Data Driven Physics: Tutorial 8

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Bayesian inference and Quantum Non-Demolition (QND) photon counting

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Serge Haroche has been awarded the Nobel Prize in Physics in 2012 for "for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems". Haroche and his collaborators have designed and realised experiments to study fundamental quantum phenomena when matter and light interact at the level of individual quantum particles: atoms and photons. The research team has been able to capture photons using a cavity made of two opposing mirrors which reflect the photons back and forth. The number of trapped photons can be then deduced by letting them interact with individual atoms passing through the trap and detected latter. This method of Quantum Non-Demolition photon counting is presented in detail in the lectures of S. Haroche given at the Collège de France in 2008 [1].

The goal of this tutorial is to apply the Bayes theorem to deduce the number of photons stored in the cavity from the repeated measurement of the spins states of atoms passing through the cavity. For this purpose we will re-analyse the data presented in Ref. [2] and kindly provided to us by Igor Dotsenko (LKB, Collège de France).

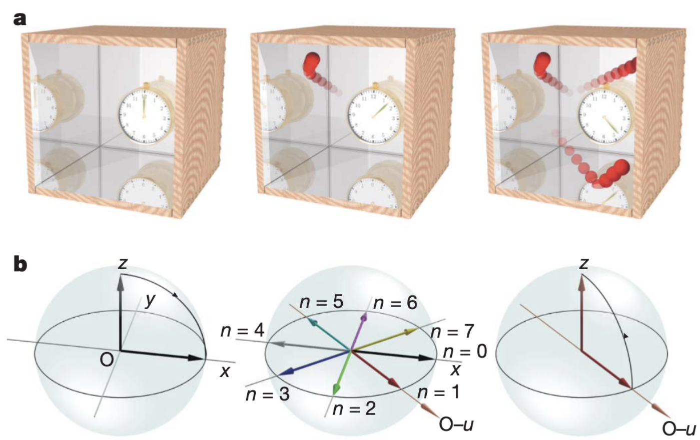

Figure 1 | Principle of QND photon counting. a, Thought experiment with a clock in a box containing n photons. The hand of the clock undergoes a $\pi/4$ phase-advance per photon ($n = 0, 1, 3$ represented). **b,** Evolution of the atomic spin on the Bloch sphere in a real experiment: an initial pulse R_1 rotates the spin from O–z to O–x (left). Light shift produces a $\pi/4$ phase shift per photon of the spin's precession in the equatorial plane. Directions associated with $n = 0$ to 7 end up regularly distributed over 360° (centre). Pulse R_2 maps the direction O–u onto O–z, before the atomic state is read out (right).

Figure 1: Sketch and description of the photon-number measurement of Ref. [2].

Problem

The number of photons trapped in the cavity is measured by a sequence of individual Rydberg atom passing through the cavity and interacting with its field one by one. Each photon produces the same known phase-shift Φ of the atomic dipole (see Fig. 1). The dipole thus gains a phase $n\Phi$ which is then measured by a Ramsey interferometer, allowing us to deduce the number of photons n. Note that the underlying physical interaction is based on the so-called "dispersive" interaction which prohibits any energy exchange between the atoms and the photons and thus keeps the photon number constant (hence the name "non-destructive" for this type of measurement).

In the case of a perfect interferometer, the probability to detect the k-th atom in its ground state g (denoted by $s = 1$) or excited state e ($s = 0$) after encountering n photons is

$$P(s|\phi_k, n) = (1 + \cos(n\Phi + \phi_k + s\,\pi))/2. \tag{1}$$

Here, ϕ_k is the phase of the Ramsey interferometer set for this atom. The realistic interferometer used in [2] has a reduced contrast of $B = 0.698$ and an offset of $A = 0.551$, resulting in the following conditional probabilities:

$$\begin{aligned} P(g|\phi_k, n) &= A - B/2\ \cos(n\Phi + \phi_k), \\ P(e|\phi_k, n) &= (1 - A) + B/2\ \cos(n\Phi + \phi_k). \end{aligned} \tag{2}$$

The experimental dephasing per photon $\Phi = 0.233\pi$ is set to be close to its optimal value of $\pi/4$ allowing to count photon numbers from $n = 0$ to 7. There are four Ramsey phases $\phi_k = -0.836, 0.033, 0.905, 1.442$ set for optimal detection of different values of n and changed cyclically along the atomic sequence.

Data: Two files resuming the quantum detections in spin state g and e can be downloaded from the webpage, (Tutorial 7). Each file contains a $M \times T$ matrix of detection results $\mathbf{D}$. The rows of the matrix correspond to $M = 2000$ independent repetitions of the experiment. Each row is a "quantum trajectory" of $T = 10000$ measurements. Its entries $d(m, t)$ indicate the number of atoms detected in the state g and e for the two files, respectively, at the detection time t, and with Ramsey Phases number $k(t)$. The 4 possible Ramsey phases are cyclically changed to $\phi_0, \phi_1, \phi_2, \phi_3, \phi_0 \ldots$ along each trajectory, so $k(t) = 0, 1, 2, 3, 0 \ldots$. Atoms are sent in packets, containing at most 3 atoms. The values $d(m, t)$ range therefore from 0 (no detected atom) and 3. Note that in the dispersive regime, each atom interacts independently with the cavity.

Questions:

1. (a) Write the probability of each set of detections along a quantum trajectory, given a number of photons in the cavity and the protocol of Ramsey phases $k(t)$ used in the experiment. Deduce, using Bayes rule, the probability of the photon number n given the detections along a quantum trajectory. Use a uniform prior in the interval $n = 0 \ldots 7$.

   (b) Re-express the log-probability of the photon number n using the counts c_g^k for detections in the state g and c_e^k in the state e for each Ramsey phase ($k = 0 \ldots 3$).

2. Focus on the trajectory $m = 1163$ (in numbering starting from m=0) and consider detections starting from $t_{in} = 400$ to $t_f = t_{in} + dt$ with $dt = 400$ detections. Count the number of atoms detected during this interval in the state e and g for each Ramsey phase.

3. Focus on the trajectory $m = 1163$. Plot the probability of n after dt detections starting from $t_{in} = 400$ and for $dt = 0, 12, 100, 200, 400$. Describe how the posterior evolves as a function of the number of measurements. Compare qualitatively to Fig. 2 of [2]. Compute the entropy of the posterior after dt detections. What is its expected value with no measurement ($dt = 0$)? Plot it as a function of the number of measures dt, how does it decay? Deduce how the posterior behaves as a function of dt. Give the most probable value of n (n_{opt}) after $dt = 400$ measurements.

4. Start by the trajectory $m = 1163$ (and repeat for trajectories 1293, 1146, 479, and 407). Plot the most probable value of n as a function of t_{in} starting from $t_{in} = 0$ and in sliding windows (sliding at each step by 4 detections to take into account all 4 phases) of $dt = 400$ detections. Convert the detection's number into time if the detections are performed every 83.3 μs. Plot the sequences of n_{opt} as a function of t_{in}. Observe the quantum jumps in n_{opt} and the collapse to the vacuum state $n = 0$. Compare with the Fig. 4 of [2] (the order of the trajectories given above corresponds to those of the panels in Fig. 4) What is the peculiarity of the trajectory 407?

5. Plot the distribution of n_{opt} over all M independent repetitions of the experiments from $dt = 400$ detections starting at $t_{in} = 16, 400, 800, 1600$. How does the average value evolve? Compare with the distribution given in Fig. 2 of [2] and the value $n_{av} = 3.46$ given in the paper.

6. Optional: Compute the probability for a single detection in g and e for $n = 0 \dots 7$ for two cases: the ideal settings with $A = 0.5$, $B = 1$, $\Phi = \pi/4$), and Ramsey phases $\phi_k = q\pi/4$ with $q \in \{-2, -1, 0, 1\}$ and for the real settings determined from the experimental limitation of the interferometer.

References

[1] S. Haroche, *Cours 2007-2008: Sixième Leçon 3 Mars 2008: Comptage QND de plusieurs photons et génération d'états de Fock de la lumière* (2008).

[2] C. Guerlin et al., *Progressive field-state collapse and quantum non-demolition photon counting*, Nature 448, **889** (2007).