Finding error bars/PDF of model parameters

Last Time

- We learned how to find the “best-fit” parameter
  - Using least squares fitting
  - calculating chi2, finding where chi2 is at its min
- Today
  - We’ll go further with our chi2 statistics
  - Find error PDFs / bars around our “best-fit”

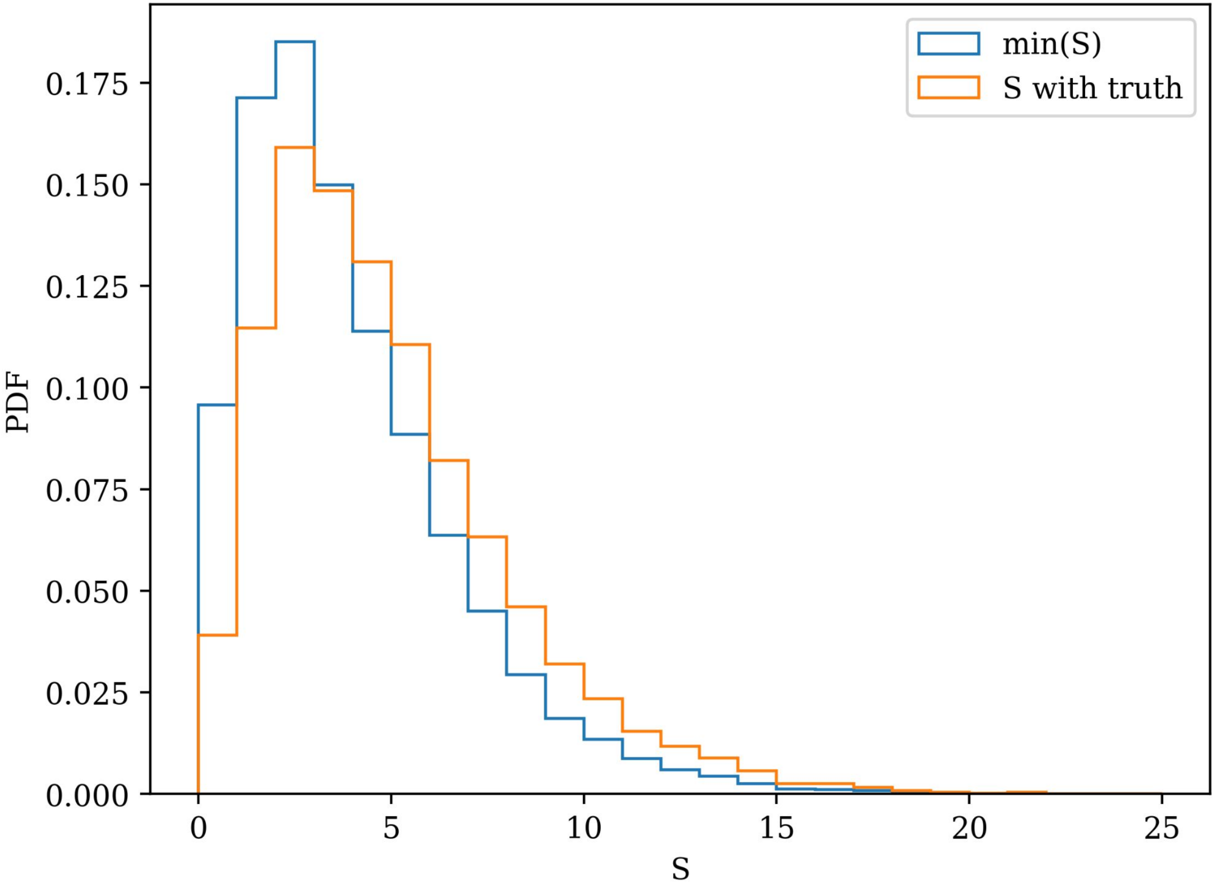
Previously we looked at the distribution of chi2 or S values assuming a = 10 m/s2, the true value

$S = \Sigma_i \left((v(t_i) - v_i) / \sigma_i \right)^2$

$v(t) = a*t$

What would happen if we didn't know the true value of a, and we instead used the best fit?

S got smaller

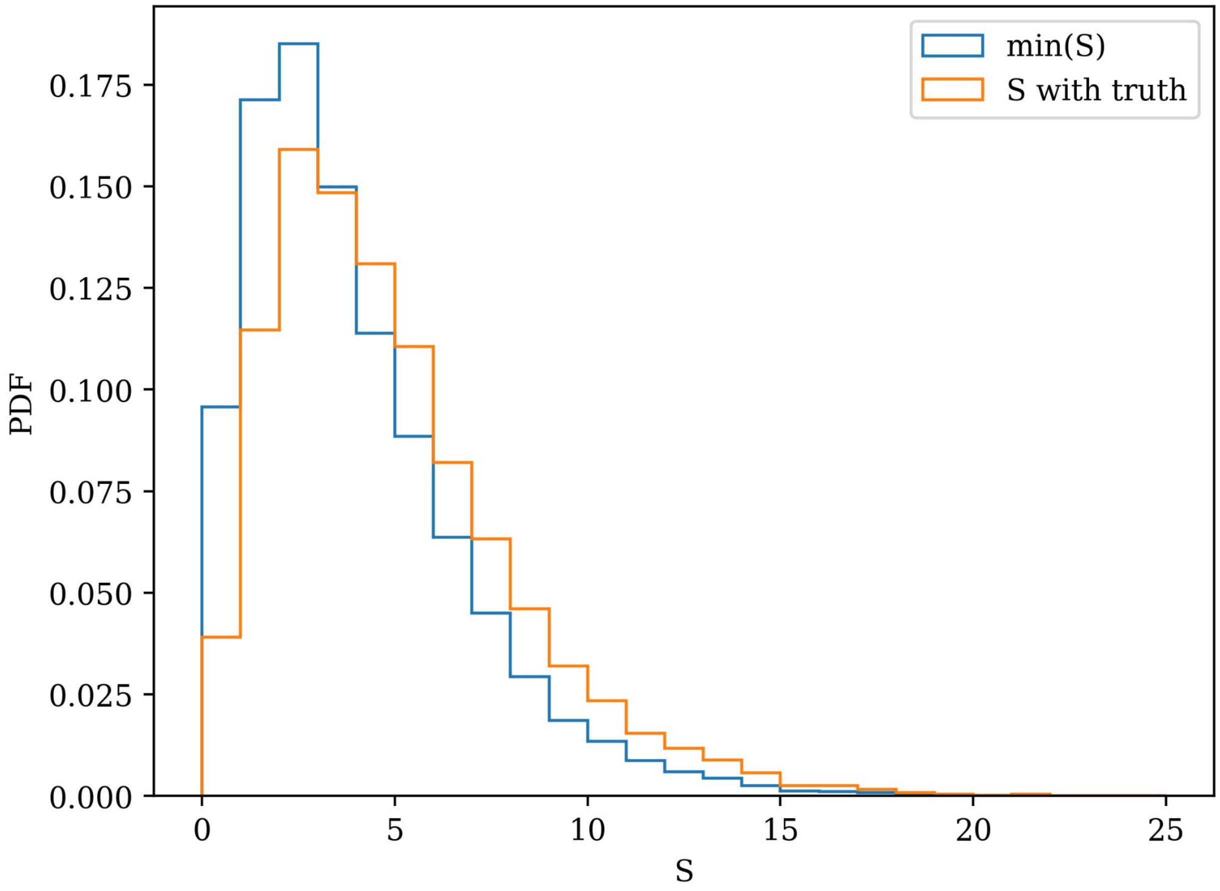


S got smaller

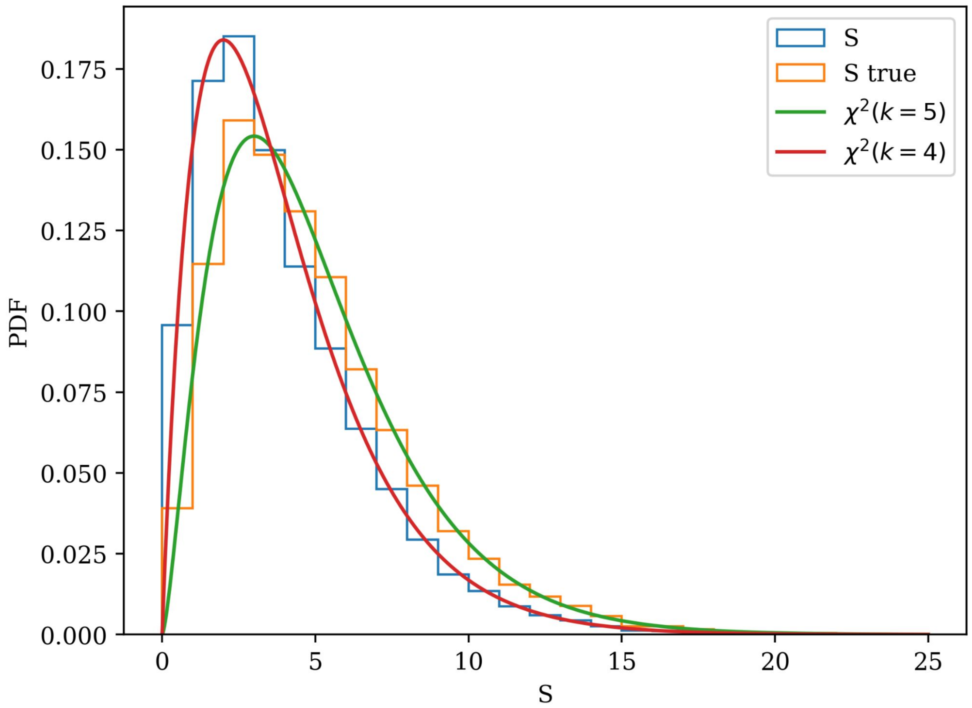
Makes sense

- we minimized it

But what happened to the distribution?



It now follows a chi2 with k = 4



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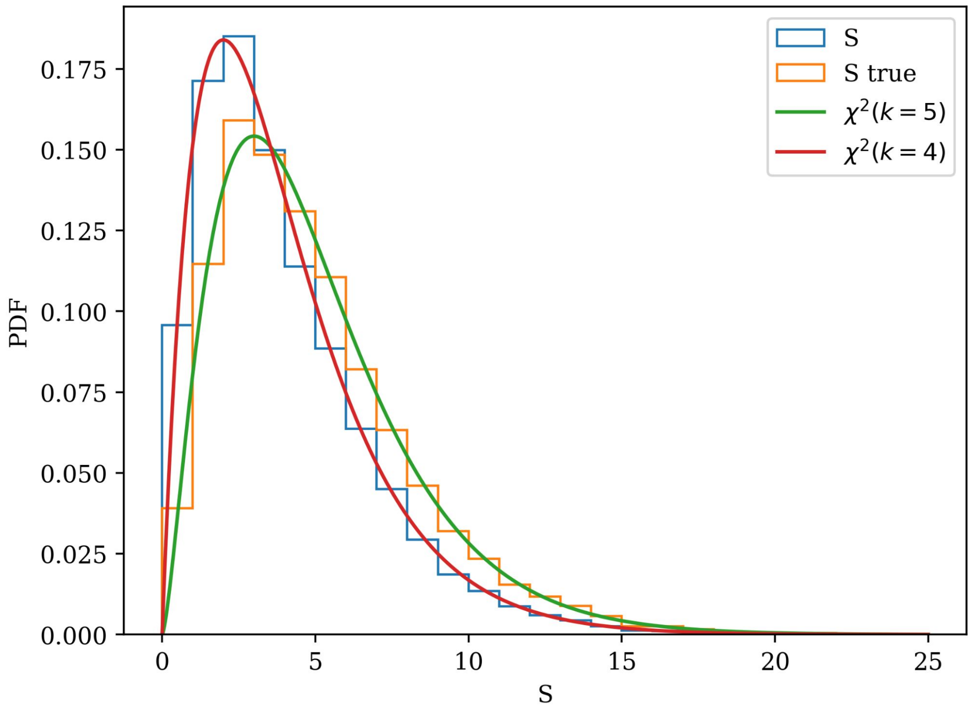

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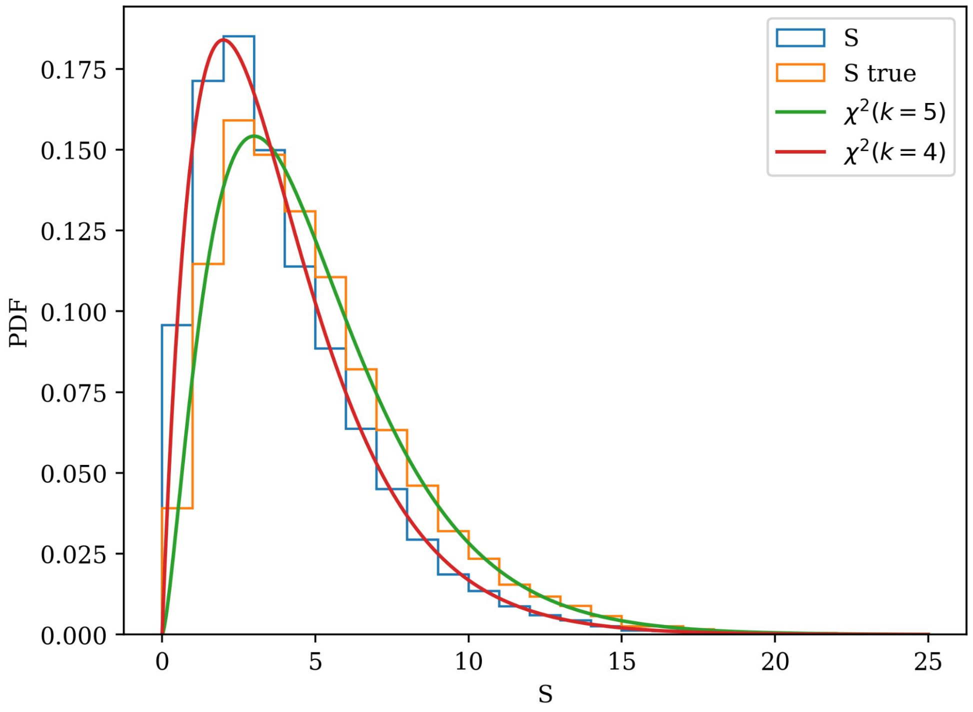
Why?

S is a sum of random, independent normally distributed variables

If instead we first know the best fit slope,

It is no longer completely independent, we used the data to calculate m



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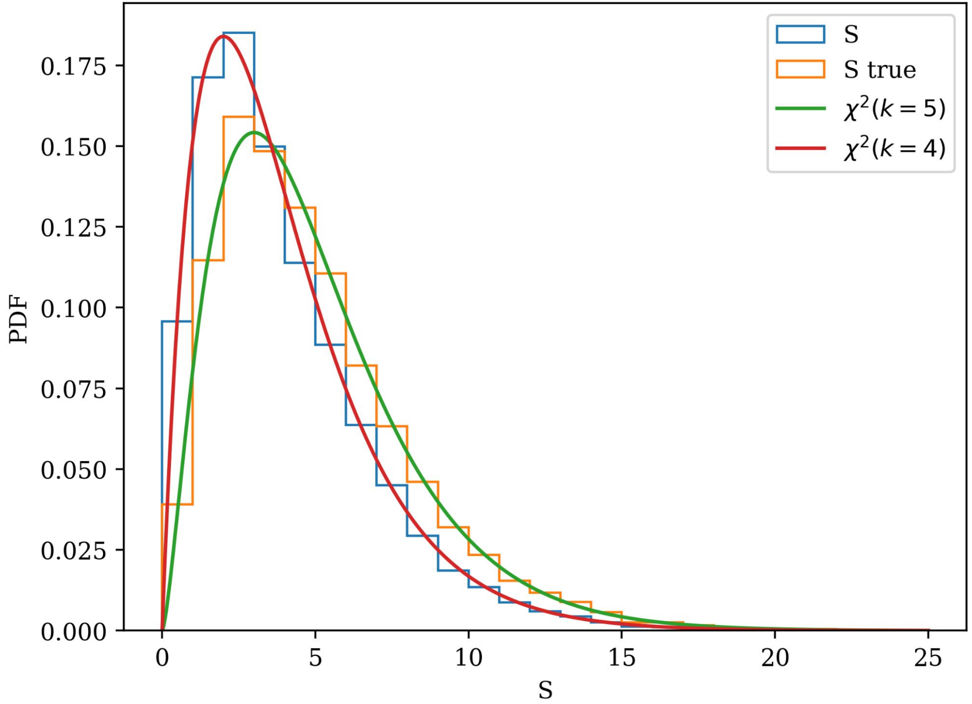

If we already know best fit m, and the first 4 data point, could we calculate the 5th data point?

Statistical degrees of freedom

k is often called the degrees of freedom

If we optimize (minimize of chi2 or S) over free parameters we constrain the number of degrees of freedom

Statistical dof or k = N - m

N = independent data points

m = number of parameters we fit

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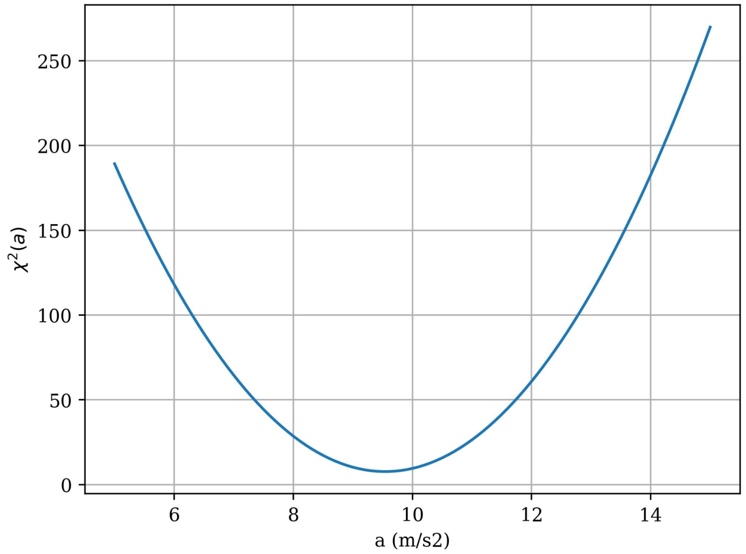
Last time we had a chi2 curve as a function of a

chi2(a= best a) is at the min

chi2(a = Truth) is at some larger value



What if we looked at the difference

chi2(a = Truth) - min(chi2)

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Run a bunch of the same experiment

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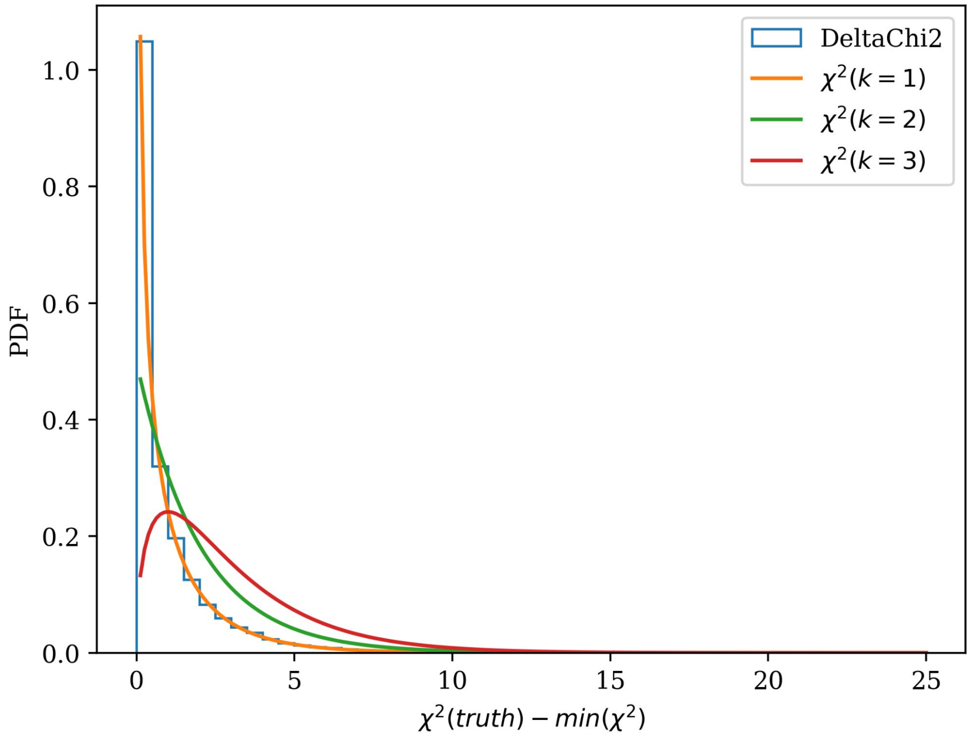
It looks like a chi2 dist with k = 1!



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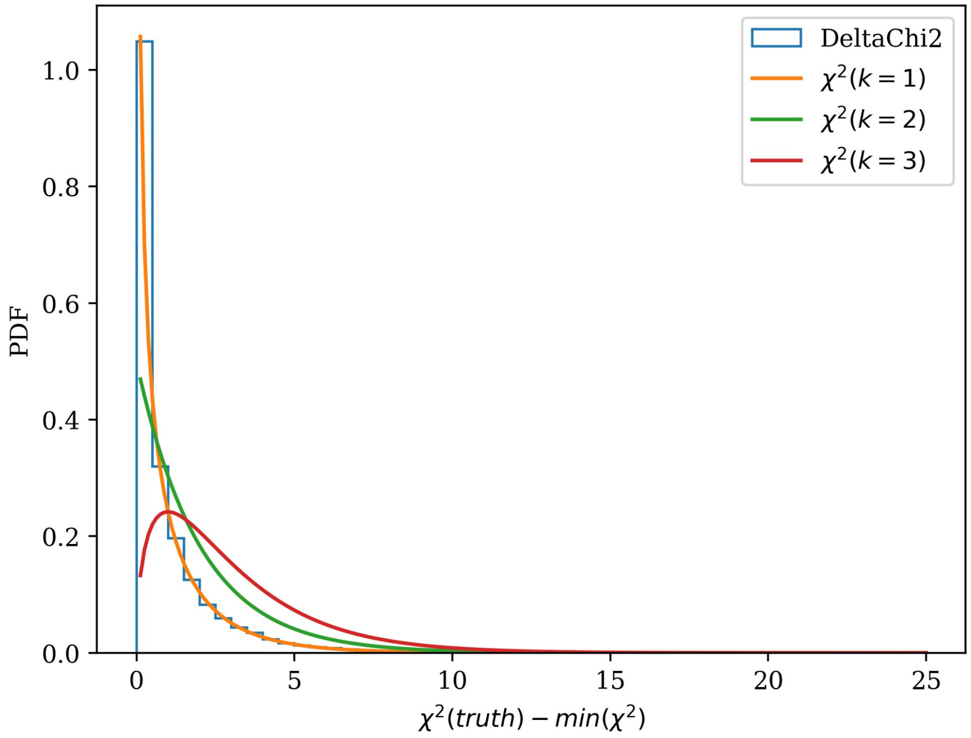

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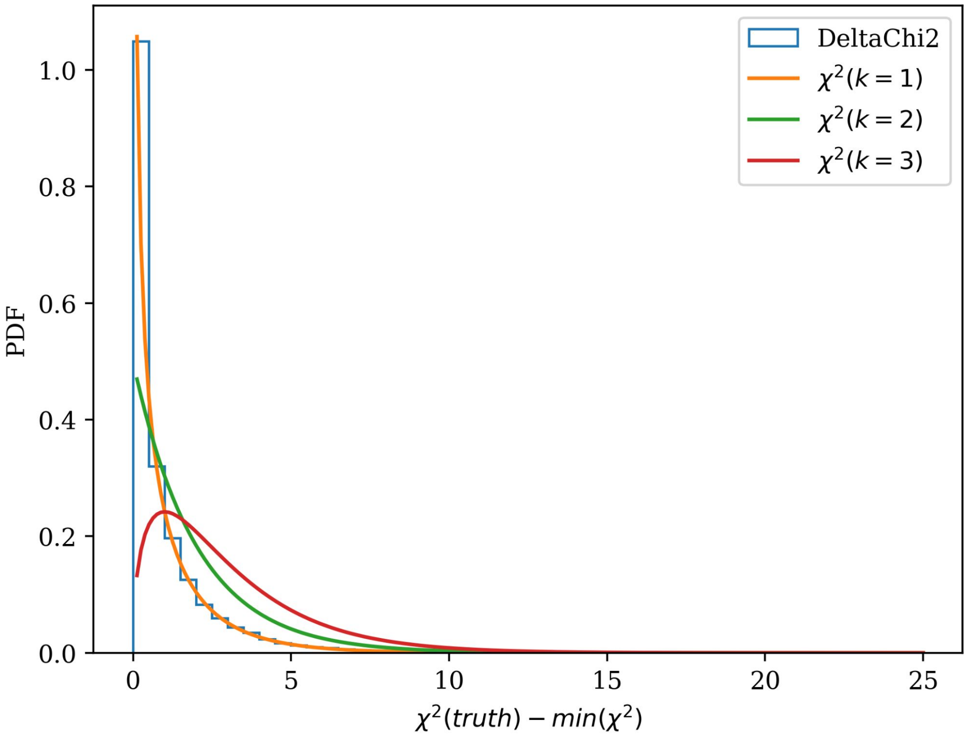
Why?

Both chi2’s are calculated using the same data

the only remaining degree of freedom is the best fit a



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- Can find this with stats.ppf(0.68, 1)

68% chance chi2(truth) is within (min(chi2), min(chi2) + 1)

Let’s look at that on our plot

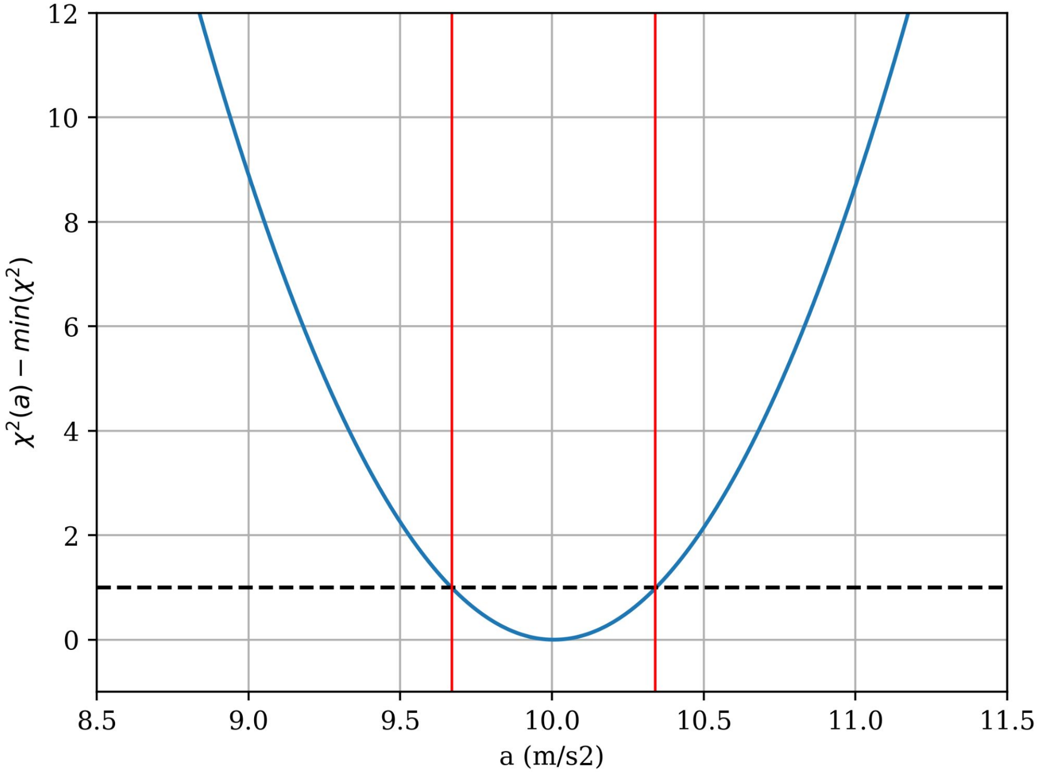
Then prob that a is in the region where

chi2 <= min(chi2) + 1

is also 68%

68% confidence that a is between 9.67 and 10.34 m/s2

a = 10 +/- 0.33 m/s2



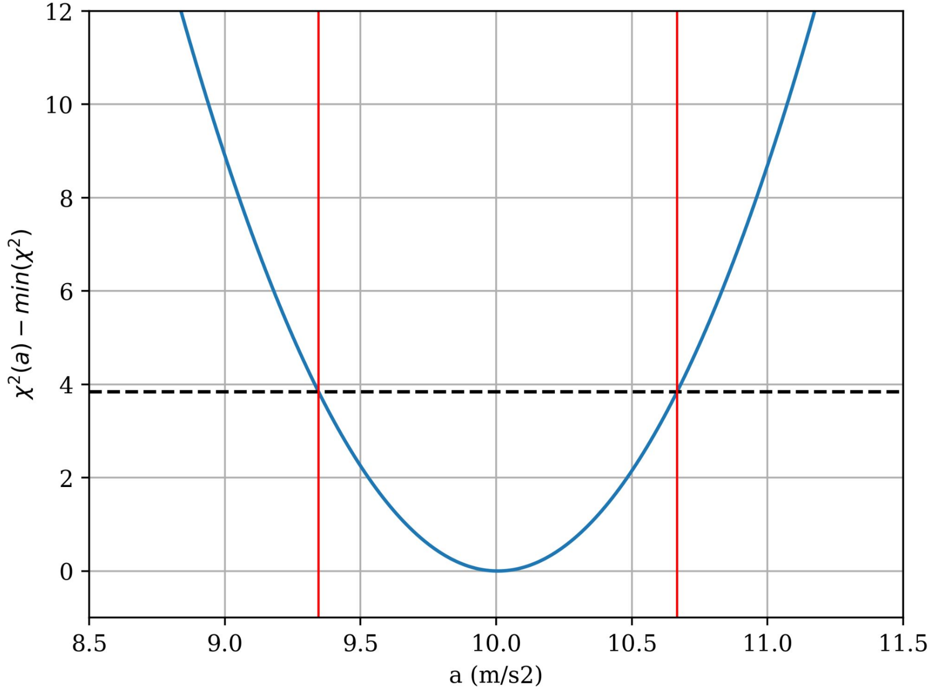
Different data than last lecture
Best-fit a ~ 10 m/s2

How about 95%?

stats.chi2.ppf(0.95, 1) = 3.84

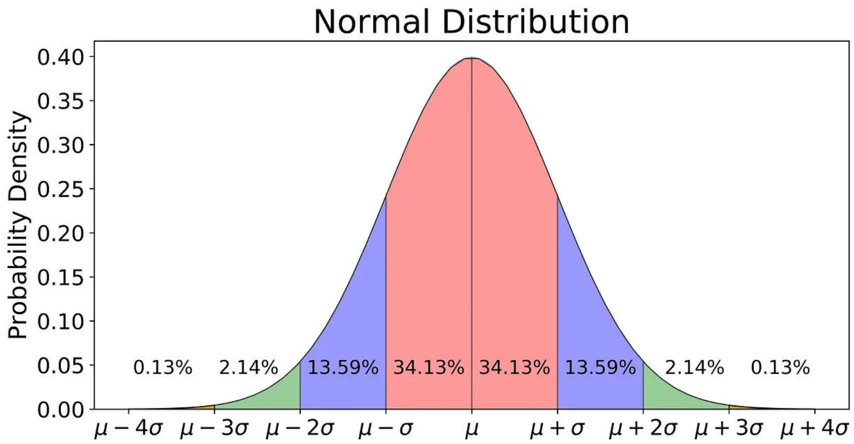
95% confidence 9.34 - 10.67

a = 10 +/- 0.67 m/s2



68% error -> 1 sigma confidence

95% error -> 2 sigma confidence



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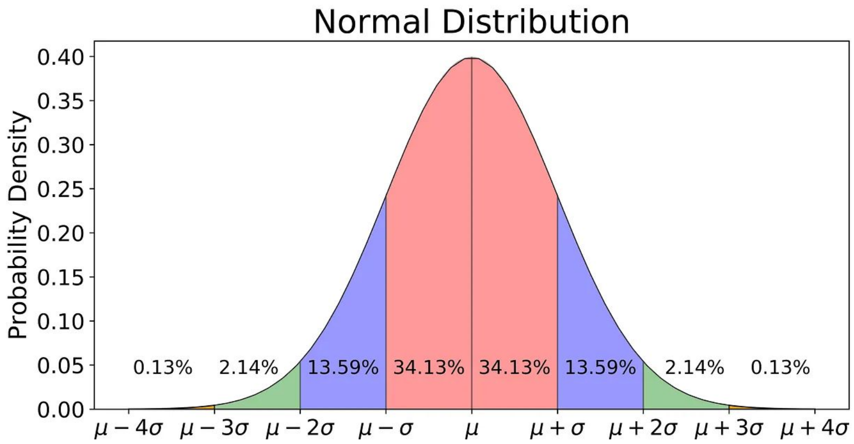
Our 68% error = +/- 0.33 m/s2

Our 95% error = +/- 0.66 m/s2

Scaling like a Gaussian!

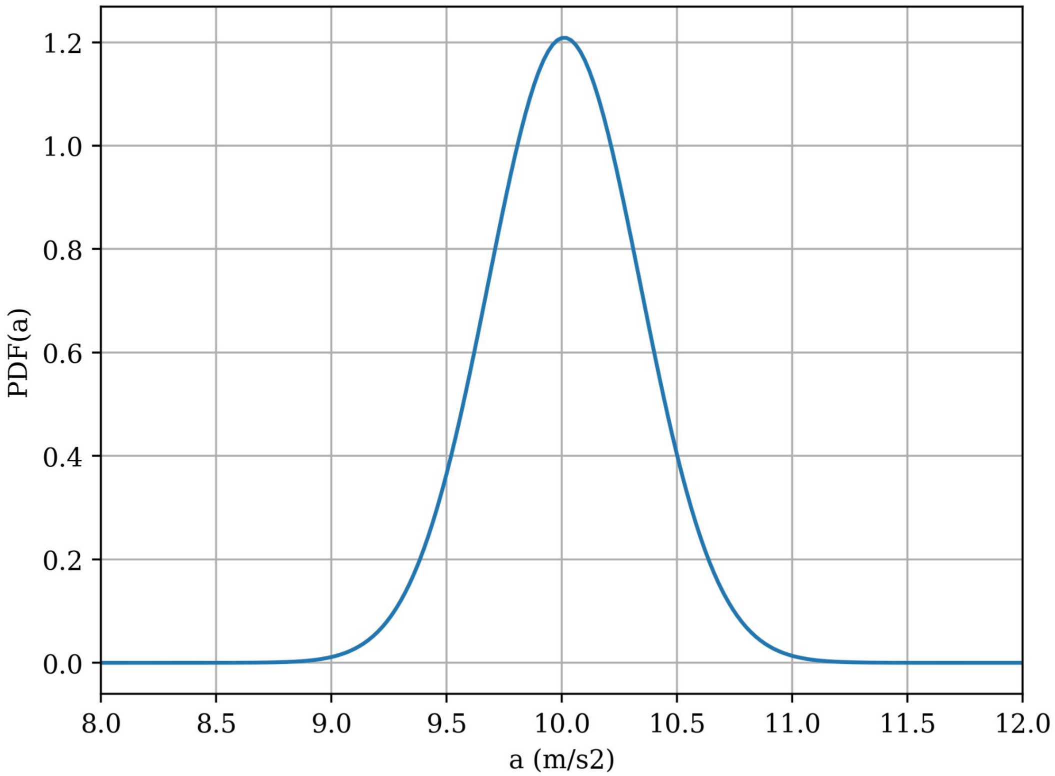


Sigma can be found from chi2(a)

Error PDF can than be a Gaussian with mu = best a

This will not always be the case, but is safe enough to assume with

- enough data
- Gaussian data errors



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chi2(a, v0)

Now it’s a 2D parameter space

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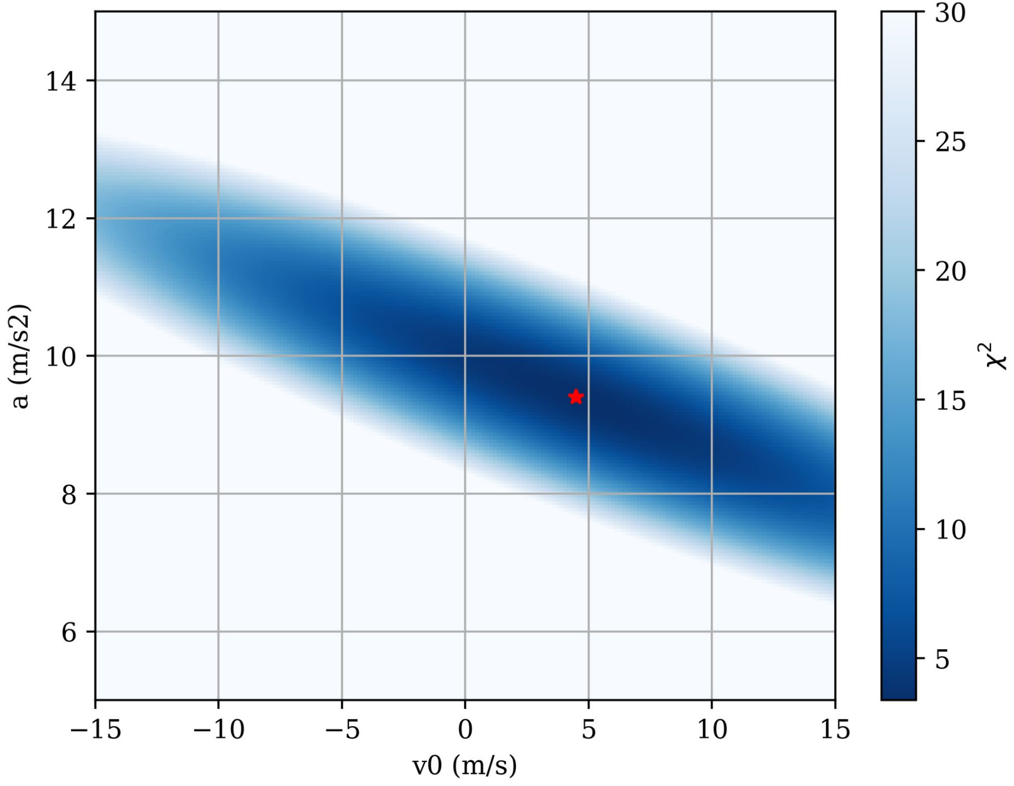

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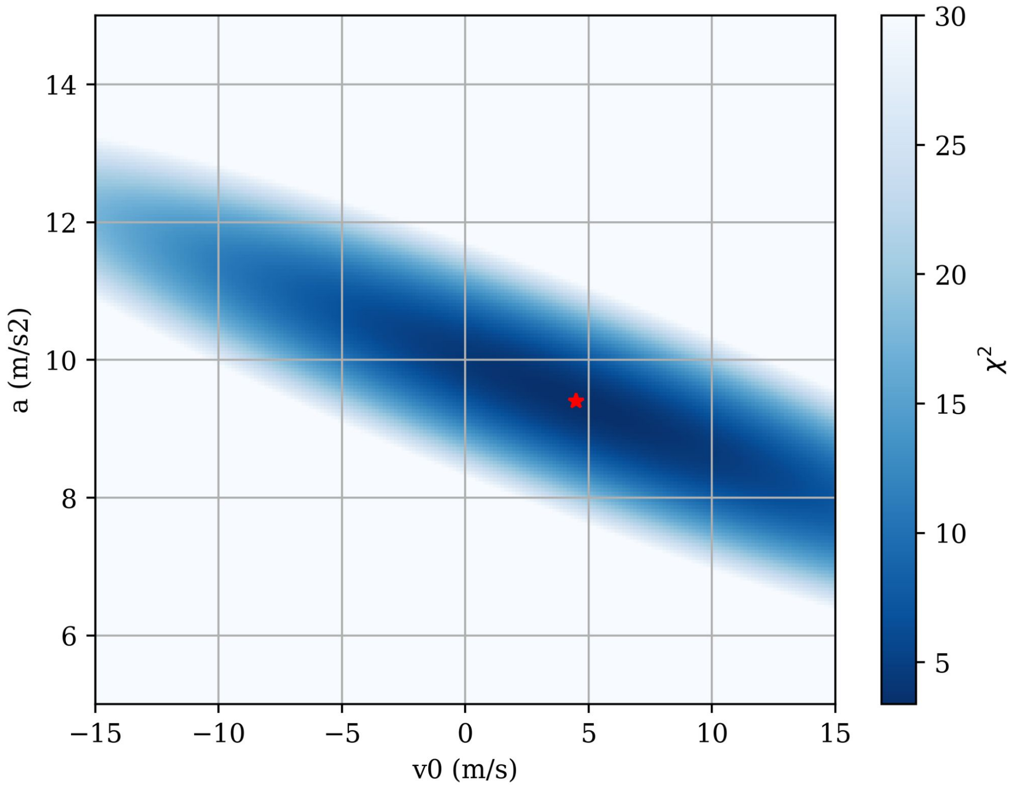
Best a = 9.4 m/s2

Best v0 = 4.5 m/s



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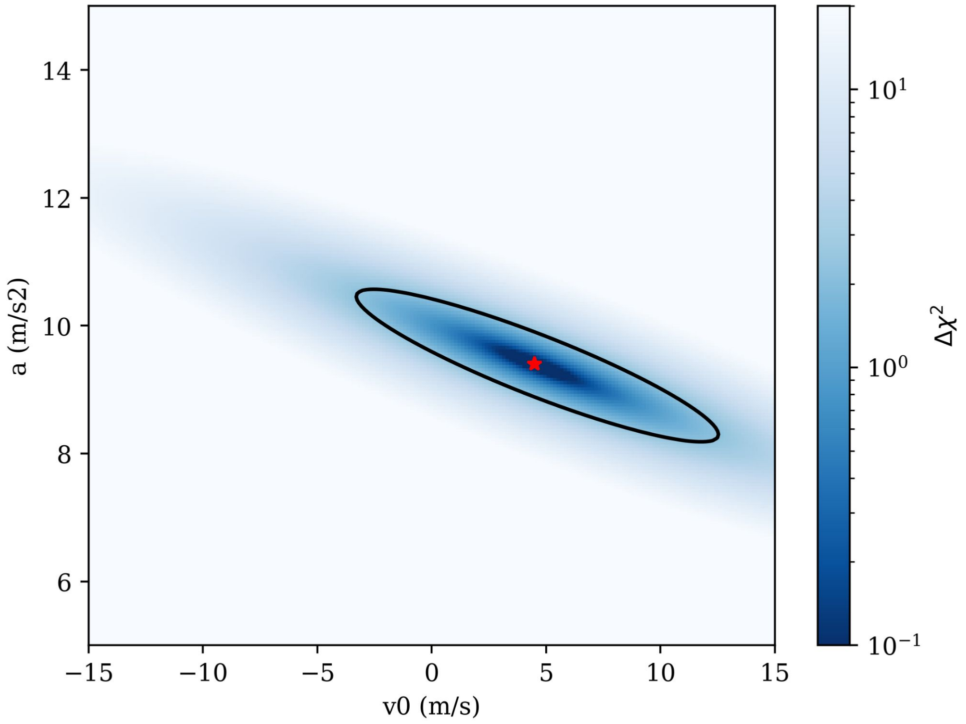
But how do we map the 1D bounds to 2D?

- A 2D contour!

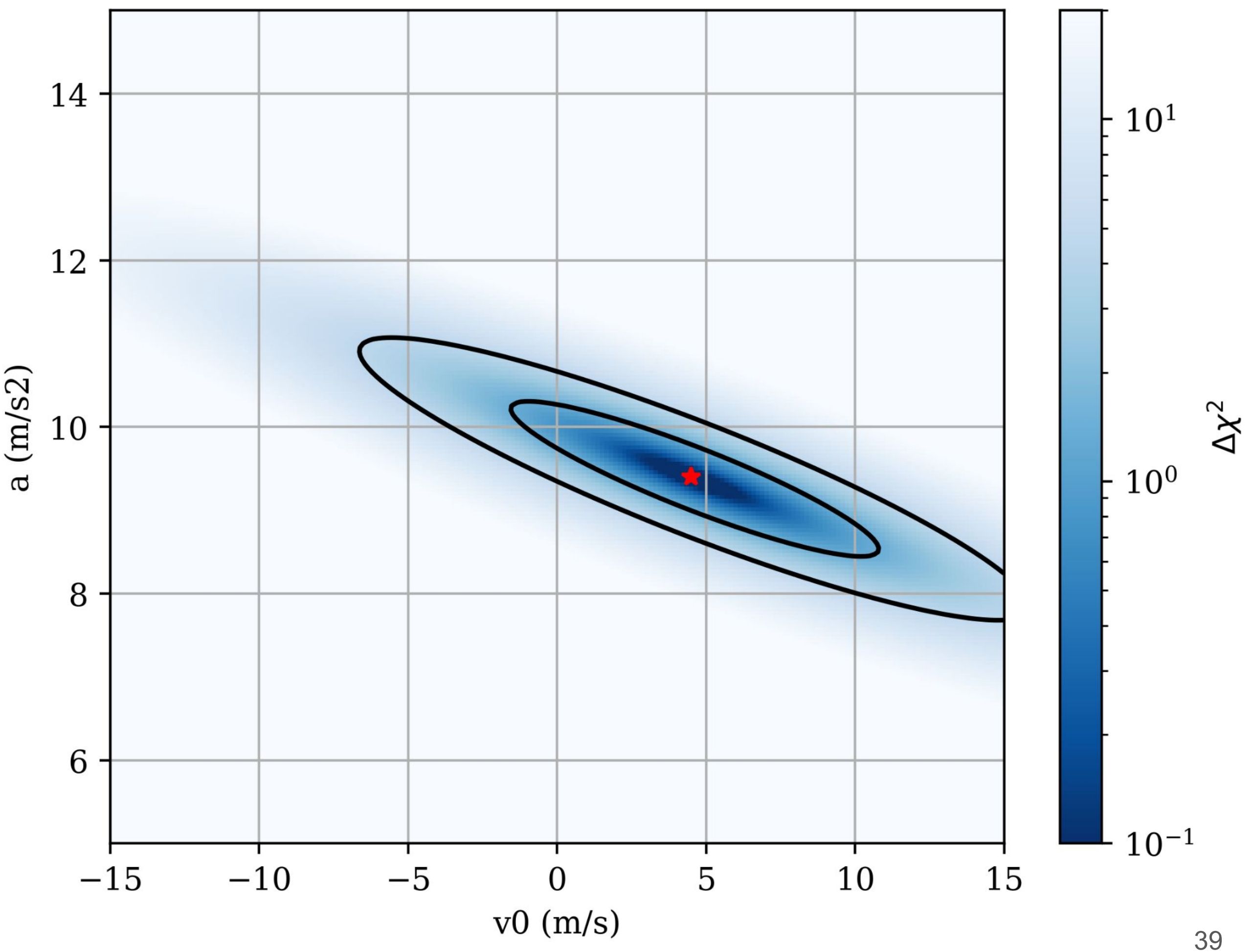


68% contour

stats.chi2.ppf(0.5, 2) = 1.39

stats.chi2.ppf(0.9, 2) = 4.6

90% probability true a and v0 is inside the 90% contour



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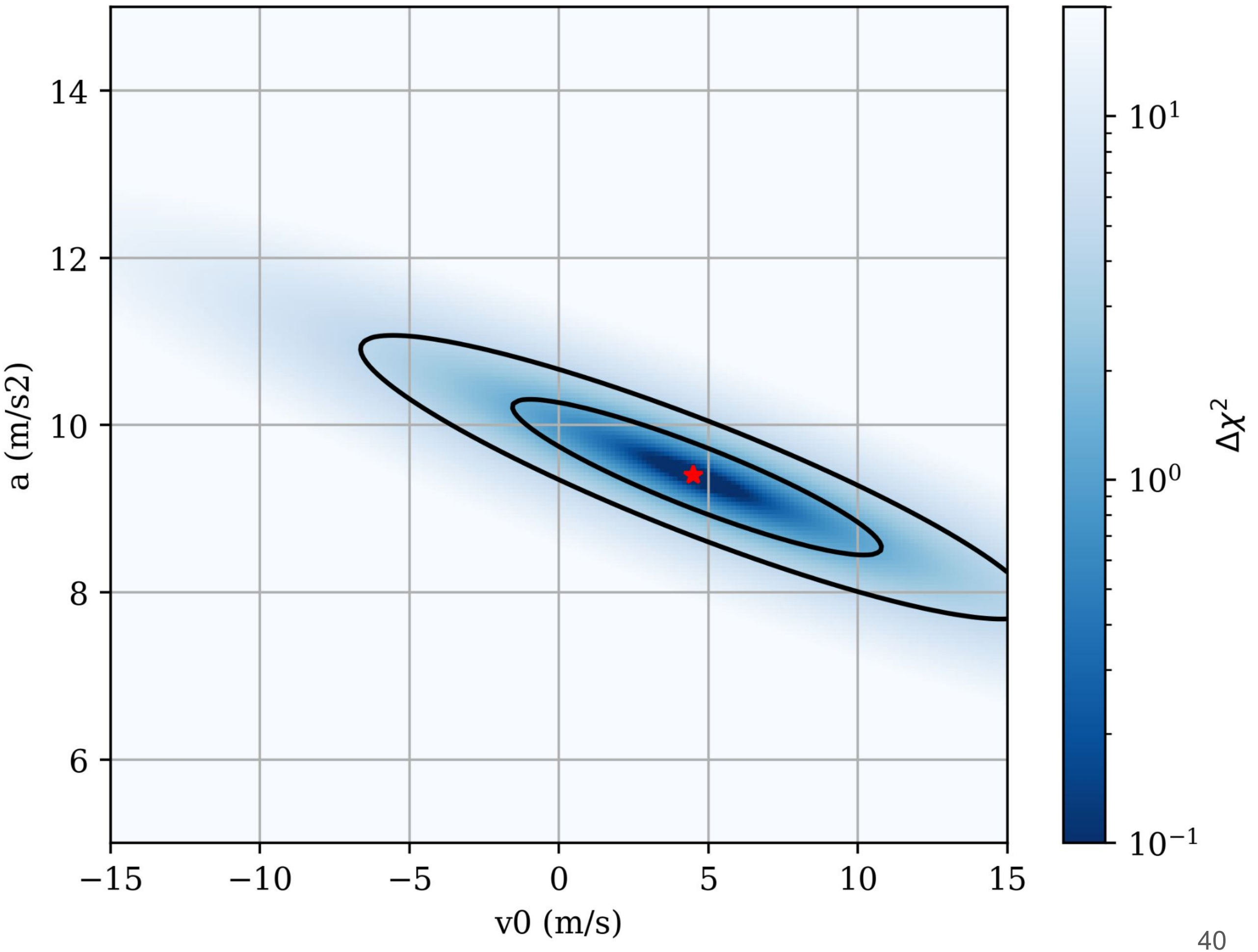

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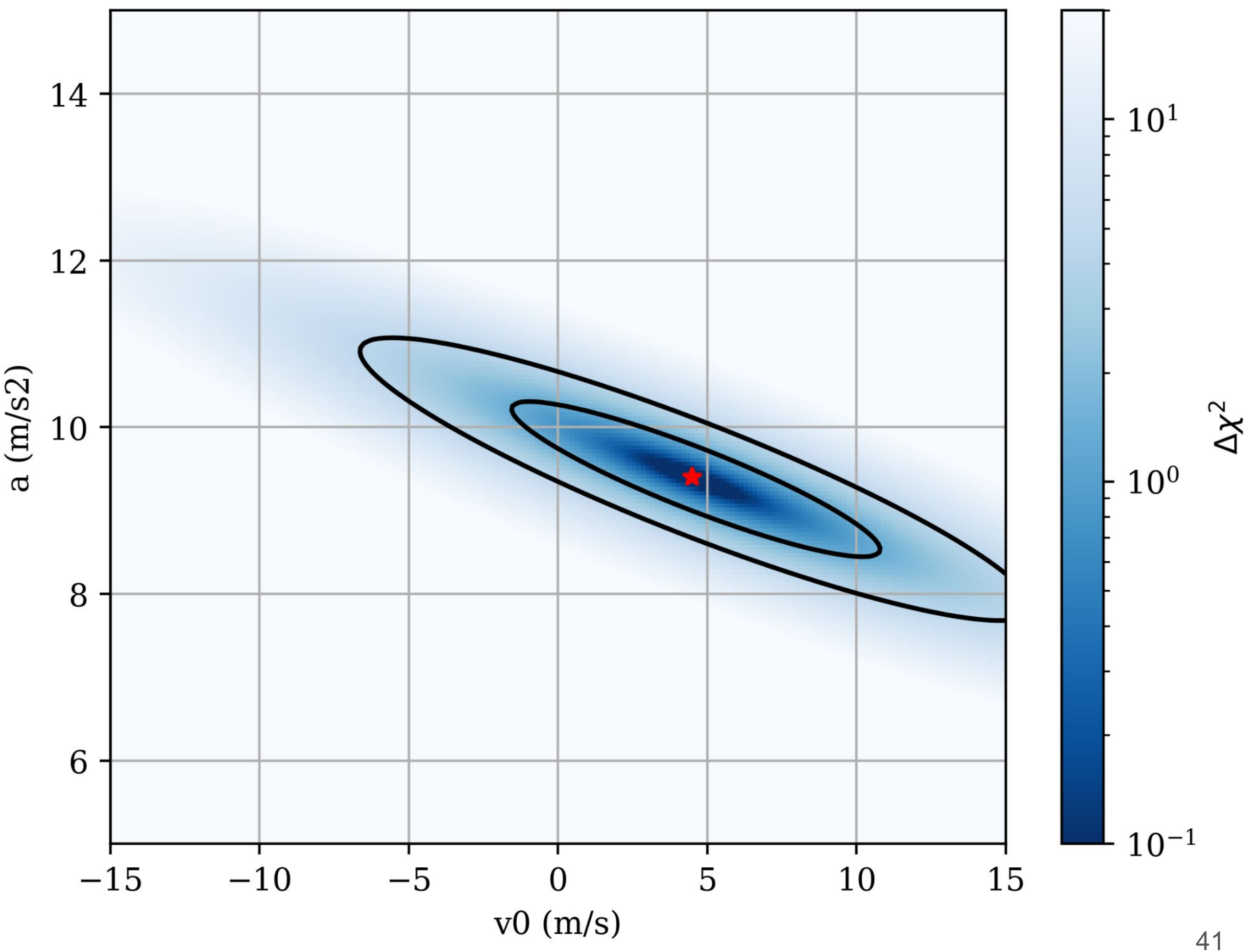
The a and v0 values that give lines that can describe the data are correlated



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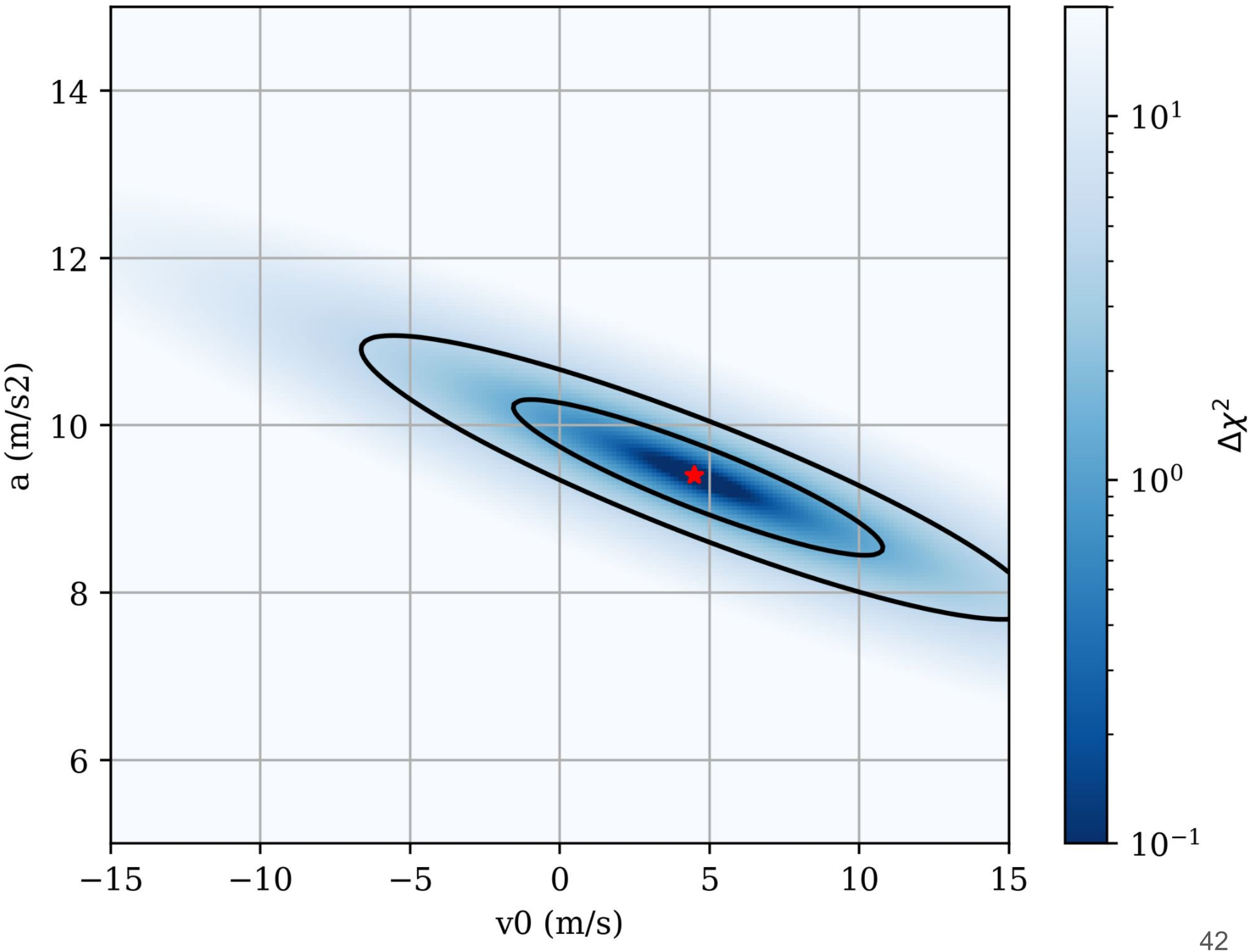

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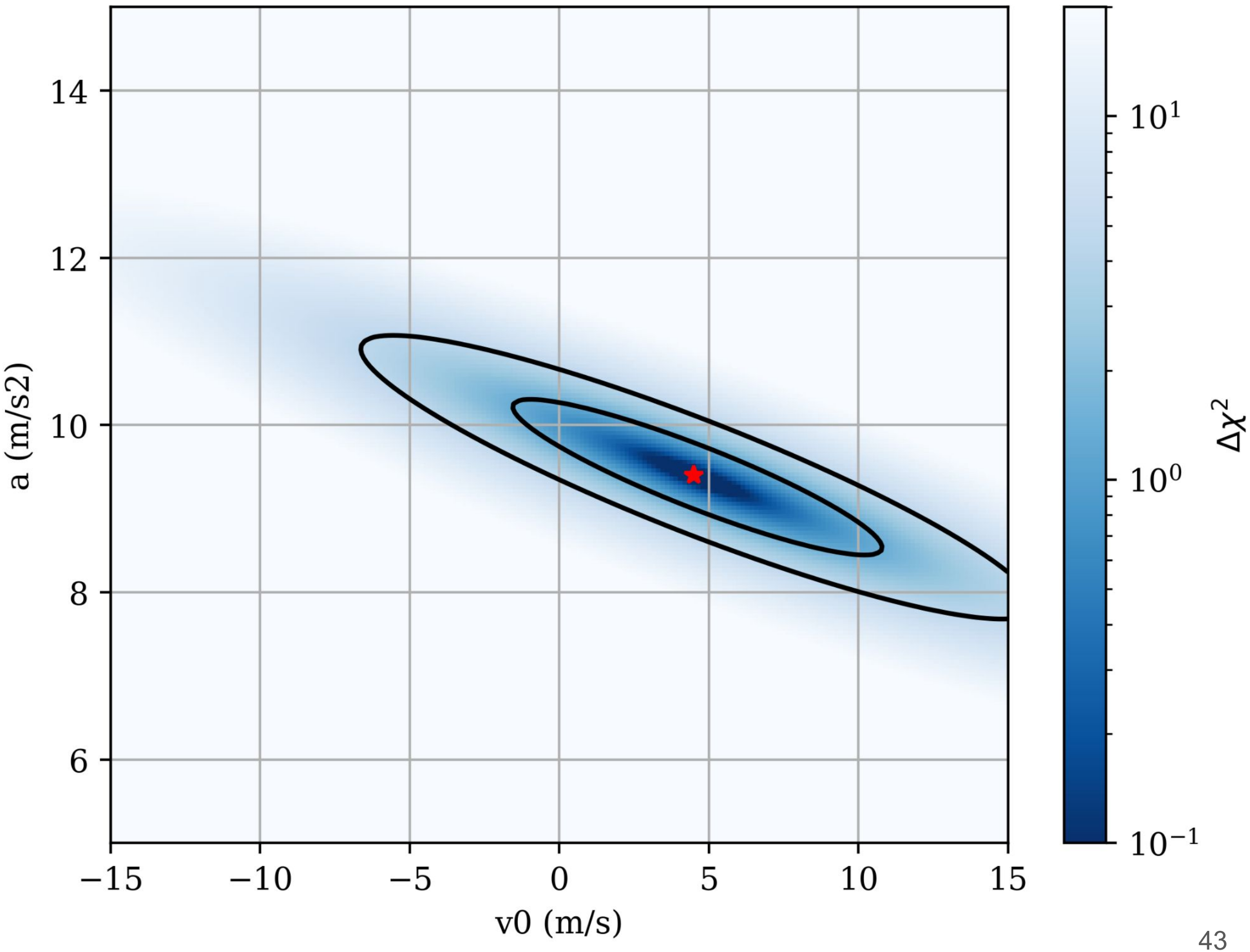

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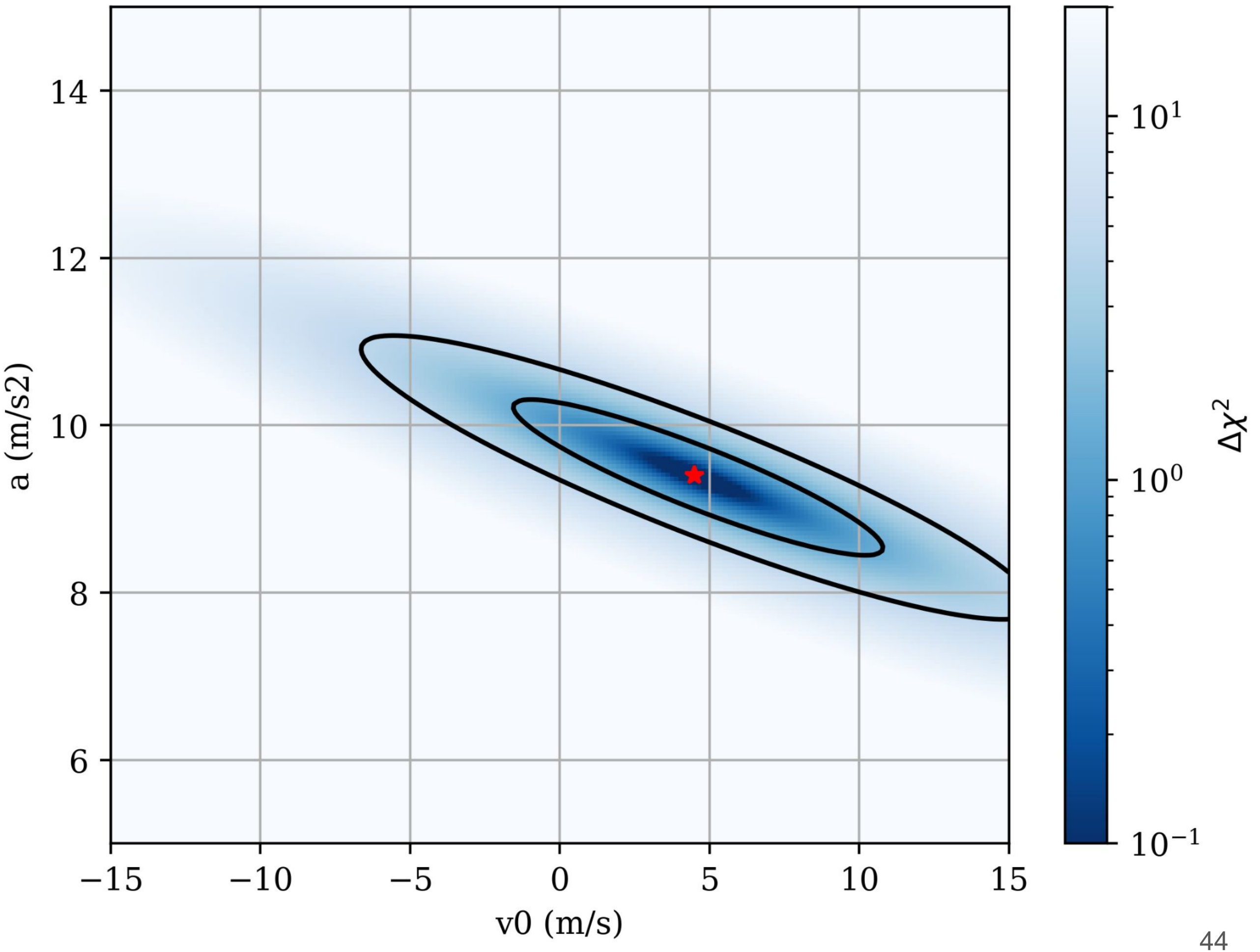
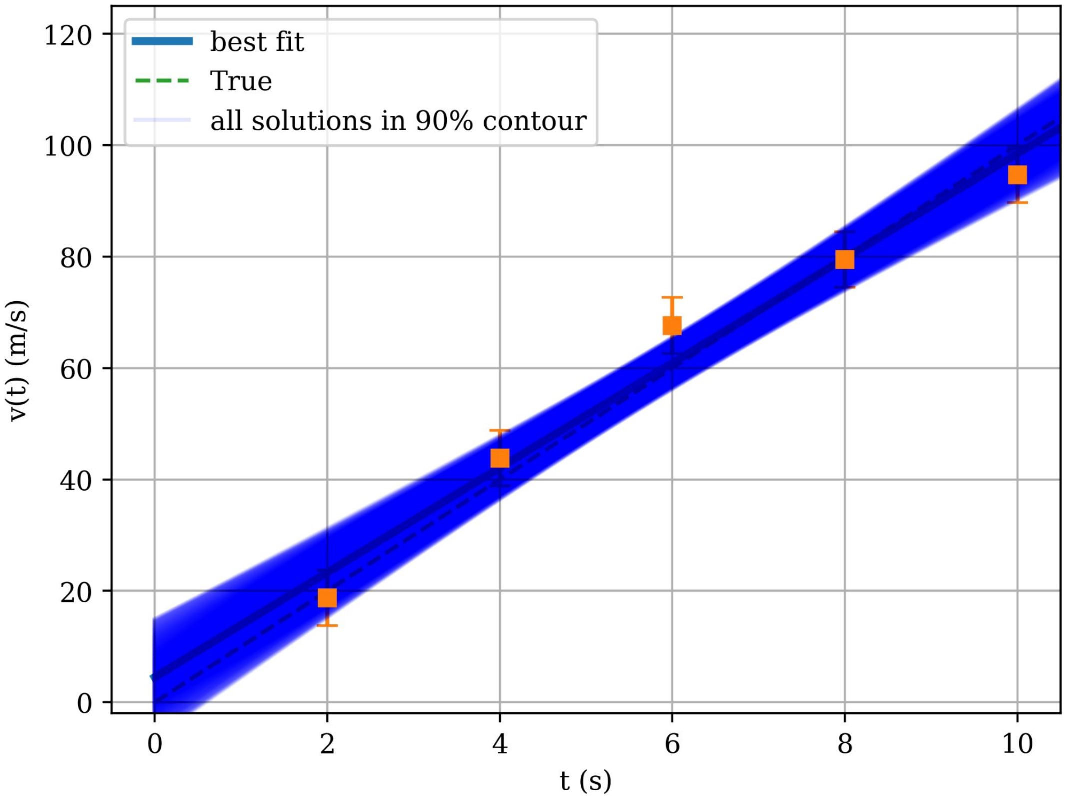
This shows us all the allowed parameter space



We can also map this to our v(t) vs t plot

What are the allowed v values as a function of t



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Say we don't care what a is, we only care about what v0 is

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In this case a is what's called a **nuisance parameter**

- An unknown free parameter in our model that we are not interested in

Getting rid of a nuisance parameter

To “get rid” of a nuisance parameter you do what’s called profiling.

You have a 2D parameter space, but you can reduce that by minimizing chi2 over a, for each v0.

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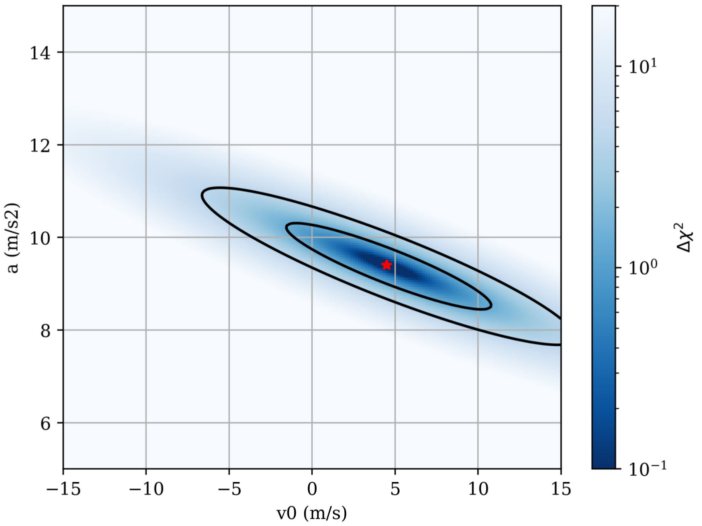

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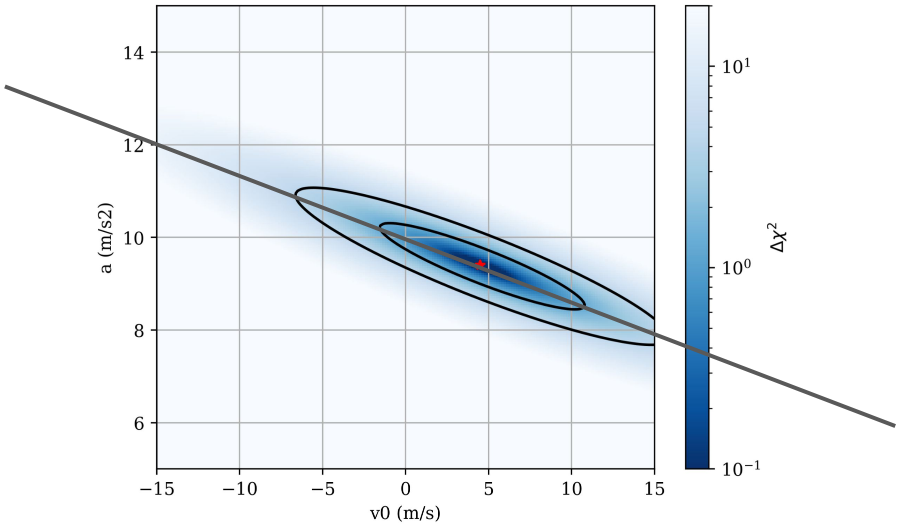

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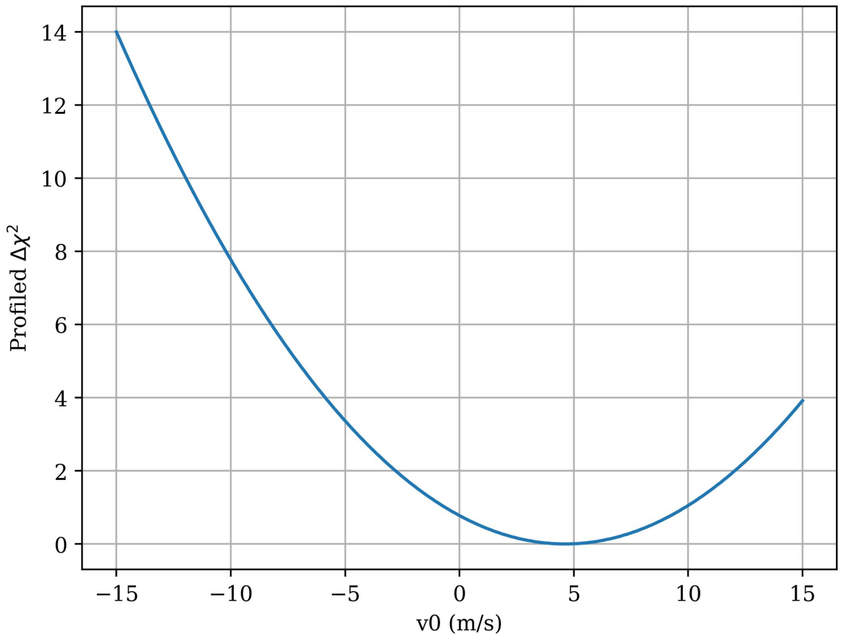
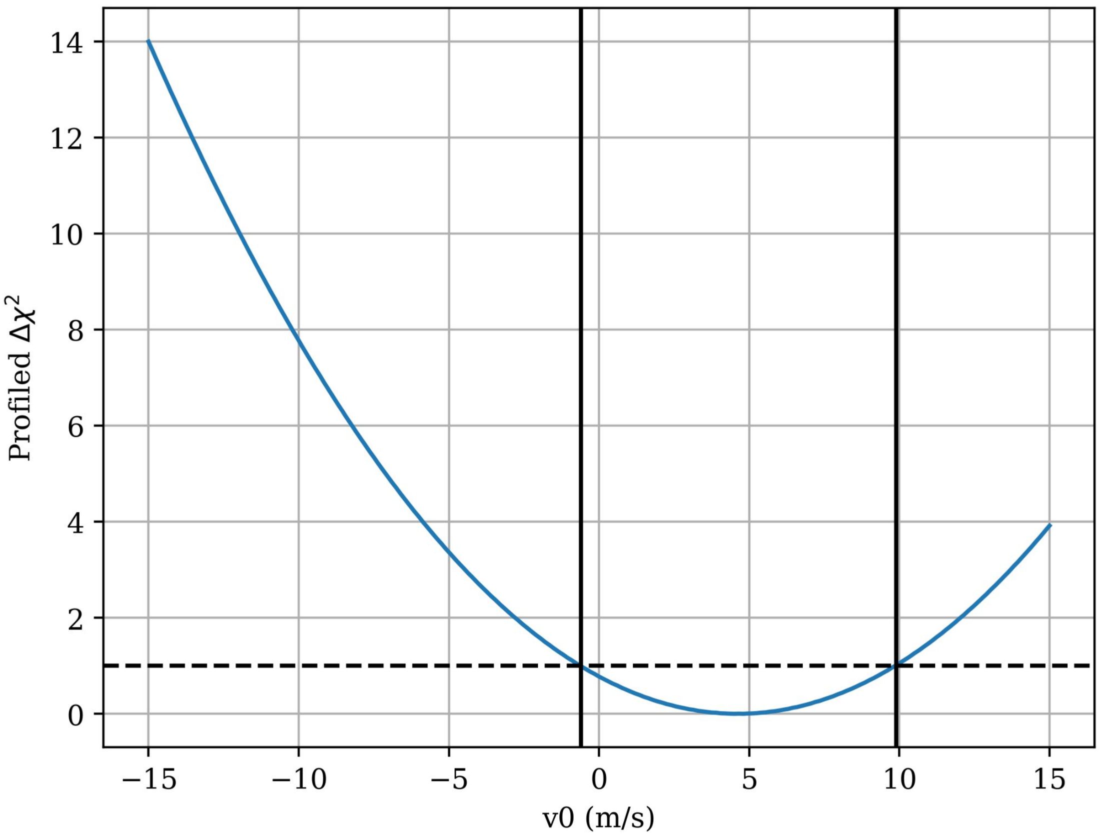


Getting rid of a nuisance parameter

We can use our chi2 with 1 dof again to find our 1D 68% error bar

- We reduced the dof by constraining 1 of them

v0 = 4.5 +/- 5.2 m/s



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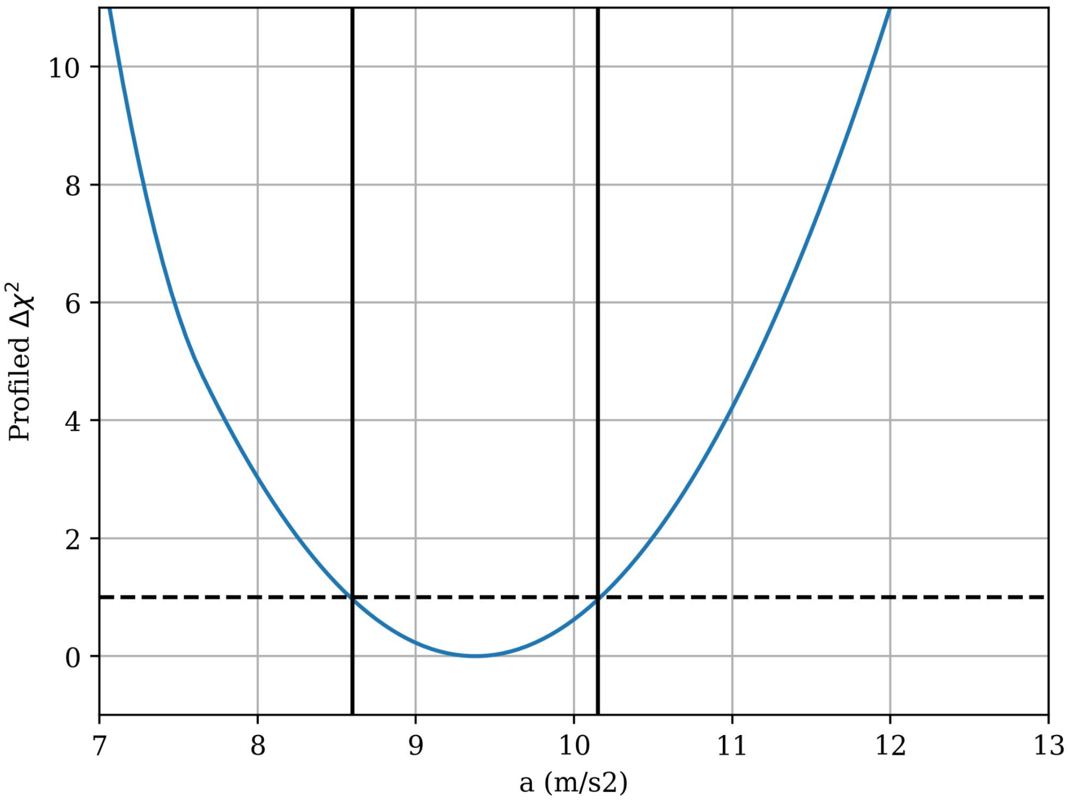

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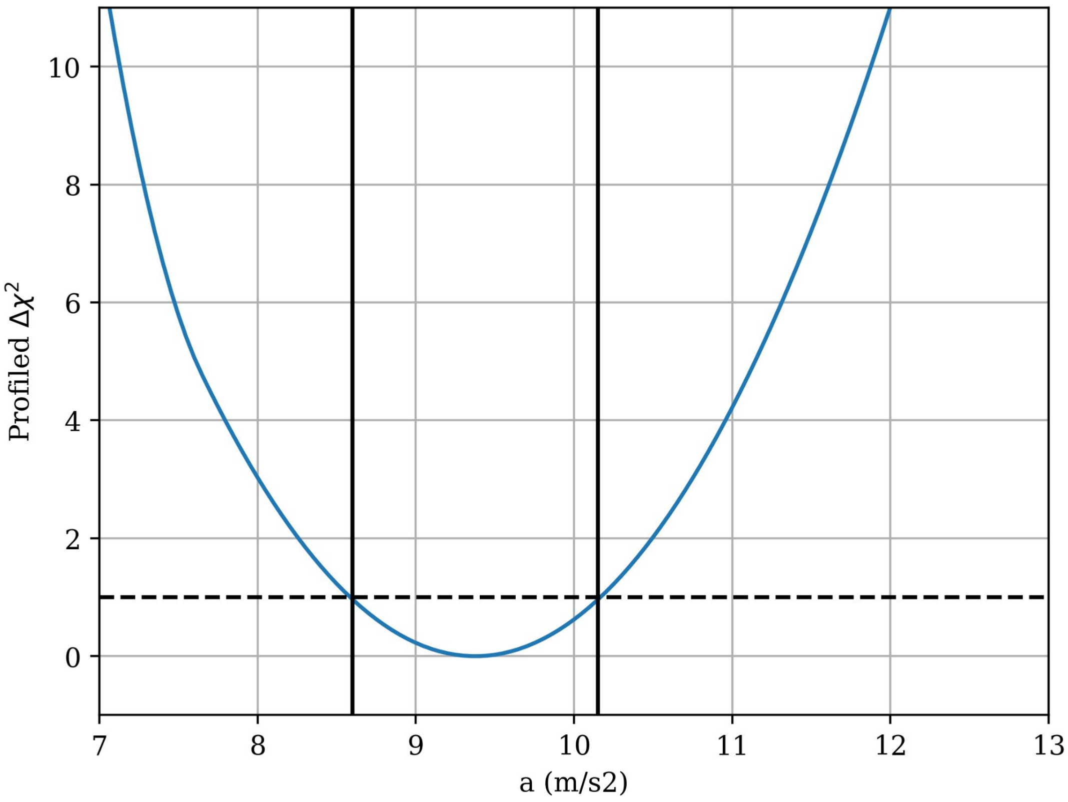

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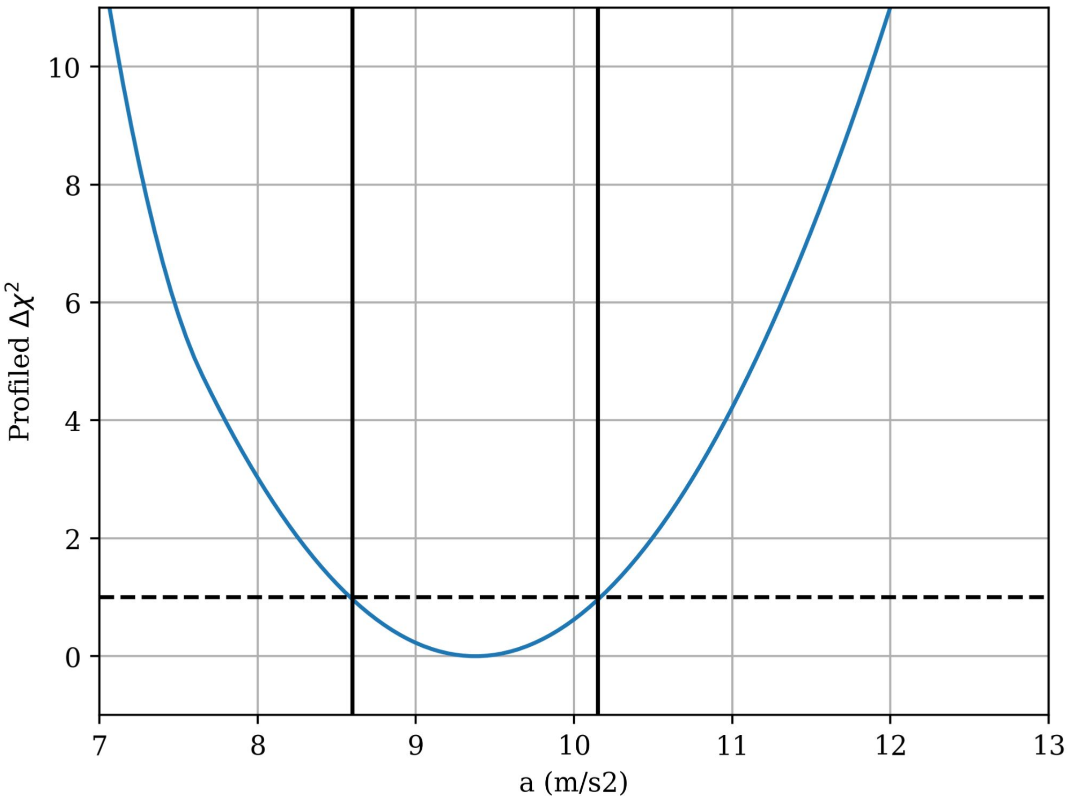

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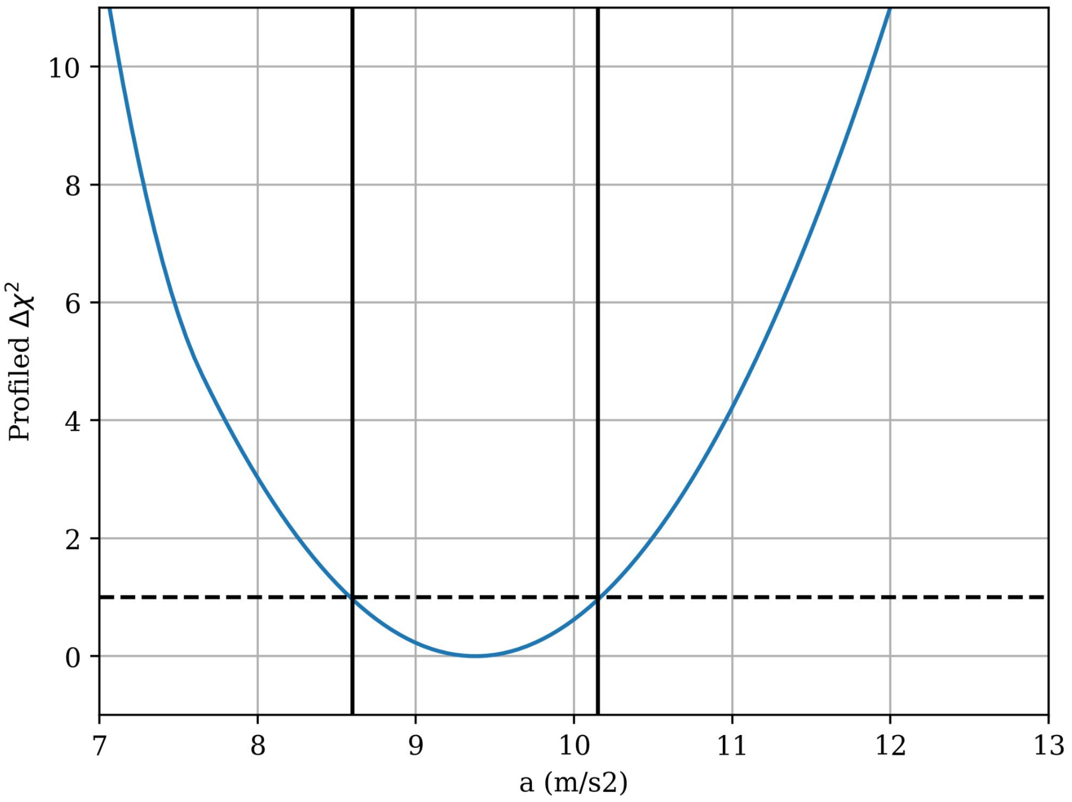
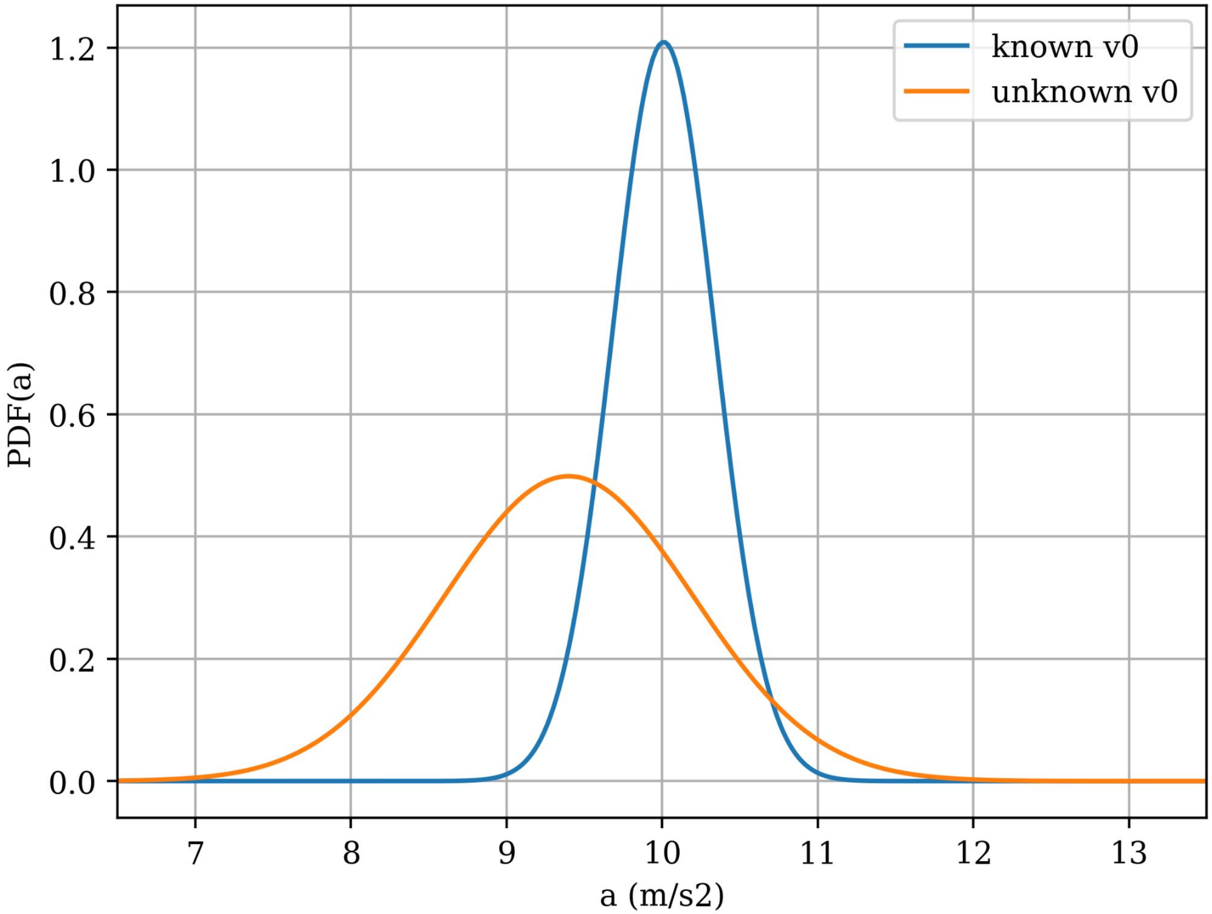
Why did the error bar get bigger?

We have less information

- v0 could be anything





known v0
unknown v0
PDF(a)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
7
8
9
10
11
12
13
a (m/s2)