$$T(x) \quad \mathbb{R}^n \longrightarrow \mathbb{R}^n$$

$$T(a+b) = T(a) + T(b)$$

$$T\left(\sum_{i=1}^{n} x_i\right) = \sum_{i=1}^{n} T(x_i)$$

- for all $x_i = 0$, $T(0) = nT(0)$

  so $T(0) = 0$

- for all x_i to be some x, $T(nx) = nT(x)$

  so homogeneity holds for all n

$$T(\underbrace{x_1 + (-x_1)}_{0}) = T(x_1) + T(-x_1)$$

$$T(0) = 0 \Rightarrow T(x_1) + T(-x_1) = 0$$

$$\Rightarrow \quad T(-x_1) = -T(x_1)$$

$$T(x) = T(1x) \Rightarrow T\left(\frac{1}{c}x\right) = \frac{1}{c}T(x)$$

applies to all $c = \frac{p}{q}$

for $c \in \mathbb{R}$ $\mathbb{Q}$ is dense in $\mathbb{R}$

there exists $\{c_n\}$ such that

$c_n \to c$ as $n \to \infty$

for any real scalar $T(cv)$

$$T(cv) = \lim_{n \to \infty} T(c_n v)$$

$$\Rightarrow T(cv) = \lim_{n \to \infty} c_n T(v) = cT(v)$$

Thus homogeneity holds for all real scalar numbers if additivity holds