A Gazebo Simulator for Continuum Parallel Robots

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- Serial robots
  - Simpler and more used
  - Limited by precision and inertia
- Parallel robots
  - Less inertia, high velocities
  - More joints involved

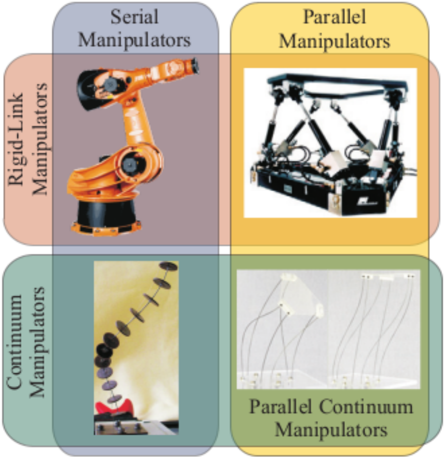

Figure: Different robot architectures

Continuum parallel robots

- Continuum parallel robots
  - May anhance safety
  - Cheaper components
  - Possible to miniturize
- Model and stability problems
  - More unstable configurations
  - Not analytical solution
- Definition of a general simulator
  - Gazebo plugin

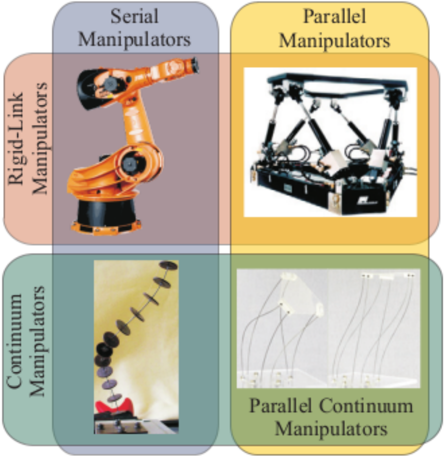

Figure: Different robot architectures.

Geometric modelling

- Rod as 1D body
- Function of the arc-lenght s
  - Centerline position $p_{(s)} \in \mathbb{R}^3$
  - Cross-section orientation $R_{(s)} \in se(3)$

- Define transformation

$$T_{(s)} = \begin{bmatrix} R_{(s)} & p_{(s)} \\ 0 & 1 \end{bmatrix} \in SE(3) \tag{1}$$

- Derivative wrt arc-lenght

$$x' = \frac{\mathrm{d}x}{\mathrm{d}s}$$

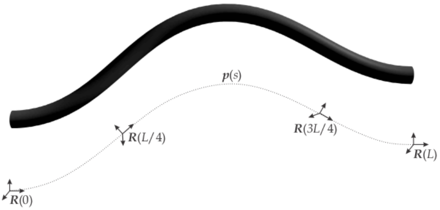

Figure: Rod geometric modelling

Equilibrium Equations

- Equilibrium consideration
  - Distributed forces/moments
  - Internal forces/moments

$$n'_{(s)} = -f_{(s)} \tag{2}$$

$$m'_{(s)} = -p'_{(s)} n_{(s)} - l_{(s)} \tag{3}$$

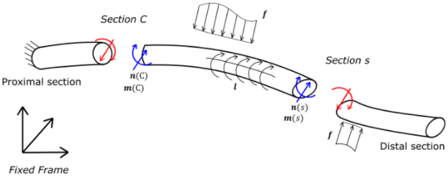

Figure: Sections of the beam considered for the static equilibrium.

Boundary Value Problem

- Constraints at the distal plate
  - External wrench $\Psi_{ext} = \begin{bmatrix} F \\ M \end{bmatrix}$
  - Rod contribution $\Psi_i = \begin{bmatrix} n_{i(L_i)} \\ m_{i(L_i)} \end{bmatrix}$
- Constraints at the base
  - Actuations Ψ_{a_i}
  - Joints and geometry

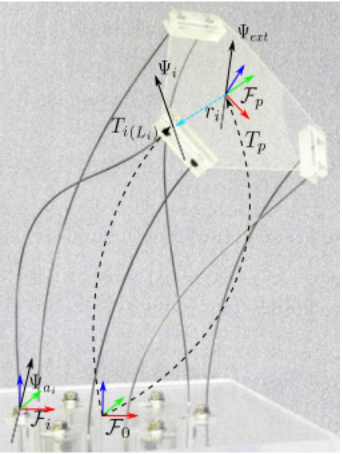

Figure: Geometrical and actuation constraints for a Stewart-Gough CPR.

Shooting Method in statics (Stewart-Gough CPR)

- ODE system in statics
  - Equilibrium equations
  - Material properties
  - Geometrical considerations
- Recursive solution
  - Needs an intial guess
  - Evaluation on a cost function

$$\boldsymbol{f} = \begin{bmatrix} \sum n_{i(L_i)} - F \\ \sum \left[p_{i(L_i)} n_{i(L_i)} + m_{i(L_i)} \right] - p_d F - M \\ p_d + R_d r_i - p_{i(L_i)} \\ \left[R^T_{i\,(L_i)} R_d - R_{i(L_i)} R^T_d \right]^V \end{bmatrix} \quad (4)$$

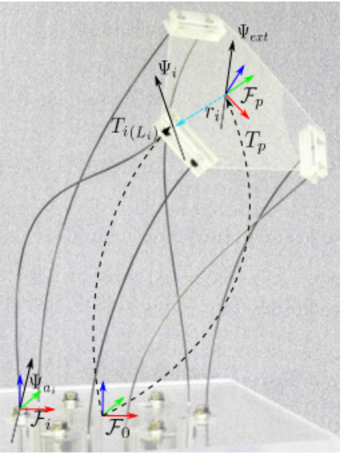

Figure: Geometrical and actuation constraints for a Stewart-Gough CPR.

Shooting Method in dynamics

- PDE system
  - Derivative wrt to arc-lenght $x^{'} = \frac{\partial x}{\partial s}$
  - Derivative wrt to time $\dot{x} = \frac{\partial x}{\partial t}$
- From PDE to ODE
  - Implicit discretization

$$\frac{\partial x}{\partial t} = c_0 x^{(i)} + \sum_{k=1}^{\infty} \left[c_k x^{(i-k)} + d_k \dot{x}^{(i-k)} \right] \tag{5}$$

$$\frac{\partial x}{\partial t} = c_0 x^{(i)} + c_1^{(i-1)} x^{(i-1)} + c_2^{(i-2)} x^{(i-2)} + d_1^{(i-1)} \frac{\partial x^{(i-1)}}{\partial t} \tag{6}$$

Non linear solver: Levenberg-Marquardt algorithm

- Iterative algorithm
- Evaluates influence of parameter vector u

$$J = \frac{\mathrm{d}f}{\mathrm{d}u} \tag{7}$$

- Updates the parameter vector

$$u_{k+1} = u_k + \left(J_k^T J_k + \mu I\right)^{-1} J_k^T f_k \tag{8}$$

Strain approach, modelling

- Modelling of a continuum body in space
  - Internally actuated Cosserat beam
  - With its configuration space $C = SE(3) \times S$
- From assumption on rod deformation
  - Allowed ξ_a, prohibited ξ_c twists
  - Strain generalized coordinates $q_{[n \times 1]}$
  - Basis functions $\Phi_{[n_a \times n]}$

$$\xi_{a(s,t)} = \xi_{a0(s)} + \Phi_{(s)} q_{(t)} \tag{9}$$

- Configuration space discretization $C = SE(3) \times \mathbb{R}^n$

Strain approach, Lagrangian model of continuum manipulator

- Strain Approach

$$\begin{bmatrix} 0 \\ Q_{ad} \end{bmatrix} = \begin{bmatrix} M_0 & M_{0\epsilon} \\ M_{\epsilon 0} & M_{\epsilon\epsilon} \end{bmatrix} \begin{bmatrix} \dot{\eta}_0 \\ \ddot{q}_{(t)} \end{bmatrix} + \begin{bmatrix} F_{v(q,\dot{q},\eta_0)} \\ Q_{v(q,\dot{q},\eta_0)} \end{bmatrix} + \begin{bmatrix} F_{c(q,g_0)} \\ Q_{c(q,g_0)} \end{bmatrix} + \begin{bmatrix} 0 \\ K_{\epsilon\epsilon} q_{(t)} + D_{\epsilon\epsilon} \dot{q}_{(t)} \end{bmatrix} \tag{10}$$

- Virtual serial mechanism analogy
    - Lagrangian model

$$\begin{bmatrix} F_0 \\ Q_a \end{bmatrix} = \begin{bmatrix} M_0 & M_{0\epsilon} \\ M_{\epsilon 0} & M_{\epsilon\epsilon} \end{bmatrix} \begin{bmatrix} \dot{\eta}_0 \\ \ddot{q}_{(t)} \end{bmatrix} + \begin{bmatrix} F_{v(q,\dot{q},\eta_0)} \\ Q_{v(q,\dot{q},\eta_0)} \end{bmatrix} + \begin{bmatrix} F_{c(q,g_0)} \\ Q_{c(q,g_0)} \end{bmatrix} \tag{11}$$

    - Recursive reconstruction

Introduction to the Isogeometric Collocation Method

- NURBS curves represent vector field
  - Control point as degree of freedom
  - Basis functions relate influence
- Cost function
  - Equilibrium equation evaluated at collocation points

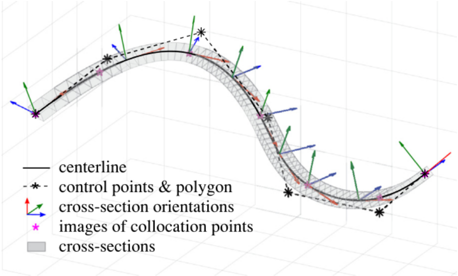

Figure: Rod centerline position and orientation represented with NURBS curves

Properties of the Isogeometric Collocation Method

- Less integrations
  - In statics no integration
  - ODE in dynamics
- Introduces possibility of modelling
  - Contact between rods
  - Changes in shape and or material
  - Rods coupling

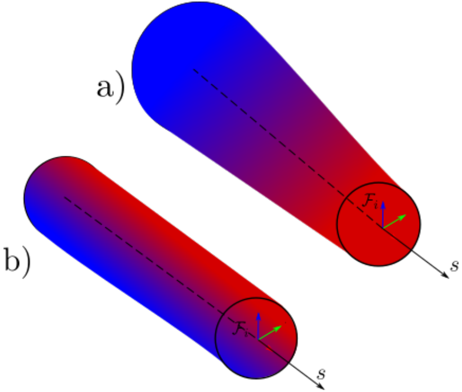

Figure: Rod with properties changing a) axially and b) transversally

Three different models

- Shooting Method
  - Integrates along the arc-lenght
  - Evaluates a cost function
  - Implicit time discretization in dynamics
- Strain Approach
  - Integrates along the arc-lenght
  - Evaluates a cost function
  - Implicit time discretization in dynamics
- Isogeometric Collocation Method
  - No integration in arc-lenght
  - Equilibrium equation in collocation points
  - Consider additional features
  - Implicit time discretization in dynamics

Expected work

- Model selection
  - Previusly presented
  - Combination
- Solve the modelling
  - Rod statics
  - Robot assembly
  - Visual interface
  - Robot dynamics

- TODO
    - TODO