Exercises

Convex Analysis and Optimization

Prof. Dr. Peter Ochs

www.mop.uni-saarland.de/teaching/CA019

— Winter Term 2019 / 2020 —

Submission Instructions: Submit your solutions in the lecture hall before or directly after the lecture. *Clearly* write your *name* on the first sheet. Please use *A4 paper format* and *staple* all sheets together. Solutions that get separated and cannot be identified will not be evaluated.

— Assignment 2 —

Exercise 1. [6 points]
Let $C \subset \mathbb{R}^N$ be a non-empty, closed, convex set. Let $T\colon \mathbb{R}^N \to \mathbb{R}^N$ be a similarity transformation, i.e., $T(x) = sRx + t$ for all $x \in \mathbb{R}^N$, with orthogonal transformation matrix $R \in \mathbb{R}^{N\times N}$ such that $\det(R) = 1$ and $R^\top R = I$ (with identity matrix I), translation $t \in \mathbb{R}^N$, and scaling $s \neq 0$. Define $D = T(C)$ and suppose that a formula for $\mathrm{proj}_C(\bar{x})$ is known for all $\bar{x} \in \mathbb{R}^N$. Derive a formula for the projection $\mathrm{proj}_D(\bar{y})$ for any point $\bar{y} \in \mathbb{R}^N$. (The formula may include $\mathrm{proj}_C(\bar{x})$).

Exercise 2. [4 points]
Let $C \subset \mathbb{R}^N$ be closed convex with $0 \in C$ and symmetric ($C = -C$). Then $\mathrm{int}\, C \neq \emptyset$ if and only if $0 \in \mathrm{int}\, C$.

Exercise 3. [3 + 3 = 6 points]
Define the following set

$$S := \left\{ x \in \mathbb{R}^N : x \geq 0\,, \sum_{i=1}^N x_i = 1 \right\} .$$

Compute the relative interior and the relative boundary of S?.

Exercise 4. [5 + 5 = 10 points]
Modify the single Python 3 file `ex02_01.py`, in which you do the following tasks.

(a) Code the 3d plot of $f(x, y) = (1 - xy)^2$ where $x \in \mathbb{R}$ and $y \in R$, for which you must choose an appropriate intervals of x, y for better visualization, for e.g., $x \in [-10, 10]$ and $y \in [-10, 10]$.

(b) Create the 3d plot of the above task, in both png and pdf formats with appropriate labels and legends using matplotlib package.

The submission instructions for this question are given below.

Exercise 5. [7+ 7 = 14 points]
In this exercise, we consider a simple convex feasibility problem introduced in Section 2.2.4 (Projection Theorem, also see Section A.3) and Section A.4 (Convex Feasibility Problems) of the lecture notes. This is an important problem with many applications such as medical imaging, computerized tomography, electron microscopy and many others, which is described in Section A.4. This can be solved by different algorithms and the most classic ones are alternating minimzation and Dykstra's projection method, which is focus of this exercise. Both the algorithms are provided in Section A.4.

The description of the exercise is given in the file `ex02_02.py`. The submission instructions for this question are given below.

Submission Instructions for the Coding Exercise:

- Create a `README.md` with your group and matriculation info.
- Use the `ex02_01.py` and `ex02_02.py` files provided.
- Make sure that the code can be executed using `python3 ex02_01.py` and `python3 ex02_02.py`.
  - *Don't use exotic packages! (we check only with python3)*
- Compress the files to `zip` or `tar.gz` format on a standard Linux machine.
  - *Submissions that cannot be unpacked on a standard Linux machine will receive no points.*
  - *Compress the files using* `tar -czvf Ex02_Surname1_Surname_2.tar.gz FOLDER`.
- Send a *single* eMail *before the end of the lecture* on the submission date to the tutor

  Mahesh Chandra Mukkamala: `mukkamala@math.uni-sb.de`.

  - *Only the first eMail will be considered!*
  - *You won't get points for late submissions!*