Sequence & Series.

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Sequence – Sequence is a bijective mapping/function from set of ~~m~~ natural number ($\mathbb{N}$) to any arbitrary subset (S) of real number ($\mathbb{R}$).

It is denoted as $\{x_n\}$; $\{x_n\}_n$; $\{x_n\}_{n \in \mathbb{N}}$

where the sequence is $\{x_1, x_2, x_3, \dots, x_n\}$

Ex: $\left\{\frac{1}{n}\right\}_{n \in \mathbb{N}}$

$\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}$

Monotonically increasing sequence – A sequence $\{a_n\}_n$ is said to increasing monotonically if :

$a_1 \leq a_2 \leq a_3 \dots$

Monotonically decreasing sequence – A sequence $\{a_n\}_n$ is said to monotonically decreasing if :

$a_1 \geq a_2 \geq a_3 \dots$

Strictly increasing sequence – A sequence $\{a_n\}_n$ is said to strictly increasing if :-

$a_1 < a_2 < a_3 \dots$

Strictly decreasing sequence – A sequence $\{a_n\}_n$ is said to strictly decreasing if :

$a_1 > a_2 > a_3 \dots$

Note :- A sequence is said to be monotonic if it is either monotonically increasing or monotonically decreasing.

Exception → $\{1, 0, 1, 0, 1, 0\}$ → Neither increasing nor decreasing.

classmate

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Bounded sequence

A sequence $\{a_n\}$ is said to be bounded above if there exists a real number $(M > 0)$ such that $a_n \leq M$ (for all), $\forall n$

A sequence is said to be bounded below if there exists a real number $(m > 0)$ such that $a_n \geq m$, $\forall n$ $(\exists)$

A sequence is said to be bounded if it is both bounded above and bounded below, i.e, in this case we have $m \leq a_n \leq M$.

Limit point of a sequence :-

A point $l \in \mathbb{R}$ is said to limit point, when $\exists$ $\epsilon > 0$ and $N \in \mathbb{N}$ such that $|a_n - l| < \epsilon$, $\forall n > N$

This is denoted as :

$$\lim_{n \to \infty} a_n = l.$$

If l is finite, then the sequence is said to be convergent and if l is infinite then the sequence is said to be divergent.