



## Exercises

# Convex Analysis and Optimization

Prof. Dr. Peter Ochs

[www.mop.uni-saarland.de/teaching/CA019](http://www.mop.uni-saarland.de/teaching/CA019)



— Winter Term 2019 / 2020 —

**Submission Instructions:** Submit your solutions in the lecture hall before or directly after the lecture. *Clearly* write your *name* on the first sheet. Please use *A4 paper format* and *staple* all sheets together. Solutions that get separated and cannot be identified will not be evaluated.

## — Assignment 11 —

### Exercise 1. [10 points]

Let  $C \subset \mathbb{R}^N$  be a non-empty compact and convex set,  $f: \mathbb{R}^N \rightarrow \mathbb{R}$  be a continuously differentiable convex function, and  $\bar{x} \in C$ . If  $y$  is a solution of the following problem

$$\min_{y \in C} \langle \nabla f(\bar{x}), y \rangle ,$$

then  $v = y - \bar{x}$  is a descent direction of the function  $f$  at  $\bar{x}$  (i.e.,  $f(\bar{x} + \tau v) < f(\bar{x})$  for  $\tau > 0$  sufficiently small) or  $\bar{x}$  is a minimizer of  $f$ .

### Exercise 2. [10 points]

Let  $B \in \mathbb{R}^{M \times N}$ . Solve the following problem:

$$\min_{X \in \mathbb{R}^{M \times N}} \text{tr}(B^\top X) \quad \text{s.t. } \forall j = 1, \dots, N: \sum_{i=1}^M X_{i,j} \leq 1 \text{ and } \forall i, j: X_{i,j} \geq 0 ,$$

where  $\text{tr}(A) := \sum_{i=1}^N A_{i,i}$  is the trace of a matrix  $A \in \mathbb{R}^{N \times N}$ .

### Exercise 3. [20 points]

Let  $A \in \mathbb{R}^{M \times N}$  and  $b \in \mathbb{R}^M$ . Derive the update step of the Conditional Gradient Method for solving the following problem:

$$\min_{x \in \mathbb{R}^N} \frac{1}{2} \|Ax - b\|_2^2 \quad \text{s.t. } \forall i = 1, \dots, N: |x_i| \leq 1 .$$

The second part of the exercise involves completing the missing code in `ex11_01.py`. We aim to obtain comparisons between Conditional Gradient Method and Projected Gradient Descent Method. Briefly describe the advantages of Conditional Gradient Method.

### Submission Instructions for the Coding Exercise:

- Create a `README.md` with your group and matriculation info.
- Use the `ex11_01.py` file provided.
- Make sure that the code can be executed using `python3 ex11_01.py`.

— Don't use exotic packages! (we check only with python3) —

- Compress the files to `zip` or `tar.gz` format on a standard Linux machine.
  - *Submissions that cannot be unpacked on a standard Linux machine will receive no points.*
  - *Compress the files using `tar -czvf Ex11_Surname1_Surname_2.tar.gz FOLDER`.*
- Send a *single* eMail *before the end of the lecture* on the submission date to the tutor

Mahesh Chandra Mukkamala: `mukkamala@math.uni-sb.de`.

- *Only the first eMail will be considered!*
- *You won't get points for late submissions!*