Exercises

Convex Analysis and Optimization

Prof. Dr. Peter Ochs

www.mop.uni-saarland.de/teaching/CAO19

— Winter Term 2019 / 2020 —

Submission Instructions: Submit your solutions in the lecture hall before or directly after the lecture. *Clearly* write your *name* on the first sheet. Please use *A4 paper format* and *staple* all sheets together. Solutions that get separated and cannot be identified will not be evaluated.

— Assignment 6 —

Exercise 1. [5 + 10 + 10 = 25 points]

Let $a := t_1 < \ldots < t_n =: b$, for some $n \in \mathbb{N}$, be breakpoints in some interval $[a, b]$. We associate a value $f_i \in \mathbb{R}$ with each of the breakpoints t_i, for $i = 1, \ldots, n$. Now, define a (piecewise affine) function $f \colon \mathbb{R} \to \overline{\mathbb{R}}$ by $f(t_i) = f_i$, for all $i = 1, \ldots, n$, and f is an affine function on (t_i, t_{i+1}), for $i = 1, \ldots, n-1$, which is extended to $\mathbb{R}$ by $f(t) = f_1 + s_-(t - t_1)$ on $(-\infty, t_1)$ and $f(t) = f_n + s_+(t - t_n)$ on $(t_n, +\infty)$, where $s_-, s_+ \in \overline{\mathbb{R}}$ are possibly "infinite" slopes.

(a) Define a convexity test for the function f on $\mathbb{R}$ that only require the breakpoints $t_1, \ldots, t_n$ and the associated values $f_1, \ldots, f_n$ and the slopes s_- and s_+. Verify the correctness of the test.

(b) Show that f^* is a convex piecewise affine function and derive an explicit formula (using the breakpoints $t_1, \ldots, t_n$, the associated values $f_1, \ldots, f_n$ and the slopes s_- and s_+).

(c) Write a python program that uses the results of this exercise to visualize the convex conjugate of a piecewise affine function. Your task is to fill in the missing pieces of the code given in `ex06_01.py`.

Exercise 2. [7 points]

Clearly draw (!) the construction of the convex conjugate for the function given by the following plot on the left into the right coordinate system (note the orientation of the vertical axes).

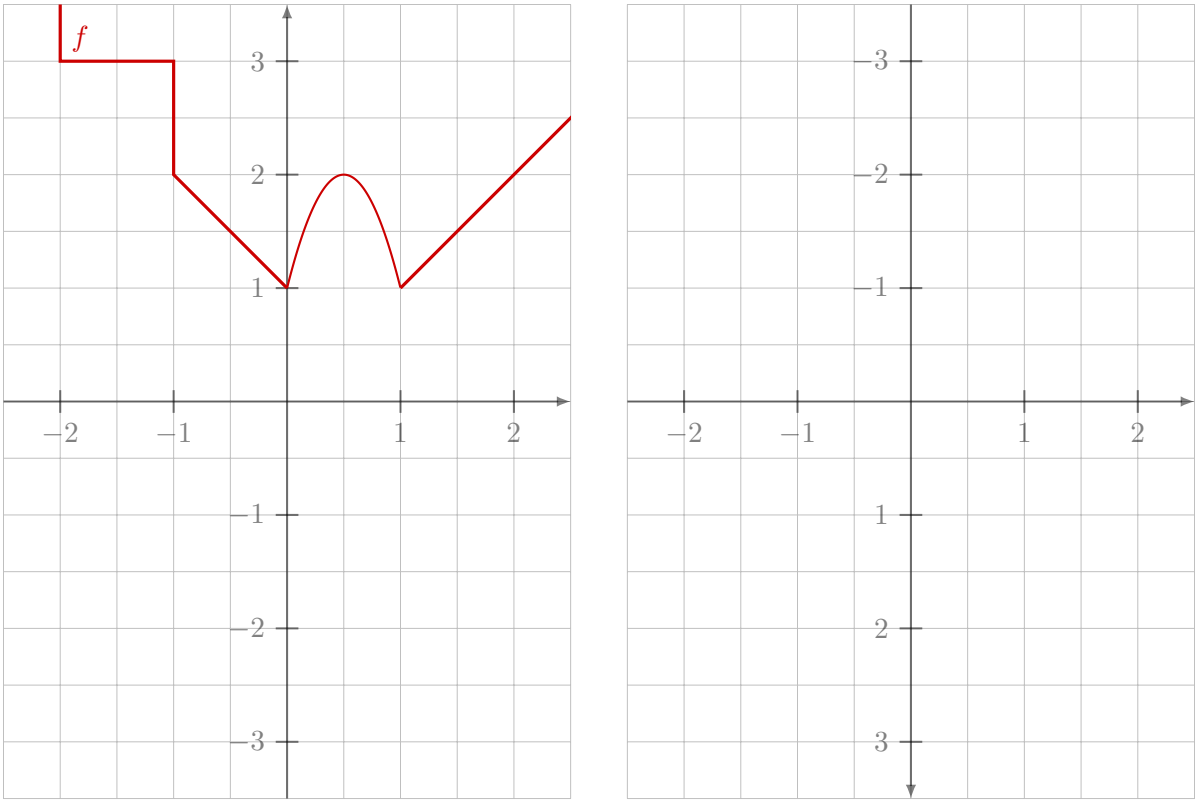

Fig. 1

Exercise 3. [8 points]
Let $h\colon \mathbb{R}^N \to \overline{\mathbb{R}}$ be a convex function, A a one-to-one linear transformation of $\mathbb{R}^N$ with adjoint A^*, $a, a' \in \mathbb{R}^N$, and $\alpha \in \mathbb{R}$. The conjugate of

$$f(x) = h(A(x-a)) + \langle x, a' \rangle + \alpha$$

is given by

$$f^*(y) = h^*((A^*)^{-1}(y-a)) + \langle y, a \rangle + \alpha' \quad \text{where } \alpha' = -\alpha - \langle a, a' \rangle \ .$$

Submission Instructions for the Coding Exercise:

- Create a `README.md` with your group and matriculation info.
- Use the `ex06_01.py` file provided.
- Make sure that the code can be executed using `python3 ex06_01.py`.
  - *Don't use exotic packages! (we check only with python3)*
- Compress the files to `zip` or `tar.gz` format on a standard Linux machine.
  - *Submissions that cannot be unpacked on a standard Linux machine will receive no points.*
  - *Compress the files using* `tar -czvf Ex06_Surname1_Surname_2.tar.gz FOLDER`.
- Send a *single* eMail *before the end of the lecture* on the submission date to the tutor

  Mahesh Chandra Mukkamala: `mukkamala@math.uni-sb.de`.

  - *Only the first eMail will be considered!*

– *You won't get points for late submissions!*