

# A Gazebo Simulator for Continuum Parallel Robots

Author Gotelli Andrea

Advisors: Sébastien Briot, Olivier Kermorgant, Federico Zaccaria,  
Antonio Sgorbissa

22/02/2021

## Index

---

- ➊ Introduction to Continuum Parallel Robots (CPR)
- ➋ Modelling of deformable links
- ➌ Methods
- ➍ Conclusions

## Serial and parallel robots

- Serial robots
  - Simpler and more used
  - Limited by weight and inertia
- Parallel robots
  - Less inertia, high speed
  - More joints involved

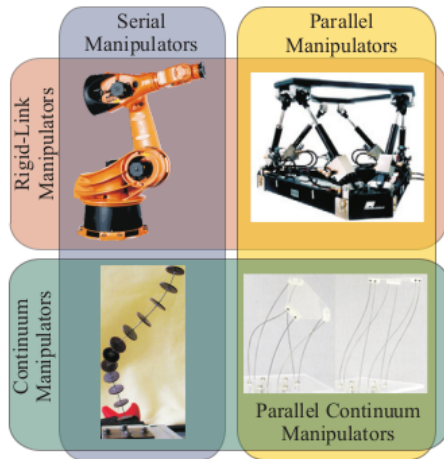


Figure: Different robot architectures [ 2].

## Continuum parallel robots

### • Advantages

- May enhance safety
- Cheaper components
- Possible to miniturize

### • Drawbacks

- More unstable configurations
- Lower payload
- Not analytical solution

### • Intention

- General simulator
- Gazebo plugins

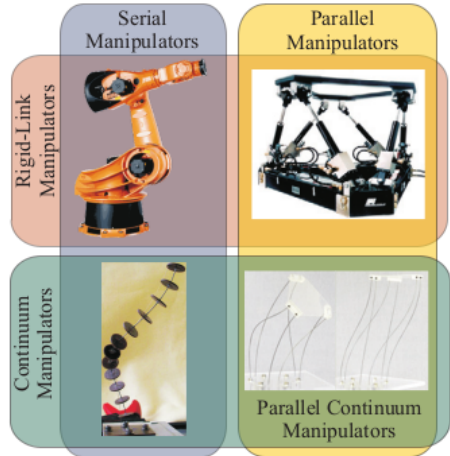


Figure: Different robot architectures [ 2].

## Geometric modelling: Cosserat rod theory

- Rod as 1D body
- Function of the arc-length  $s$ 
  - Centerline position  $p_{(s)} \in \mathbb{R}^3$
  - Cross-section orientation  $R_{(s)} \in se(3)$
- Define transformation

$$T_{(s)} = \begin{bmatrix} R_{(s)} & p_{(s)} \\ 0 & 1 \end{bmatrix} \in SE(3) \quad (1)$$

- Derivative wrt arc-length

$$x' = \frac{dx}{ds}$$

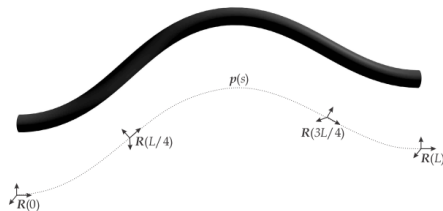


Figure: Rod geometric modelling [ 4].

## Equilibrium Equations

- Equilibrium consideration
  - Distributed forces/moments
  - Internal forces/moments

$$n'_{(s)} = -f_{(s)} \quad (2)$$

$$m'_{(s)} = -p'_{(s)} n_{(s)} - l_{(s)} \quad (3)$$

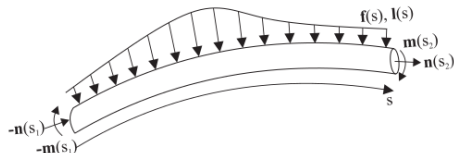


Figure: Sections of the beam considered for the static equilibrium [ 1].

## Boundary Value Problem

- Constraints at the distal plate

- External wrench  $\Psi_{ext} = \begin{bmatrix} F \\ M \end{bmatrix}$

- Rod contribution

$$\Psi_i = \begin{bmatrix} n_{i(L_i)} \\ m_{i(L_i)} \end{bmatrix}$$

- Constraints at the base

- Actuations  $\Psi_{a_i}$
- Joints and geometry

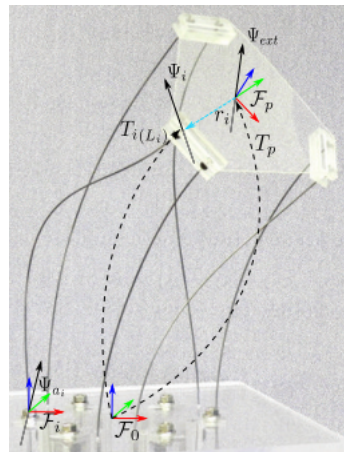


Figure: Geometrical and actuation constraints for a Stewart-Gough CPR.

## Shooting Method in statics (Stewart-Gough CPR)

- ODE system in statics
  - Equilibrium equations
  - Material properties
  - Geometrical considerations
- Recursive solution
  - Needs an initial guess
  - Evaluation on a cost function

$$\mathbf{f} = \begin{bmatrix} \sum n_{i(L_i)} - F \\ \sum \left[ p_{i(L_i)} n_{i(L_i)} + m_{i(L_i)} \right] - p_d F - M \\ p_d + R_d r_i - p_{i(L_i)} \\ \left[ R_{i(L_i)}^T R_d - R_{i(L_i)} R_d^T \right]^v \end{bmatrix} \quad (4)$$

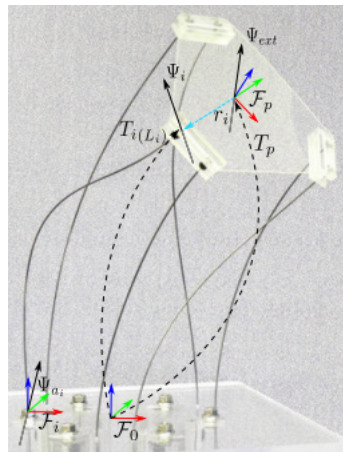


Figure: Geometrical and actuation constraints for a Stewart-Gough CPR.



## Shooting Method in dynamics

- PDE system

- Derivative wrt to arc-length  $x' = \frac{\partial x}{\partial s}$
- Derivative wrt to time  $\dot{x} = \frac{\partial x}{\partial t}$

- From PDE to ODE

- Implicit discretization

$$\frac{\partial x}{\partial t} = c_0 x^{(i)} + \sum_{k=1}^{\infty} \left[ c_k x^{(i-k)} + d_k \dot{x}^{(i-k)} \right] \quad (5)$$

$$\frac{\partial x}{\partial t} = c_0 x^{(i)} + c_1^{(i-1)} x^{(i-1)} + c_2^{(i-2)} x^{(i-2)} + d_1^{(i-1)} \frac{\partial x^{(i-1)}}{\partial t} \quad (6)$$

## Non linear solver: Levenberg-Marquardt algorithm

---

- Iterative algorithm
  - From initial conditions
  - Very sensitive
- Evaluates influence of parameter vector  $u$

$$J = \frac{df}{du} \quad (7)$$

- Updates the parameter vector

$$u_{k+1} = u_k + \left( J_k^T J_k + \mu I \right)^{-1} J_k^T f_k \quad (8)$$

## Strain approach

---

- Modelling of a continuum body in space
  - Internally actuated Cosserat beam
  - In its dynamics

$$\eta = \left( T^{-1} \dot{T} \right)^V \qquad \xi = \left( T^{-1} T' \right)^V \qquad (9)$$

- From assumption on rod deformation
  - Allowed  $\xi_a$ , prohibited  $\xi_c$  twists
  - Strain generalized coordinates  $q_{[n \times 1]}$
- Define its dynamic model

## Strain approach, Lagrangian model of continuum manipulator

- Lagrangian model of continuum

$$\begin{bmatrix} 0 \\ Q_{ad} \end{bmatrix} = \begin{bmatrix} M_0 & M_{0\epsilon} \\ M_{\epsilon 0} & M_{\epsilon\epsilon} \end{bmatrix} \begin{bmatrix} \dot{\eta}_0 \\ \ddot{q}(t) \end{bmatrix} + \begin{bmatrix} F_v(q, \dot{q}, \eta_0) \\ Q_v(q, \dot{q}, \eta_0) \end{bmatrix} + \begin{bmatrix} F_c(q, g_0) \\ Q_c(q, g_0) \end{bmatrix} + \begin{bmatrix} 0 \\ K_{\epsilon\epsilon} q(t) + D_{\epsilon\epsilon} \dot{q}(t) \end{bmatrix} \quad (10)$$

- Virtual serial mechanism analogy

- The Lagrangian model

$$\begin{bmatrix} F_0 \\ Q_a \end{bmatrix} = \begin{bmatrix} M_0 & M_{0\epsilon} \\ M_{\epsilon 0} & M_{\epsilon\epsilon} \end{bmatrix} \begin{bmatrix} \dot{\eta}_0 \\ \ddot{q}(t) \end{bmatrix} + \begin{bmatrix} F_v(q, \dot{q}, \eta_0) \\ Q_v(q, \dot{q}, \eta_0) \end{bmatrix} + \begin{bmatrix} F_c(q, g_0) \\ Q_c(q, g_0) \end{bmatrix} \quad (11)$$

- Recursive reconstruction

## Introduction to the Isogeometric Collocation Analysis

- NURBS curves represent vector field
  - Control point as degree of freedom
  - Basis functions relate influence
- Cost function
  - Equilibrium equation evaluated at collocation points

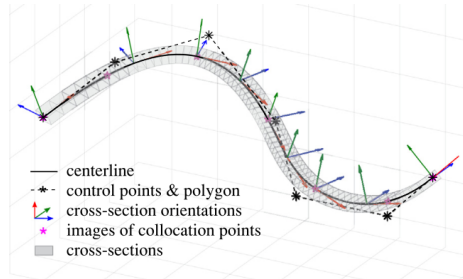


Figure: Rod centerline position and orientation represented with NURBS curves [ 3].

## Properties of the Isogeometric Collocation Analysis

- Less integrations
  - In statics no integration
  - ODE in dynamics
- Introduces possibility of modelling
  - Contact between rods
  - Changes in shape and or material
  - Rods coupling

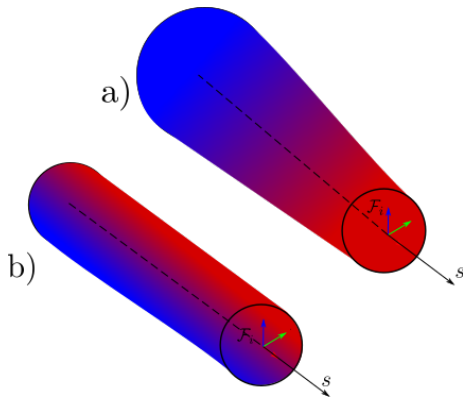


Figure: Rod with properties changing a) axially and b) transversally

## Conclusions

---

- Types of robots
  - Serial robots
  - Parallel robots
  - Continuum parallel robots
- Model of deformable links
  - Geometric
  - Equilibrium
  - Assembly
- Methods
  - Shooting Method
  - Strain Approach
  - Isogeometric Collocation Method

## Expected work

---

- A Gazebo Simulator for Continuum Parallel Robots
- Immediate terms: Method(s) selection
  - Previously presented
  - One or combination
- Short terms: Solve the modelling
  - Rod statics
- Long terms
  - Robot assembly
  - Visual interface
  - Robot dynamics



*Thank you for your attention*



Black, C. B., J. Till, and D. C. Rucker (Feb. 2018). “Parallel Continuum Robots: Modeling, Analysis, and Actuation-Based Force Sensing”. In: *IEEE Transactions on Robotics* 34.1. Conference Name: IEEE Transactions on Robotics, pp. 29–47.



Bryson, C. E. and D. C. Rucker (May 2014). “Toward parallel continuum manipulators”. In: *2014 IEEE International Conference on Robotics and Automation (ICRA)*. 2014 IEEE International Conference on Robotics and Automation (ICRA). ISSN: 1050-4729, pp. 778–785.



Piegl, L. A. and W. Tiller (1997). *The NURBS book*. 2nd ed. Monographs in visual communications. Berlin ; New York: Springer. 646 pp.



Till, J. (May 1, 2019). “On the Statics, Dynamics, and Stability of Continuum Robots: Model Formulations and Efficient Computational Schemes”. In: *Doctoral Dissertations*.