NLP HW1

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6.1

I take the phrase "Assume...and equal prior probabilities" to mean that $P(+) = P(-)$.

Thus for each class we just multiply the word probabilities to get the probability of that class.

$$\begin{aligned} P(+) &= 0.09 \times 0.07 \times 0.29 \times 0.04 \times 0.08 = 5.8464 \times 10^{-6} \\ P(-) &= 0.16 \times 0.06 \times 0.06 \times 0.15 \times 0.11 = 9.5040 \times 10^{-6} \end{aligned} \tag{1}$$

As we can see, the Naive Bayes predicts the class to be negative.

6.2

NOTE: It seems that I used binary naive bayes in this exercise already, I hope that's okay

We'll denote the "comedy" class probability with $P(C)$ and the "action" with $P(A)$.

The priors are

$$\begin{aligned} P(C) &= \frac{2}{5} \\ P(A) &= \frac{3}{5} \end{aligned} \tag{2}$$

The total number of unique words that appear in all the documents is $3 + 3 + 1 + 0 + 0 = 7$. The number of unique words in comedy is $3 + 2 = 5$. In action $3 + 1 + 2 = 6$

The equation for determining the class probability of a word is

$$P(\text{word}|\text{class}) = \frac{\text{occurrences} + 1}{\text{unique occurrences in the class} + \text{sum of all unique occurrences}} \tag{3}$$

This probability is then multiplied by the prior to get the final probability.

The test document is

```
D = fast, couple, shoot, fly
```

For comedy

$$\begin{aligned} P(\text{fast}|C) &= \frac{1+1}{5+7} = \frac{2}{12} \\ P(\text{couple}|C) &= \frac{1+1}{5+7} = \frac{2}{12} \\ P(\text{shoot}|C) &= \frac{0+1}{5+7} = \frac{1}{12} \\ P(\text{fly}|C) &= \frac{1+1}{5+7} = \frac{2}{12} \end{aligned} \tag{4}$$

For action

$$\begin{aligned}
P(\text{fast}|A) &= \frac{1+1}{6+7} = \frac{2}{13} \\
P(\text{couple}|A) &= \frac{0+1}{6+7} = \frac{1}{13} \\
P(\text{shoot}|A) &= \frac{1+1}{6+7} = \frac{2}{13} \\
P(\text{fly}|A) &= \frac{1+1}{6+7} = \frac{2}{13}
\end{aligned} \tag{5}$$

Final probabilities

$$\begin{aligned}
P(C)P(D|C) &= \frac{2}{5} \times \frac{2^3 \times 1}{12^4} = \frac{1}{6480} \approx 1.5432 \times 10^{-4} \\
P(A)P(D|A) &= \frac{3}{5} \times \frac{2^3 \times 1}{13^4} = \frac{24}{142805} \approx 1.6806 \times 10^{-4}
\end{aligned} \tag{6}$$

So the predicted genre for the test document is action.

6.3

Priors are the same with both methods.

$$\begin{aligned}
P(+) &= \frac{2}{5} \\
P(-) &= \frac{3}{5}
\end{aligned} \tag{7}$$

The test sentence:

```
A good, good plot and great characters, but poor acting
```

Because our training set only contains the words "good", "poor" and "great", the test sentence is modified to the form:

```
good good great poor
```

Naive Bayes

Total word occurrences: 23
Positive word occurrences: 9
Negative word occurrences: 14
For the positive class

$$\begin{aligned}
P(\text{good}|+) &= \frac{3+1}{9+23} = \frac{4}{32} \\
P(\text{great}|+) &= \frac{5+1}{9+23} = \frac{6}{32} \\
P(\text{poor}|+) &= \frac{1+1}{9+23} = \frac{2}{32}
\end{aligned} \tag{8}$$

For the negative class

$$\begin{aligned}
P(\text{good}|-) &= \frac{2+1}{14+23} = \frac{3}{37} \\
P(\text{great}|-) &= \frac{2+1}{14+23} = \frac{3}{37} \\
P(\text{poor}|-) &= \frac{10+1}{14+23} = \frac{11}{37}
\end{aligned} \tag{9}$$

Final probabilities (noting that "good" occurs twice)

$$\begin{aligned} P(+)P(S|+) &= \frac{2}{5} \times \frac{4 \times 4 \times 6 \times 2}{32^4} = \frac{3}{40960} \approx 7.3242 \times 10^{-5} \\ P(-)P(S|-) &= \frac{3}{5} \times \frac{3 \times 3 \times 3 \times 11}{37^4} \approx 9.5083 \times 10^{-5} \end{aligned} \tag{10}$$

Which means that the naive bayes recognizes this as a negative review.

Binarized Naive Bayes

Binarized NB is almost the same, both we clip word counts in a document at 1.
Total word occurrences: 11
Positive word occurrences: 4
Negative word occurrences: 7
For the positive class

$$\begin{aligned} P(\text{good}|+) &= \frac{1+1}{4+11} = \frac{2}{15} \\ P(\text{great}|+) &= \frac{2+1}{4+11} = \frac{3}{15} \\ P(\text{poor}|+) &= \frac{1+1}{4+11} = \frac{2}{15} \end{aligned} \tag{11}$$

For the negative class

$$\begin{aligned} P(\text{good}|-) &= \frac{2+1}{7+11} = \frac{3}{18} \\ P(\text{great}|-) &= \frac{1+1}{7+11} = \frac{2}{18} \\ P(\text{poor}|-) &= \frac{3+1}{7+11} = \frac{4}{18} \end{aligned} \tag{12}$$

Final probabilities (noting that "good" occurs twice)

$$\begin{aligned} P(+)P(S|+) &= \frac{2}{5} \times \frac{2 \times 2 \times 3 \times 2}{15^4} = \frac{16}{84375} \approx 1.8963 \times 10^{-4} \\ P(-)P(S|-) &= \frac{3}{5} \times \frac{3 \times 3 \times 2 \times 4}{18^4} = \frac{1}{2430} \approx 4.1152 \times 10^{-4} \end{aligned} \tag{13}$$

So, even the Binarized NB recognizes this as a negative review. I would guess this to be because of the prior bias to negative reviews, and the fact that good occurs twice in the negative reviews, and twice in the test review.