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Homework 1

MAT 167
January 8, 2020

Problem 1

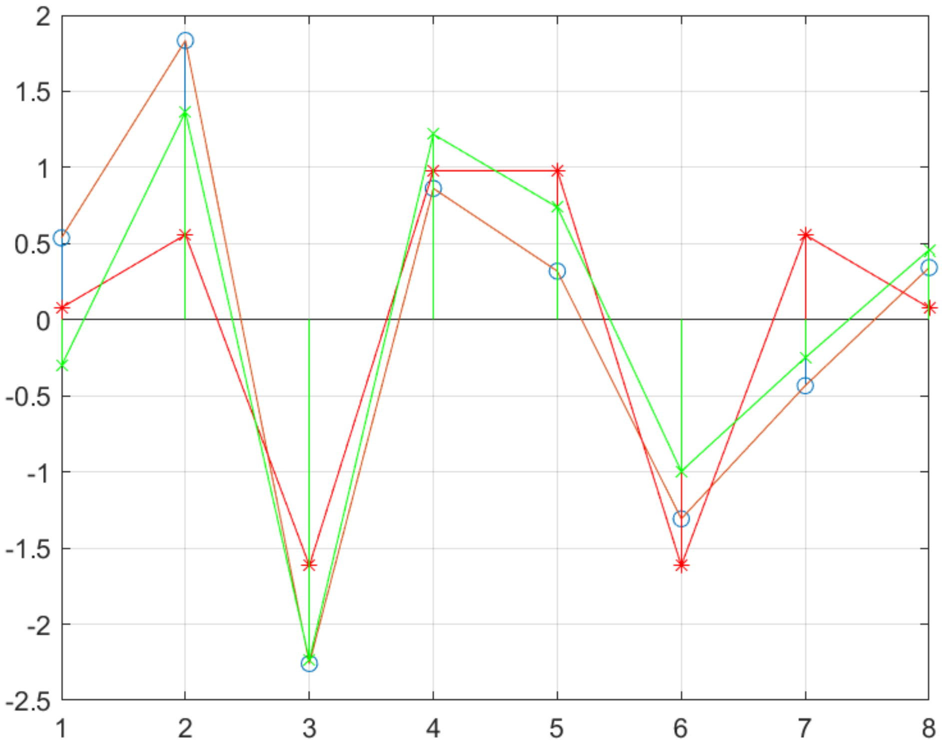

Figure 1: Plot of the original signal vector x (depicted by orange line) along with two approximations. First approximation of the original using the two largest coefficients (depicted by red line).Second approximation of the original using the four largest coefficients (depicted by green line). Coefficients were calculated using the transpose of the matrix U and the original signal vector x. The figure shows how well the approximations replicate the original data. As we can see, when compared to the original signal vector, the approximation using four coefficients does an alright job. The approximation using two coefficients however, does a really poor job.

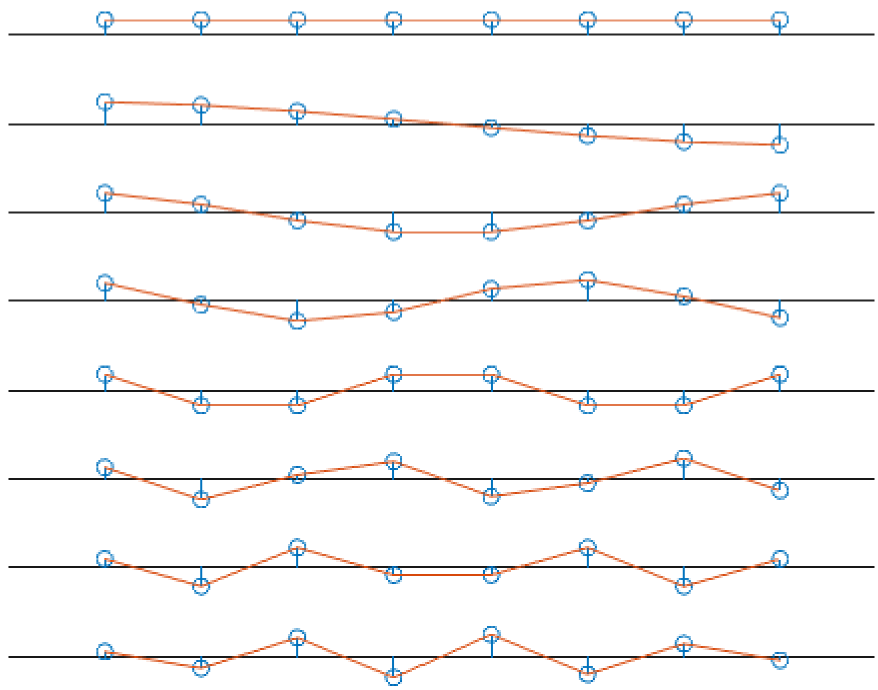

Figure 2: Plot of the 8 basis vectors of the matrix U.

(h)

Relative Error x8	1.7991e-16
Relative Error x4	0.3440
Relative Error x2	0.5757

The relative error for each approximation is listed in the table above. Intuitively this makes sense because the relative error from using all coefficients is extremely low (close to machine zero). As less coefficients are used to approximate the data, the relative error becomes higher. This is because the approximations are becoming less accurate due to making approximations with less data.

(i)

This MATLAB program first creates a plot (figure 1) of the original signal vector x. Next it creates another plot (figure 2) of the basis vectors of the matrix U. Next, the program uses the idea of Discrete Cosine Transformation (DCT) to compress/compactly approximate the signal vector x. To do this, a coefficient vector 'a' is calculated by multiplying the inverse of the basis vector matrix U with the original signal vector x. Using the coefficient vector calculated, two approximation signals are generated using either only two or four of the largest coefficient values. The coefficient values indicate how important that particular point in the data is when generating the original signal vector x. The two approximation signals are then plotted on to figure 1 to visualize the differences between the three signal vectors. It is clear from the plot that using less coefficients result in a less accurate approximation of the original signal vector x. Next, the program calculates the full reconstruction of the signal x using all coefficients. Lastly, the relative error for each signal approximation is calculated. From the results of the relative error calculations, it is clear that using less coefficients results in a less accurate approximation. To be precise, reconstruction using all coefficients resulted in a relative error close to machine zero. Reconstruction using only two coefficients resulted in a large relative error 0.5757.

Problem 2a

8 x 11 Term Document Matrix

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11
T1	0	1	0	0	1	1	0	0	0	0	0
T2	0	0	0	1	0	0	0	0	0	1	0
T3	0	1	0	0	0	0	0	1	0	0	0
T4	0	0	1	1	0	0	0	0	0	0	0
T5	0	0	0	1	0	0	0	0	0	1	0
T6	1	1	0	0	0	0	0	0	0	0	1
T7	0	0	0	0	0	1	1	0	0	0	0
T8	0	0	1	0	0	0	0	0	1	0	0

Problem 2B

Query ”Integral Equation” (Terms 2 and 4)

$\vec{a} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$

Problem 2C

Based on the query vector a, the three closest documents for the query are documents 3, 4, and 10.

Problem 3

CA Pop = 36,453,973

USA Pop = 271,491,582

Moved to CA = 458,682

Left CA = 545,921

Final CA Pop = 36,453,973 - 545,921 + 458,682 = 36,366,734

Final USA Pop = 271,491,582 - 458,682 + 545,921 = 271,678,821

$$\begin{bmatrix} CAPop & LeftCA & MovedToCA \\ USAPop & MovedToCA & LeftCA \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 36,366,734 \\ 271,678,821 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} 36,453,973 & 545,921 & 458,682 \\ 271,491,582 & 458,682 & 545,921 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 36,366,734 \\ 271,678,821 \end{bmatrix} \tag{2}$$

Problem 4A

$$\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{v_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad \vec{v_3} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v_4} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

First, find the orthonormal basis for $\overrightarrow{v_1}$

$$||\overrightarrow{v_1}|| = \sqrt{1^2 + 0^2 + 1^2 + 0^2} = \sqrt{2} \tag{1}$$

$$\overrightarrow{\mu_1} = \frac{1}{||\overrightarrow{v_1}||} * \overrightarrow{v_1} \tag{2}$$

$$\overrightarrow{\mu_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Second, find the orthonormal basis for $\overrightarrow{v_2}$

$$\overrightarrow{y_2} = \overrightarrow{v_2} - \mathrm{proj}_{\overrightarrow{\mu_1}}(\overrightarrow{v_2}) \tag{1}$$

$$\overrightarrow{y_2} = \overrightarrow{v_2} - (\overrightarrow{v_2} \bullet \overrightarrow{\mu_1})\overrightarrow{\mu_1}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} - \left(\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \bullet \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right) \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$||\vec{y_2}|| = \sqrt{0^2 + 1^2 + 0^2 + (-1)^2} = \sqrt{2} \tag{2}$$

$$\vec{\mu_2} = \frac{1}{||\vec{y_2}||} * \vec{y_2} \tag{3}$$

$$\vec{\mu_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

Third, find the orthonormal basis for $\vec{v_3}$

$$\vec{y_3} = \vec{v_3} - \text{proj}_{\vec{\mu_1}}(\vec{v_3}) - \text{proj}_{\vec{\mu_2}}(\vec{v_3}) \tag{1}$$

$$\vec{y_3} = \vec{v_3} - (\vec{v_3} \bullet \vec{\mu_1})\vec{\mu_1} - (\vec{v_3} \bullet \vec{\mu_2})\vec{\mu_2}$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \bullet \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right) \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \left(\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \bullet \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \right) \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{-1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$||\vec{y_3}|| = \sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2 + (\frac{-1}{2})^2 + (\frac{1}{2})^2} = 1 \tag{2}$$

$$\vec{\mu_3} = \frac{1}{||\vec{y_3}||} * \vec{y_3} \tag{3}$$

$$\vec{\mu_3} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{-1}{2} \\ \frac{1}{2} \end{bmatrix}$$

Lastly, find the orthonormal basis for $\vec{v_4}$

$$\vec{y_4} = \vec{v_4} - \text{proj}_{\vec{\mu_1}}(\vec{v_4}) - \text{proj}_{\vec{\mu_2}}(\vec{v_4}) - \text{proj}_{\vec{\mu_3}}(\vec{v_4}) \tag{1}$$

$$\vec{y_4} = \vec{v_4} - (\vec{v_4} \bullet \vec{\mu_1})\vec{\mu_1} - (\vec{v_4} \bullet \vec{\mu_2})\vec{\mu_2} - (\vec{v_4} \bullet \vec{\mu_3})\vec{\mu_3}$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \left(\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \bullet \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right) \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \left(\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \bullet \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \right) \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

$$- \left(\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{-1}{2} \\ \frac{1}{2} \end{bmatrix} \right) \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{-1}{2} \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$||\overrightarrow{y_4}|| = \sqrt{(\frac{-1}{2})^2 + (\frac{1}{2})^2 + (\frac{1}{2})^2 + (\frac{1}{2})^2} = 1 \tag{2}$$

$$\overrightarrow{\mu_4} = \frac{1}{||\overrightarrow{y_4}||} * \overrightarrow{y_4} \tag{3}$$

$$\overrightarrow{\mu_4} = \begin{bmatrix} \frac{-1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

From the above Gram-Schmidt calculations, the orthonormal basis are as follows:

$$\overrightarrow{\mu_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \overrightarrow{\mu_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad \overrightarrow{\mu_3} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{-1}{2} \\ \frac{1}{2} \end{bmatrix}, \quad \overrightarrow{\mu_4} = \begin{bmatrix} \frac{-1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

Problem 4B

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v_3} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v_4} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

First, find the orthonormal basis for $\overrightarrow{v_1}$

$$||\overrightarrow{v_1}|| = \sqrt{1^2 + 0^2 + 0^2 + 1^2} = \sqrt{2} \tag{1}$$

$$\overrightarrow{\mu_1} = \frac{1}{||\overrightarrow{v_1}||} * \overrightarrow{v_1} \tag{2}$$

$$\overrightarrow{\mu_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Second, find the orthonormal basis for $\overrightarrow{v_2}$

$$\overrightarrow{y_2} = \overrightarrow{v_2} - \text{proj}_{\overrightarrow{\mu_1}}(\overrightarrow{v_2}) \tag{1}$$

$$\overrightarrow{y_2} = \overrightarrow{v_2} - (\overrightarrow{v_2} \bullet \overrightarrow{\mu_1})\overrightarrow{\mu_1}$$

$$\begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \left(\begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} \bullet \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right) \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ -2 \end{bmatrix}$$

$$||\overrightarrow{y_2}|| = \sqrt{2^2 + 1^2 + 0^2 + (-2)^2} = \sqrt{9} = 3 \tag{2}$$

$$\overrightarrow{\mu_2} = \frac{1}{||\overrightarrow{y_2}||} * \overrightarrow{y_2} \tag{3}$$

$$\overrightarrow{\mu_2} = \frac{1}{3}\begin{bmatrix} 2 \\ 1 \\ 0 \\ -2 \end{bmatrix}$$

Third, find the orthonormal basis for $\overrightarrow{v_3}$

$$\overrightarrow{y_3} = \overrightarrow{v_3} - \text{proj}_{\overrightarrow{\mu_1}}(\overrightarrow{v_3}) - \text{proj}_{\overrightarrow{\mu_2}}(\overrightarrow{v_3}) \tag{1}$$

$$\overrightarrow{y_3} = \overrightarrow{v_3} - (\overrightarrow{v_3} \bullet \overrightarrow{\mu_1})\overrightarrow{\mu_1} - (\overrightarrow{v_3} \bullet \overrightarrow{\mu_2})\overrightarrow{\mu_2}$$

$$\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix} - \left(\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix} \bullet \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right)\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \left(\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix} \bullet \frac{1}{3}\begin{bmatrix} 2 \\ 1 \\ 0 \\ -2 \end{bmatrix}\right)\frac{1}{3}\begin{bmatrix} 2 \\ 1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

$$||\overrightarrow{y_3}|| = \sqrt{(0^2 + 0^2 + 2^2 + 0^2} = \sqrt{4} = 2 \tag{2}$$

$$\overrightarrow{\mu_3} = \frac{1}{||\overrightarrow{y_3}||} * \overrightarrow{y_3} \tag{3}$$

$$\overrightarrow{\mu_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Lastly, find the orthonormal basis for $\overrightarrow{v_4}$

$$\overrightarrow{y_4} = \overrightarrow{v_4} - \text{proj}_{\overrightarrow{\mu_1}}(\overrightarrow{v_4}) - \text{proj}_{\overrightarrow{\mu_2}}(\overrightarrow{v_4}) - \text{proj}_{\overrightarrow{\mu_3}}(\overrightarrow{v_4}) \tag{1}$$

$$\overrightarrow{y_4} = \overrightarrow{v_4} - (\overrightarrow{v_4} \bullet \overrightarrow{\mu_1})\overrightarrow{\mu_1} - (\overrightarrow{v_4} \bullet \overrightarrow{\mu_2})\overrightarrow{\mu_2} - (\overrightarrow{v_4} \bullet \overrightarrow{\mu_3})\overrightarrow{\mu_3}$$

$$\begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} - \left(\begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \bullet \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right) \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} - \left(\begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \bullet \frac{1}{3} \begin{bmatrix} 2 \\ 1 \\ 0 \\ -2 \end{bmatrix} \right) \frac{1}{3} \begin{bmatrix} 2 \\ 1 \\ 0 \\ -2 \end{bmatrix}$$

$$- \left(\begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right) \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} \\ 2 \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

$$||\overrightarrow{y_4}|| = \sqrt{(\frac{-1}{2})^2 + 2^2 + 0^2 + (\frac{1}{2})^2} = \sqrt{4.5} \tag{2}$$

$$\overrightarrow{\mu_4} = \frac{1}{||\overrightarrow{y_4}||} * \overrightarrow{y_4} \tag{3}$$

$$\overrightarrow{\mu_4} = \frac{1}{\sqrt{4.5}} \begin{bmatrix} \frac{-1}{2} \\ 2 \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

From the above Gram-Schmidt calculations, the orthonormal basis are as follows:

$$\overrightarrow{\mu_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \overrightarrow{\mu_2} = \frac{1}{3} \begin{bmatrix} 2 \\ 1 \\ 0 \\ -2 \end{bmatrix}, \quad \overrightarrow{\mu_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \overrightarrow{\mu_4} = \frac{1}{\sqrt{4.5}} \begin{bmatrix} \frac{-1}{2} \\ 2 \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

Problem 4C

The gram-schmidt function I wrote resulted in approximately the same vectors as the ones calculated by hand from problem 4a and 4b. The results of gram-schmidt function I wrote can be viewed by running the attached file kvnchuGRAMSCHMIDT.m

Problem 5

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 5 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

First find the rref of A and $A|I$ (will be used for later problems)

$$rref(A) = \begin{bmatrix} 1 & 1 & 1 & 1 & | & 0 \\ 0 & 5 & 1 & 1 & | & 0 \\ 0 & 0 & 2 & 1 & | & 0 \end{bmatrix} \tag{1}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & | & 0 \\ 0 & 1 & \frac{1}{5} & \frac{1}{5} & | & 0 \\ 0 & 0 & 1 & \frac{1}{2} & | & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \frac{4}{5} & \frac{4}{5} & | & 0 \\ 0 & 1 & \frac{1}{5} & \frac{1}{5} & | & 0 \\ 0 & 0 & 1 & \frac{1}{2} & | & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & \frac{2}{5} & | & 0 \\ 0 & 1 & 0 & \frac{1}{10} & | & 0 \\ 0 & 0 & 1 & \frac{1}{2} & | & 0 \end{bmatrix}$$

$$rref(A|I) = \begin{bmatrix} 1 & 1 & 1 & 1 & | & 1 & 0 & 0 \\ 0 & 5 & 1 & 1 & | & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & | & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$= \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{array}\right]$$

$$= \left[\begin{array}{cccc|ccc} 1 & 0 & \frac{4}{5} & \frac{4}{5} & 1 & \frac{-1}{5} & 0 \\ 0 & 1 & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} & \frac{-1}{10} \\ 0 & 0 & 1 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{array}\right]$$

$$= \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & \frac{2}{5} & 1 & \frac{-1}{5} & \frac{-2}{5} \\ 0 & 1 & 0 & \frac{1}{10} & 0 & \frac{1}{5} & \frac{-1}{10} \\ 0 & 0 & 1 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{array}\right]$$

Problem 5A

$\mathbf{Dim_A}$ = 3x4

Domain $= \mathbb{R}^4$

Co-Domain $= \mathbb{R}^3$

Problem 5B

$$C(A) = span(\vec{c_1}, ..., \vec{c_4})$$

where $\vec{c_1}, ..., \vec{c_4}$ are the columns of matrix A. In other words, the column space is the span of all linearly independent columns within the matrix A (i.e. all columns with a pivot in rref(A)).

Problem 5C

There are three pivots in the result of rref(A). This indicates that the first three columns of matrix A are the basis of A.

$$C(A) = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\}$$

Problem 5D

$$A\vec{x} = \vec{0}$$

$$N(A) = \left\{ \vec{x} : A\vec{x} = \vec{0} \right\}$$

Problem 5E

From the results of rref(A), we have a linear system with four equations as follows

$$x_1 = \frac{-2x_4}{5}, \quad x_2 = \frac{-x_4}{10}, \quad x_3 = \frac{-x_4}{2}, \quad x_4 = \text{free variable}$$

Since there is a free variable in the system of equations, this indicates that there is 1 column without a pivot. Because of this, we can conclude that that the matrix is not completely linearly independent. From this, we can also conclude that the nullity of the matrix is 1 which indicates that the dimension of the null space is 1.

Problem 5F

$$C(A^T) = span(\vec{r_1}, \vec{r_2}, \vec{r_3})$$

where $\vec{r_1}, \vec{r_2}, \vec{r_3}$ are the rows of matrix A. In other words, the row space is the set of all non-zero rows in rref(A).

Problem 5G

From the results of rref(A), the basis for the row space of A are all three rows of A.

$$C(A^T) = \left\{ \begin{bmatrix} 1 & 0 & 0 & \frac{2}{5} \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & \frac{1}{10} \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \end{bmatrix} \right\}$$

Problem 5H

$$A^T \vec{x} = \vec{0}$$

$$N(A^T) = \left\{ \vec{x} : A^T \vec{x} = \vec{0} \right\}$$

Problem 5I

From rref(A| I), we obtain the elimination matrix

$$E = \begin{bmatrix} 1 & \frac{-1}{5} & \frac{-2}{5} \\ 0 & \frac{1}{5} & \frac{-1}{10} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

Left nullspace = last m-r rows of E, where r is the rank. Here, m = 3 and r = 3.

$$m - r = 3 - 3 = 0$$

$$N(A^T) = \left\{ \vec{0} \right\}$$

Problem 5J

We see that there are three pivots in the final result of rref(A). Thus we can conclude the rank of the matrix A is 3.