Bornoulli's Equation.

(non-linear ODE)

form
$$dy + P(x)y = Q(x)y^n$$

or,
$$\frac{1}{y^n} \frac{dy}{dx} + P(x) \cdot \frac{1}{y^{n-1}} = Q(x) - C$$

now, let
$$y^{1-n} = Z$$

$$\frac{1}{2} = (1-n) y^{-n} dy$$

or,
$$\frac{1}{(1-n)} \frac{dz}{dx} = \frac{1}{y^n} \frac{dy}{dx} = \frac{1}{(1-n)}$$

Plugging (1) into (1);

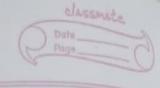
$$\frac{1}{(1-n)} \frac{dz}{dz} + \frac{P(\alpha)}{Z} = Q(\alpha)$$

or,
$$\frac{dz}{dx} + (1-n)P(x)Z = (1-n)Q(x)$$

or,
$$\frac{dz}{dx} + P'(x)z = Q'(x)$$

where,
$$P'(\alpha) = (1-n) P(\alpha)$$

 $Q'(\alpha) = (1-n) Q(\alpha)$



now, If = esp'(x) da

: Solution :-

$$Z \times If = \int (Q(x) \times If) dx + C$$

where B= y 1-n

Question Solve: 2 dy = y + y2

dx = x + x2

 $\frac{dy}{dz} = \frac{1}{2z} \cdot \frac{1}{3} \cdot \frac$

This is a Bernoulli's Equation. We can rewrite this equation as:

1 dy 1 y 1 y2 da 2a y2 2a2

or, $y^{-2} dy = \frac{1}{2\alpha} y^{-1} - \frac{1}{2\alpha^2}$

mow, let y' = 2 $\frac{dz}{da} = -y^2 dy$ $\frac{dz}{da} = -y^2 dy$

Plugging (a) Int (1); (a)

=> - dz _-11 z= 1 mg

or,
$$dz$$
 + $\frac{1}{2\alpha}z = -\frac{1}{2\alpha^2}$ Comparing to $dz + P(\alpha)z = R(\alpha)$

Solution:
$$\frac{1}{y}\sqrt{x} = -\int \left(\frac{1}{2\alpha^2} \cdot \sqrt{x}\right) d\alpha + C$$

$$= -\frac{1}{2}\left(\frac{x^{1/2}}{\alpha^2}\right) d\alpha + C$$

$$= -\frac{1}{2} \int (x^{1/2} - 2) dx + c$$

$$=-\frac{1}{2}\frac{x^{-1/2}}{x^{-1/2}}+c$$

or,
$$\frac{\alpha}{y} = 1 + C\sqrt{\alpha}$$

dy , 2 y = 2 Jy Question Jy dy + x Jy = x - 0 Comparing to y let Jy = Z $\frac{dz}{dz} = \frac{d}{dx} \left(\frac{y'h}{y'} \right) = \frac{1}{2} \frac{y'^2}{dy} = \frac{1}{2} \frac{dy}{dx}$ Multiplying 1 by 1/2 $\frac{1}{2\sqrt{y}} \frac{dy}{dx} + \frac{q}{2(1-x^2)} \sqrt{\frac{q}{y}} = \frac{q}{2}$ $\frac{dz}{dt} + \frac{z}{2(1-x^2)} = \frac{2}{2}$ now, $P(\alpha) = \frac{\alpha}{2(1-\alpha^2)}$ $S(\alpha) = \frac{\alpha}{2}$ If = e Sp(n) dx = e \(\frac{2}{2(1-n^2)} \, dx