Statistics!

Probability

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- What exactly is probability?
  - Well it’s how probable something is
- Let’s think about it a different way
  - Probability is how likely something is to occur
  - You can think of probability as the % chance something happens or is in a certain state

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    - There’s a probability of each outcome happening?

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  - We just logic’ed out a fundamental theorem to probability theory
  - Law of total probability

Let’s keep going

- What’s the lowest the probability of something happening can be?

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The probability of something must be between 0 and 1!

And the sum of the probability of all outcomes must be 1

Let’s use this to solve a problem

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P(red or not red) = 1 = P(red) + P(not red) **Law of total probability**

P(not red) = 1 - P(red) = 1 - 0.4 = 0.6

Probability of discrete outcomes

- Discrete outcomes are things that are specific
  - Red or not red
  - Heads or tails for a coin
  - 1-6 for a dice
  - Also, counting numbers, or integers

- The function that describes the probability of each outcome is called the Probability Mass Function

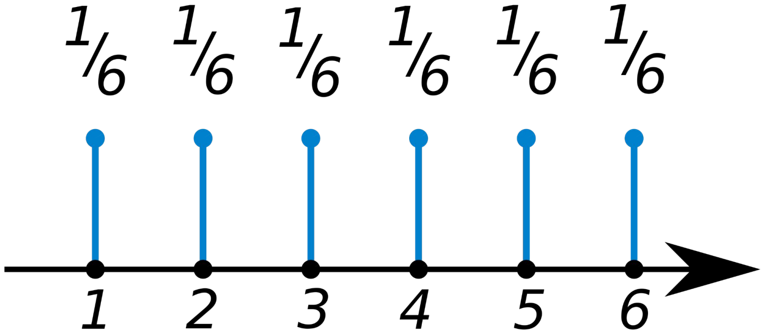
Probability Mass Function (PMF)

PMF for a coin flip

Heads = 1, tails = 0

$$p_X(x) = \begin{cases} \frac{1}{2}, & x = 0, \\ \frac{1}{2}, & x = 1, \\ 0, & x \notin \{0, 1\}. \end{cases}$$

PMF for a dice roll



What about outcomes that are not discrete?

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What is the probability that I could press stop and stop it exactly at 3 seconds?

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Ex: all decimal numbers

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It has no numbers, remember

But if it did have numbers then the probability would be > 0

If probability is only non-zero over intervals, how do we know how probable something is around some value?

- Probability density function (PDF)
    - $p(x) = dP/dx$
    - The derivative of the probability

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The probability over an interval is then the definite integral

$$P(-2 \leq x \leq 2) = \int_{-2}^{2} p(x)\, dx$$

Statistics time

Statistics is the study of data

Data is a set of observations

Let’s say we have a sample of data,

$x_i \sim \{x_1, x_2, .. x_N\}$

What are some statistical measures you know?

Statistics time

Let's say we have a sample of data,

$x_i \sim \{x_1, x_2, .. x_N\}$

$$mean = \bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$x_i \sim \{1, 5, 3\}$

Mean = (1 + 5 + 3) / 3 = 3

$$Variance = \sigma^2 = \frac{1}{N}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2$$

Var = $((1-3)^2 + (5-3)^2 + (3-3)^2)$ / 3 = (4 + 4 + 0)/3 = 8/3

What is variance?

$$Variance = \sigma^2 = \frac{1}{N}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2$$

It’s the average deviation from the average squared

- The average of the value $(x - <x>)^2$

You may more often here the term standard deviation

standard deviation = σ = √(Variance)

This is the average distance to the mean

These are measures of how spread out your data is

Going one further there's also a measure called skewness

The average deviation from the mean cubed

$$skew = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{x_i - \bar{x}}{\sigma}\right)^3$$

This is a measure of how asymmetric your data is

Moments

The mean, variance, and skew of your data are also known as

the first, second, and third moments

These give you the

- Location
- Spread
- Asymmetry

Of the underlying distribution of your data

Data taking from the perspective of probability theory

When you make a measurement for a data point you are sampling from an underlying distribution

As you take more data the underlying distribution becomes more clear

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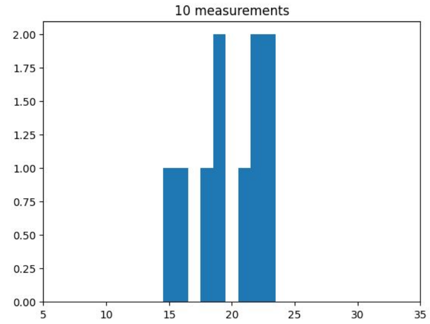

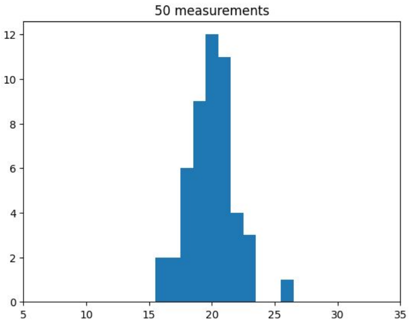

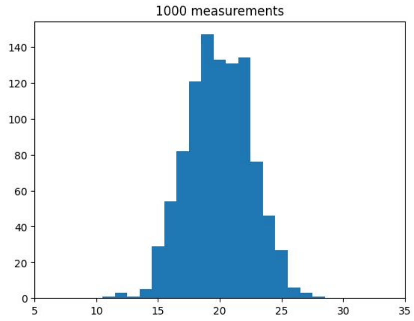

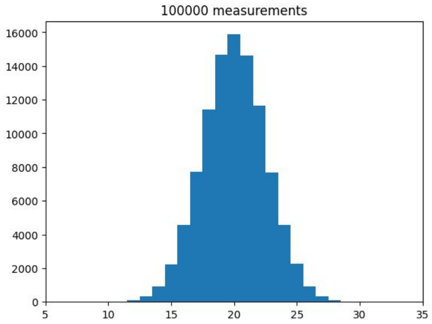

Histograms

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- A histogram is the number of values in each bin

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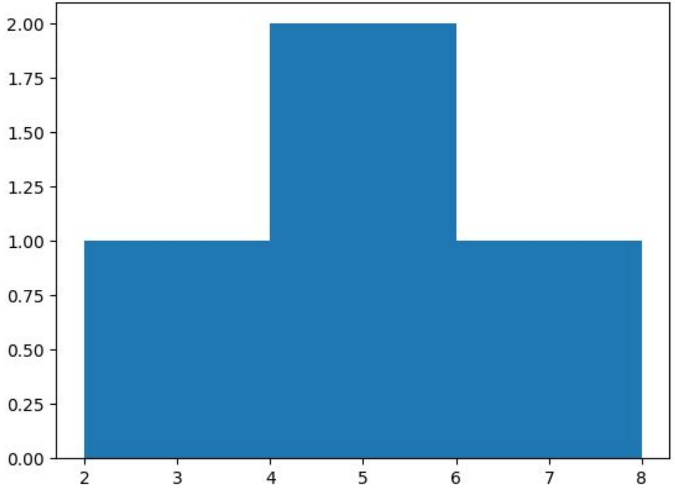
x = [2.3, 4.5, 5.5, 7.1]

Bins = [2, 4, 6, 8]

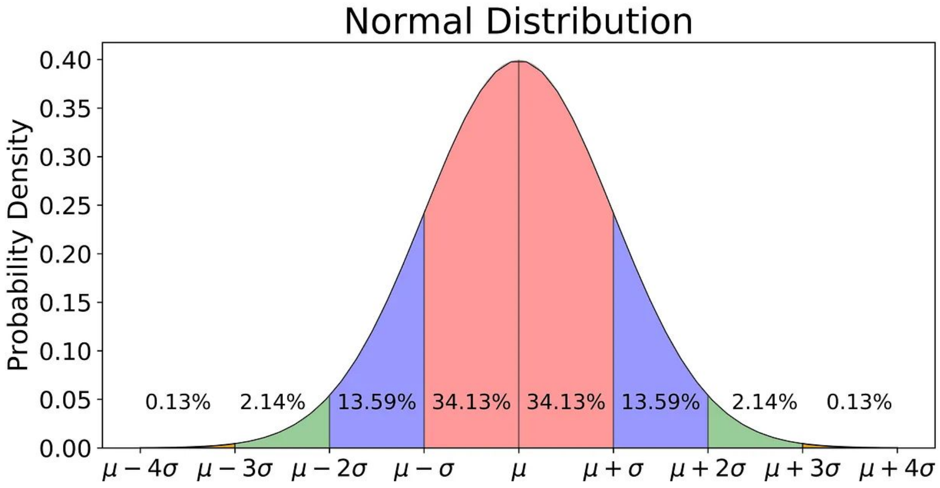


Normal distribution

One of the most common probability distributions is a Normal distribution

Sometimes also known as a Gaussian

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



Normal distribution

Normal distributions are so common due to something called the

Central Limit Theorem -

If you take the average of many data samples all from the same underlying distribution, those averages will follow a Normal distribution

Let’s move on to the tutorial to see an example of this