CSE332:
Design and Analysis of Algorithms
Project

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Introduction

In this report we have solved 6 puzzles using the required algorithms and we write the implementation for these algorithms then we calculate the complexity for each algorithm and finally we compared the required algorithm with the optimal algorithm and also we wrote its implementation and we calculated its complexity.

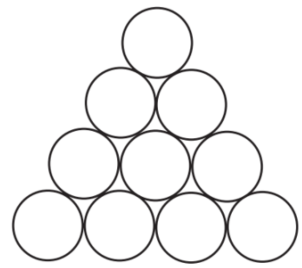
1. PROJECT REQUIREMENTS

1.1 Task 1

1.1.1 Problem description

Task 1

Inverting a Coin Triangle Consider an equilateral triangle formed by closely packed pennies or other identical coins like the one shown in the figure below. (The centers of the coins are assumed to be at the points of the equilateral triangular lattice.)
Use iterative improvement method to design an algorithm to flip the triangle upside down in the minimum number of moves if on each move you can slide one coin at a time to its new position.



1.1.2 Solution

1.1.2.1 Pseudocode

Algorithm Invert a Triangle of Coins (int no_of_rows)

//input no_of_rows “Integer specified by the user”

//output Inverted Triangle of coins

Class rows (no_of_coins,totalrows)

{

addcoins(addcoins)

{

no_of_coins ← 0

no_of_coins ← no_of_coins + addcoins

updatespaces()

}

removecoins(removeCoins)

```
{
    no_of_coins ← 0
    no_of_coins ← no_of_coins - removeCoins
    updatespaces()
  }
  updatespaces()
{
    Spaces ← 0
    Spaces ← totalrows - no_of_coins
}
}
Class pyramid (total_rows)
{
  Create a list called "rowlist"
  Create a list called "rowlist2"
  Create a list called "manipulatedrows"
  for  i ← 1  to (total_rows+1) do
      rowlist.append(rows(i,total_rows))

   no_of_iterations()
{
    coins_number ← 0
    for i ← 1 to (total_rows+1) do
      coins_number ← coins_number + i
```

```
        iterations ← (floor(coins_number/3))

        print("number of iterartions are: ")

        print(iterations)

}

    showPyramid(self)

    {

        Create a list called "temparr"

        for row ← 0 to rowlist do

            for i ← 0 to row.spaces do

                temparr.append("")

            for i ← 0 to row.no_of_coins do

                temparr.append("1")

            for i ← 0 to row.spaces do

                temparr.append("")

            for i ← 0 to  len(temparr)  do

                 print(i, end =" ")

            print("\n")

            temparr.clear()

}

getmanipulated()

{

        manipulatedrows.append(rowlist[0])

        int max ← 0

        if (len(rowlist)%2 = 0)
```

```
         {
           max ← ((len(rowlist)/2)+1)
           for i ← max to len(rowlist) do
              manipulatedrows.append(rowlist[i])
         }
         else
           {
           manipulatedrows.append(rowlist[0])
           max ← floor(int((len(rowlist)/2)+1))
           for i  ← max to len(rowlist)) do
              manipulatedrows.append(rowlist[i])
}
 updaterowlist()
{
        index ← self.total_rows - 2
        s ← 1
        for k ← 0 to  (len(rowlist2)) do
           for i ← 0 to  (len(rowlist)-s) do
                      rowlist[i].removecoins(1)
           firstrow ← self.rowlist2[index]
           rowlist.append(firstrow)
           showPyramid()
           index ← index - 1
           s ← s + 1
```

```
}

}
```

1.1.2.2 Code

```
from math import ceil, floor
from mimetypes import init
from tempfile import tempdir


class pyramid:
    rowlist=[]
    rowlist2 =[]
    manipulatedrows=[]
    def __init__(self,total_rows):
        self.total_rows = total_rows
        for i in range(1,total_rows+1):
            self.rowlist.append(rows(i,total_rows))
        for i in range(1,total_rows):
            self.rowlist2.append(rows(i,total_rows))


    def no_of_iterations(self):
        coins_number = 0
        for i in range(1,(self.total_rows+1)):
            coins_number = coins_number+i
        iterations = int (floor(coins_number/3))
        return iterations
        """print("number of iterartions are: ")
        print(iterations)"""

    def showPyramid(self):
        temparr=[]
        for row in self.rowlist:
            for i in range(row.spaces):
                temparr.append("")

            for i in range(row.no_of_coins) :
                temparr.append("1")
            for i in range(row.spaces):
                temparr.append("")
            for i in temparr:
                 print(i, end =" ")
            print("\n")
            temparr.clear()

    def getmanipulated(self):
```

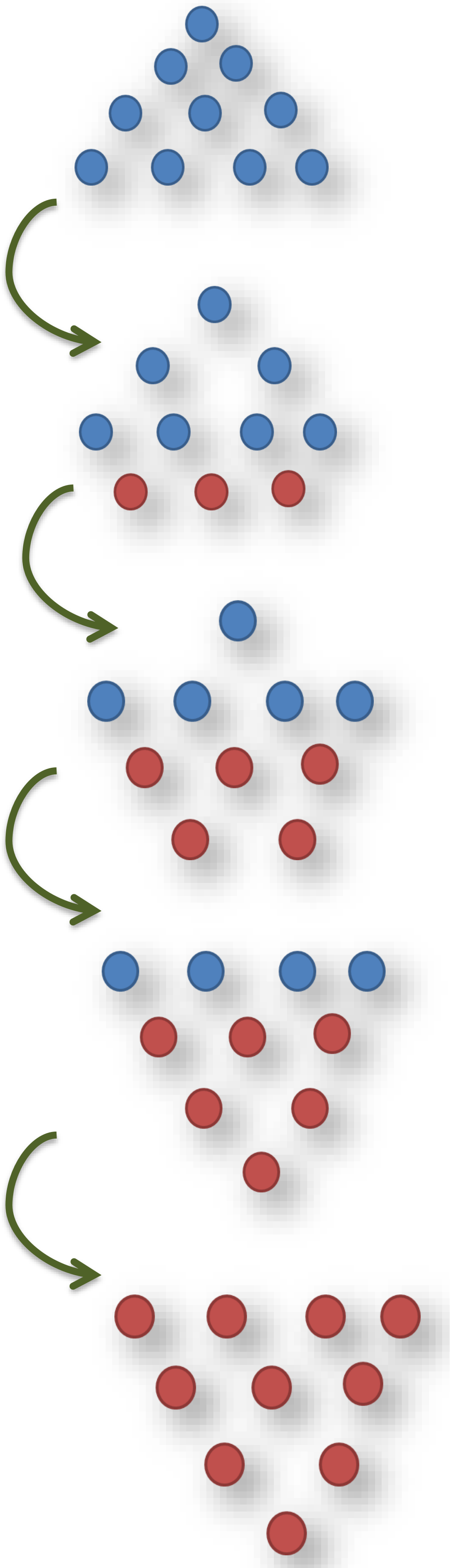
```
        self.manipulatedrows.append(self.rowlist[0])
        if (len(self.rowlist)%2 == 0):
            max=int((len(self.rowlist)/2)+1)
            for i in range(max,len(self.rowlist)):
                self.manipulatedrows.append(self.rowlist[i])
        else:
            self.manipulatedrows.append(self.rowlist[0])
            max= floor(int((len(self.rowlist)/2)+1))
            for i in range(max,len(self.rowlist)):
                self.manipulatedrows.append(self.rowlist[i])

    def updaterowlist(self):
        index = self.total_rows - 2
        s = 1
        for k in range (len(self.rowlist2)):

            for i in range (len(self.rowlist)-s):
                self.rowlist[i].removecoins(1)
            #self.showPyramid()
            firstrow=self.rowlist2[index]
            self.rowlist.append(firstrow)
            self.showPyramid()
            index = index - 1
            s = s + 1



class rows:
     # init method or constructor
    def __init__(self,no_of_coins,totalrows):
        self.no_of_coins = no_of_coins
        self.spaces = totalrows - no_of_coins
        self.totalrows=totalrows
    def addcoins(self,addcoins):
        self.no_of_coins=self.no_of_coins+addcoins
        self.updatespaces()
    def removecoins(self,removeCoins):
        self.no_of_coins=self.no_of_coins-removeCoins
        self.updatespaces()
    def updatespaces(self):
        self.spaces=self.totalrows-self.no_of_coins
```

```python
if __name__=="__main__":
    no_of_rows = int (input("Enter number of rows: "))
    mypyramid = pyramid(no_of_rows)
    mypyramid.showPyramid()
    mypyramid.getmanipulated()
    mypyramid.updaterowlist()
    print ('\n')
    print ("Final Result")
    mypyramid.showPyramid()
```

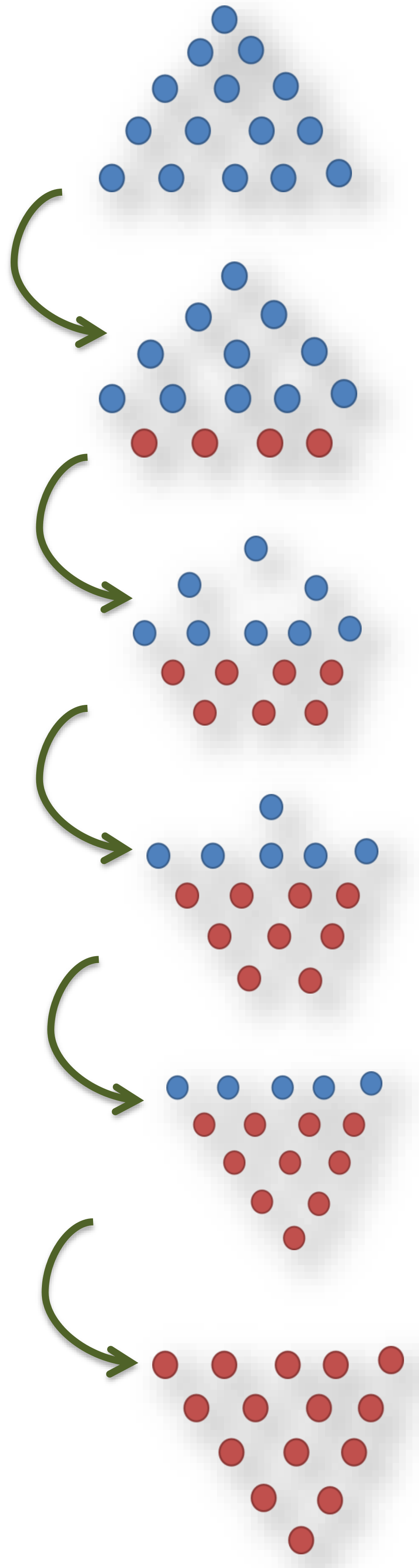
1.1.2.3 Solution description

1- The user enter number of rows
2- We used this number to draw a triangle of coins
3- First we draw triangle of coins Note: every coin is represented by "1" in the code
4- Our goal is to invert this triangle of coins:
    - In the beginning we need to know that we have symmetric shape in the middle and we will slide only the edge coins
    - Firstly we will slide number of coins equal to "Specific Number"
      Specific Number = ((the number of coins in the last row)-1)
      Note: We slide only one coin at a time to its new position
    - Second we will slide number of coins equal to "Specific Number - 1", our notes is still applicable
    - Third we will repeat step number 2 until we reach that the "Specific Number = 0"
    - Fourth we will find that we have got the inverted triangle
    - Finally I will found that these steps helps me to achieve the minimum number of moves

5- To Solve this puzzle I will use iterative improvement algorithm: Iterative Improvement Meaning: It means that you repeat your logic for number of iterations; in each iteration your problem is getting better and get nearer to the final answer.
6- These figures can help in understanding my solution:



For number of rows = 4



For number of rows = 5

1.1.2.4 Complexity analysis

The total number of moves made by the algorithm, M(k), is obviously the minimum needed to make the kth row the base of the inverted triangle, because each coin move increases the number of coins in a row that must be lengthened and simultaneously decreases the number of coins in a row that must be shortened.

M(k) can be computed as follows:

M(k) = $\sum_{j=0}^{\lfloor (n-k)/2 \rfloor}(n-k-2j) + \sum_{j=1}^{k-1} j$ = $\sum_{j=0}^{\lfloor (n-k)/2 \rfloor}(n-k)$ - $\sum_{j=0}^{\lfloor (n-k)/2 \rfloor}(2j)$ + $\sum_{j=1}^{k-1} j$

= (n-k) ($\lfloor \frac{n-k}{2} \rfloor$ + 1) - $\lfloor \frac{n-k}{2} \rfloor$(($\lfloor \frac{n-k}{2} \rfloor$ + 1) + $\frac{(k-1)k}{2}$

= ($\lfloor \frac{n-k}{2} \rfloor$ + 1) $\lceil \frac{n-k}{2} \rceil$ + $\frac{(k-1)k}{2}$

If n-k is odd, the formula can be simplified to:

M(k) = ($\frac{n-k}{2}$+1)$\frac{n-k}{2}$ + $\frac{(k-1)k}{2}$ = $\frac{3k^2-(2n+4)k+ n^2+2n}{4} = \mathrm{O}(k^2)$

If n-k is even, the formula can be simplified to:

M(k) = ($\frac{n-k-1}{2}$+1)$\frac{n-k+1}{2}$ + $\frac{(k-1)k}{2}$ = $\frac{3k^2-(2n+4)k+(n+1)^2}{4} = \mathrm{O}(k^2)$

Where

k = (n + 2)/3.

Tn = n(n + 1)/2 is the total number of coins in the triangle.

1.1.2.5 Comparison between another algorithm

Iterative improvement achieved the minimum number of iterations which is achieved using this formula T_n= n(n + 1)/2 coins is $\lfloor T_n/3 \rfloor$.

1- We can also solve this puzzle using another algorithm “brute force” but is won’t be optimized because the number if iterations will equal the number of coins
    - Brute Force Algorithm meaning: straightforward method of solving a problem by trying every possibility rather than advanced techniques to improve efficiency.

2- We can solve this puzzle using "Dynamic Programming" algorithm

- Dynamic Programming Algorithm meaning: is a technique in computer programming that helps to efficiently solve a class of problems that have overlapping subproblems and optimal substructure property.
- Solution Description for applying dynamic programming on this puzzle:
  - First I will get the number of rows from the user
  - Second I will draw triangle of coins. Note: every coin is represented by "1" in the code
  - Third I will divide my triangle to subproblems "Smaller triangles"
  - Fourth I will memorize the result of the subproblems to use them in solving the other subproblems. Note: In each time we call the solving function we first check the memorized results to use them directly and after that we complete our answer.
  - Finally we will get our inverted triangle
  - Note: The Code take into consideration sliding the coins and sliding only one coin at a time to its new position
- Sample of the output: "Theses screenshots shows the first step and the final result it doesn't show the intermediate steps "

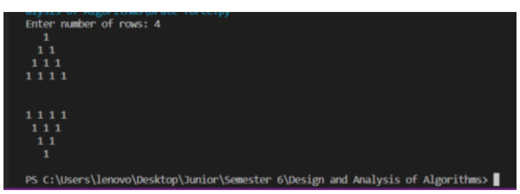


Figure 1: output for n = 4

Figure 2: output for n =7

Pseudocode for brute force algorithm

```
Algorithm Invert a Triangle of Coins (int no_of_rows)
//input no_of_rows "Integer specified by the user"
//output Inverted Triangle of coins
Class rows (no_of_coins,totalrows)
{
   addcoins(addcoins)
{
      no_of_coins ← 0
      no_of_coins ← no_of_coins + addcoins
          updatespaces()
}
   updatespaces()
{
      Spaces ← 0
      Spaces = totalrows - no_of_coins
}
}
Class pyramid (total_rows)
{
   Create a list called "rowlist"
   Create a list called "manipulatedrows"
   for  i ← 1  to (total_rows+1) do
         rowlist.append(rows(i,total_rows))
```

```
no_of_iterations()
{
    coins_number ← 0
    for i ← 1 to (total_rows+1) do
        coins_number ← coins_number + i
    print("number of iterartions are: ")
    print(coins_number)
}
showPyramid(self)
{
    Create a list called "temparr"
    for row ← 0 to rowlist do
        for i ← 0 to row.spaces do
            temparr.append("")
        for i ← 0 to row.no_of_coins do
            temparr.append("1")
        for i ← 0 to row.spaces do
            temparr.append("")
        for i ← 0 to  len(temparr)  do
             print(i, end =" ")
        print("\n")
        temparr.clear()
}
```

```
getmanipulated()
{
    for i ← 0  to (len(rowlist)) do
        manipulatedrows.append(rowlist[i])
}
updaterowlist()
{
        j ← 0
        if (j <= (len(manipulatedrows)))
      {
            firstrow ← manipulatedrows[j]
            rowlist.remove(firstrow)
            rowlist.append(firstrow)
            manipulatedrows.remove(firstrow)
            showPyramid()
            j ← j +1
      }
}
```

Pseudocode for dynamic programming algorithm

```
Algorithm Invert a Triangle of Coins (int no_of_rows)

//input no_of_rows "Integer specified by the user"

//output Inverted Triangle of coins

Class rows (no_of_coins,totalrows)

{

   addcoins(addcoins)

{

      no_of_coins ← 0

      no_of_coins ← no_of_coins + addcoins

          updatespaces()

}

    removecoins(removeCoins)

{

      no_of_coins ← 0

      no_of_coins ← no_of_coins - removeCoins

      updatespaces()

   }

   updatespaces()

{

      Spaces ← 0

      Spaces ← totalrows - no_of_coins

}

}
```

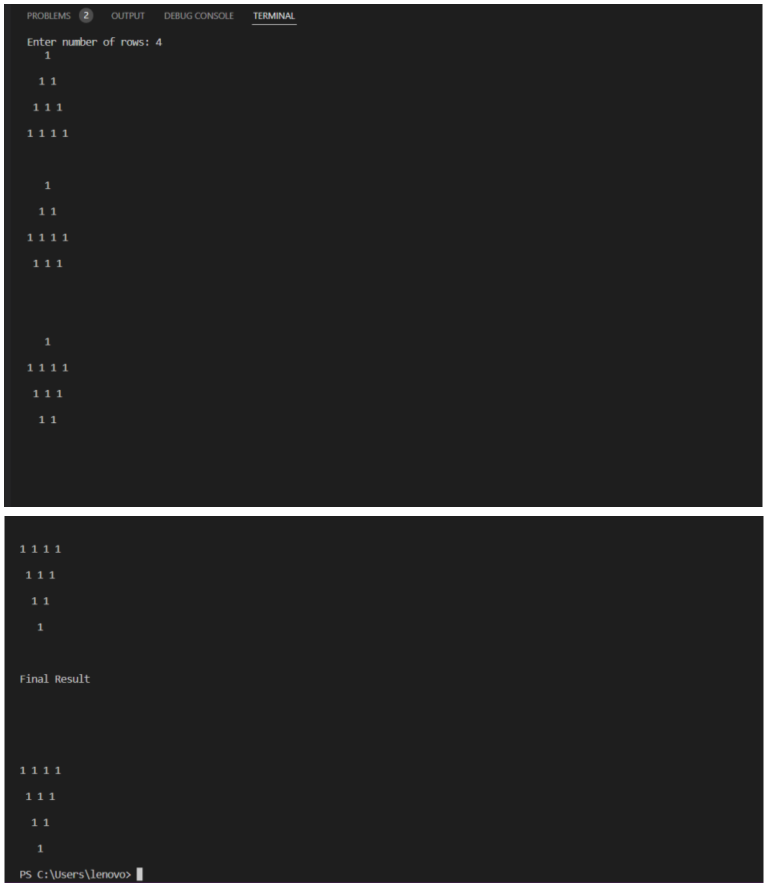
```
Class pyramid (total_rows)

{

   Create a list called “rowlist”

   Create a list called “rowlist2”

   Create a list called “savingList”

   Create a list called “manipulatedrows”

   for  i ←1  to (total_rows+1) do

         rowlist.append(rows(i,total_rows))


   showPyramid(self)

   {

      Create a list called “temparr”

      for row ← 0 to rowlist do

         for i ← 0 to row.spaces do

            temparr.append("")

         for i ← 0 to row.no_of_coins do

            temparr.append("1")

         for i ← 0 to row.spaces do

            temparr.append("")

         for i ← 0 to  len(temparr)  do

             print(i, end =" ")

         print("\n")

         temparr.clear()

}
```

```
getmanipulated()
{
    manipulatedrows.append(rowlist[0])
    int max ← 0
    if (len(rowlist)%2 = 0)
     {
      max ← int((len(rowlist)/2)+1)
      for i ← max to len(rowlist) do
         manipulatedrows.append(rowlist[i])
     }
    else
      {
      manipulatedrows.append(rowlist[0])
      max ← floor(int((len(rowlist)/2)+1))
      for i  ← max to len(rowlist)) do
         manipulatedrows.append(rowlist[i])
}
manipulatingFunction()
{
    for i ← 1 to (total_rows+1) do
      savingList =  mypyramid.updaterowlist(i)
      fsMemoizer(i, savingList)
}
```

```
   fsMemoizer (i, savingList[])
{
       dict = {i: savingList}
}
updaterowlist(input_i)
{
       Create a list called "list"
        for s ← 0 to (len(fsMemoizer.dict)) do
          i ← fsMemoizer.dict(i)
          list ← fsMemoizer.dict.(savingList)
          rowlist.append(list)


       for i ← 0 to (input_i - i) do
          index ← total_rows - 2
          s ← 1
          for k ← 0 to (len(rowlist2)) do
             for i ← 0 to  (len(self.rowlist)-s) do
                rowlist[i].removecoins(1)
             firstrow ← rowlist2[index]
             rowlist.append(firstrow)
             showPyramid()
             index ← index - 1
             s ← s + 1
}}
```

1.1.2.6 Sample of the output

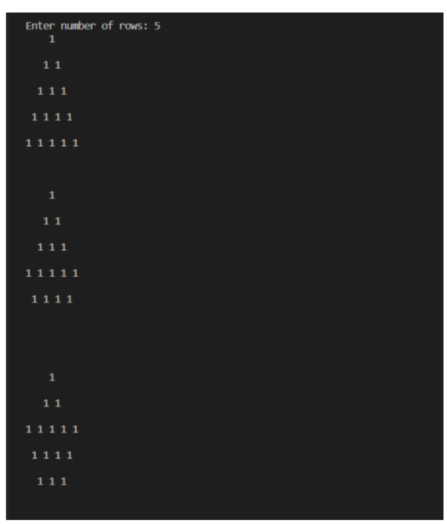
Screenshots for the Iterative improvement algorithm implementation: “This Code output shows the triangle every time it has clear shape”



```
PROBLEMS 2   OUTPUT   DEBUG CONSOLE   TERMINAL

Enter number of rows: 4
   1

  1 1

 1 1 1

1 1 1 1


   1

  1 1

1 1 1 1

 1 1 1



   1

1 1 1 1

 1 1 1

  1 1
```

```
1 1 1 1

 1 1 1

  1 1

   1


Final Result



1 1 1 1

 1 1 1

  1 1

   1

PS C:\Users\lenovo>
```

Figure 3: Output for number of rows = 4



```
Enter number of rows: 5
    1

   1 1

  1 1 1

 1 1 1 1

1 1 1 1 1


    1

   1 1

  1 1 1

1 1 1 1 1

 1 1 1 1



    1

   1 1

1 1 1 1 1

 1 1 1 1

  1 1 1
```

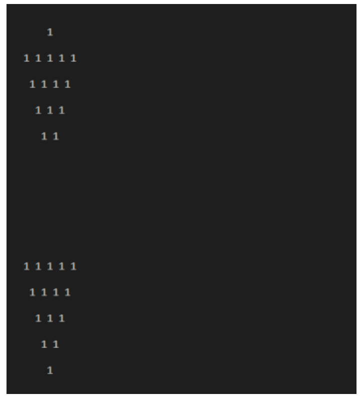
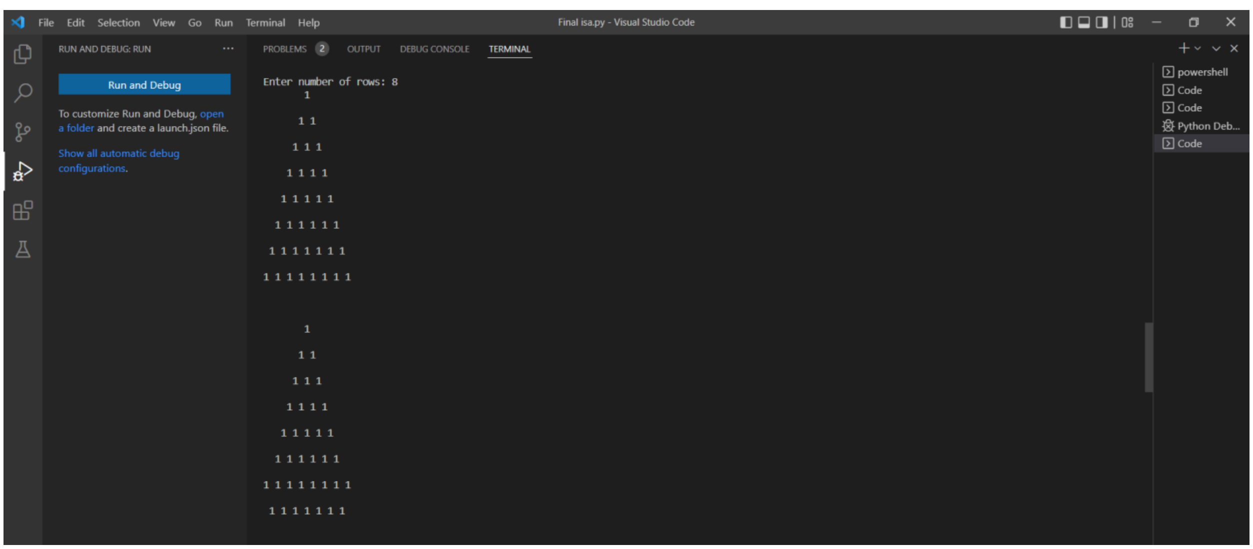


Figure 4: Output for number of rows = 5




Run and Debug
To customize Run and Debug, open a folder and create a launch.json file.
Show all automatic debug configurations.
1
1 1
1 1 1
1 1 1 1
1 1 1 1 1
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1
1 1 1 1 1 1
1
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1 1 1
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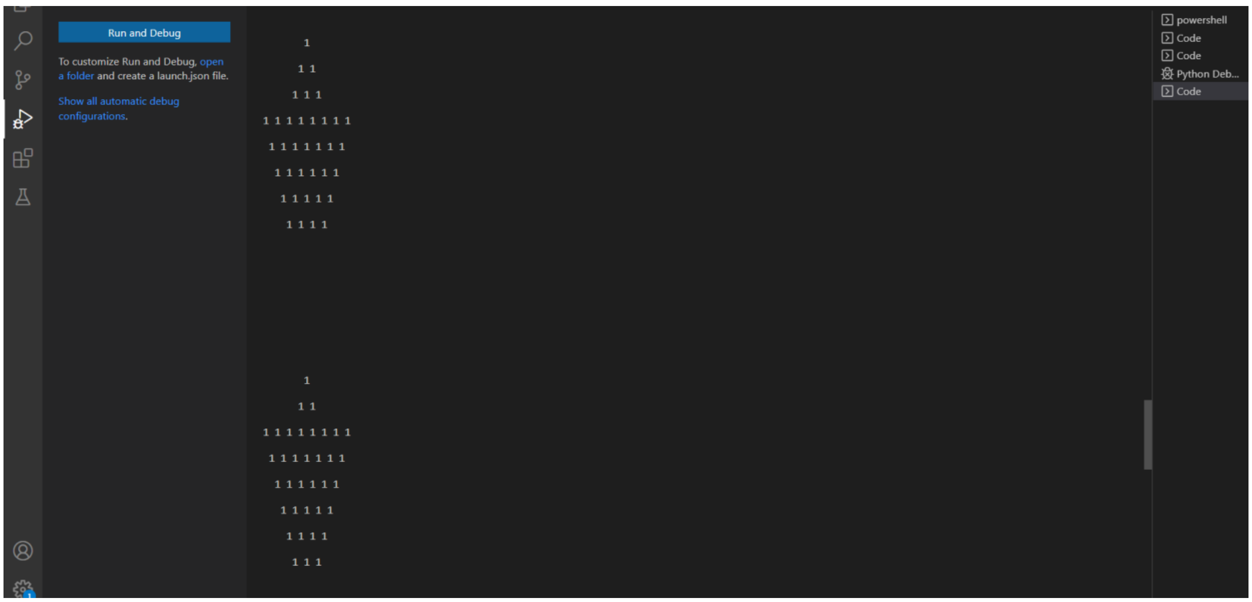
1 1 1 1 1 1 1 1
1 1 1 1 1 1 1
1 1 1 1 1 1
1 1 1 1 1
powershell
Code
Code
Python Deb...
Code


Run and Debug
To customize Run and Debug, open a folder and create a launch.json file.
Show all automatic debug configurations.
1
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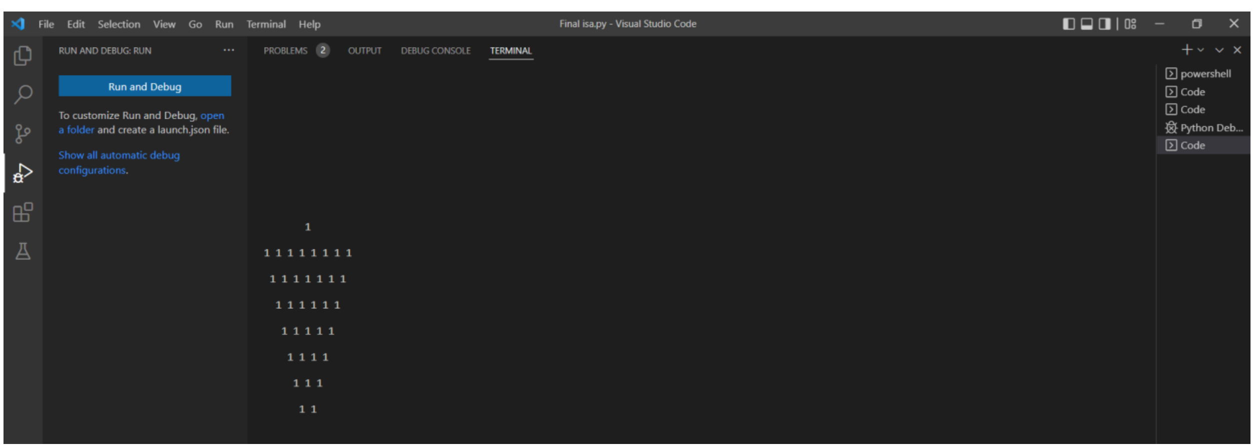
1 1 1
powershell
Code
Code
Python Deb...
Code


File Edit Selection View Go Run Terminal Help
Final isa.py - Visual Studio Code
RUN AND DEBUG: RUN
PROBLEMS 2 OUTPUT DEBUG CONSOLE TERMINAL
Run and Debug
To customize Run and Debug, open a folder and create a launch.json file.
Show all automatic debug configurations.
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1 1
powershell
Code
Code
Python Deb...
Code

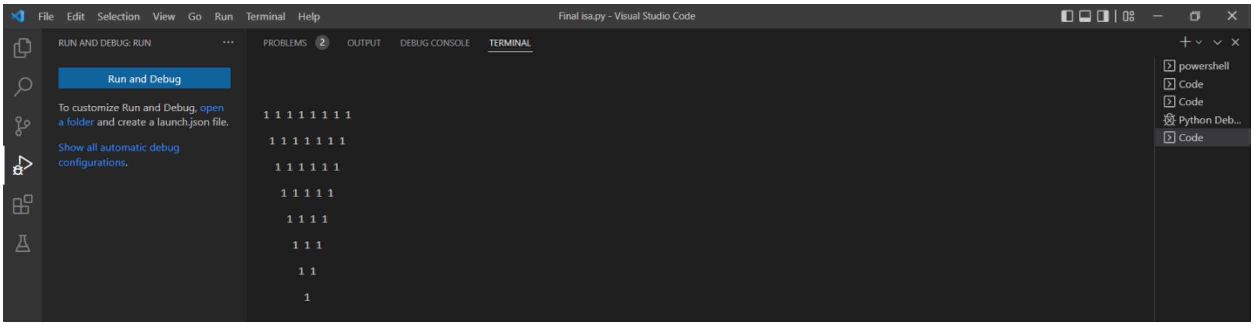
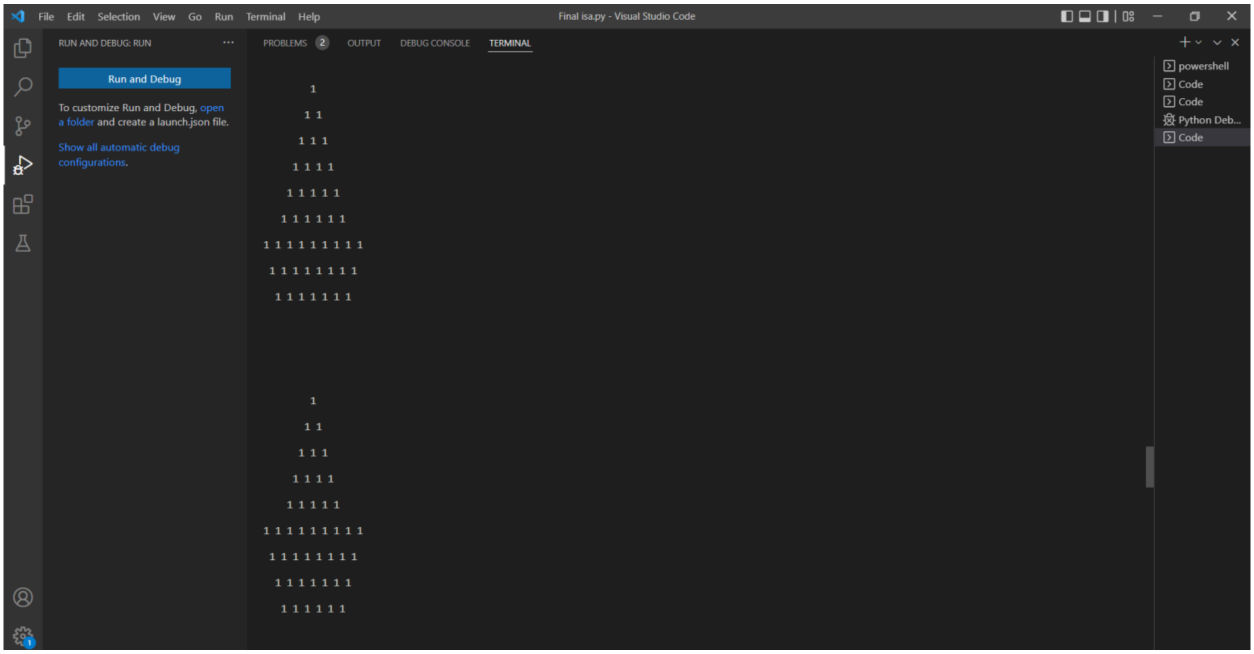


Figure 5: Output for number of rows = 8


File Edit Selection View Go Run Terminal Help
Final isa.py - Visual Studio Code
RUN AND DEBUG: RUN
Run and Debug
To customize Run and Debug, open a folder and create a launch.json file.
Show all automatic debug configurations.
PROBLEMS 2 OUTPUT DEBUG CONSOLE TERMINAL
powershell
Code
Code
Python Deb...
Code
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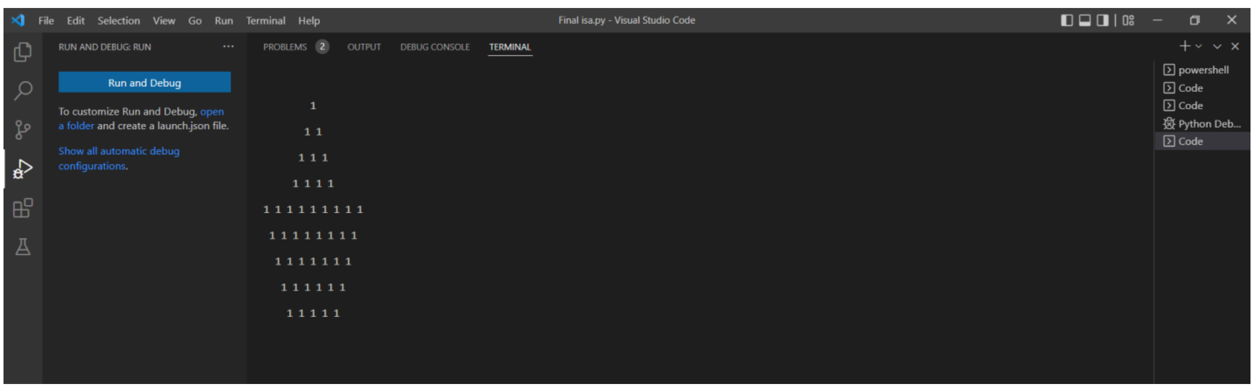
File Edit Selection View Go Run Terminal Help
Final isa.py - Visual Studio Code
RUN AND DEBUG: RUN
Run and Debug
To customize Run and Debug, open a folder and create a launch.json file.
Show all automatic debug configurations.
PROBLEMS 2 OUTPUT DEBUG CONSOLE TERMINAL
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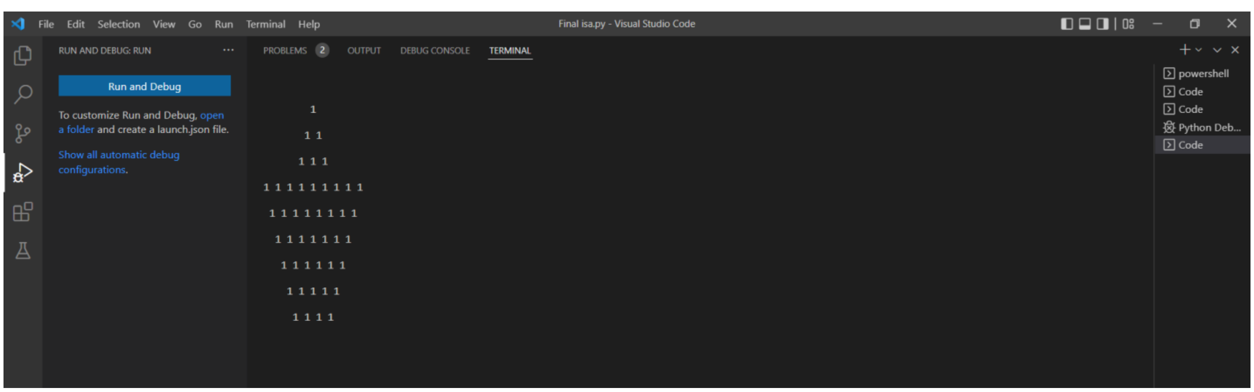
File Edit Selection View Go Run Terminal Help
Final isa.py - Visual Studio Code
RUN AND DEBUG: RUN
Run and Debug
To customize Run and Debug, open a folder and create a launch.json file.
Show all automatic debug configurations.
PROBLEMS 2 OUTPUT DEBUG CONSOLE TERMINAL
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1 1 1 1 1 1 1
1 1 1 1 1 1
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1 1 1 1

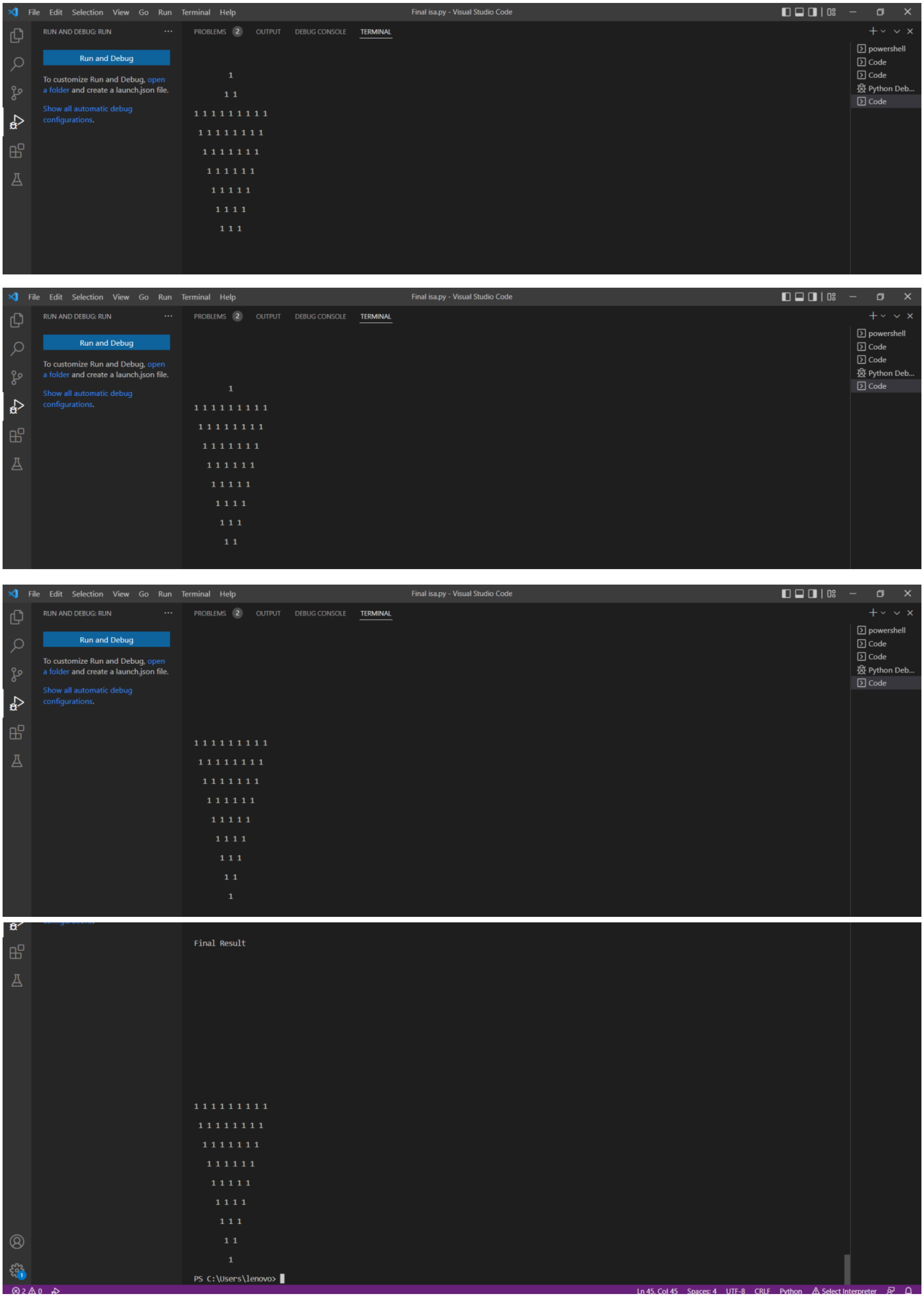


Figure 6: Output for number of rows = 9

1.2 Task 2

1.2.1 Problem description

Task 2

Consider the one-dimensional version of peg solitaire played on an array of n cells, where n is even and greater than 2. Initially, all but one cell are occupied by some counters (pegs), one peg per cell. On each move, a peg jumps over its immediate neighbor to the left or to the right to land on an empty cell; after the jump, the jumped-over neighbor is removed from the board.

Using dynamic programming methodology to

a) write an algorithm that remove all but one peg by a sequence of such moves.
b) Find all the locations of the empty cell in the initial setup for which the puzzle can be solved and the corresponding locations of the single remaining peg.

1.2.2 Solution

1.2.2.1 Pseudocode

```
action when left has 2 1's and right has even number of 1's which is
greater than 2
def l2rlargerequal2even(board , firstzeroindex,start):
    board[start]
    board[start+2] ←board[start+2]
    board[start]
    board[start+1] ←0
    print(board)
    board[firstzeroindex+1]
    board[firstzeroindex+3] ←board[firstzeroindex+3]
    board[firstzeroindex+1]
    board[firstzeroindex+2] ←0
    print(board)

#action when left has no 1's and right has 2 1's
def l0r2(board , start , end):
    board[end]
    board[start+1] ←board[start+1]
    board[end]
    board[start+2] ←0
    print(board)

#action when right has no 1's and left has 2 1's
def r0l2(board , start ):
    board[start]
    board[start+2] ←board[start+2]
    board[start]
    board[start+1] ←0
```

```
    print(board)




# actions to do when empty cell position wont lead to a board with only
one peg
def restofstates():
    print("The choice of empty cell doesn't allow board to be reduced to
only 1 peg","\n", "For a board to be reduced choices of empty cell must be
2 , 5 , n-1 and n-4 only" )

# action to do when there is 2 1's on the right and left side of 2 zeros
has even number of 1's and greater than 2
def r2llargerequal2even(board , firstzeroindex,end):
    board[end]
    board[end-2]=board[end-2]
    board[end]
    board[end-1]=0
    print(board)
    board[firstzeroindex]
    board[firstzeroindex-2] ←board[firstzeroindex-2]
    board[firstzeroindex]
    board[firstzeroindex-1]=0
    print(board)


# dictionary that stores the states the peg board will be in and also
stores
# the appropriate action for each state
# l = left , r = right , and number donates how many 1's
solDict={
    'l0r2': l0r2,
    'l2r>=2even':l2rlargerequal2even ,
    'r0l2': r0l2,
    'r2l>=2even': r2llargerequal2even,
    'rest':restofstates
    # 'l0r>2': ,
    # 'l1r>=1': ,
    # 'l>2r>=1': ,
    # 'r0l>2': ,
    # 'r1l>=1': ,
    # 'r>2l>=1':
}


# recursive function that solves according to the above constraints
def solve(board , start , end):
    left = 0
    right = 0
    index = start
    zeroindex = 0


    #stop when reached the end of the board
    if (start >= end ):
```

```
        return

    # count how many 1's on the left of the 2 zeros

    for i to range(start,end+1) do
        if board[i] == 1:
            left ← left + 1
        elif board[i] == 0:
            zeroindex = index
            break
        index ← index + 1

    #  count how many 1's on the right of the 2 zeros

    for i to range(zeroindex, end+1) do
        if board[i] == 1:
            right ← right + 1


    # choose from Dictionary the appropriate action to make

    if(left==0 and right==2):
        findInDict = 'l0r2'
        solDict[findInDict](board,start,end)
    else (left==2 and right >= 2 and right % 2 == 0):
        findInDict = 'l2r>=2even'
        solDict[findInDict](board,zeroindex,start)
        # recursive call to the solve function sending it a subproblem
        solve(board,start+2,end)
    else (right == 0 and left == 2):
        findInDict = 'r0l2'
        solDict[findInDict](board, start)
    else (right==2 and left >= 2 and left % 2 == 0):
        findInDict = 'r2l>=2even'
        solDict[findInDict](board, zeroindex, end)
        # recursive call to the solve function sending it a subproblem
        solve(board, start, end-2)
    else:
        findInDict = 'rest'
        solDict[findInDict]()


# intial move to place 2 zeros next to eachother to begin solution then
call solve function
def initialmove(board , empty , n):

    if (empty == n-4):
        board[empty + 2]
       board[empty] ← board[empty]
       board[empty + 2]
        board[empty + 1] = 0

    else empty == 1 or empty == 0 :
        board[empty + 2]
        board[empty] ← board[empty]
```

```
        board[empty + 2]
        board[empty + 1] = 0
    else:
        board[empty-2],board[empty]=board[empty],board[empty-2]
        board[empty-1]=0

    print(board)
    solve(board, 0, len(board) - 1)



# check if user entered odd number or 2 or 0
def checknumberofcells(n):
    if n % 2 != 0 or n == 0 or n == 2 :
        return True
```

1.2.2.2 Code

```
2. from time import perf_counter
   # action when left has 2 1's and right has even number of 1's which is
   greater than 2
   def l2rlargerequal2even(board , firstzeroindex,start):
       board[start],board[start+2]=board[start+2],board[start]
       board[start+1]=0
       print(board)

   board[firstzeroindex+1],board[firstzeroindex+3]=board[firstzeroindex+3],bo
   ard[firstzeroindex+1]
       board[firstzeroindex+2]=0
       print(board)

   #action when left has no 1's and right has 2 1's
   def l0r2(board , start , end):
       board[end],board[start+1]=board[start+1],board[end]
       board[start+2]=0
       print(board)

   #action when right has no 1's and left has 2 1's
   def r0l2(board , start ):
       board[start],board[start+2]=board[start+2],board[start]
       board[start+1]=0
       print(board)

   # actions to do when empty cell position wont lead to a board with only
   one peg
   def restofstates():
       print("The choice of empty cell doesn't allow board to be reduced to
   only 1 peg","\n", "For a board to be reduced choices of empty cell must be
   2 , 5 , n-1 and n-4 only" )
```

```
# action to do when there is 2 1's on the right and left side of 2 zeros
has even number of 1's and greater than 2
def r2llargerequal2even(board , firstzeroindex,end):
    board[end],board[end-2]=board[end-2],board[end]
    board[end-1]=0
    print(board)
    board[firstzeroindex],board[firstzeroindex-2]=board[firstzeroindex-
2],board[firstzeroindex]
    board[firstzeroindex-1]=0
    print(board)


# dictionary that stores the states the peg board will be in and also
stores
# the appropriate action for each state
# l = left , r = right , and number donates how many 1's
solDict={
    'l0r2': l0r2,
    'l2r>=2even':l2rlargerequal2even ,
    'r0l2': r0l2,
    'r2l>=2even': r2llargerequal2even,
    'rest':restofstates
    # 'l0r>2': ,
    # 'l1r>=1': ,
    # 'l>2r>=1': ,
    # 'r0l>2': ,
    # 'r1l>=1': ,
    # 'r>2l>=1':
}


# recursive function that solves according to the above constraints
def solve(board , start , end):
    left = 0
    right = 0
    index = start
    zeroindex = 0


    #stop when reached the end of the board
    if (start >= end ):
        return

    # count how many 1's on the left of the 2 zeros

    for i in range(start,end+1):
        if board[i] == 1:
            left = left + 1
        elif board[i] == 0:
            zeroindex = index
            break
        index = index + 1

    #  count how many 1's on the right of the 2 zeros

    for i in range(zeroindex, end+1):
        if board[i] == 1:
```

```
            right = right + 1

        # choose from Dictionary the appropriate action to make

        if(left==0 and right==2):
            findInDict = 'l0r2'
            solDict[findInDict](board,start,end)
        else (left==2 and right >= 2 and right % 2 == 0):
            findInDict = 'l2r>=2even'
            solDict[findInDict](board,zeroindex,start)
            # recursive call to the solve function sending it a subproblem
            solve(board,start+2,end)
        else (right == 0 and left == 2):
            findInDict = 'r0l2'
            solDict[findInDict](board, start)
        else (right==2 and left >= 2 and left % 2 == 0):
            findInDict = 'r2l>=2even'
            solDict[findInDict](board, zeroindex, end)
            # recursive call to the solve function sending it a subproblem
            solve(board, start, end-2)
        else:
            findInDict = 'rest'
            solDict[findInDict]()


    # intial move to place 2 zeros next to eachother to begin solution then
    call solve function
    def initialmove(board , empty , n):

        if (empty == n-4):
            board[empty + 2], board[empty] = board[empty], board[empty + 2]
            board[empty + 1] = 0

        else empty == 1 or empty == 0 :
         board[empty + 2]
         board[empty] ← board[empty]
         board[empty + 2]
         board[empty + 1] = 0
     else:
         board[empty-2],board[empty]=board[empty],board[empty-2]
         board[empty-1]=0

     print(board)
     solve(board, 0, len(board) - 1)



# check if user entered odd number or 2 or 0
def checknumberofcells(n):
    if n % 2 != 0 or n == 0 or n == 2 :
        return True
```

```
def main():
    n = int(input("Enter Number Of Cells : "))
    if checknumberofcells(n):
        print("N should be Even and Greater than 2")
    else:
        # initialize board and put empty cell in its place
        board = [1]*n
        print(board)
        empty = int(input("Choose Position Of Empty Cell : "))
        board[empty-1]=0
        print(board)
        initialmove(board,empty-1 ,n-1)
```

1.2.2.3 Solution description

The board contains even number of cells from 1 to n where the empty cell is located between 2 and 5 where its symmetrically as it could be n-1 or n-4. The solution of the problem occurs in 4 cases to apply dynamic programming and as the array consist of 0 and 1.

First case: if the left side is 0 and the right is 2 as [0011] or vise versa and will only one single peg will be on the board so it's a success case

Second case: if l =1 and r ≥ 1 as [101---1] or vise versa as r=1 and l ≥ 1 by solving the puzzle the remaining will be more than one peg so it will be a dead-end case

Third case: if l = 2 and r ≥ 2 and r is an even number or vise versa as [11001-1] by solving the puzzle one peg will remain so it's another success case

Fourth case: if l > 2 and r ≥ 1 as [1----110011-1] where the array is divided into two arrays

A: [1-----110] and B: [011-----1] and with each implementation reduce the two 0's in the furthermost in the arrays or vise versa but this will occur when both A and B is joined it will contain two ones so it's a dead-end case.

1.2.2.4 Complexity analysis

1.2.2.5 Comparison between another algorithm

```
def generateBoard(n):
    return [1]*n


def solve(board):
    if checkBoard(board):
        return True
    elif checkUnsolvable(board):
        return False


    moves = []
    for i in range(len(board)):
        if i < len(board)-2:
            if board[i] and board[i+1] and not board[i+2]:
                moves.append((i, 'right'))
        if i > 1:
            if board[i] and board[i-1] and not board[i-2]:
                moves.append((i, 'left'))


    for move in moves:
        newBoard = makeMove(board, move)
        if solve(newBoard):
```

```
            return True
        continue

    return False


def makeMove(board, move):
    index, direction = move
    b = [element for element in board]
    if direction == 'right':
        b[index] = 0
        b[index+1] = 0
        b[index+2] = 1
    elif direction == 'left':
        b[index] = 0
        b[index-1] = 0
        b[index-2] = 1
    return b


def checkBoard(board):
    if sum(board) == 1:
        return True
    return False
```

```
def checkUnsolvable(board):
    expression1 = '1000+1' #RE for a proven to be unsolvable board
    expression2 = '00100'  #RE for a proven to be unsolvable board
    string = ''.join([str(element) for element in board])
    if re.search(expression1, string) or re.search(expression2, string):
        return True
    return False


def countSolutions(board):
    indices = []
    for i in range(len(board)):
        b = [element for element in board]
        b[i] = 0
        if solve(b):
            indices.append(i+1)
    return indices



n = int(input())
print(countSolutions(generateBoard(n)))
```

1.2.2.6 Sample of the output

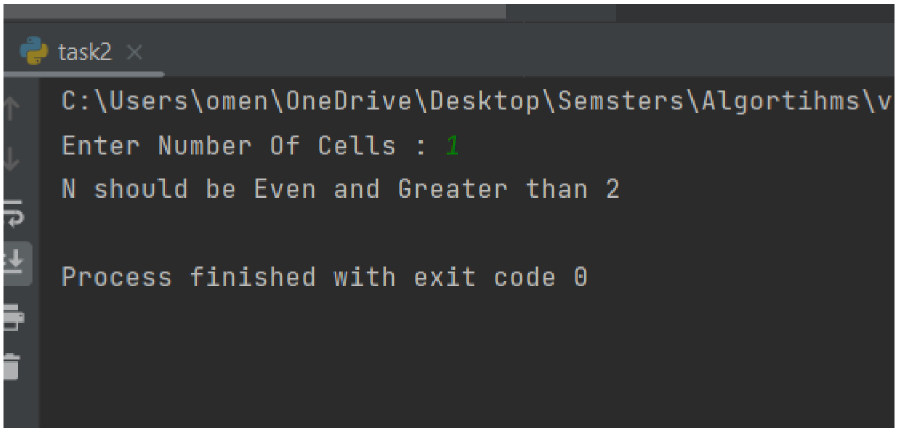


Figure 7: Output for number of cells = 1

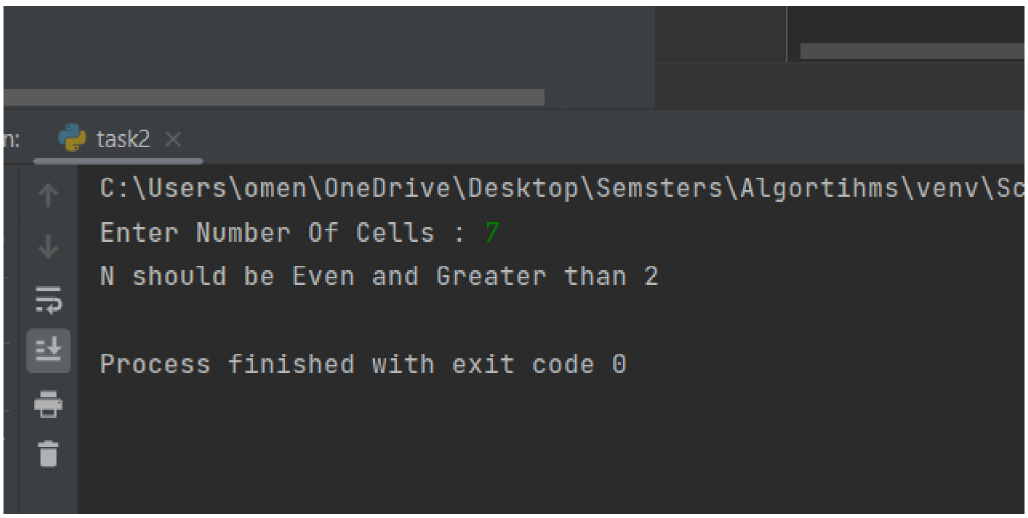
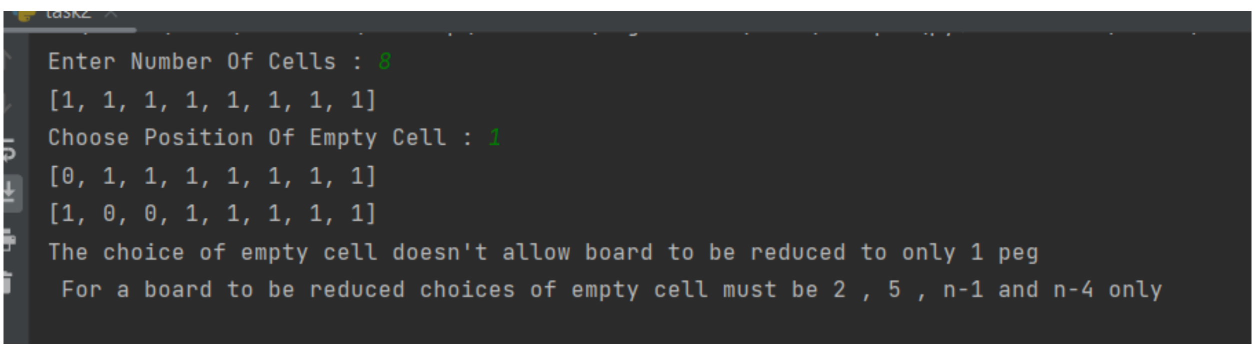


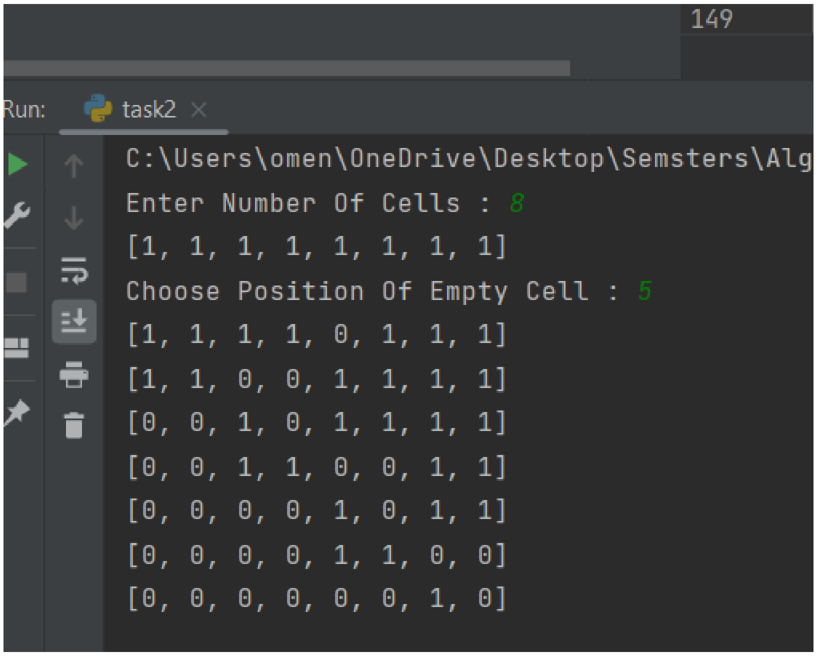
Figure 8: Output for number of cells = 7



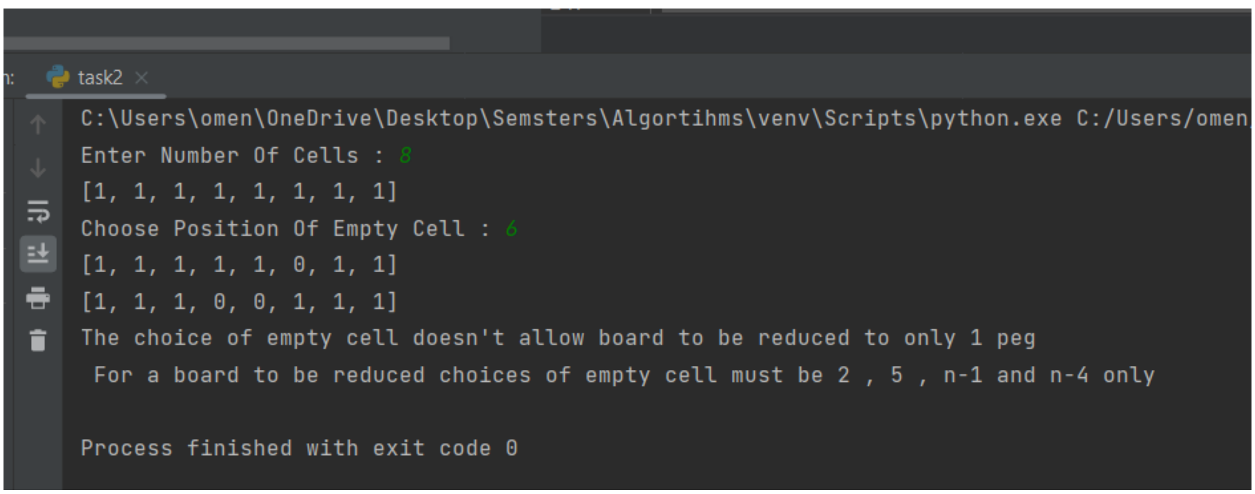
```
task2
Enter Number Of Cells : 8
[1, 1, 1, 1, 1, 1, 1, 1]
Choose Position Of Empty Cell : 2
[1, 0, 1, 1, 1, 1, 1, 1]
[1, 1, 0, 0, 1, 1, 1, 1]
[0, 0, 1, 0, 1, 1, 1, 1]
[0, 0, 1, 1, 0, 0, 1, 1]
[0, 0, 0, 0, 1, 0, 1, 1]
[0, 0, 0, 0, 1, 1, 0, 0]
[0, 0, 0, 0, 0, 0, 1, 0]
```

```
Enter Number Of Cells : 8
[1, 1, 1, 1, 1, 1, 1, 1]
Choose Position Of Empty Cell : 3
[1, 1, 0, 1, 1, 1, 1, 1]
[0, 0, 1, 1, 1, 1, 1, 1]
The choice of empty cell doesn't allow board to be reduced to only 1 peg
 For a board to be reduced choices of empty cell must be 2 , 5 , n-1 and n-4 only
```

```
task2
Enter Number Of Cells : 8
[1, 1, 1, 1, 1, 1, 1, 1]
Choose Position Of Empty Cell : 4
[1, 1, 1, 0, 1, 1, 1, 1]
[1, 1, 1, 1, 0, 0, 1, 1]
[1, 1, 1, 1, 0, 1, 0, 0]
[1, 1, 0, 0, 1, 1, 0, 0]
[0, 0, 1, 0, 1, 1, 0, 0]
[0, 0, 1, 1, 0, 0, 0, 0]
[0, 0, 0, 0, 1, 0, 0, 0]
```



```
149
Run:  task2
C:\Users\omen\OneDrive\Desktop\Semsters\Alg
Enter Number Of Cells : 8
[1, 1, 1, 1, 1, 1, 1, 1]
Choose Position Of Empty Cell : 5
[1, 1, 1, 1, 0, 1, 1, 1]
[1, 1, 0, 0, 1, 1, 1, 1]
[0, 0, 1, 0, 1, 1, 1, 1]
[0, 0, 1, 1, 0, 0, 1, 1]
[0, 0, 0, 0, 1, 0, 1, 1]
[0, 0, 0, 0, 1, 1, 0, 0]
[0, 0, 0, 0, 0, 0, 1, 0]
```



```
n:  task2
C:\Users\omen\OneDrive\Desktop\Semsters\Algortihms\venv\Scripts\python.exe C:/Users/omen
Enter Number Of Cells : 8
[1, 1, 1, 1, 1, 1, 1, 1]
Choose Position Of Empty Cell : 6
[1, 1, 1, 1, 1, 0, 1, 1]
[1, 1, 1, 0, 0, 1, 1, 1]
The choice of empty cell doesn't allow board to be reduced to only 1 peg
 For a board to be reduced choices of empty cell must be 2 , 5 , n-1 and n-4 only

Process finished with exit code 0
```

```
task2
C:\Users\omen\OneDrive\Desktop\Semsters
Enter Number Of Cells : 8
[1, 1, 1, 1, 1, 1, 1, 1]
Choose Position Of Empty Cell : 7
[1, 1, 1, 1, 1, 1, 0, 1]
[1, 1, 1, 1, 0, 0, 1, 1]
[1, 1, 1, 1, 0, 1, 0, 0]
[1, 1, 0, 0, 1, 1, 0, 0]
[0, 0, 1, 0, 1, 1, 0, 0]
[0, 0, 1, 1, 0, 0, 0, 0]
[0, 0, 0, 0, 1, 0, 0, 0]
```

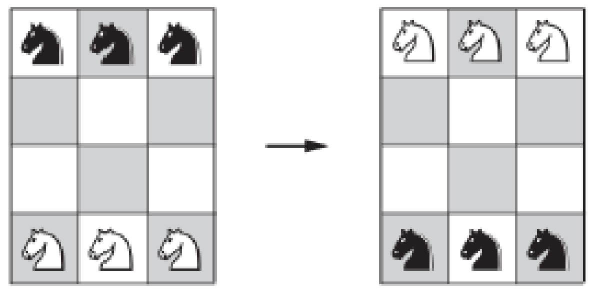
```
149
task2
C:\Users\omen\OneDrive\Desktop\Semsters\Algorti
Enter Number Of Cells : 8
[1, 1, 1, 1, 1, 1, 1, 1]
Choose Position Of Empty Cell : 8
[1, 1, 1, 1, 1, 1, 1, 0]
[1, 1, 1, 1, 1, 0, 0, 1]
The choice of empty cell doesn't allow board to
 For a board to be reduced choices of empty cel

Process finished with exit code 0
```

Figure 9: Output for number of cells = 8

1.3 Task 3

1.3.1 Problem description

Task 3
There are six knights on a 3 × 4 chessboard: the three white knights are at the bottom row, and the three black knights are at the top row.
Design a divide and conquer algorithm to exchange the knights to get the position shown on the right of the figure in the minimum number of knight moves, not allowing more than one knight on a square at any time.



1.3.2 Solution

1.3.2.1 Pseudocode

//initialize the board with knights in their places

board <-- [(1, 'b1'), (2, 'b2'), (3, 'b3'), (4, 'null'), (5, 'null'), (6, 'null'), (7, 'null'), (8, 'null'),

(9, 'null'), (10, 'w1'), (11, 'w2'), (12, 'w3')

create an object of class task3 with its board = board

object.divide1()

object.divide2()

class task 3:

task3(board){

this.board=board

}//initialization constructor

```
//this function is responsible to move the knights correctly according to the adjacency grapgh
//of the knights' movements and ensures that no knights overlap, returns true if move was
//sucessful, false otherwise

function move(knight,destination){

pos = self.position(knight) // a function that returns the position of a given knight

// 12 if conditions to ensure that the movements are according to the adjacency grapgh

 if pos == 1

           if destination not in (6, 8)

              return False

        if pos == 2

           if destination not in (9, 7)

              return False

    .

         .

         .

        if pos == 12

           if destination not in (5, 7)

              return False


 if board[destination][2] == 'null'{ //if destination is empty

           board[pos][2] ='null' // set current position to be empty

           board[destination][2] =  k // move knight to destination

           return True

}
```

```
        else
            return False
}

// function to return a knight's position
position(k){
        for i in board:
            if i[1] == k
                return i[0]
        return 0
}



//first subproblem, 2knights and 6positions
     divide1(){
        anticlk = [9, 4, 11, 6, 7, 2] // the 6 positions which the function is going to maneuver
        move('w2', 6) // initial move to easilymove the rest
        for i in anticlk: // move the black knight until it is in the right position
            if position('b2') == 11
                break
            move('b2', i)
        for i in anticlk: // move the white knight until it is in the right position
            if position('w2') == 2
                break
```

```
            else:
                if move('w2', i)
                    continue
    }


// second sub problem, 4knights and 10 positions
divide2(){
        anticlk = [1, 6, 7, 12, 5, 10, 9, 4, 3, 8, 1, 6] // the 10 positions with the first 2 repeated to avoid double loops
        for i in anticlk: // move the first black knight to allow the wight knights to take its place
            if position('b1') == 7
                break
            move('b1', i)
        for i in anticlk: // move the second black knight to allow the wight knights to take its place
            if position('b3') == 6
                break
            move('b3', i)
        for i in anticlk: // move the white knights into place
            if position('w1') == 1
                break
            move('w1', i)
        for i in anticlk: // move the white knights into place
            if position('w3') == 3
                break
```

move('w3', i)

for i in anticlk: // move the black knights into place

if position('b1') == 10

break

move('b1', i)

for i in anticlk: // move the black knights into place

if position('b3') == 12

break

move('b3', i)

}

1.3.2.2 Code

```
class task3:

    def __init__(self, board):
        self.board = board
        self.numberofsteps = 0

    def position(self, k):
        board = self.board
        for i in board:
            if i[1] == k:
                return i[0]

        return 0

    def move(self, k, destination):
        pos = self.position(k)
        if pos == 1:
            if destination not in (6, 8):
                print('move failed : ' + k + ' to ' + str(destination))
                return False
        if pos == 2:
            if destination not in (9, 7):
                print('move failed : ' + k + ' to ' + str(destination))
                return False
        if pos == 3:
            if destination not in (8, 4):
                print('move failed : ' + k + ' to ' + str(destination))
                return False
```

```
        if pos == 4:
            if destination not in (3, 9, 11):
                print('move failed : ' + k + ' to ' + str(destination))
                return False
        if pos == 5:
            if destination not in (10, 12):
                print('move failed : ' + k + ' to ' + str(destination))
                return False
        if pos == 6:
            if destination not in (1, 7, 11):
                print('move failed : ' + k + ' to ' + str(destination))
                return False
        if pos == 7:
            if destination not in (2, 6, 12):
                print('move failed : ' + k + ' to ' + str(destination))
                return False
        if pos == 8:
            if destination not in (1, 3):
                print('move failed : ' + k + ' to ' + str(destination))
                return False
        if pos == 9:
            if destination not in (2, 4, 10):
                print('move failed : ' + k + ' to ' + str(destination))
                return False
        if pos == 10:
            if destination not in (5, 9):
                print('move failed : ' + k + ' to ' + str(destination))
                return False
        if pos == 11:
            if destination not in (4, 6):
                print('move failed : ' + k + ' to ' + str(destination))
                return False
        if pos == 12:
            if destination not in (5, 7):
                print('move failed : ' + k + ' to ' + str(destination))
                return False

        if self.board[destination][1] == 'null':
            self.board[pos] = (pos, 'null')
            self.board[destination] = (destination, k)
            print('knight ' + k + ' was moved to ' + str(destination))
            self.numberofsteps += 1
            return True
        else:
            print('move failed (not empty) : ' + k + ' to ' +
str(destination))
            return False

    def divide1(self):
        self.move('w2', 6)
        anticlk = [9, 4, 11, 6, 7, 2]
        for i in anticlk:
            if self.position('b2') == 11:
                break
            self.move('b2', i)
        for i in anticlk:
            if self.position('w2') == 2:
```

```
                    break
                else:
                    self.move('w2', i)

        print(self.board)

    def divide2(self):
        anticlk = [1, 6, 7, 12, 5, 10, 9, 4, 3, 8, 1, 6]
        for i in anticlk:
            if self.position('b1') == 7:
                break
            self.move('b1', i)
        for i in anticlk:
            if self.position('b3') == 6:
                break
            self.move('b3', i)
        for i in anticlk:
            if self.position('w1') == 1:
                break
            self.move('w1', i)
        for i in anticlk:
            if self.position('w3') == 3:
                break
            self.move('w3', i)
        for i in anticlk:
            if self.position('b1') == 10:
                break
            self.move('b1', i)
        for i in anticlk:
            if self.position('b3') == 12:
                break
            self.move('b3', i)
        print(self.board)



brd = [(0, 'null'), (1, 'b1'), (2, 'b2'), (3, 'b3'), (4, 'null'), (5,
'null'), (6, 'null'), (7, 'null'), (8, 'null'),
       (9, 'null'), (10, 'w1'), (11, 'w2'), (12, 'w3')]
chess = task3(brd)
print(chess.board)
chess.divide1()
chess.divide2()
print(chess.numberofsteps)
```

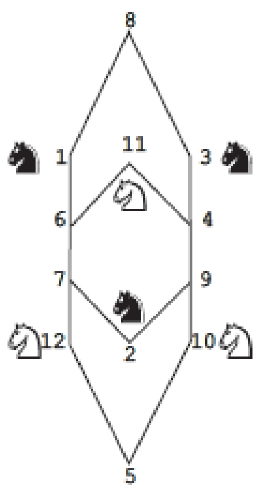
1.3.2.3 Solution description

Firstly, we index our 4x3 chess board numerically for better representation :

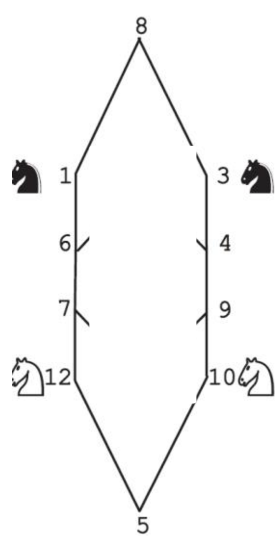
1	2	3
4	5	6
7	8	9
10	11	12

with 3 black knights at postitons 1,2,3 and 3 white knights at positions 10,11,12

Then we draw an adjacency graph, which is the key for the solution, this undirected graph represents all the possible movements from each positions according to the correct movement of a knight in chess (ie. 2 squares in 1 directions,1 square in another , or L-shaped movements)



this leaves us with the following adjacency graph , which concludes that no there is only one way to divide our problem into subproblems, by taking the 2 middle knights in the inner graph loop and swap them , then do the same with the outer graph loop and the 4 corner knights



1.3.2.4 Complexity analysis

-The time complexity of the position function is

P(n) = n while n is the number of squares in our board (12)

-The time complexity of the move function is

N the time complexity of the position function

1*11 +2 the time complexity of the 12 of the guard conditions that ensure movement follows adjacency graph

5 the time complexity for successfully moving the knight

M(n) = n+ 18

-The time complexity of the divide1 function is:

The first call of the function move: n+18

k Is the number of elements in the anticlk list (6)

$$\sum_{i=1}^{k} 1 + n + 18 = 19 + n + 6 = 25 + n$$

This has total complexity of 2n+43

-The time complexity of the divide2 function is:

The loops have time complexity:

k Is the number of elements in the anticlk list (12)

$$\sum_{i=1}^{k} 1 + n + 18 = 19 + n + 12 = 31 + n$$

Total time complexity is: 6*(31+n) = 6n + 186

The total time complexity of the algorithm is 6n + 186 + 2n+43 = 8n + 229

1.3.2.5 Comparison between another algorithm

Another algorithm is the backtracking algorithm which searches for the optimal solution (16 steps) which is 14 steps faster than the divide and conquer code, it pseudo code:

Initialize board = { (1='B1'), (2= 'B2'), (3= 'B3'), (4 = ' '), (5 = ' '), (6 = ' '), (7 = ' '), (8 = ' '), (9 = ' '), (10='W1'), (11='W2'), (12='W3')}

Initialize solution board = { (1='W1'), (2= 'W2'), (3= 'W3'), (4 = ' '), (5 = ' '), (6 = ' '), (7 = ' '), (8 = ' '), (9 = ' '), (10='B1'), (11='B2'), (12='B3')}

initialize solution vector

If solution positions are reached print the chessboard Else

check if the next movement is valid (according to adjacency graph and destination is empty)

Add the next move to the solution vector and recursively check if this move would lead to a proper correct solution.

if the move chosen doesn't lead to a solution, then remove this move from the solution vector and try one of the other connected nodes,

If the alternative moves don't work, return false. This will remove the previously added node in recursion. If false is returned by the very first call of recursion,

then "no solution exists".

The python code of backtracking:

(note that the code found for backtracking solves for n*n chessboard meaning only 3*3 or 4*4)

```
# Python3 program to solve Knight Tour problem using Backtracking


# Chessboard Size

n = 3



def isSafe(x, y, board):
```

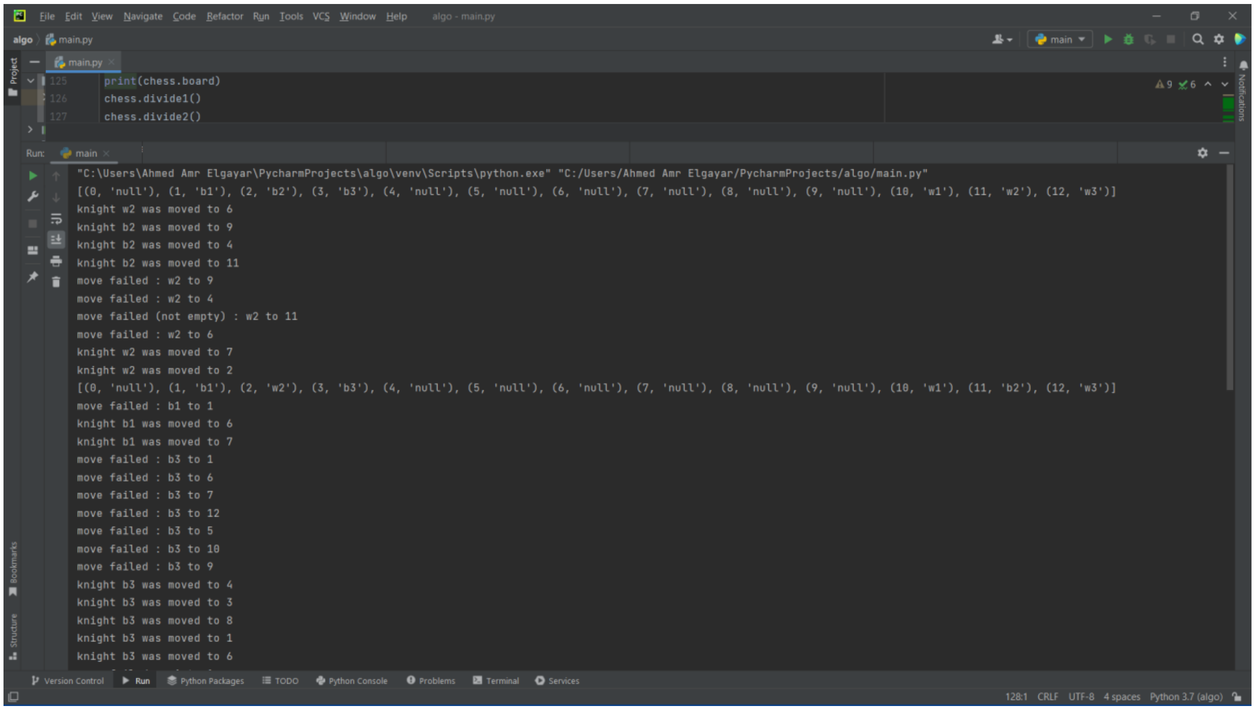
```
    '''
        A utility function to check if i,j are valid indexes
        for N*N chessboard
    '''
    if(x >= 0 and y >= 0 and x < n and y < n and board[x][y] == -1):
        return True
    return False


def printSolution(n, board):
    '''
        A utility function to print Chessboard matrix
    '''
    for i in range(n):
        for j in range(n):
            print(board[i][j], end=' ')
        print()


def solveKT(n):
    '''
        This function solves the Knight Tour problem using
        Backtracking. This function mainly uses solveKTUtil()
        to solve the problem. It returns false if no complete
```

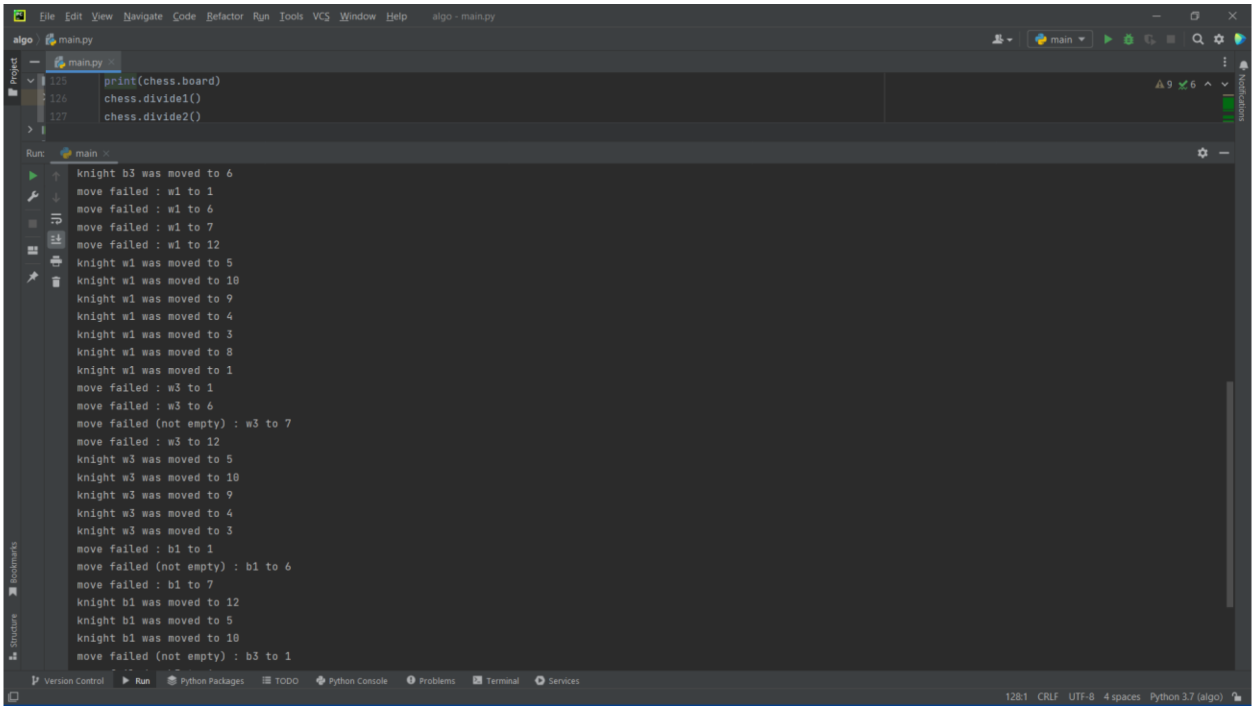
```
	tour is possible, otherwise return true and prints the
	tour.
	Please note that there may be more than one solutions,
	this function prints one of the feasible solutions.
'''

# Initialization of Board matrix
board = [[-1 for i in range(n)]for i in range(n)]

# move_x and move_y define next move of Knight.
# move_x is for next value of x coordinate
# move_y is for next value of y coordinate
move_x = [2, 1, -1, -2, -2, -1, 1, 2]
move_y = [1, 2, 2, 1, -1, -2, -2, -1]

# Since the Knight is initially at the first block
board[0][0] = 0

# Step counter for knight's position
pos = 1

# Checking if solution exists or not
if(not solveKTUtil(n, board, 0, 0, move_x, move_y, pos)):
	print("Solution does not exist")
```

```
	else:
		printSolution(n, board)


def solveKTUtil(n, board, curr_x, curr_y, move_x, move_y, pos):
	'''
		A recursive utility function to solve Knight Tour
		problem
	'''

	if(pos == n**2):
		return True

	# Try all next moves from the current coordinate x, y
	for i in range(3):
		new_x = curr_x + move_x[i]
		new_y = curr_y + move_y[i]
		if(isSafe(new_x, new_y, board)):
			board[new_x][new_y] = pos
			if(solveKTUtil(n, board, new_x, new_y, move_x, move_y, pos+1)):
				return True

			# Backtracking
			board[new_x][new_y] = -1
```

return False

1.3.2.6 Sample of the output





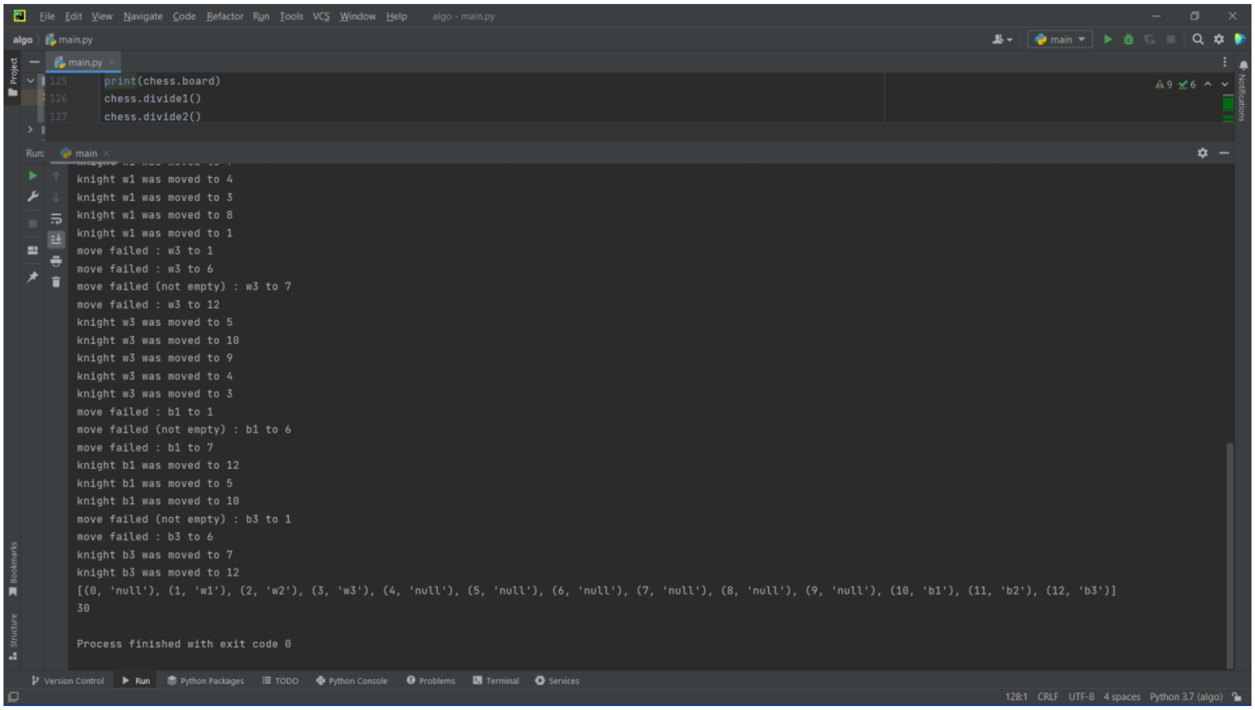


Figure 10: Output for the Code

1.4 Task 4

1.4.1 Problem description

A “machine” consists of a row of boxes. To start, one places n pennies in the leftmost box. The machine then redistributes the pennies as follows.
On each iteration, it replaces a pair of pennies in one box with a single penny in the next box to the right. The iterations stop when there is no box with more than one coin. For example, see the figure that shows the work of the machine in distributing six pennies by always selecting a pair of pennies in the leftmost box with at least two coins.
Design an algorithm using greedy method automate the machine, then answer the following questions.

(a) Does the final distribution of pennies depend on the order in which the machine processes the coin pairs?
(b) What is the minimum number of boxes needed to distribute n pennies?
(c) How many iterations does the machine make before stopping?

6				
4	1			
2	2			
0	3			
0	1	1		

1.4.2 Solution

1.4.2.1 Pseudocode

```
Donecheck ( Boxes list)

    Doneflag = false

    For i=0 to list’s size do

        If ( Boxes[i] > 1 )

            Doneflag = true

            Break

return Doneflag


Maintask(Boxes list)

Index ← 0

Loop ← true

While loop== true do

    If( Boxes[index] > 1)
```

If (Boxes[index] %2 ==0)

Increase boxes list size by 1

Boxes[index+1] = Boxes[index] / 2

Boxes[index] = 0

Increment index by 1

Loop = Donecheck(Boxes list)

Continue

If (Boxes[index] %2 != 0)

Increase boxes list size by 1

Boxes[index+1] = Boxes[index] / 2

Boxes[index] = 1

Increment index by 1

Loop= Donecheck(Boxes list)

Continue

1.4.2.2 Code

```
def donecheck(boxes):
    doneFlag= False
    for x in boxes:
        if x > 1 :
            doneFlag=True
            break
    return doneFlag


def maintask(boxes:list):
    index=0
    loop=True
    while (loop == True):
        if (boxes[index]>1):
            if (boxes[index]%2==0 ):
                boxes.append(0)
                boxes[index+1]=boxes[index]//2
                boxes[index]=0
                index=index+1
                loop = donecheck(boxes)
```

```
                print(boxes)
                continue
            elif (boxes[index]%2 != 0 ):
                boxes.append(0)
                boxes[index+1]=boxes[index]//2
                boxes[index]=1
                index=index+1
                loop = donecheck(boxes)
                print(boxes)
                continue


boxes = []
print("Enter Number of Pennies:")
n = input()
boxes.append(int(n))
print(boxes)
maintask(boxes)
```

1.4.2.3 Solution description

The problem solution is to combine 2 pennies into 1 penny and transfer it to the box on the right

1 transfer at a time, this will cost too many steps. So the greedy solution to this problem is to reduce the number of steps by doing the following steps:

1- Start with one box with all n pennies in it and the while condition is true (loop == True)

2- Check the number of pennies n in this box[i] if it even or odd

3- If even divide the number of pennies by 2 , add another box[i+1] to the right, add the n/2 pennies to the new box[i+1] and add the value 0 to that box[i] (even numbers contain n/2 couple of pennies)

4- If odd divide the number of pennies by 2 , add another box[i+1] to the right, add the n/2 pennies to the new box[i+1] and add the value 1 to that box[i] (odd numbers contain n/2 couple of pennies and 1 extra penny uncoupled)

5- Increment the indexes of boxes to start the while loop from that box (index = index+1]

6- Before starting the next iteration , check whether or not all boxes have values greater than 1 (by calling the function "donecheck(boxes)",if any of the boxes have values equal 0 or 1, doneFlag is set to be to be false making the while condition false (loop=donecheck(boxes)) to stop iterating and by this way we reached to the end of the solution

7- If values in any of the boxes have a value greater than 1 iterate again with the last recent value of index until donecheck returns false

1.4.2.4 Complexity analysis

O(log n)

1.4.2.5 Comparison between another algorithm

Optimal Algorithm 1: "output is readable from right to left"

Pseudocode:

i = 5 bits

While(i > 0)

If ((n % i) not equal 0)

Print (1)

Else

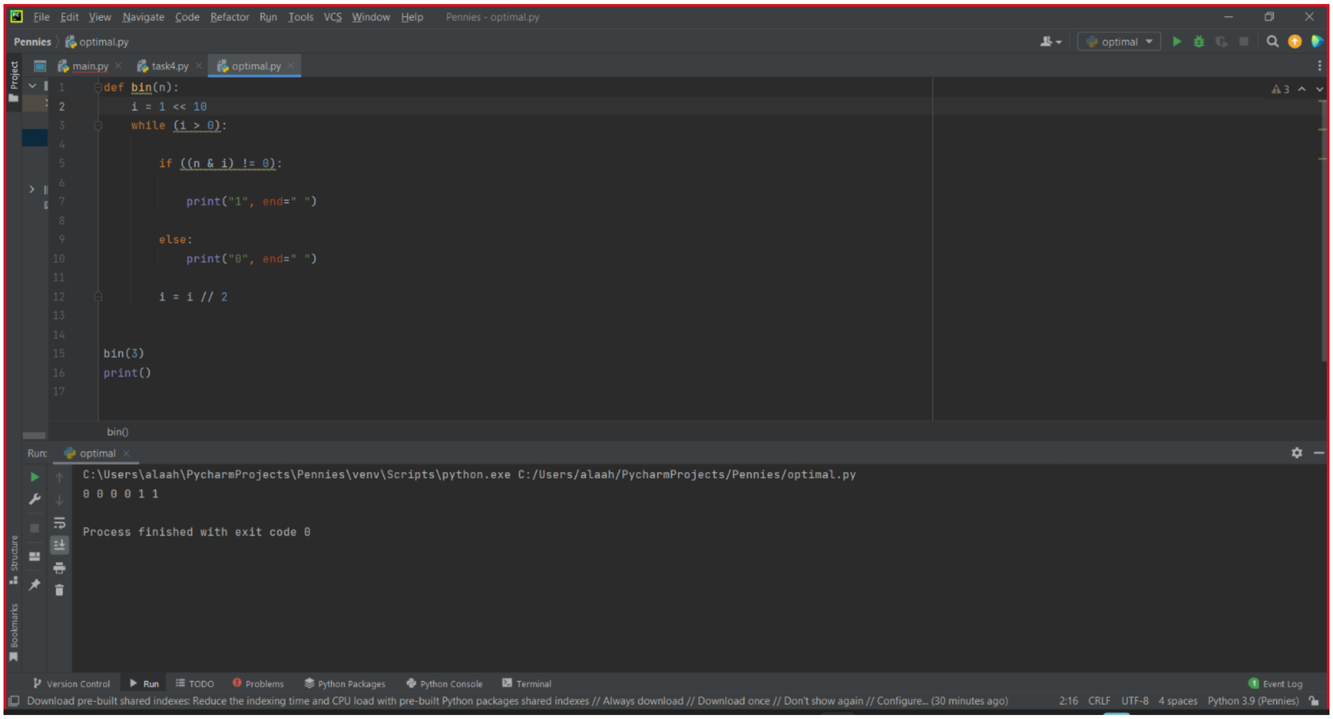
Print(0)

i = i/2

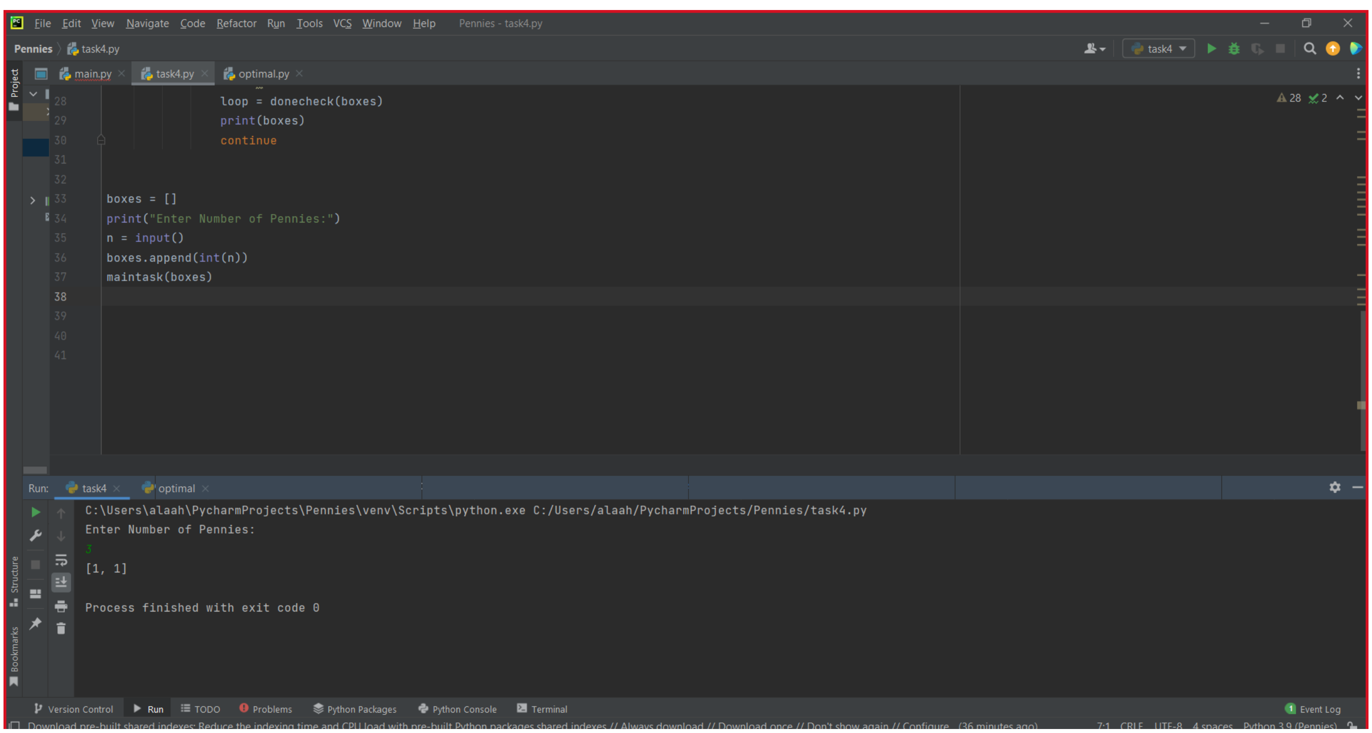
Code:

```
    i = 1 << 5
    while (i > 0):

        if ((n & i) != 0):

            print("1", end=" ")

        else:
            print("0", end=" ")

        i = i // 2


bin(3)
print()
```

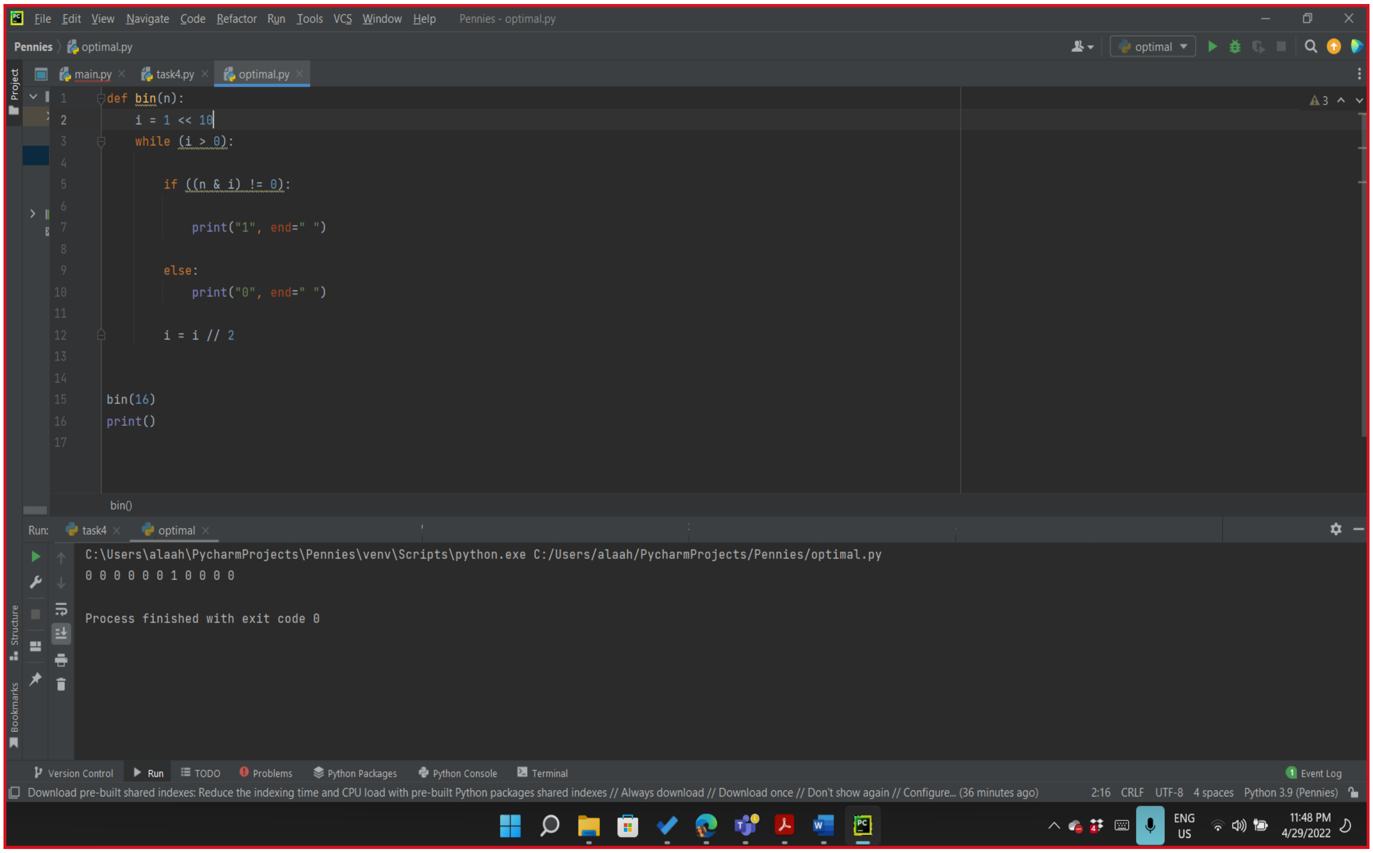
Optimal Code



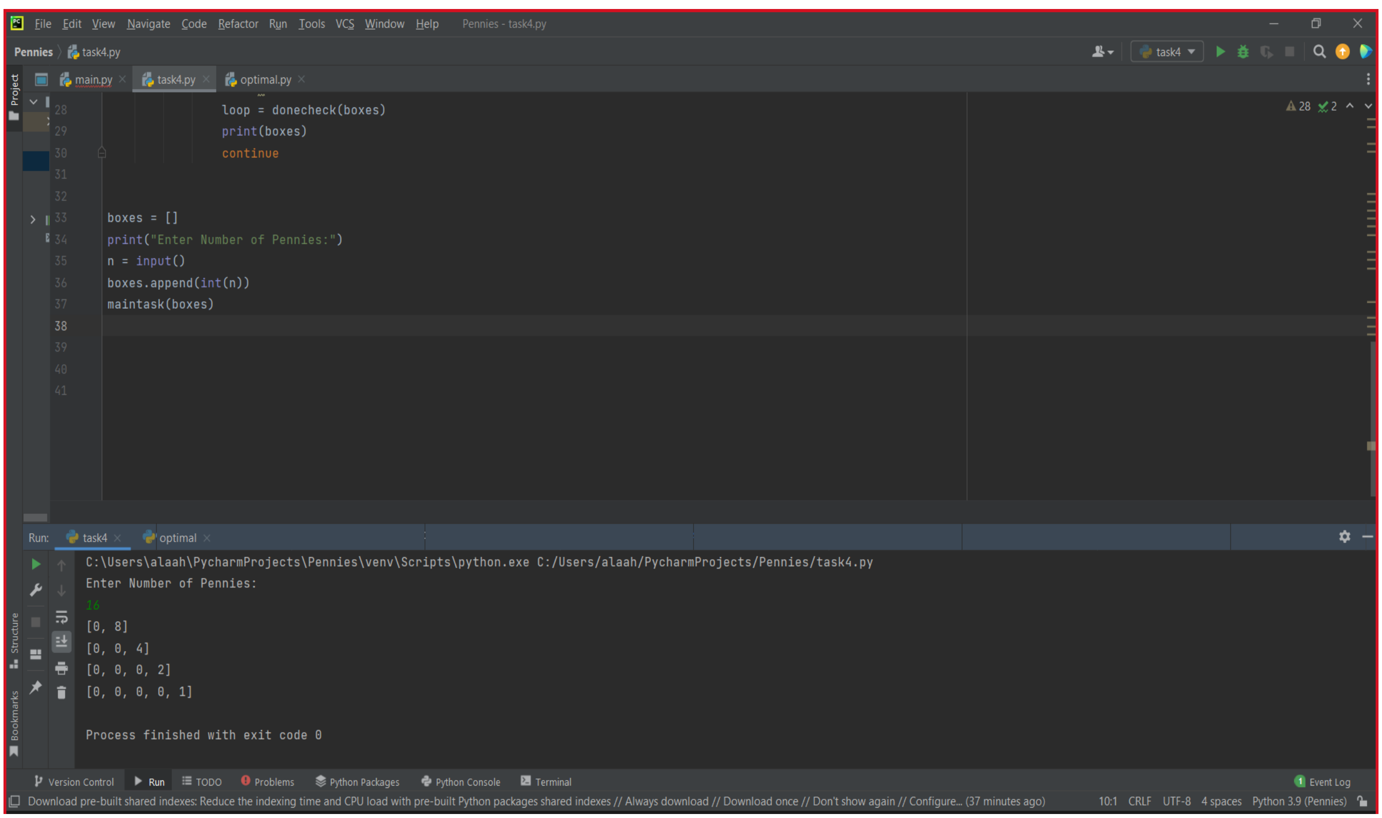
Our code:



Optimal Code:



Our Code:



Optimal Algorithm 2:

Pseudo code:

Bin(n)

If n>1

Bin(n / 2)

Print (n % 2)

Code:

```
def bin(n):
    if n > 1:
        bin(n // 2)


    print(n % 2, end=" ")


# Driver Code
if _name_ == "_main_":
    bin(8) #trying any number
```

Optimal Code"output is readable from right to left"

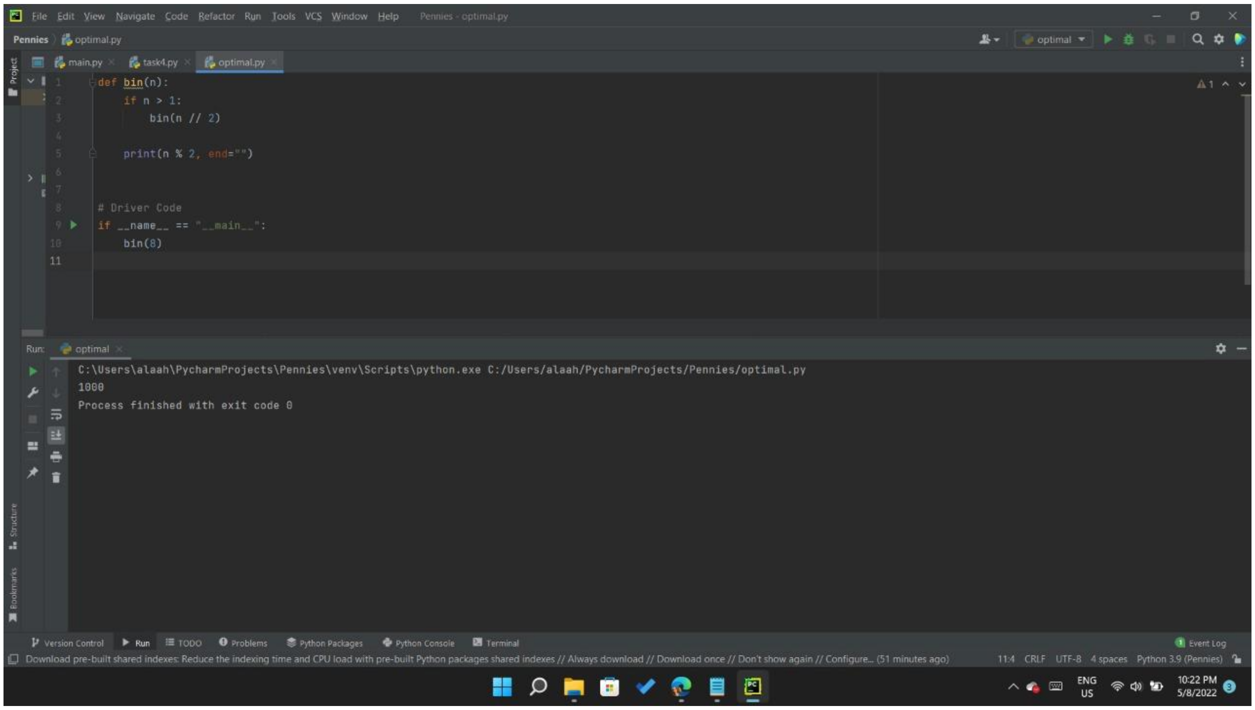


Figure 11: Output for number (8)

1.4.2.6 Sample of the output

Our code: "output is readable from left to right"

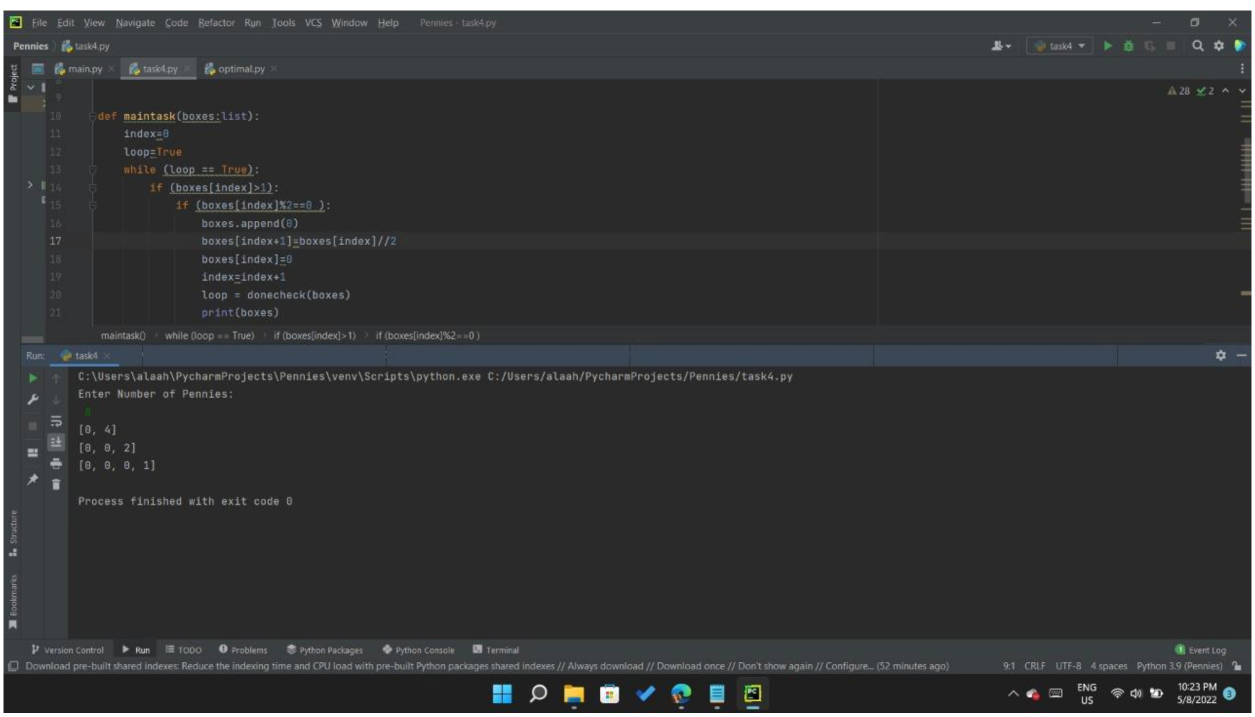


Figure 12: Output for number (8)

Optimal Code:

Our Code:

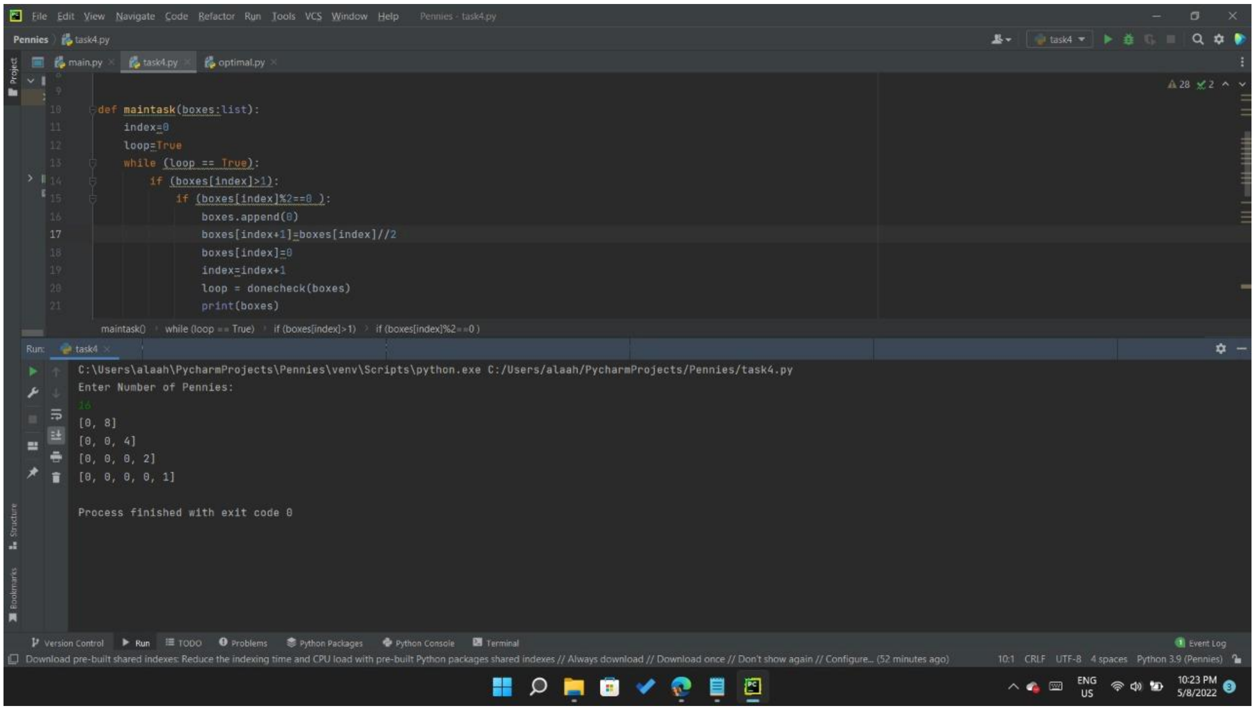


Figure 13: Output for number (16) using optimal code

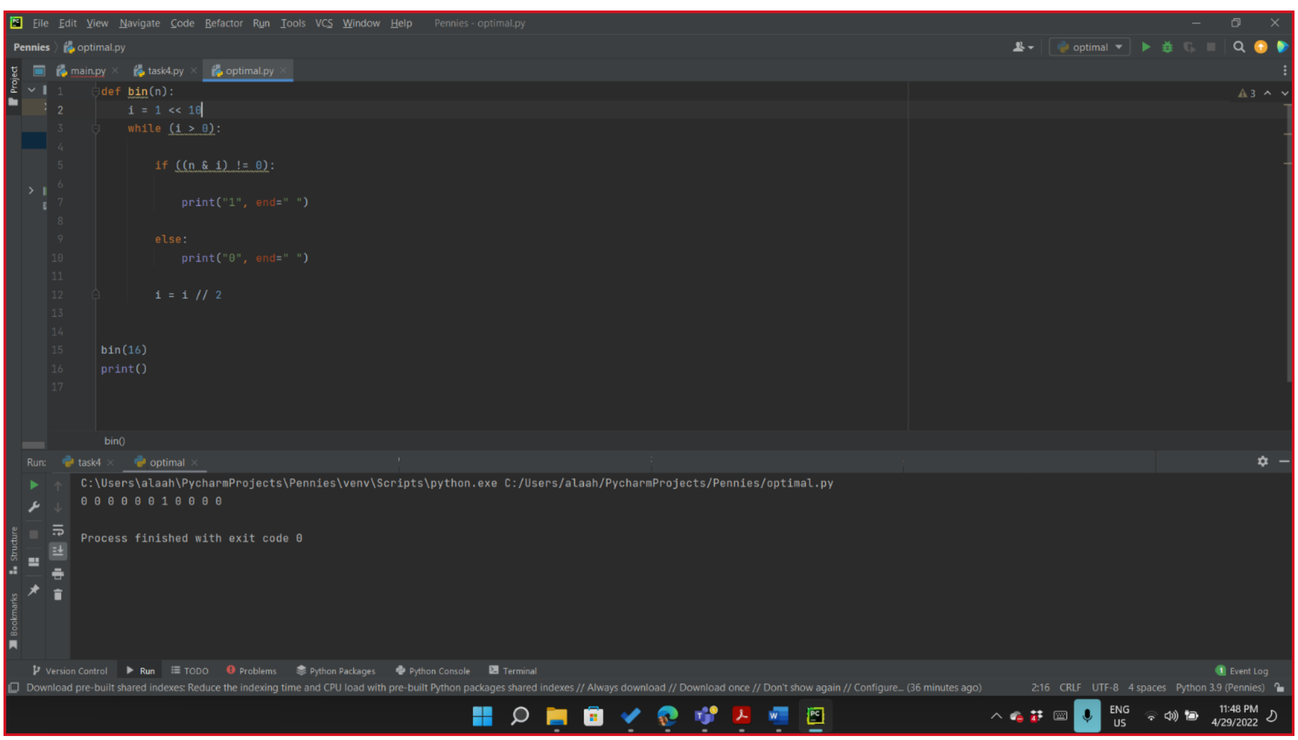


Figure 14: Output for number (16) using greedy algorithm

Sample output of the solution for the different cases of the algorithms:

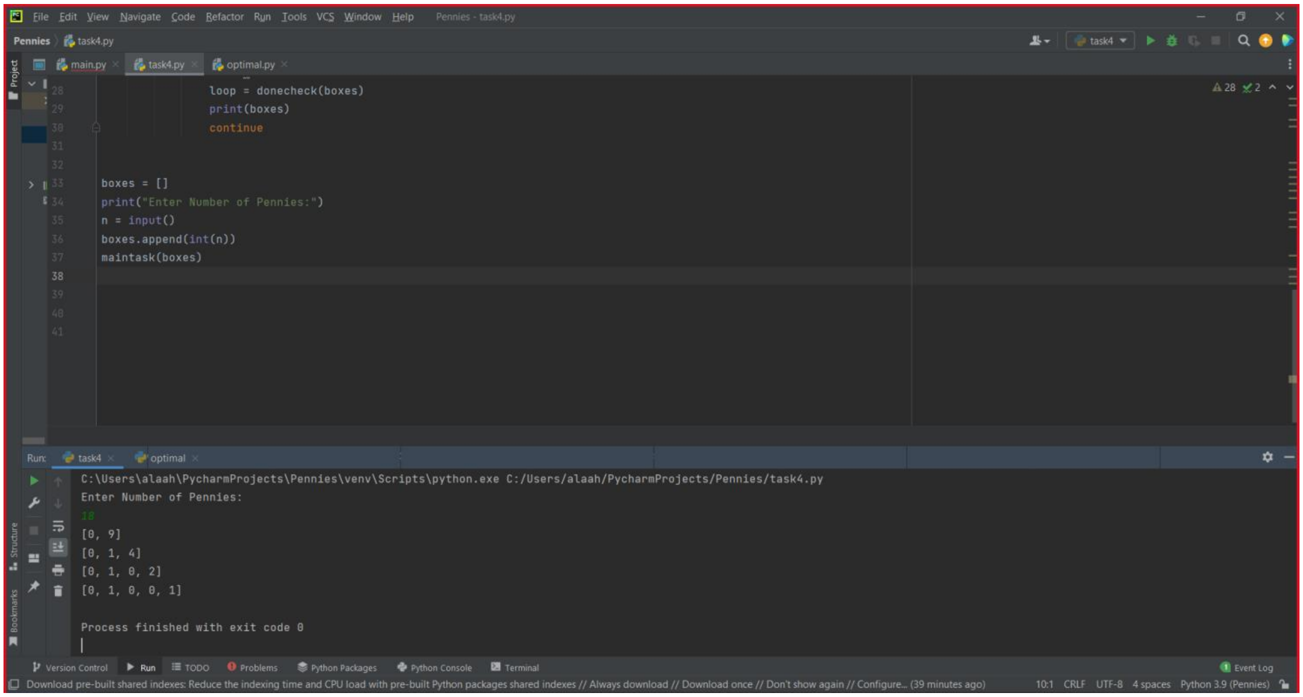


Figure 15: Output for number (18)

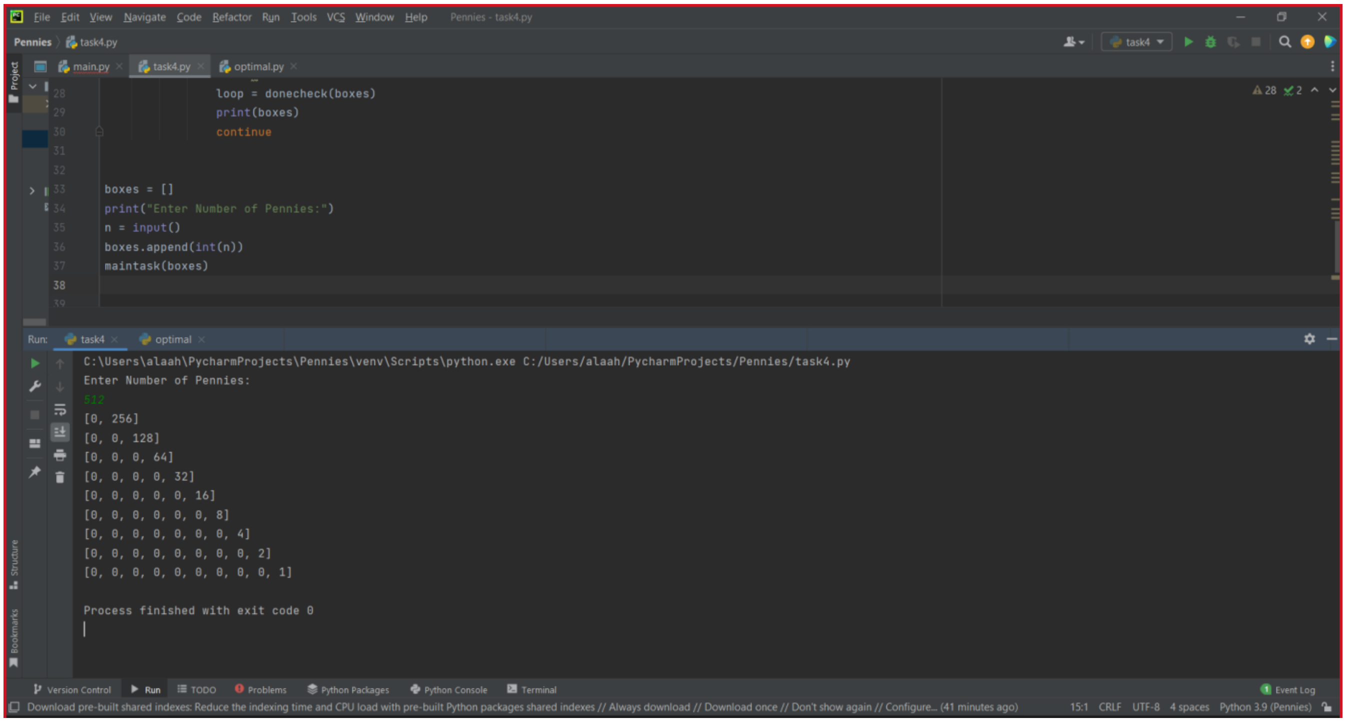


Figure 16: Output for number (512)

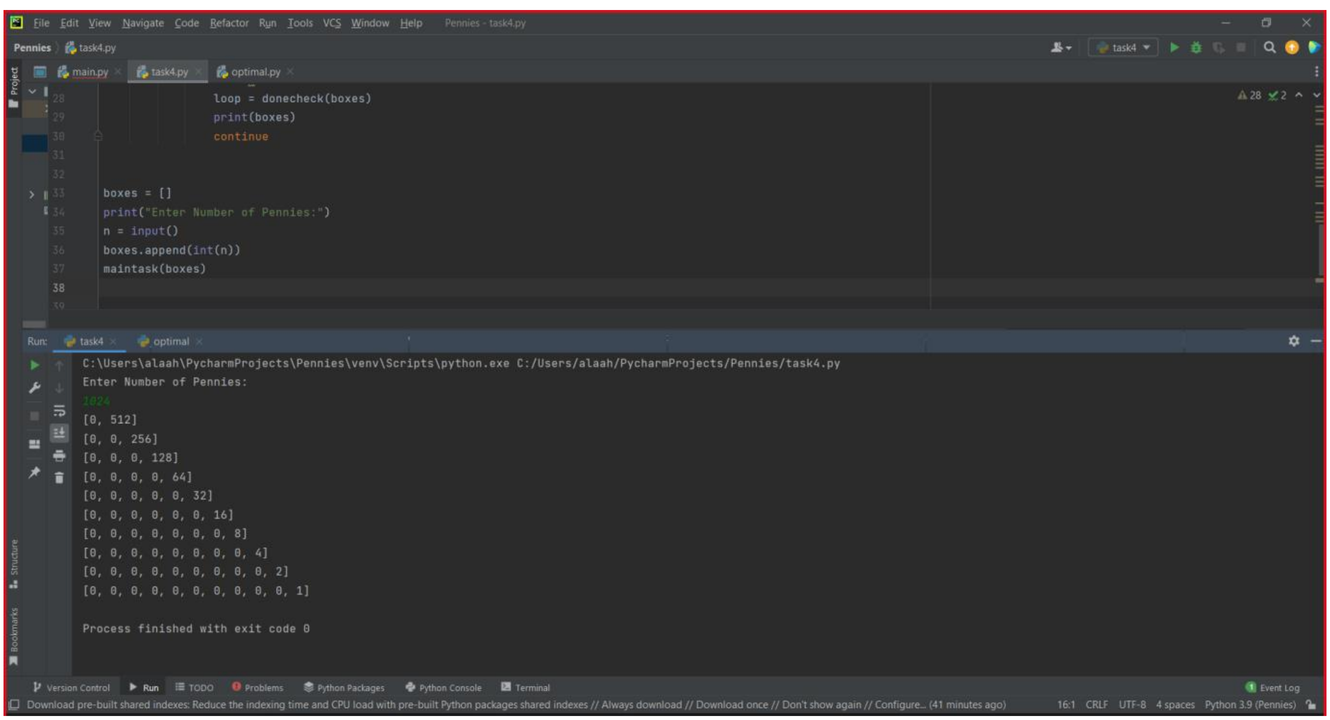


Figure 17: Output for number (1024)

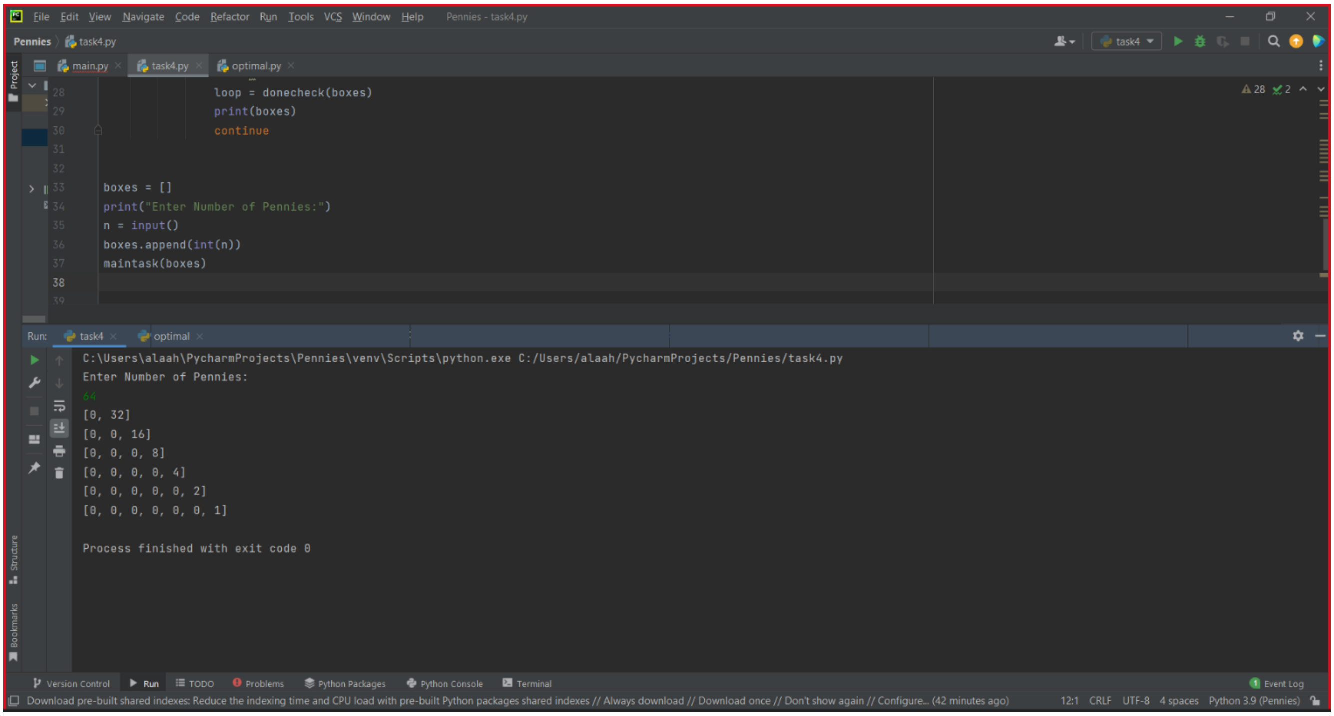


Figure 18: Output for number (64)

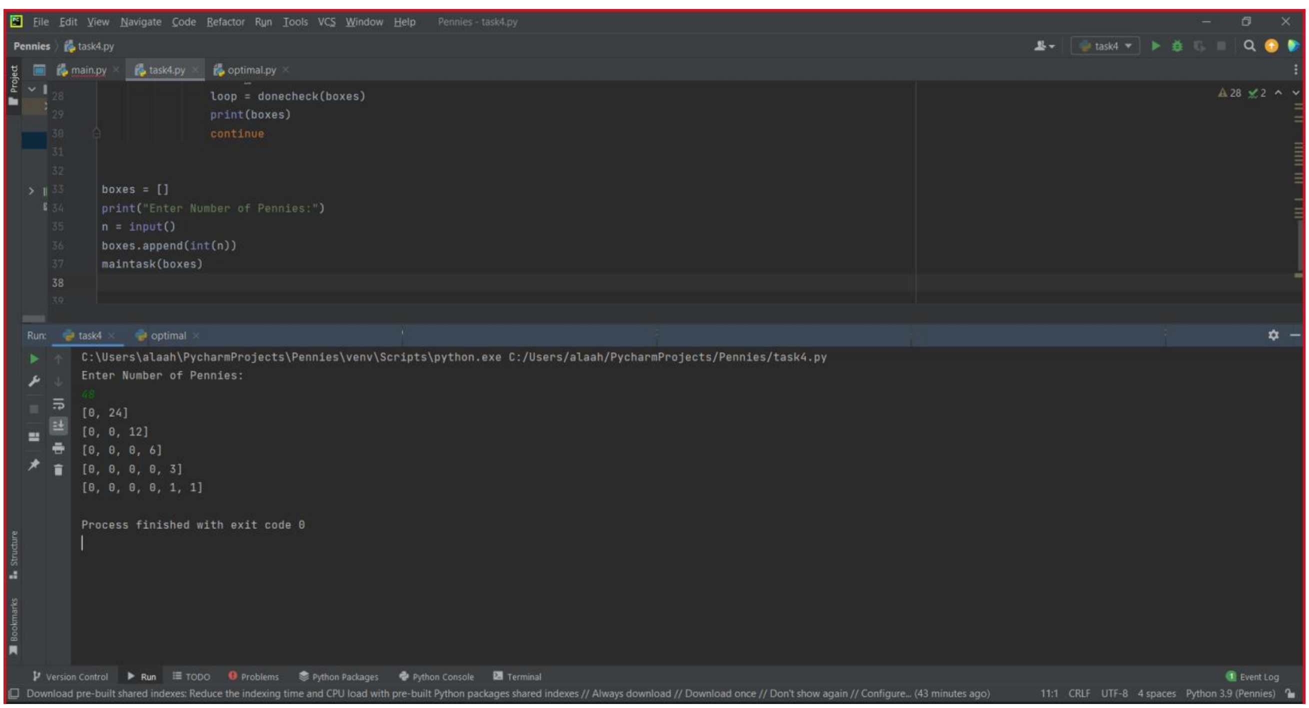


Figure 19: Output for number (48)

Task 4 part A:

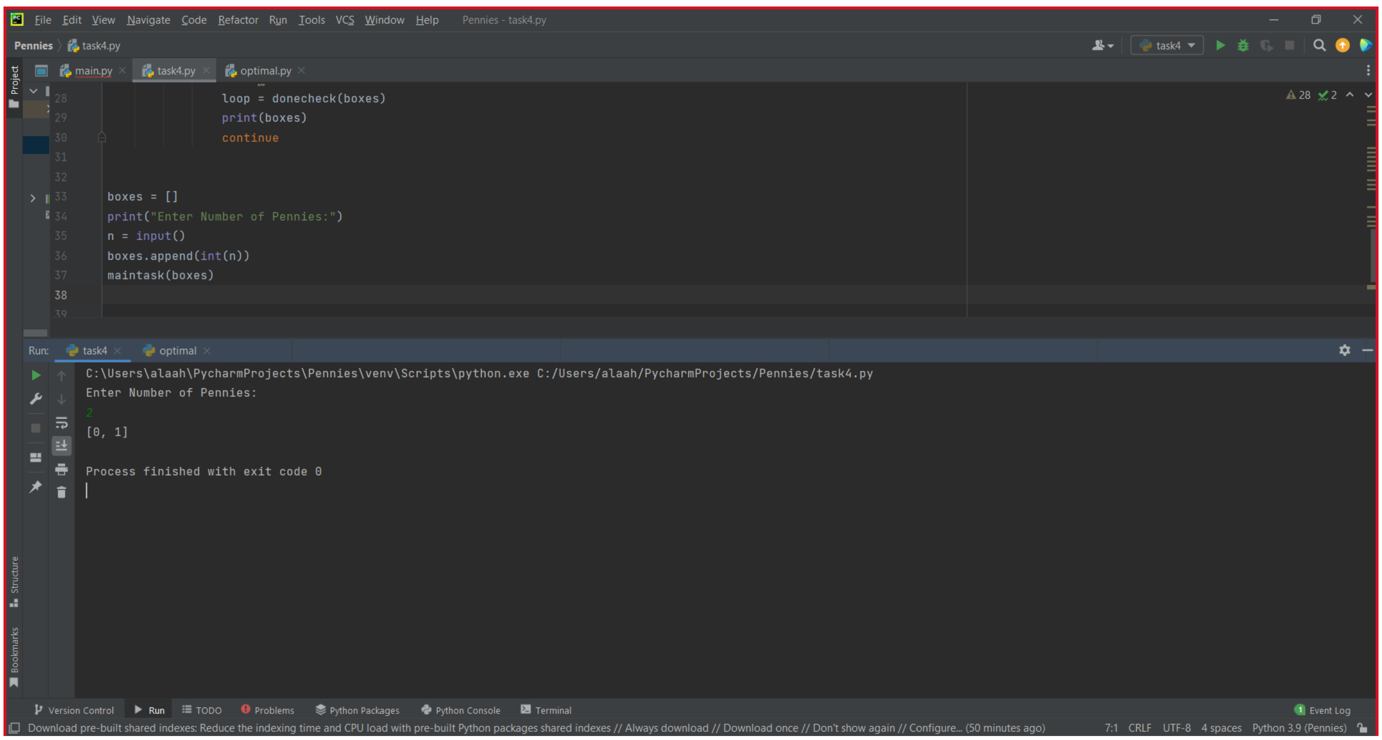
Does the final distribution of pennies depend on the order in which the machine processes the coin pairs?

-The algorithm will run the same way but the binary representation of the number will be read from right to left instead from left to right.

Task 4 part B:

What is the minimum number of boxes needed to distribute n pennies?

-for n>=2 minimum number of boxes needed is 2 because in the first iteration you will take 2



pennies

Figure 20: Output for number (2)

Task 4 part C:

How many iterations does the machine make before stopping?

if we start with 17 pennies in the leftmost box the final distribution of pennies will be

1st iteration 15 , 2nd iteration 13, 3rd iteration 11, and so on

-Log n iterations

1.5 Task 5

1.5.1 Problem description

There is a row of n security switches protecting a military installation entrance. The switches can be manipulated as follows:

(i) The rightmost switch may be turned on or off at will.

(ii) Any other switch may be turned on or off only if the switch to its immediate right is on and all the other switches to its right, if any, are off.

(iii) Only one switch may be toggled at a time.

Design a divide and conquer algorithm to turn off all the switches, which are initially all on, in the minimum number of moves. (Toggling one switch is considered one move.) Also find the minimum number of moves.

1.5.2 Solution

1.5.2.1 Pseudocode

```
Create new Array Switches

OFF ( start , end)

    N = end – start +1

    If ( N ==1)

        Toggle(end)

    Else if ( N== 2)

        Toggle ( start)

        Toggle ( end )

    Else

        OFF( start +2 , end )

        Toggle ( start )

        ON ( start + 2, end)

        OFF ( start + 1 , end)

ON ( start , end)

    N = end – start +1
```

IF(N == 1)

Toggle(end)

Else if (N == 2)

Toggle(end)

Toggle(start)

Else

ON (start + 1, end)

OFF (start + 2, end)

Toggle(start)

ON (start +2, end)

Toggle (i)

If (switches[i] == 1)

switches [i] = 0

Else

Switches[i] = 1

Print Switches array

1.5.2.2 Code

```
switches = []


def Off(start, end):
    n = end - start + 1
    if n == 1:
        toggle(end)
    elif n == 2:
        toggle(start)
        toggle(end)
```

```
    else:
        Off(start + 2, end)
        toggle(start)
        On(start + 2, end)
        Off(start + 1 , end)

def On(start, end):
    n = end - start + 1
    if n == 1:
        toggle(end)
    elif n == 2:
        toggle(end)
        toggle(start)
    else:
        On(start + 1, end)
        Off(start + 2, end)
        toggle(start)
        On(start +2, end)

def toggle(i):
    if switches[i] == 1:
        switches[i] = 0
    else:
        switches[i] = 1
    print(switches)


def main():
    global switches
    n = int(input("Enter number of switches: "))
    switches = [1] * n
    print(switches)
    Off(0, n - 1)

if __name__ == "__main__":
    main()
```

1.5.2.3 Solution description

This decrease and conquer approach manages to make this problem easier to solve, where the user will input n which is the number of switches to turn off and output is every step taken towards the solution. Once n is entered function Off() is called to start executing;

- If n is either 1 or 2, it will be handled as base case in which toggling is known and standard in all cases
- If n > 2 then it will enter a state where a few steps are repeated until solved.
- Off function executes as follows
  - Turn off starting from index (n+2)
  - Then, toggle index n
  - Afterwards turn on switches starting from index (n+2)
  - Finally, turn off the switches from index (n+1)
- On function executes as follows
  - Turn On starting from index (n+1)
  - Afterwards turn off switches starting from index (n+1)
  - Then, toggle index n
  - Finally, turn on the switches from index (n+2)

1.5.2.4 Complexity analysis

<u>First algorithm time complexity</u>

1) Recurrence relationship

$$F(n) = 2\,F(n-2) + F(n-1) + 1$$

$$F(1) = 1$$

$$F(2) = 2$$

$$F(n-1) = [F(n-2) + 2F(n-3) + 1)] + 2[F(n-3) + 2F(n-4) + 1] + 1$$

After simplification

$$\boldsymbol{F(n-1) = F(n-2) + 4[F(n-3) + F(n-4) + 1]}$$

$$F(n-2) = [F(n-3) + 2\,F(n-4) + 1\,] + 4[F(n-4) + 2F(n-5) + 1] + 4[F(n-5) + 2\,F(n-6) + 1] + 4$$

After simplification

$$\boldsymbol{F(n-2) = F(n-3) + 6[F(n-4) + F(n-5)] + 2\,F(n-6) + 13}$$

$$\boldsymbol{F(n-3)} = [F(n-4) + 2\,F(n-5) + 1\,] + 6[F(n-5) + 2F(n-6) + 1] + 6[F(n-6) + 2F(n-7) + 1] + 2[F(n-7) + 2\,F(n-8) + 1] + 13$$

After simplification

$$\boldsymbol{F(n-3) = F(n-4) + 8F(n-5) + 18F(n-6)] + 14F(n-7) + 4F(n-8) + 27}$$

Solving the recurrence relationship

$F(n) = \frac{2}{3}2^n - \frac{1}{6}(-1)^n - \frac{1}{2}$ for n>=1 $O(2^n)$

1.5.2.5 Comparison between another algorithm

Second Algorithm Pseudocode

```
SwitchOff(int n)

{

        PatternCount = [-1] * n

        If n -> even

                PatternCount[n-1] = 0

                PatternCount[n-2] = 3

        Else

                PatternCount[n-1] = 1

                PatternCount[n-2] = 0

        While (!Solved(switches))

                For i = 0 -> n

                        If PatternCount[i] = -1

                                If switches[i+1] = 1 and switches[i+2….n-1] =0

                                        Toggle switches[i]

                                        Print(switches)

                        Else


}

Boolean Solved(int [] array)

{

        Bool solved = true

        For i = 0 in switches

                If switches[i] = 1

                        Solved = false

        Return solved

}
```

Second algorithm complexity

$O(n^2)$

1.5.2.6 Sample of the output

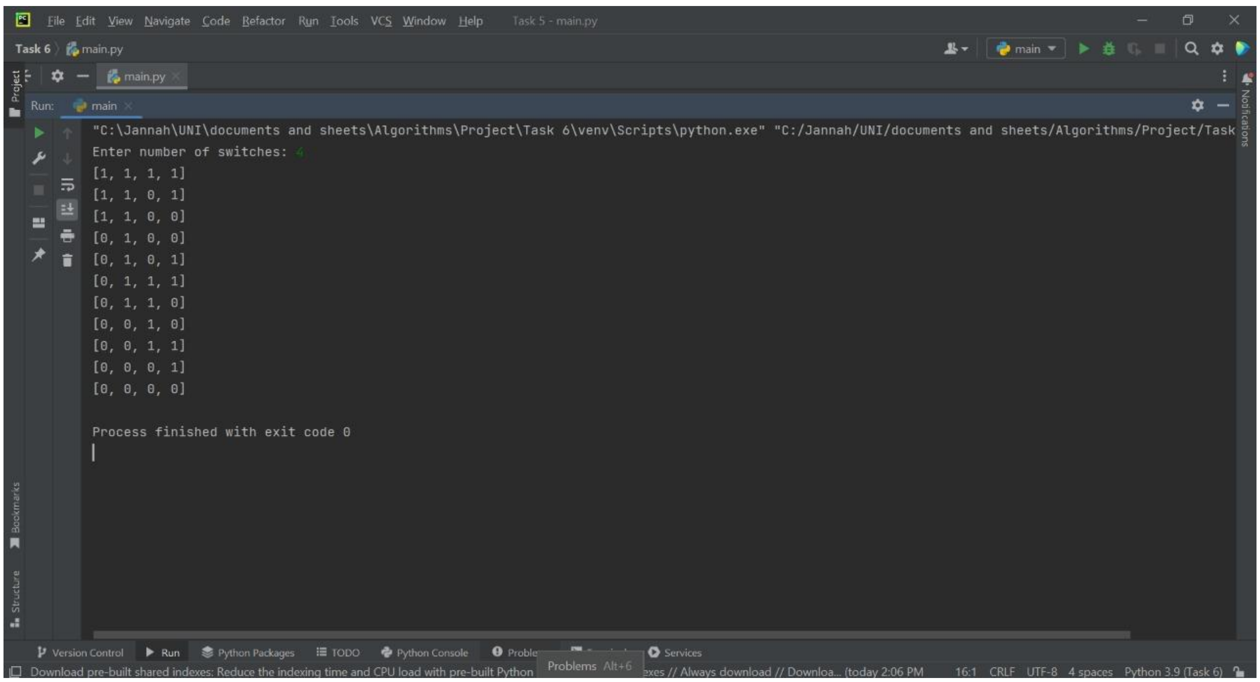


Figure 21: Output for number (4)

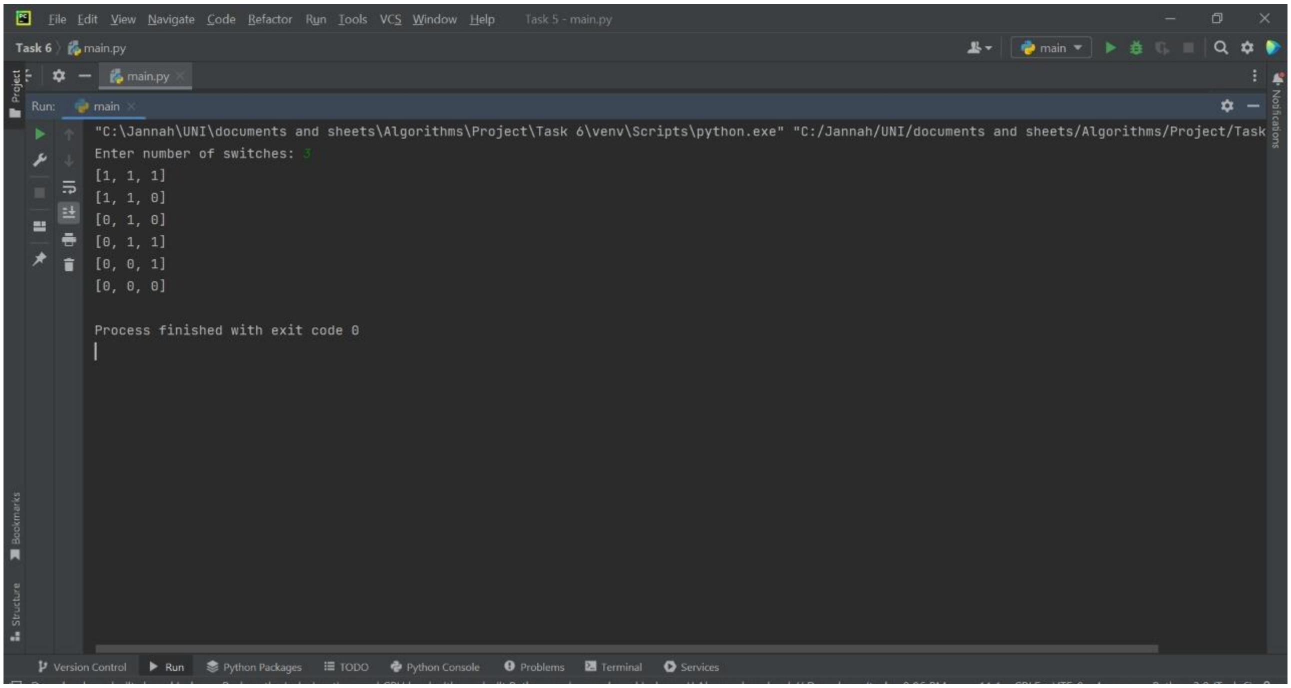
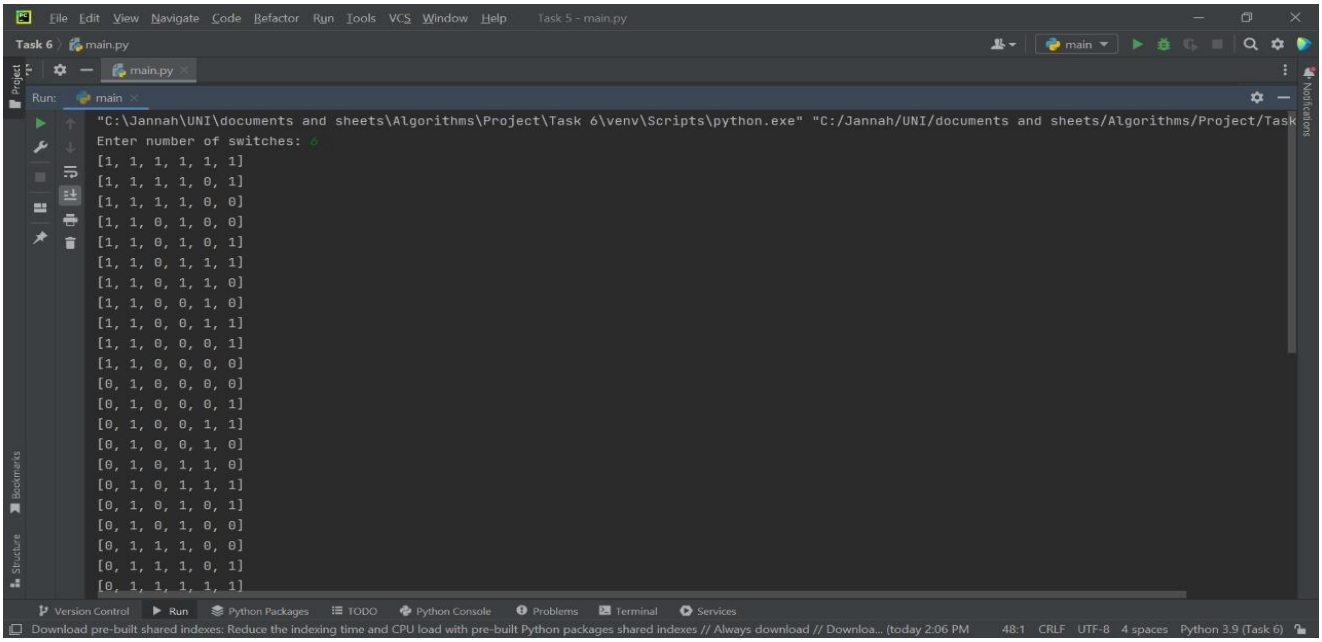


Figure 22: Output for number (3)



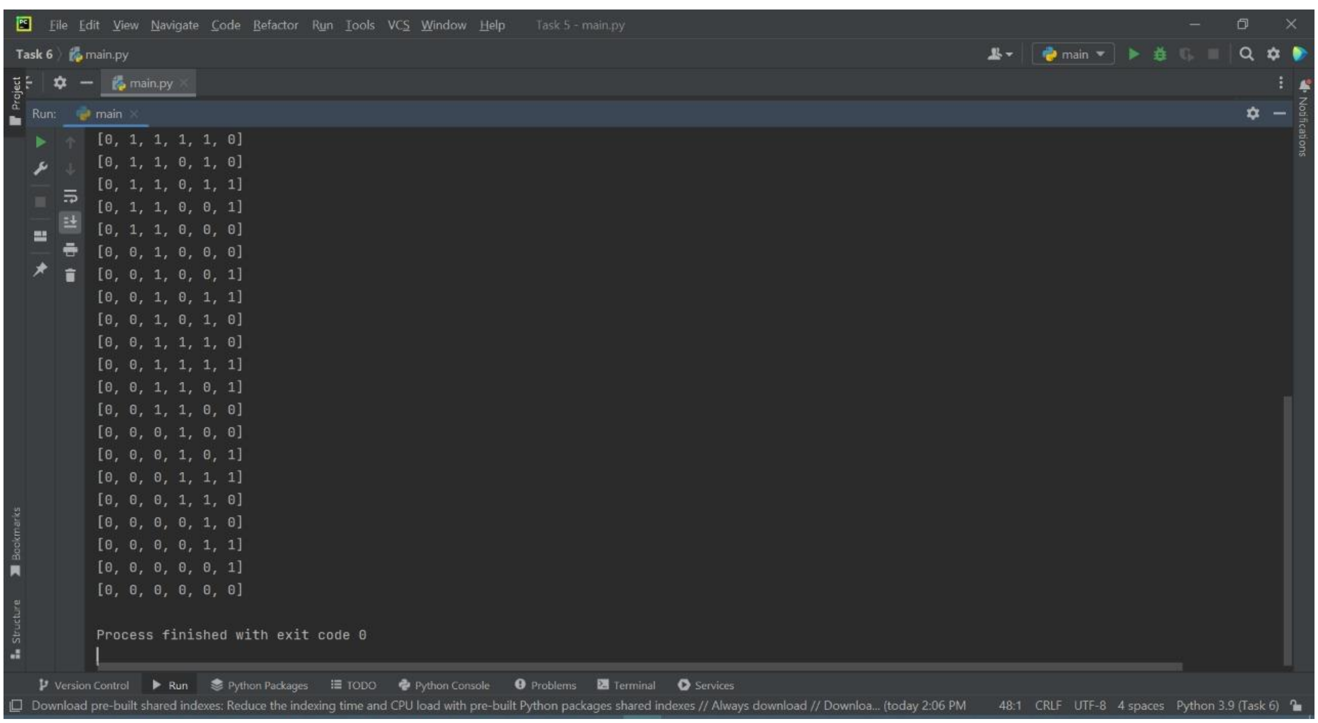


Figure 23: Output for number (6)

1.6 Task 6

1.6.1 Problem description

Task 6

There are eight disks of different sizes and four pegs. Initially, all the disks are on the first peg in order of size, the largest on the bottom and the smallest on the top.

Use dynamic programming method to transfer all the disks to another peg by a sequence of moves. Only one disk can be moved at a time, and it is forbidden to place a larger disk on top of a smaller one.

Does the dynamic programming method can solve the puzzle in 33 moves? If not then design an algorithm that solves the puzzle in 33 moves.

1.6.2 Solution

1.6.2.1 Pseudocode

//call the FrameStewartSolution function giving it atleast number of disks on initial peg

@fsmemoizer //decorator for memoizer

FrameStewartSolution (ndisks , , start=1, end=4, pegs=set([1, 2, 3, 4]))

{

If ndisks == zero then return nothing

If ndisks == 1 and number of pegs >1 {move that disk to the end peg}

If number of pegs == 3

{

call the towers3 function which solves the normal towers of Hanoi of 3 pegs

}

```
If number of pegs >= 3 and ndisks > 0 :

        {

        For each K:

                Find helper peg

                //3 Recursive calls to FrameStwewartSolution

                LHSmoves = FrameStewartSolution(kdisks, start, helper_peg, pegs)

                Mymoves = FrameStewartSolution(ndisks - kdisks, start, end,
                pegs_for_my_moves)

                RHSmoves = FrameStewartSolution(kdisks, helper_peg, end, pegs)

                Check if any of the 3 moves returned NONE that means it was is bad path so this
                K is ignored

                movelist = LHSMoves + MyMoves + RHSMoves

                if moveslist is shorter than Bestscore

                 {

                Bestsolution = movelist

                Bestscore = Length of movelist

        Return BestSolution

                }

If there was no solution{ return NONE}

} // end of FrameStewartSolution

//fsmemoizer function

fsmemoizer(f)

{

        //initialize dictionary to as a memory
```

```
        Cx{}

        f2(*args)

        {

                //compute the key of dictionary entry by taking the values of the argument sent
                to the function as keyvalue

                key= json.dumps(args)

                if arguments found in dictionary{ return value}

                else {store it in the Cx for later use}

        }

} // end of memoizer


Towers3 (ndisks ,start=1 ,target=3 ,peg_set=set([1 ,2 ,3]))

{

        If ndisks == 0  or start == end {return no moves}

        Mymove = move first disk form initial to final peg

        If ndisks == 1 {return mymove only}

        Determine helperpeg

        // 2 recursive calls to Towers3

        Movestomymoves =  towers3(ndisks -1 ,start ,helper_peg,peg_set)

        Movesaftermymoves = towers3(ndisks -1 ,helper_peg ,target,peg_set)

        Return sum of 3 moves made


} //end of Towers3 function
```

1.6.2.2 Code

```
import json


# tower3 solves normal towers of hanoi of 3 pegs
def towers3(ndisks ,start=1 ,target=3 ,peg_set=set([1 ,2 ,3])):
    if ndisks == 0 or start == target:  # if there are no disks, or no move
to make
        return []  # no moves
    my_move = "move(%s,%s) " %(start ,target)
    if ndisks == 1:  # trivial case if there is only one disk, just move it
        return [my_move]
    helper_peg = peg_set.difference([start ,target]).pop()
    moves_to_my_move = towers3(ndisks -1 ,start ,helper_peg,peg_set)
    moves_after_my_move = towers3(ndisks -1 ,helper_peg ,target,peg_set)
    return moves_to_my_move + [my_move] + moves_after_my_move

# memoizer used to store solutions to be used later , this is the dynamic
programming enhancement
# if you wish to see its importance comment out the memoizer function and the
decorator then call the
# framestewartsolution with 16 disks ( it will take almost 60 seconds) but
with memoizer almost instantly under 1 second
def fsMemoizer(f):  # just a junky quick memoizer
    cx = {}
    def f2(*args):
        try:
            key= json.dumps(args)
        except:
            key =json.dumps(args[:-1] + (sorted(list(args[-1])),))
        if key not in cx:
            cx[key] = f(*args)
        return cx.get(key)

    return f2


@fsMemoizer
def FrameStewartSolution(ndisks, start=1, end=4, pegs=set([1, 2, 3, 4])):
    if ndisks == 0 or start == end:  # zero disks require zero moves
        return []
    if ndisks == 1 and len(pegs) > 1:  # if there is only 1 disk it will only
take one move
        return ["move(%s,%s)" % (start, end)]
    if len(pegs) == 3:  # 3 pegs is well defined optimal solution of 2^n-1
        return towers3(ndisks, start, end, pegs)
    if len(pegs) >= 3 and ndisks > 0:
        best_solution = float("inf")
        best_score = float("inf")
        for kdisks in range(1, ndisks):
            helper_pegs = list(pegs.difference([start, end]))
```

```
            LHSMoves = FrameStewartSolution(kdisks, start, helper_pegs[0],
pegs)
            pegs_for_my_moves = pegs.difference([helper_pegs[0]])# cant use
the peg our LHS stack is sitting on
            MyMoves = FrameStewartSolution(ndisks - kdisks, start, end,
pegs_for_my_moves)  # misleading variable name but meh
            RHSMoves = FrameStewartSolution(kdisks, helper_pegs[0], end,
pegs)  # move the intermediat stack to
            if any(move is None for move in [LHSMoves, MyMoves, RHSMoves]):
continue  # bad path :(
            move_list = LHSMoves + MyMoves + RHSMoves
            if (len(move_list) < best_score):
                best_solution = move_list
                best_score = len(move_list)
        if best_score < float("inf"):
            return best_solution
    # all other cases where there is no solution (namely one peg, or 2 pegs
and more than 1 disk)
    return None




if __name__ == '__main__':
        # this will show a list of 33 moves performed
        print(FrameStewartSolution(8))
```

1.6.2.3 Solution description

- Since the problem is an obvious extension of the Tower of Hanoi puzzle , it is natural to use a similar recursive approach.
- Namely, if ndisks > 2, transfer k smallest disks to an intermediate peg recursively using help of all four pegs (LHSmove), then
- move the remaining n − k disks to the destination peg by the classic recursive algorithm for the three-peg Tower of Hanoi puzzle (MYmove)
- finally , transfer the k smallest disks to the destination peg recursively using all four pegs.(RHSmove)
- If n = 1 or 2, solve the trivial instances of the problem in one and three moves , respectively, as it is done in the three-peg Tower of Hanoi solution.
- The value of parameter k must be selected to minimize the total number of disk moves made by the algorithm. That why we try all possible values of K then choose the K which results in the minimum number of total moves
- For dynamic programming enhancement we use a simple memoizer which stores the the parameters and their return values of a call to the main function so when this expensive function is called again with same parameter instead of calculating all over again the memoizer return the stored value of the return value of such prameters.
- The effect of dynamic programming time enhancements can be noticed mostly if the same function was called for 16 initial disks and comment out the memoizer and then using a memorize , the effect is huge . with a memoizer it takes less than a second while without it , it takes almost 60 seconds!!

1.6.2.4 Complexity analysis

Recurrence relation for the number of

moves, *R*(*n*), made by this algorithm to move *n* disks:

$R(n) = min_{1\leq k<n}$ $[2R(k) + 2^{n-k} - 1]$ for $n > 2$, $R(1) = 1$, $R(2) = 3$.

- $2^{n-k} - 1$ is the recurrence relation for Towers of Hanoi with 3 pegs
- 2 R(k) is the steps taken to move K disks initially to a certain helper peg (LHSmove) then in then end to move the same K disks the target peg (RHSmove)

- $min_{1 \le k < n}$ since we try all K values then choose the K which produces the least number of moves

Towers of Hanoi recurrence analysis:

T(n) = 2*T(n-1) + 1

T(n) = 2 * (2 * T(n-2) + 1) + 1

T(n) = (2 ^ 2) * T(n-2) + 2^1 + 2^0

T(n) = (2^k) * T(n-k) + 2^(k-1) + 2^(k-2) + ... + 2^0

Solving this the closed from comes out to be

T(n) = (2^n) - 1 with T(0) = 0

Asymptotic notation for Towers of Hanoi is $2^n - 1$ so $\Theta(2^n)$

IN OUR CASE :

The initial number of disks is 8 so

The code tries all K values and each try is $\Theta(2^n)$ so this search's asymptotic notation is $\Theta(2^n)$ as well

We can use this algorithm to deduce K for our case :

$$k = n - 1 - m \text{ where } m = \lfloor \sqrt{8n - 7} - 1)/2 \rfloor,$$

K is calculated to be 4 so :

R(n) = 2 $R(4)$ + $2^{n-4} - 1$ for $n > 2$, $R(1) = 1$, $R(2) = 3$.

R(4) = 2 R(1) + $2^{4-1} - 1 = 9$

R(n) = 18 + $2^{n-4} - 1$

R(n) = 2^{n-4} + 17

R(n) = $\frac{1}{16}(2^n + 273)$

So the asymptotic notation of the code is $\Theta(2^n)$

Of course this time is drastically reduced with the help of the dynamic programming enhancements

1.6.2.5 Comparison between another algorithm

Another algorithm we can use to solve this problem uses a decrease and conquer algorithm

Code:

```
# slower solution using divide and qonquer
def slowersolution(disk, source, temppeg1, temppeg2, destination):
    if disk == 1:
        print((source, destination))
    elif disk == 2:
        print((source, temppeg1))
        print((source, destination))
        print((temppeg1, destination))
    else:
        slowersolution(disk - 2, source, temppeg2, destination, temppeg1)
        print((source, temppeg2))
        print((source, destination))
        print((temppeg2, destination))
        slowersolution(disk - 2,  temppeg1, source, temppeg2, destination)
```

This algorithm takes 45 moves unlike the dynamic programming algorithm we made which takes 33 moves only to solve so it makes it the next best solution (suboptimal).

1.6.2.6 Sample of the output

Main:

```
if __name__ == '__main__':

    mylistofmoves = FrameStewartSolution(8)
    for i in mylistofmoves:
        print(i)


    slowersolution(8,'a','b','c','d')
```

Figure 24: main to run the Output

OUR CODE (optimal):

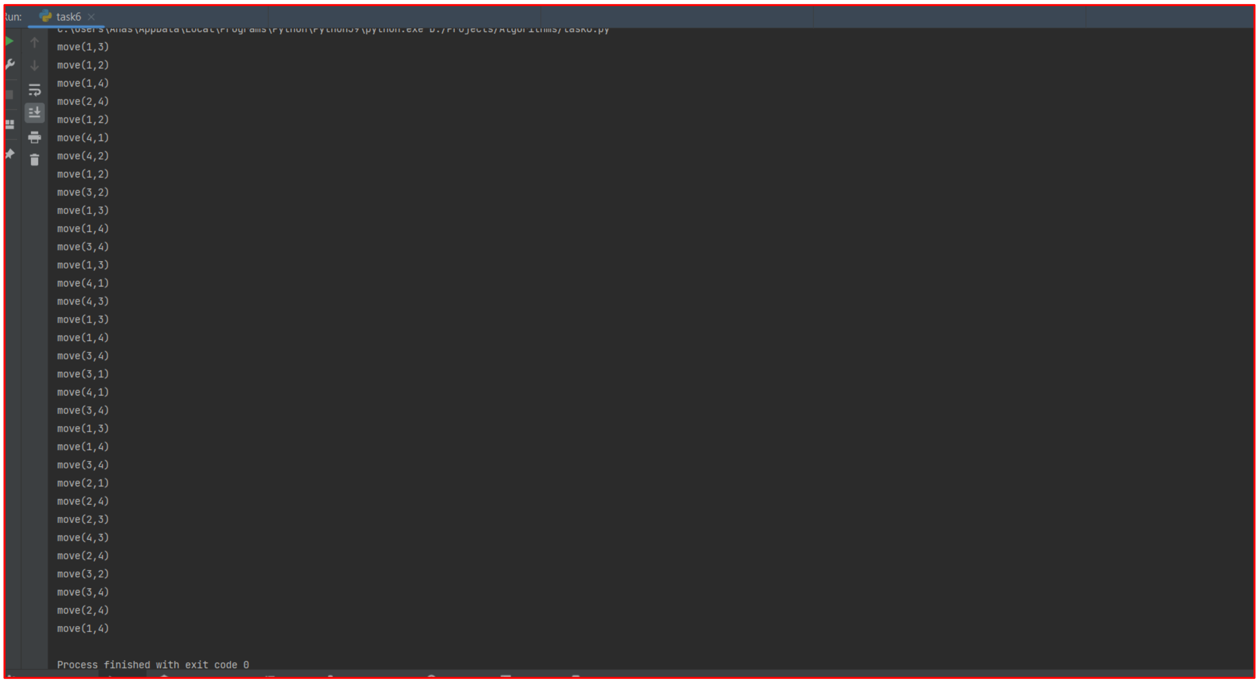


Figure 25: Output for number (8) using dynamic programming

Slower code (suboptimal):

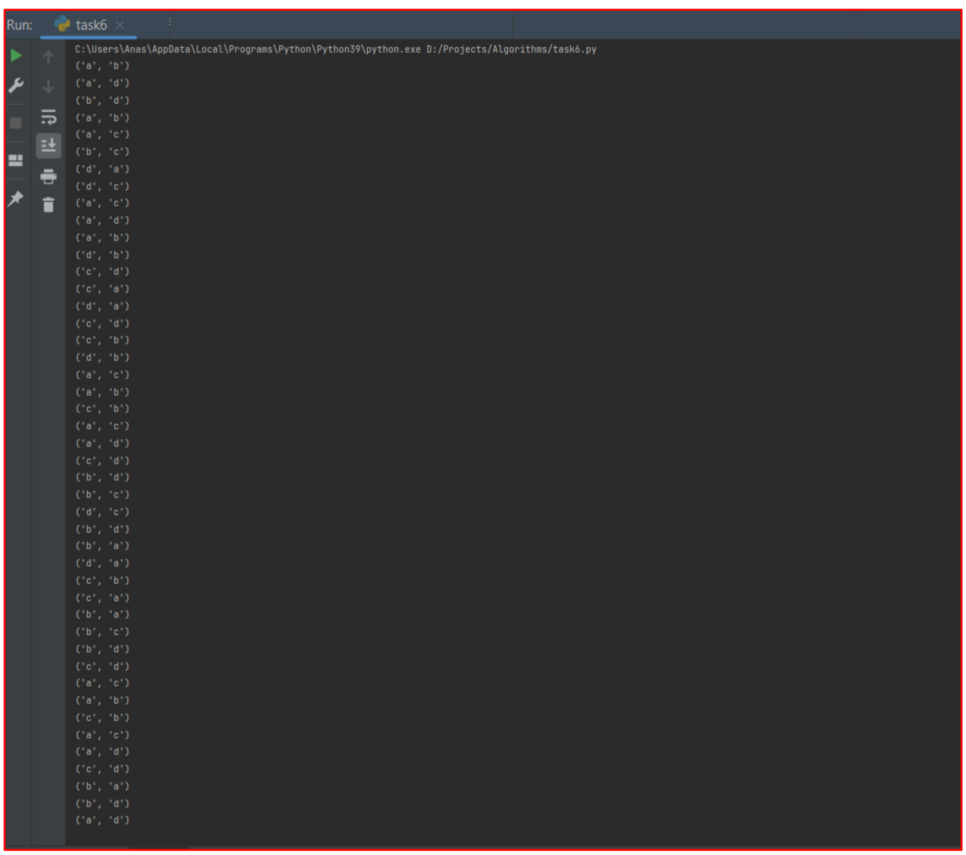


Figure 26: Output for number (8) using decrease and conquer

2. REFRENCES

[1] Levitin, A. and Levitin, M., 2011. ALGORITHMIC PUZZLES. Oxford University Press; Illustrated edition, p.280.

[2] Levitin, A., 2006. Introduction to the Design and Analysis of Algorithms. 2nd ed. Addison Wesley, p.592.