INEXACT DIFFERENCIAL EQUATION 20.03.23

form: Mdx + Ndy = 0

for exactness: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial z}$

Inexact DE: DM + DN Dy + DR

Ex: yda-ady=0-0

Here, M=y N=2xpritate

now dy dy

and, $\partial N = \partial (-x) = -1$. $\partial N = \partial (-x) = -1$. $\partial N = \partial (-x) = -1$. $\partial N = \partial (-x) = -1$.

: y da - ady = 0 is Texact.

Ex: now, multiplying 1/2 to 0;

yda-2dy = 0

=> \frac{1}{y} da - \frac{2}{y^2} dy = 0

 η_{00} , $\frac{\partial M}{\partial y} = \frac{\partial (y^{-1})}{\partial y^{-1}} = -\frac{1}{y^{2}}$

 $\frac{\partial N}{\partial x} = \frac{\partial}{\partial x} \left(-\frac{x}{y^2} \right) = -\frac{1}{y^2}$

mow, yda-ady o is exact where y is the Trategrating factor.

End of Capacitation of Total Mail Integrating factor: for the D.E of the form

Mdx + Ndy = 0

If DM + DN, then equation is a Enexact DE. To make it exact, we multiply the equation by f(x,y). Here if is known as entegrating factor.

finding IF:

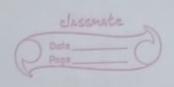
For an integrating inexact de Mdx + Ndy = 0

i) if $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = h(x)$

then, I.f = e Sh(a) da

ii) if $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = g(y)$

then, I.F = e s-g(y) dy



Procedure for solving Inexact DE:

Step!: Compare the given DE with std. form Mdz + Ndy = 0 and obtain the value of M2N.

Stop: 2: Calculate the value of DM & DN and get the value of Dy Dx.

Step:3: Find the appropriate Integrating factor by dividing the difference and and by Mor N.

Then, multiply the IF with the DE to and Hence obtain an exact DE, as M'dx + N'dy =0.

where M' = M. IF and N'= N. IF

Stop-4 Solve the exact DE M'dx + N'dy = 0 using earlier procedione.

Ex: (4xy + 3y2-1x) dx + x (x+2y) dy = 0

Soln: Here, comparing fluor DE to Mdx + Ndy = 0

M = 4xy + 3y^2 - 2

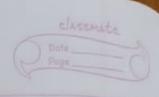
N = x^2 + 2xy

 $\frac{\partial M}{\partial y} = \frac{\partial}{\partial y} \left(\frac{4\pi y}{3} + \frac{3y^2 - \alpha}{3} \right) = \frac{4\pi}{6} + \frac{6y}{6}$

 $\frac{\partial N}{\partial x} = \frac{\partial}{\partial x} \left(x^2 + 2xy \right) = 2x + 2y$

: 2M + 200, given DE is inexact

 $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \frac{4x + 6y - 2x + 2y}{2x + 2y} = \frac{2x + 4y}{2(x + 2y)}$



now, and and house of same and same and

 $= \frac{2(9+2y)}{2(2+2y)} = \frac{2}{2}$

 $\frac{1}{2} \int_{-\infty}^{\infty} \frac{1}{2} d\alpha = 2 \ln \alpha = 2 \ln \alpha^{2} = \alpha^{2}$

2. The exact DE will be:

22 (4ay + 3y2)-a) da + 23 (2+2y) dy = 0

M'= x2 (4ay + 3y2-a)

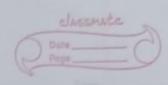
N'= 23(2+24)

900, $\int M' dx = \int x^2 (4xy + 3y^2 - x) dx$ $= \int x^2 (4x^3y + 3x^2y^2 - x^3) dx$

 $= \frac{4yx^{4} + 3y^{2}x^{3} - x^{4}}{3} - \frac{x^{4}}{4} = x^{4}y + \frac{3}{4}y^{2}x^{3} - \frac{x^{4}}{4}$

and, $\int N' da = \int (x^4 + 2x^3y) dy$ $= x^4y + 2x^3y^2 - x^4y + x^3y^2$

: Solution of DE 16: x4y + 23y2 - x4 = C



Integrating factor

Swestian you + (-2-ye)dy = 0

-> Here, comparing given DE to Mdx + Ndy = D

M=y N=(-x-y2)

 $\frac{\partial M}{\partial y} = \frac{\partial y}{\partial y} = \frac{1}{2} \cdot \frac{\partial N}{\partial x} = \frac{\partial}{\partial x} \left(-x - y^2\right)$

= -1

: 2M != 2N given Df is inexact

now, $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 1 - (-1) = 2$.

also, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \frac{2}{y}$

! $Pf = e^{-\int (\frac{2}{y}) dy} = e^{-2 \ln y} = e^{\ln y^{-2}} = \frac{1}{y^2}$

moso, M'= M x PF = y x \frac{1}{y^2} = \frac{1}{y}

 $N' = N \times IF = -(x+y^2) \times \frac{1}{y^2} = -\frac{x+y^2}{y^2}$

mas, M'da + N'dy = 0

or, y da - 2+y2 dy = 0. - 1

 $M' = \frac{1}{y}$ $N' = -\frac{x-y^2}{y^2}$ $\int M' dx = \int \int dx = \frac{x}{9}$ $\int N' dy = \int \left(-\frac{\alpha}{y^2} - \frac{y^2}{y^2}\right) dy = \frac{\alpha}{y} - y$: Solution of 1) is 3-y= C. (3y-ex) dx + 2 dy = 0 M= 3y-e2 N= 2 2M. 2 (3y-e3): 3. $\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x) = 1.$: 2M 1= 2N given DF is inexact. now, 2M - 2N = 3-1=2 alou, am - an ay an : If= e \ \frac{2}{2} dx



M': 322y - 22e2 N': 23

SM'da = s(3x²y - x²e²) dα = x³y - (x²e² - 2(e²z - e²))

In'dy = Jx3day = x3y

: General solution = x3y-x2ex-2(e2x-ex)

 $\int x^2 e^{x} dx \rightarrow integration by parts.$

 $\Rightarrow x^2e^{x}-2(e^{2}x-e^{2}) \rightarrow \text{while doing this } \int x \cdot e^{x} dx = (x-1)e^{x}$ $(x^2-2x+2)e^{x}$ $= \begin{cases} (x^2-2x+2)e^{x} \end{cases}$ $= \begin{cases} (x^2-2x+2)e^{x} \end{cases}$ $= \begin{cases} (x^2-2x+2)e^{x} \end{cases}$ $= \begin{cases} (x^2-2x+2)e^{x} \end{cases}$

0.4

5 3 th 2-23 3 46 Call 3

DA 45 (100) - D2

3 2 66 (3 2 2)