Physics
A Reference Manual

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Fundamentals of Electricity and Magnetism

1. Coulomb's Law

Equation:

$$F = k_e \frac{|q_1 \cdot q_2|}{r^2}$$

Explanation: Coulomb's Law calculates the electric force F between two charges, q_1 and q_2, separated by a distance r. k_e is Coulomb's constant. The force is attractive if the charges are opposite and repulsive if they are the same. **Example:** If two charges, +1 C and -1 C, are 1 meter apart, the force between them is calculated using $k_e = 8.987 \times 10^9$ N m^2/C^2.

2. Electric Field

Equation:

$$E = \frac{F}{q}$$

Explanation: The electric field E at a point in space is defined as the electric force F experienced by a small positive test charge q placed at that point, divided by the magnitude of the charge. **Example:** If a 1 C charge experiences a force of 10 N, the electric field at that point is 10 N/C.

3. Ohm's Law

Equation:

$$V = IR$$

Explanation: Ohm's Law relates the voltage across a conductor V, the current flowing through it I, and its resistance R. It states that the current is directly proportional to the voltage and inversely proportional to the resistance. **Example:** If a 10 V battery is connected across a resistor of 5 ohms, the current flowing through the circuit is $I = \frac{V}{R} = 2$ A.

4. Faraday's Law of Electromagnetic Induction

Equation:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Explanation: Faraday's Law states that the induced electromotive force (EMF) $\mathcal{E}$ in any closed circuit is equal to the negative rate of change of the magnetic flux Φ_B through the circuit. **Example:** If the magnetic flux through a loop changes by 0.02 Wb in 0.01 seconds, the induced EMF is $\mathcal{E} = -2$ V.

5. Ampere's Law

Equation:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

Explanation: Ampere's Law relates the integrated magnetic field $\vec{B}$ around a closed loop to the electric current I_{enc} passing through any surface bounded by the loop. μ_0 is the permeability of free space. **Example:** For a long straight wire carrying a current of 5 A, the magnetic field at a distance of 2 meters from the wire can be calculated using Ampere's Law.

6. Maxwell's Equations

Equation:

- Gauss's Law for Electricity: $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}$
- Gauss's Law for Magnetism: $\oint \vec{B} \cdot d\vec{A} = 0$
- Faraday's Law: $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$
- Ampere's Law with Maxwell's addition: $\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_{\text{enc}} + \varepsilon_0 \frac{d\Phi_E}{dt})$

Explanation: Maxwell's Equations are a set of four equations that form the foundation of classical electromagnetism. They describe how electric and magnetic fields are generated by charges, currents, and changes of the fields. **Example:** Using Gauss's Law for Electricity, the electric field outside a uniformly charged sphere can be found by considering a Gaussian surface outside the sphere.

1. Magnetic Force on a Moving Charge

Equation:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Explanation: A charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force $\vec{F}$. This force is perpendicular to both the velocity and the magnetic field. **Example:** A charge of +1 C moving at 1 m/s perpendicular to a magnetic field of 1 T experiences a force of 1 N.

2. Magnetic Field Due to a Long Straight Wire

Equation:

$$B = \frac{\mu_0 I}{2\pi r}$$

Explanation: The magnetic field B at a distance r from a long straight wire carrying current I is given by this formula, where μ_0 is the permeability of free space. **Example:** A wire carrying a current of 2 A will produce a magnetic field of 4×10^{-7} T at a distance of 1 m from the wire.

3. Biot-Savart Law

Equation:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$$

Explanation: The Biot-Savart Law calculates the differential magnetic field $d\vec{B}$ produced at point P by a small segment of current-carrying wire $Id\vec{s}$, where $\hat{r}$ is the unit vector from the wire to point P. **Example:** Calculating the magnetic field at a point away from a small segment of current-carrying wire requires integrating this expression over the length of the wire.

4. Magnetic Flux

Equation:

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Explanation: Magnetic flux Φ_B through a surface is the integral of the magnetic field $\vec{B}$ over that surface $d\vec{A}$. It represents the number of magnetic field lines passing through the surface. **Example:** If a magnetic field of 2 T passes uniformly through a 1 m^2 area, the magnetic flux is 2 Weber.

5. Ampere's Circuital Law

Equation:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

Explanation: Ampere's Circuital Law relates the integrated magnetic field $\vec{B}$ around a closed loop to the total electric current I_{enc} passing through the loop. **Example:** For a long, straight wire carrying a current of 5 A, the magnetic field can be calculated at any point in a circular path around the wire.

6. Lorentz Force Law

Equation:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Explanation: The Lorentz force law gives the total force $\vec{F}$ experienced by a charge q moving with velocity $\vec{v}$ in the presence of electric $\vec{E}$ and magnetic $\vec{B}$ fields. **Example:** A +1 C charge moving with a velocity of $1\,\mathrm{m/s}$ in a region with both a 1 T magnetic field and a 1 V/m electric field will experience a force.

7. Magnetic Moment

Equation:

$$\vec{\mu} = I\vec{A}$$

Explanation: The magnetic moment $\vec{\mu}$ of a current loop is the product of the current I and the area vector $\vec{A}$ of the loop. It determines the torque the loop experiences in a magnetic field. **Example:** A loop carrying 2 A of current with an area of 1 m^2 has a magnetic moment of 2 A m^2.

8. Magnetic Field of a Solenoid

Equation:

$$B = \mu_0 n I$$

Explanation: The magnetic field B inside a long solenoid with n turns per unit length carrying current I is uniform and given by this equation. **Example:** A solenoid with 1000 turns per meter carrying 0.5 A will have a magnetic field of 6.28×10^{-4} T inside it.

9. Force between Two Parallel Currents

Equation:

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi d}$$

Explanation: This formula calculates the force per unit length F/L between two long parallel wires carrying currents I_1 and I_2 separated by distance d. The force is attractive for currents in the same direction and repulsive for opposite directions. **Example:** Two parallel wires 1 m apart carrying 2 A and 3 A respectively experience a force per meter of $4 \times 10^{-7}\,\mathrm{N/m}$.

10. Magnetic Materials and Susceptibility

Equation:

$$\chi_m = \frac{M}{H}$$

Explanation: The magnetic susceptibility χ_m of a material is the ratio of its magnetization M to the applied magnetic field H. It indicates how easily the material can be magnetized. **Example:** A material with a magnetization of $4\,\mathrm{A/m}$ in a $2\,\mathrm{T/m}$ field has a susceptibility of 2.

Motional emf

Equation:

$$\mathcal{E} = B\ell v \sin(\theta)$$

Explanation: Motional emf ($\mathcal{E}$) is the electromotive force induced in a conductor moving through a magnetic field. This occurs due to the Lorentz force acting on the charges within the conductor. Here, B is the magnetic field strength, ℓ is the length of the conductor within the magnetic field, v is the velocity of the conductor relative to the magnetic field, and θ is the angle between the velocity and the magnetic field. The $\sin(\theta)$ component determines the effective component of velocity that is perpendicular to the magnetic field.

Example: Consider a rod of length 1 m moving at a speed of 2 m/s perpendicular ($\theta = 90°$) to a magnetic field of strength 0.5 T. The motional emf induced in the rod is calculated as $\mathcal{E} = 0.5 \times 1 \times 2 \times \sin(90°) = 1$ V.

Forces and Torques on Currents in Electricity and Magnetism

1) Force on a Straight Current Segment

Equation:

$$\vec{F} = I\vec{L} \times \vec{B}$$

Explanation: A straight segment of wire carrying a current I in a magnetic field $\vec{B}$ experiences a force $\vec{F}$. The length of the wire in the field is represented by $\vec{L}$, and the force is perpendicular to both $\vec{L}$ and $\vec{B}$.

2) Force on a Curved Current Segment

Equation:

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

Explanation: For a curved segment of current-carrying wire, the differential force $d\vec{F}$ on a small segment $d\vec{l}$ is given by this equation. The total force is the integral of $d\vec{F}$ over the length of the wire.

3) Force on a Current Loop

Equation:

$$\vec{F} = \oint (Id\vec{l} \times \vec{B})$$

Explanation: The net force on a current loop in a uniform magnetic field is often zero because the forces on opposite segments of the loop cancel out. However, in a non-uniform field, the loop can experience a net force.

4) Torque on a Current Loop

Equation:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Explanation: A current loop in a magnetic field experiences a torque $\vec{\tau}$, which tends to rotate the loop. Here, $\vec{\mu} = I\vec{A}$ is the magnetic dipole moment of the loop with area $\vec{A}$, and $\vec{B}$ is the magnetic field.

5) Dipole Moment of Current Loop

Equation:

$$\vec{\mu} = I\vec{A}$$

Explanation: The magnetic dipole moment $\vec{\mu}$ of a current loop is the product of the current I and the area vector $\vec{A}$ of the loop. It represents the strength and orientation of the loop's magnetic effect.

6) Potential Energy of Dipole in Magnetic Field

Equation:

$$U = -\vec{\mu} \cdot \vec{B}$$

Explanation: The potential energy U of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ is given by this equation. It represents the work done to rotate the dipole from its stable equilibrium position to the current orientation.