Course: Engineering Mathematics II (MTH11502) Group: B.Tech (All) (Semester II)

Instructor-in-charge: Dr. Supriyo Mazumder, Dr. Avik Pradhan, & Dr. Sumit Som

Submission Date: April 02, 2023

1. Formulate differential equations corresponding to the following functions and hence find the order, degree, and nature (linear or non-linear) of the equation (a and b are arbitrary constants):

a.
$$y^2 = 4a(x + a)$$

b.
$$ax^2 + by^2 = 1$$

c.
$$y = ae^{3x} + be^x$$

2. Using appropriate methodology, solve the following first order differential equations:

a.
$$x \frac{dy}{dx} - y = 2x^2y$$

$$b. \quad ydx + \left(-x - y^2\right)dy = 0$$

c.
$$\frac{dy}{dx} = \frac{y(x-y)}{x(2x+3y)}$$

d.
$$(x^2 - 1)\frac{dy}{dx} + 2xy = 2x - 2$$

e.
$$\frac{dy}{dx} = \frac{2x+3y+4}{4x+6y+1}$$

f.
$$e^x \sin y \, dx + (e^x + 1) \cos y \, dy = 0$$
.

g.
$$(x^2 + y^2)dx - 2xy dy = 0$$
.

h.
$$ylogy dx + (x - logy) dy = 0$$
.

3. Find the convergence of the following series:

i)
$$\frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \cdots$$

ii)
$$\frac{1}{1+1} + \frac{1}{1+2} + \frac{1}{1+2^2} + \frac{1}{1+2^3} + \cdots$$

iii)
$$\left(\frac{1}{2}\right)^2 + \left(\frac{1.3}{2.4}\right)^2 + \left(\frac{1.3.5}{2.4.6}\right)^2 + \cdots$$

iv)
$$\frac{3}{5} + \frac{4}{5^2} + \frac{3}{5^3} + \frac{4}{5^4} + \cdots$$

v)
$$\sum \frac{(-1)^n n!}{n^n}$$

vi)
$$1 + \frac{2^p}{2!} + \frac{3^p}{3!} + \frac{4^p}{4!} + \cdots (p > 0)$$

vii)
$$\sum_{n=1}^{\infty} \sqrt[3]{n^3+1} - n$$

$$viii)\sum_{n=2}^{\infty} \frac{1}{\sqrt{n(n-1)}}$$

ix)
$$\sum_{n=1}^{\infty} (\sqrt{n^4 + 1} - \sqrt{n^4 - 1})$$

$$x) \sum_{n=1}^{\infty} (\frac{n^2}{2^n} + \frac{1}{n^2}).$$

Note: Marks (2+4+5)=11