Exercises

Convex Analysis and Optimization

Prof. Dr. Peter Ochs

www.mop.uni-saarland.de/teaching/CA019

— Winter Term 2019 / 2020 —

Submission Instructions: Submit your solutions in the lecture hall before or directly after the lecture. *Clearly* write your *name* on the first sheet. Please use *A4 paper format* and *staple* all sheets together. Solutions that get separated and cannot be identified will not be evaluated.

— Assignment 10 —

Exercise 1. [6 points]
Let $f\colon \mathbb{R}^N \to \mathbb{R}$ be finite-valued convex. Then, f is m-Lipschitz continuous if and only if $\sup_{v\in\partial f(x)} \|v\|_2 \leq m$ for all $x \in \mathbb{R}^N$.

Exercise 2. [6 points]
Construct a simple example that shows Proximal Subgradient Descent is not a descent method. (*For any step size $\tau > 0$, the objective increases along the current step direction.*)

Exercise 3. [8 points]
Let $h(x) = |x|$ and $x^{(0)} = 1$. Show that the iterates of Subgradient Descent with constant step size $\tau > 0$ do not converge to a minimizer, for any $\tau \in [0,1] \setminus \mathbb{Q}$, although the objective is differentiable at all iterates. (*Hint: Choose 0 as subgradient at 0.*)

Exercise 4. [20 points]
The goal is minimize the following optimization problem

$$\min_{u\in[-1,1]^N} \langle c, u\rangle + ||Ku||_1 \,, \tag{1}$$

where $K \in \mathbb{R}^{M\times N}$ and $c \in \mathbb{R}^N$ for certain positive integers M and N. We try to solve the problem using proximal subgradient method. Your task is to fill in the missing parts of the code given in `ex10_01.py`. Briefly describe the issues involved in applying proximal subgradient method?

Appendix: The above problem arises in the context of binary image segmentation. The task essentially involves an image and the goal is to extract certain objects of interest. In the data given, the objects of interest are flowers. The likelihood of a pixel belonging to the foreground region (flowers) or background is represented by a cost value. The cost for pixels are represented in vector notation c. For binary segmentation of the image u, we optimize over relaxed constraint set which is $[-1, 1]$ for each pixel. Once the relaxed segmentation is obtained, we can apply thresholding based on pixel value being positive or negative. The motivation of the minimization problem comes from the minimal partition problem. In this context, the regularization term $||Ku||_1$ can be seen as a relaxation for measuring the length of the boundary between foreground and background region, which explains the name: partitioning of the image with minimal length of the interface between the regions.

Submission Instructions for the Coding Exercise:

- Create a `README.md` with your group and matriculation info.
- Use the `ex10_01.py` file provided.
- Make sure that the code can be executed using `python3 ex10_01.py`.
  - *Don't use exotic packages! (we check only with python3)*
- Compress the files to `zip` or `tar.gz` format on a standard Linux machine.
  - *Submissions that cannot be unpacked on a standard Linux machine will receive no points.*
  - *Compress the files using* `tar -czvf Ex10_Surname1_Surname_2.tar.gz FOLDER`.
- Send a *single* eMail *before the end of the lecture* on the submission date to the tutor

  Mahesh Chandra Mukkamala: `mukkamala@math.uni-sb.de`.

  - *Only the first eMail will be considered!*
  - *You won't get points for late submissions!*