Physics
A Reference Manual

Khalil Gatto

December, 2023

Fundamentals of Electricity and Magnetism

Coulomb's Law

Equation:

$$F = k_e \frac{|q_1 \cdot q_2|}{r^2}$$

Explanation: Coulomb's Law calculates the electric force F between two charges, q_1 and q_2, separated by a distance r. k_e is Coulomb's constant. The force is attractive if the charges are opposite and repulsive if they are the same.

Electric Field

Equation:

$$E = \frac{F}{q}$$

Explanation: The electric field E at a point in space is defined as the electric force F experienced by a small positive test charge q placed at that point, divided by the magnitude of the charge.

Ohm's Law

Equation:

$$V = IR$$

Explanation: Ohm's Law relates the voltage across a conductor V, the current flowing through it I, and its resistance R. It states that the current is directly proportional to the voltage and inversely proportional to the resistance.

Gauss' Law and Symmetry

Gauss' Law is a fundamental principle that relates the distribution of electric charge to the resulting electric field. The law is expressed as:

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \tag{1}$$

where the total electric flux Φ_E through a closed surface is equal to the charge q_{enc} enclosed by the surface divided by the vacuum permittivity ε_0.

Example: Spherical Symmetry

For a point charge, the electric field is radially outward and the Gaussian surface is a sphere:

$$E(4\pi r^2) = \frac{q}{\varepsilon_0} \quad \Longrightarrow \quad E = \frac{q}{4\pi\varepsilon_0 r^2} \tag{2}$$

where r is the radius of the Gaussian sphere and q is the enclosed charge.

Charges on Conductors

For a conductor in electrostatic equilibrium, the charge resides on the surface, the electric field inside is zero, and on the surface, it's perpendicular to the surface.

Induced Charges on Conductors

When a conductor is placed in an external electric field, charges within the conductor redistribute until the field inside is zero, creating induced charges on the surface.

Induced Charges on Conductors

When a conductor is placed in an external electric field, charges within the conductor redistribute until the field inside is zero, creating induced charges on the surface.

Example: Solid Infinite Cylindrical Conductor

For a long cylindrical conductor with charge per unit length λ, Gauss' Law can be applied with a cylindrical Gaussian surface:

$$E(2\pi r L) = \frac{\lambda L}{\varepsilon_0} \quad \Longrightarrow \quad E = \frac{\lambda}{2\pi\varepsilon_0 r} \tag{3}$$

where L is the length of the cylinder and r is the radial distance from the axis.

Infinite Sheet of Charge

For an infinite sheet of charge with surface charge density σ, the electric field is:

$$E = \frac{\sigma}{2\varepsilon_0} \tag{4}$$

directed perpendicularly away from the sheet.

Superposition

The principle of superposition states that the total electric field created by multiple charges is the vector sum of the electric fields created by each charge individually:

$$\vec{E}_{\text{total}} = \sum_{i=1}^{n} \vec{E}_i \tag{5}$$

where $\vec{E}_i$ is the electric field due to the ith charge.

Faraday's Law of Electromagnetic Induction

Equation:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Explanation: Faraday's Law states that the induced electromotive force (EMF) $\mathcal{E}$ in any closed circuit is equal to the negative rate of change of the magnetic flux Φ_B through the circuit.

Ampere's Law

Equation:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

Explanation: Ampere's Law relates the integrated magnetic field $\vec{B}$ around a closed loop to the electric current I_{enc} passing through any surface bounded by the loop. μ_0 is the permeability of free space.

Maxwell's Equations

Equation:

- Gauss's Law for Electricity: $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}$
- Gauss's Law for Magnetism: $\oint \vec{B} \cdot d\vec{A} = 0$
- Faraday's Law: $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$
- Ampere's Law with Maxwell's addition: $\oint \vec{B} \cdot d\vec{l} = \mu_0(I_{\text{enc}} + \varepsilon_0 \frac{d\Phi_E}{dt})$

Explanation: Maxwell's Equations are a set of four equations that form the foundation of classical electromagnetism. They describe how electric and magnetic fields are generated by charges, currents, and changes of the fields.

Magnetic Force on a Moving Charge

Equation:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Explanation: A charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences a force $\vec{F}$. This force is perpendicular to both the velocity and the magnetic field.

Magnetic Field Due to a Long Straight Wire

Equation:

$$B = \frac{\mu_0 I}{2\pi r}$$

Explanation: The magnetic field B at a distance r from a long straight wire carrying current I is given by this formula, where μ_0 is the permeability of free space.

Biot-Savart Law

Equation:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s} \times \hat{r}}{r^2}$$

Explanation: The Biot-Savart Law calculates the differential magnetic field $d\vec{B}$ produced at point P by a small segment of current-carrying wire $Id\vec{s}$, where $\hat{r}$ is the unit vector from the wire to point P.

Magnetic Field Produced by an Infinite Straight Wire

Equation:

$$B = \frac{\mu_0 I}{2\pi r}$$

Explanation: For an infinitely long straight wire carrying a steady current I, the magnetic field B at a distance r from the wire is given by this equation, where μ_0 is the permeability of free space. The direction of the field is given by the right-hand rule.

Force between Two Parallel Current Carrying Wires

Equation:

$$F = \frac{\mu_0 I_1 I_2 L}{2\pi d}$$

Explanation: Two parallel wires carrying currents I_1 and I_2 over a length L, separated by a distance d, exert a force F on each other. The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

Magnetic Field along the Axis of a Current Loop

Equation:

$$B = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{3/2}}$$

Explanation: The magnetic field B at a point along the axis of a circular loop of radius R carrying current I at a distance z from the center of the loop is given by this equation. The field is strongest at the center of the loop and decreases with distance from the loop.

The Off-Axis Magnetic Field of a Current Carrying Loop

Equation: Complex - involves elliptic integrals. **Explanation:** The off-axis magnetic field due to a current carrying loop is more complex and typically requires numerical methods or approximations for practical calculation. It involves integrating the contributions of each element of the loop over its circumference considering the position relative

Magnetic Flux

Equation:

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Explanation: Magnetic flux Φ_B through a surface is the integral of the magnetic field $\vec{B}$ over that surface $d\vec{A}$. It represents the number of magnetic field lines passing through the surface.

Ampere's Circuital Law

Equation:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

Explanation: Ampere's Circuital Law relates the integrated magnetic field $\vec{B}$ around a closed loop to the total electric current I_{enc} passing through the loop.

Lorentz Force Law

Equation:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Explanation: The Lorentz force law gives the total force $\vec{F}$ experienced by a charge q moving with velocity $\vec{v}$ in the presence of electric $\vec{E}$ and magnetic $\vec{B}$ fields.

Magnetic Moment

Equation:

$$\vec{\mu} = I\vec{A}$$

Explanation: The magnetic moment $\vec{\mu}$ of a current loop is the product of the current I and the area vector $\vec{A}$ of the loop. It determines the torque the loop experiences in a magnetic field.

Magnetic Field of a Solenoid

Equation:

$$B = \mu_0 n I$$

Explanation: The magnetic field B inside a long solenoid with n turns per unit length carrying current I is uniform and given by this equation.

Magnetic Materials and Susceptibility

Equation:

$$\chi_m = \frac{M}{H}$$

Explanation: The magnetic susceptibility χ_m of a material is the ratio of its magnetization M to the applied magnetic field H. It indicates how easily the material can be magnetized. A material with a magnetization of $4\,\mathrm{A/m}$ in a $2\,\mathrm{T/m}$ field has a susceptibility of 2.

Motional emf

Equation:

$$\mathcal{E} = B\ell v \sin(\theta)$$

Explanation: Motional emf ($\mathcal{E}$) is the electromotive force induced in a conductor moving through a magnetic field. This occurs due to the Lorentz force acting on the charges within the conductor. Here, B is the magnetic field strength, ℓ is the length of the conductor within the magnetic field, v is the velocity of the conductor relative to the magnetic field, and θ is the angle between the velocity and the magnetic field. The $\sin(\theta)$ component determines the effective component of velocity that is perpendicular to the magnetic field.

Forces and Torques on Currents in Electricity and Magnetism

Force on a Straight Current Segment

Equation:

$$\vec{F} = I\vec{L} \times \vec{B}$$

Explanation: A straight segment of wire carrying a current I in a magnetic field $\vec{B}$ experiences a force $\vec{F}$. The length of the wire in the field is represented by $\vec{L}$, and the force is perpendicular to both $\vec{L}$ and $\vec{B}$.

Force on a Curved Current Segment

Equation:

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

Explanation: For a curved segment of current-carrying wire, the differential force $d\vec{F}$ on a small segment $d\vec{l}$ is given by this equation. The total force is the integral of $d\vec{F}$ over the length of the wire.

Force on a Current Loop

Equation:

$$\vec{F} = \oint (Id\vec{l} \times \vec{B})$$

Explanation: The net force on a current loop in a uniform magnetic field is often zero because the forces on opposite segments of the loop cancel out. However, in a non-uniform field, the loop can experience a net force.

Torque on a Current Loop

Equation:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Explanation: A current loop in a magnetic field experiences a torque $\vec{\tau}$, which tends to rotate the loop. Here, $\vec{\mu} = I\vec{A}$ is the magnetic dipole moment of the loop with area $\vec{A}$, and $\vec{B}$ is the magnetic field.

Dipole Moment of Current Loop

Equation:

$$\vec{\mu} = I\vec{A}$$

Explanation: The magnetic dipole moment $\vec{\mu}$ of a current loop is the product of the current I and the area vector $\vec{A}$ of the loop. It represents the strength and orientation of the loop's magnetic effect.

Potential Energy of Dipole in Magnetic Field

Equation:

$$U = -\vec{\mu} \cdot \vec{B}$$

Explanation: The potential energy U of a magnetic dipole $\vec{\mu}$ in a magnetic field $\vec{B}$ is given by this equation. It represents the work done to rotate the dipole from its stable equilibrium position to the current orientation.