Exercises

Convex Analysis and Optimization

Prof. Dr. Peter Ochs

www.mop.uni-saarland.de/teaching/CA019

— Winter Term 2019 / 2020 —

Submission Instructions: Submit your solutions in the lecture hall before or directly after the lecture. *Clearly* write your *name* on the first sheet. Please use *A4 paper format* and *staple* all sheets together. Solutions that get separated and cannot be identified will not be evaluated.

— Assignment 4 —

Exercise 1. [6 points]

If f_1 and f_2 are convex functions on $\mathbb{R}^N$ and $\lambda_1, \lambda_2 \geq 0$, then $\lambda_1 f_1 + \lambda_2 f_2$ is convex.

Exercise 2. [6 points]

Consider a set of points $\{(x^1, y^1), (x^2, y^2), \dots, (x^m, y^m)\}$ where $x^i \in \mathbb{R}^N$ and $y^i \in \{0, 1\}$. Note that y^i takes binary values. For $\theta \in \mathbb{R}^N$, denote the following

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}} ,$$

and

$$J(\theta) = -\sum_{i=1}^{m} \left(y^i \log(h_\theta(x^i)) + (1 - y^i) \log(1 - h_\theta(x^i)) \right) .$$

Prove that the function $J(\theta)$ is convex.

Exercise 3. [6 points]

Let C be a closed subset of $\mathbb{R}^N$. Prove or disprove the following statement:
The convex hull $\operatorname{conv} C$ of the set C is closed in $\mathbb{R}^N$.

Exercise 4. [4 + 4 + 4 = 12 points]

(a) Let D be a convex set in $\mathbb{R}^{N+1}$. The function

$$f(x) := \inf\{\mu \in \overline{\mathbb{R}} : (x, \mu) \in D\}$$

is a convex function on $\mathbb{R}^N$.

(b) Let $g \colon \mathbb{R}^N \times \mathbb{R}^M \to \overline{\mathbb{R}}$ be a convex function. The function

$$f(u) := \inf_{x \in \mathbb{R}^N} g(x, u)$$

is a convex function on $\mathbb{R}^M$

(c) Let $g \colon \mathbb{R}^N \times \mathbb{R}^M \to \overline{\mathbb{R}}$ be a convex function. The set

$$C_u := \operatorname*{argmin}_{x \in \mathbb{R}^N} g(x, u)$$

is a convex set for each $u \in \mathbb{R}^M$.

Exercise 5. [6 + 4 = 10 points]
Let $a := t_1 < \ldots < t_n =: b$, for some $n \in \mathbb{N}$, be breakpoints in some interval $[a, b]$. Define a (piecewise affine) function $f : [a, b] \to \mathbb{R}$. The slope within the interval (t_i, t_{i+1}) is given by m_i and we set $f(a) = 0$. This information defines the piecewise affine function uniquely. We want to check the convexity of the entire function within interval $[a, b]$.

(a) Edit `ex04_01.py` to complete the following task. Consider the interval $[0, 100]$ and set $f(0) = 0$. Write a function `convex_check` which takes two numpy arrays as input (slopes and breakpoints) and returns `True` if the piecewise affine function generated is convex. Otherwise, it should return `False`.

(b) Provide a theoretical proof of convexity for your proposed strategy in (a).

Submission Instructions for the Coding Exercise:

- Create a `README.md` with your group and matriculation info.
- Use the `ex04_01.py` file provided.
- Make sure that the code can be executed using `python3 ex04_01.py`.
  - *Don't use exotic packages! (we check only with python3)*
- Compress the files to `zip` or `tar.gz` format on a standard Linux machine.
  - *Submissions that cannot be unpacked on a standard Linux machine will receive no points.*
  - *Compress the files using* `tar -czvf Ex04_Surname1_Surname_2.tar.gz FOLDER`.
- Send a *single* eMail *before the end of the lecture* on the submission date to the tutor

  Mahesh Chandra Mukkamala: `mukkamala@math.uni-sb.de`.

  - *Only the first eMail will be considered!*
  - *You won't get points for late submissions!*