2/23

Infinite Series

Let $\{u_n\}$ be the sequence of real number, then the expression: $u_1 + u_2 + u_3 \dots \infty$ is called infinite series and it is denoted as $\sum u_n$.

To find whether the series is convergent or not, we need to construct, $S_1 = u_1$

$S_2 = u_1 + u_2$

$\vdots$

where $\{S_n\}$ is called sequence of partial sum.

If the sequence $\{S_n\}$ is convergent, then the series is also convergent.

Let us ~~cond~~ consider a series :-

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots \infty$$

Here,

$$u_n = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

now, $S_n = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \dots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$

$$= 1 - \frac{1}{n+1}$$

$\therefore \lim\limits_{n\to\infty} = \lim\limits_{n\to\infty}\left(1 - \frac{1}{n+1}\right) = 1$

Thus, the series is convergent and $\sum u_n$ converges to 1.

Divergent Series: A series $\sum u_n$ is said to be divergent if $\lim_{n\to\infty} S_n = \pm\infty$

Ex: $S_n = 1+2+3+\dots\dots+n$

$$S_n = \frac{n(n+1)}{2}$$

$$\lim_{n\to\infty} S_n = \infty$$

Oscillatory Series: A series which is neither convergent nor divergent is called oscillatory series.

Ex: $1+(-1)+1+(-1)+1+(-1)+\dots\infty$

Geometric Series: The series $1+x+x^2+x^3+\dots\infty$ is known as geometric series. This series is convergent if $-1<x<1$ and divergent if $x \geq 1$.

If $x=-1$, then the series is oscillatory

Ex: i) $1+\frac{1}{2}+\frac{1}{2^2}+\dots$

$\therefore x = 1/2 < 1 \rightarrow$ convergent

ii) $1+2+2^2+2^3+\dots$

$\therefore x = 2 > 1 \rightarrow$ divergent

17/02/23

Series

p- series.

The series of the form $1+\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}+\dots+\infty$ is called p-series. This series is convergent if $p \geq 1$ and divergent if $p<1$.

Properties of series :

- If the series $\sum u_n$ is convergent and converges to 's' then the series $\sum K u_n$ is also convergent and converges to 'Ks'.
- If 2 series $\sum u_n$ and $\sum v_n$ converging to 's' and 't' respectively then $\sum(u_n+v_n)$ converges to 's+t'.

Note: If $\sum u_n$ is a convergent series then for its 'n^{th} term' - u_n $\lim_{n\to\infty} u_n = 0$.

→ Let us consider a series $1+\frac{1}{1\cdot 3}+\frac{1}{1\cdot 3\cdot 5}+\dots+\infty$.

Check whether it is convergent or not.

ans

$$u_n = \frac{1}{1.3.5\dots(2n-1)} < \frac{1}{1.2.2\dots(n-1)\text{ times}} = \frac{1}{2^{n-1}}$$

Let, $\frac{1}{2^{n-1}} = v_n$

$$\therefore \sum_{n=1}^{\infty} v_n = 1+\frac{1}{2}+\frac{1}{2^2}+\dots\infty$$

$\therefore \sum v_n$ is a geometric series of $x = 1/2 < 1$,
and is thus convergent.
and since, $u_n < v_n$
by process of comparision test;
$\sum u_n$ is also convergent.

→ Comparision Test :

Let $\sum u_n$ and $\sum v_n$ be 2 series of positive terms and there exist an integer 'N' such that –

$$u_n \leq k v_n \;,\; \forall n \geq N.$$

then, $\sum u_n$ is convergent if $\sum v_n$ is so and $\sum v_n$ is divergent if $\sum u_n$ is so.

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Alternative Series.

A series whose terms are alternatively positive and negative is called an alternative series.

Absolutely Convergent:

A series $\sum u_n$ is said to be absolutely convergent if $\sum |u_n|$ is convergent.

Ex: of alternative series: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \dots$