



Case 1 Second order Linear Differencial equation with constant coefficient:

P(x), Q(x) are constant and f(x) = 0

: a2 d2y + a, dy + a0y = 0 --- (1)

Solution Procedure:

let y = emx be a solution of the equation.

Then, dy = memx, d2y = m2emx

: yeem is a solution of (1), we must have:

azmemz + a, memz + aoemz = 0.

or, $(a_2 m^2 + a_1 m + a_0) e^{m\alpha} = 0$

1: agm² + a,m + ao = 0 [: em +0]

Which is known as auxiliary equation.

Solve this equation for the values of m and accordingly, we will have solution of O.

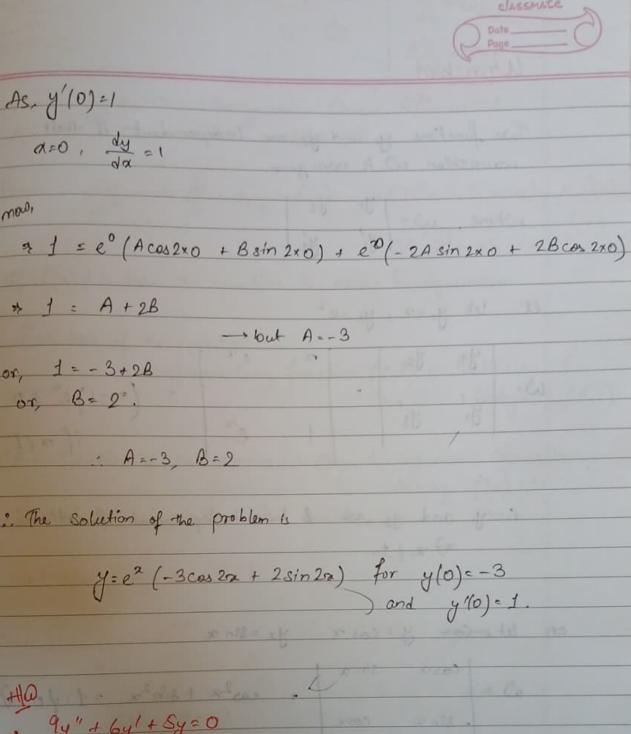
Depending on the values of m, we have 3 different case, (ase (i) Two values of m are real and distinct.

Let, the roots be m, and my : m, 7 m, 6 m, m, ER is Solution is given by :y = Aem, x + Bemzx Two values of m are real and equal. $m_1 = m_2$ $(m, m_1 \in \mathbb{R})$ Solution is given by $m_1 = m_2$ $y = e^{m_1 x} (A + Bx)$ $m_1 = m_2$ $m_2 = m_2 x$ case (iii) Two values of m are complex.

: Let $m_1 = \alpha + i\beta$: $m_2 = \alpha - i\beta$ — Conjugate of m_1 y=ema= e(a+ib)x= exx. eibx y= em2 = e (x-ip) = e an e-ipx

che kmow, eix = cos x + i sin x iba = cos Ba + isin Ba and $e^{-i\beta x} = e^{i(-\beta x)} = \cos \beta x - i \sin \beta x$. Solution is given by: y = exx (A crospx + Bsingx) In all of the above cases, A & B are arbitrary constant guestion find Solution - dy - 6 dy + 9y = 0. Sol Let y = e ma be a solution of the solution of the equation : The auxiliary equation is given by $m^2 - 6m + 9 = 0$ $m^2 - 3m - 3m + 9 = 0$ or, m(m-3) - 3(m-3) = 0 $(m-3)^2=0$. m=3,3. : Solution => y = = e3x (A + Bx)

Solve: y"-2y'+5y=0 , y(0)=-3, y'(0)=1 Question y(0)=-3 sed let y e ma be a solution of the equation for, x=0, y=-3 y'(0) = 1 Then the auxillary equation is given by: for x=0, dy = 1 $m^2 - 2m + 5 = 0$ $m_1 = -(-2) + \sqrt{4 - 20} = 1 + 2i$ $m_2 = -(-2) - \sqrt{4-20} = 1-2i$.. The solution is a y = ex (A cas 2a + B sin 2x) Li where A, B are orbitrary constants. A. y(0)=-3 .. We have, -3= e° (A cos 0 + B sin 0) nas, dy = ex (Acos 2x + Bs in 2x) + e2 (-2Asin 2x + 2Bcos 2x)



+40 9y" + 6y' + Sy=0 y(0)=6, y'(0)=0

: A=-3, B=2

As, y'(0)=1

00, dy =1

* 1 = A + 2B

or, 1 = -3 + 2B

or, B= 2.

· y"-4y'+4y=0 y(0)=3, y'(0)=1

Wronskian

Two functions y, and y, are independent if their wronskian w is non-zero

where,
$$\omega = \begin{vmatrix} y' & y_2 \\ y' & y'_2 \end{vmatrix}$$

ex! let y,= x, y2=e2

	Actd b		ea	1 ox	1	72	1 7	
2 \$ 0.	(x-1) 22	=			2		,	W=
			22	1		72	79,	
	if		ex	1		32	7	

..
$$y_1$$
 and y_2 are 2 two independent functions

 $\frac{1}{2}x + 1$

ex. Let $= \cos x$ $y_1 = \cos x$ $y_2 = \sin x$

$$\frac{1}{2}\cos x + \sin x$$

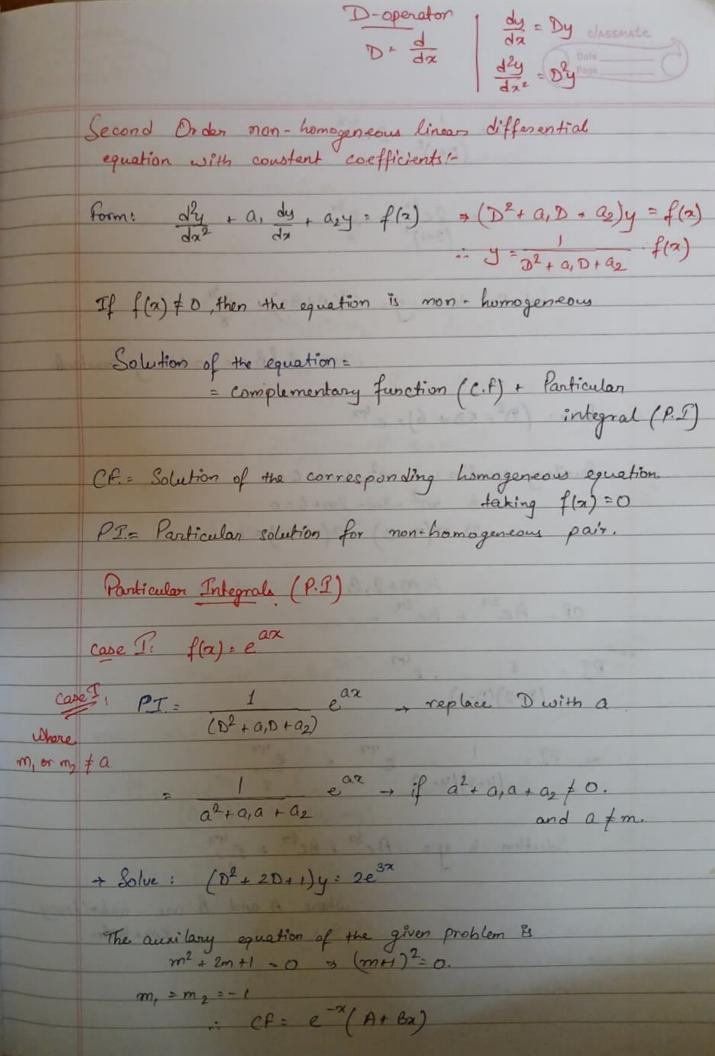
$$\frac{1}{2}\cos x + \sin x$$

$$\frac{1}{2}\cos x + \sin x + \sin x$$

$$\frac{1}{2}\cos x + \sin x + \sin x$$

: y, and y2 are independent functions. In.

is you and you are independent functions of ox.



.. PT.
$$\frac{1}{(D+1)^2} = 2e^{3x}$$
 $\frac{2e^{3x}}{(3+1)^2} = \frac{e^{3x}}{8}$

.. Solution is $y = e^{-7}(A+Bx) + e^{\frac{3x}{2}}$

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.. Solution $e^{-7}(A+Bx) + e^{-7}(A+Bx) + e^{-7}(A+B$

Suppose the auxillary equation has acots. and $a = m_1 \otimes m_2$ G par then, P.I= 1 (D-m,)(D-m2) {eax} $\left(\frac{ne^{\alpha n}}{(a-m_1)}\right)$ Solve: (D2-5D+6)=e301 Auxilary equation: m²-5m+6=0 ((1-)-1) $M_1 = a = 3$. $M_2 = a = 3$. $M_3 = a = 3$. $M_4 = a = 3$: Solution: Ae3x + Be2x + xe32 where A & B are arbitrary contents

then,
$$PP = \frac{\alpha^2}{2!} e^{\alpha \alpha}$$

Solve:
$$(D^2 + D - 2)y = 2e^{x} + 7e^{-2a} + 4e^{2x}$$
.
Let $f(x)$ be 0.

$$(m^2 + m - 2)y = 0$$

$$\Rightarrow$$
 $m^2 + m - 2 = 0$

$$\Rightarrow$$
 $m^2 + 2m - m - 2 = 0$

$$m_1 = 1$$
 $m_2 = -2$

$$PI_1 = 2 ne^{\frac{\pi}{2}} - 2 ne^{\frac{\pi}{2}}$$

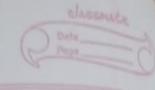
$$(1 - (-2)) 3$$

$$P\Gamma_2 = \frac{1}{4} \times \pi e^{-2\pi} = -\frac{1}{4} \pi e^{-2\pi}$$
(-2-1)

PI3 =
$$4 e^{2\pi} = 4e^{2\pi} = e^{2\pi}$$

.. Solution:
$$Ae^{x} + B + 2ne^{x} - 4ne^{-2n} + e$$

 $\frac{1}{1-D} = 1-1D + D^2 + D^3 + \dots$ 1+D = 1-D+De-D3+... $\frac{1}{(1-D)^2} = 1 + 2D + 3D^2 + 4D^3 + \dots$ $\frac{1}{(1+5)^2} = \frac{1}{(1+5)^2} + \frac{1}{35} +$ Queston (D2-2D+1) y:323 Sol Auxilary eq: m2-2m+== 0. = m-m+=10 + m(m-1) -= (m-1) = 0. = (m-1)2=0. 2. m=1,1 CF = e2 (A + B2) $\frac{P7}{D^2-2D+1} \left\{ \frac{3a^3}{3a^3} \right\} = \frac{1}{(D^2-1)^2} \left\{ \frac{3a^3}{3a^3} \right\} = \frac{1}{(1-D)^2} \left\{ \frac{3a^3}{3a^3} \right\}$



$$= \begin{cases} 1 + 2D + 3D^2 + 4D^3 + \dots \end{cases} \begin{cases} 3\alpha^3 \end{cases}$$

$$= 3\alpha^3 + 2 d (3\alpha^3) + 3 d^2 (3\alpha^3) + 4 d^3 (3\alpha^3)$$

$$= d\alpha^2 (3\alpha^3) + 3 d^2 (3\alpha^3) + 4 d^3 (3\alpha^3)$$

= 3x3+ 18x2+ 54x+72

: y = Aea + Bxea + 3x3 + 18x2 + 54x + 72.

guestion (D - 4D + 4) y = 2x + 3

Auxilary equation is given by!

=> m2 - 4m+4 =0

or, $m^2 - 2m - 2m + 4 = 0$ or, m(m-2) - 2(m-2) = 0

or, (m-2)2=0.

or, m = 2, 2.

CF = Pex (A+Bx)

 $\frac{1}{(D-2)^2}$ { 2a+3 }

 $\left[-\left(2\left(\frac{D}{2}-1\right)^{2}\right]\left\{2\alpha+3\right\}$

4/1- D)2 [la+3]

$$= \frac{1}{4} \left[\frac{1+2}{2} + \frac{D}{2} + \frac{3}{2} + \frac{(D)^2 + 4}{(D)^3} + \dots \right] \left\{ \frac{2\alpha + 3}{2} \right\}$$

$$= \frac{1}{4} \left[(2\alpha + 3) + \frac{1}{4\alpha} (2\alpha + 3) \right] = \frac{1}{4} \left[2\alpha + 3 + 2 \right] = \frac{2\alpha + 5}{4}$$

$$= \frac{1}{1 + (D^2 + D)} \left\{ 5x^3 \right\}$$

=
$$\frac{1}{1+D} = \{ Sn^3 \}$$
 where $D = D^2 + D$.

Let
$$PI = \frac{1}{0^2 + 20 - 3} \left\{ \frac{2\alpha + 7}{3} \right\}$$

$$= \frac{1}{-3\left(1-\left(\frac{2D+D^2}{3}\right)\right)}\left\{2x+7\right\}$$

$$= -\frac{1}{3} \left[1 + \frac{2D+D^2}{3} + \left(\frac{2D+D^2}{3} \right)^2 + \dots \right] \left\{ 2\alpha + 7 \right\}$$

Classmate Dute Page

Question (D2-1) y = 12x4-3x+1

$$PI = \frac{1}{5^2-1} - \left\{ 2x^4 - 3x + 1 \right\}$$

$$= \frac{1}{1-0^2} \left\{ 2x^4 - 3x + 1 \right\}$$

$$= -\left[1 + D^2 + D^4\right] \left\{2x^4 - 3x + 1\right\}$$

$$= -\left[2x^{4} - 3x + 1 + d^{2}\left(2x^{4} - 3x + 1\right) + d^{4}\left(2x^{4} - 3x + 1\right)\right]$$

$$= - \left[2\alpha^4 - 3n + 1 + 24\alpha^2 + 148 \right]$$

Margine Wanter End ! - O to

2 -23 E 18 (20 m) - (20 m) + (20 m) + 1 "

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