

1. Formulate differential equations corresponding to the following functions and hence find the order, degree, and nature (linear or non-linear) of the equation (a and b are arbitrary constants):

a. $y^2 = 4a(x + a)$

b. $ax^2 + by^2 = 1$

c. $y = ae^{3x} + be^x$

2. Using appropriate methodology, solve the following first order differential equations:

a. $x \frac{dy}{dx} - y = 2x^2 y$

b. $ydx + (-x - y^2)dy = 0$

c. $\frac{dy}{dx} = \frac{y(x-y)}{x(2x+3y)}$

d. $(x^2 - 1) \frac{dy}{dx} + 2xy = 2x - 2$

e. $\frac{dy}{dx} = \frac{2x+3y+4}{4x+6y+1}$

f. $e^x \sin y dx + (e^x + 1) \cos y dy = 0.$

g. $(x^2 + y^2)dx - 2xy dy = 0.$

h. $y \log y dx + (x - \log y) dy = 0.$

3. Find the convergence of the following series:

i) $\frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \dots$

ii) $\frac{1}{1+1} + \frac{1}{1+2} + \frac{1}{1+2^2} + \frac{1}{1+2^3} + \dots$

iii) $\left(\frac{1}{2}\right)^2 + \left(\frac{1.3}{2.4}\right)^2 + \left(\frac{1.3.5}{2.4.6}\right)^2 + \dots$

iv) $\frac{3}{5} + \frac{4}{5^2} + \frac{3}{5^3} + \frac{4}{5^4} + \dots$

v) $\sum \frac{(-1)^n n!}{n^n}$

vi) $1 + \frac{2^p}{2!} + \frac{3^p}{3!} + \frac{4^p}{4!} + \dots (p > 0)$

vii) $\sum_{n=1}^{\infty} \sqrt[3]{n^3 + 1} - n$

viii) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n(n-1)}}$

ix) $\sum_{n=1}^{\infty} (\sqrt{n^4 + 1} - \sqrt{n^4 - 1})$

x) $\sum_{n=1}^{\infty} \left(\frac{n^2}{2^n} + \frac{1}{n^2}\right).$

Note: Marks (2+4+5)=11