REMEMBER THAT YOU ARE REQUIRED TO WRITE UP YOUR HW IN LaTeX

(Post on PIAZZA if you need help.)

Problem 01 (100 points) This is a MATLAB exercise.

(a) Download the data file: HW_01.mat from CANVAS to your working directory, and load it into your MATLAB session by:

```
>> load HW_01;
```

Then, draw the signal $\mathbf{x}$ in the data file using the following commands:

```
>> figure(1);
>> stem(x); hold on; plot(x); grid;
```

Note that this signal $\mathbf{x}$ consists of only 8 points, i.e., a very short signal (vector).

(b) In a different figure window, draw the 8 basis vectors stored as column vectors of the matrix U as follows:

```
>> figure(2);
>> for k=1:8
     subplot(8,1,k);
     stem(U(:,k)); axis([0 9 -0.5 0.5]); axis off; hold on;
   end
>> for k=1:8
     subplot(8,1,k);
     plot(U(:,k));
   end
```

You may need to see the details of these 8 plots by enlarging the window to a full screen. Print this figure and attach it to your HW submission.

(c) Compute the expansion coefficients (i.e., the weights of the linear combination) of $\mathbf{x}$ with respect to the basis vectors $U(:,1),\dots,U(:,8)$ via

```
>> a=U'*x;
```

(d) Check the values of the entries of the coefficient vector $\mathbf{a}$ and create a new vector $\mathbf{a}2$ of length 8 whose only nonzero entries are the two largest entries of $\mathbf{a}$ in terms of their absolute values.

(e) Construct an approximation $\mathbf{x}2$ of $\mathbf{x}$ using $\mathbf{a}2$. Then, plot $\mathbf{x}2$ over Figure 1 as follows:

```
>> figure(1); stem(x2,'r*'); plot(x2,'r');
```

(f) Now, instead of $\mathbf{a}2$, let's construct $\mathbf{a}4$ of length 8 whose only nonzero entries are the four largest entries of $\mathbf{a}$ in terms of their absolute values. Then,

(g) Construct an approximation $\mathbf{x}4$ of $\mathbf{x}$ using $\mathbf{a}4$. Then, plot $\mathbf{x}4$ over Figure 1 as follows (note using the different color from $\mathbf{x}2$):

```
>> figure(1); stem(x4,'gx'); plot(x4,'g');
```

Then, print out Figure 1, and attach it to your HW submission.

(h) Consider now $\mathbf{x}8$, which is just a full reconstruction without throwing out any coefficients, i.e.,

```
>> x8=U*a;
```

Finally, compute the relative error of $\mathbf{x}8$ by

```
>> sqrt(sum((x-x8).^2)/sum(x.^2))
```

and report the result. Similarly compute the relative error of $\mathbf{x}4$ and $\mathbf{x}2$, and report the results.

(i) Write a detailed explanation of what this MATLAB program does.

Problem 02 FIRST: If you haven't already, read about term-document matrices in the first chapter of Eldén. Now, consider the following set of terms (words) and documents (or rather book titles):

	Terms		Documents
T1:	Book (Handbook, BOOK)	D1:	The Princeton Companion to Mathematics
T2:	Equation (Equations)	D2:	NIST Handbook of Mathematical Functions
T3:	Function (Functions)	D3:	Table of Integrals, Series, and Products
T4:	Integral (Integrals)	D4:	Linear Integral Equations
T5:	Linear	D5:	Proofs from THE BOOK
T6:	Mathematics (Mathematical)	D6:	The Book of Numbers
T7:	Number (Numbers)	D7:	Number Theory in Science and Communication
T8:	Series	D8:	Green's Functions and Boundary Value Problems
		D9:	Discourse on Fourier Series
		D10:	Basic Linear Partial Differential Equations
		D11:	Mathematical Physics, An Advanced Course

(a) Construct 8×11 term-document matrix.

(b) Suppose we want to query "Integral Equation." Construct the query vector $\mathbf{a}$.

(c) Find the three closest documents for the query in **Problem 02 (b)**.

Problem 03 (This is a review problem.) At the beginning of 2009, the population of California was 36,453,973. The population living in the United States but outside of California was 271,491,582. During that year, 458,682 people moved to California from another state. Similarly, 545,921 people moved from California to elsewhere in the United States. Set up a matrix vector multiplication problem whose solution shows the population changes in California and in the rest of the United States for 2009.

Problem 04 (This is another review problem.) Given the four vectors $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, and $\mathbf{x}_4$ in **Problem 04 (a)** use the Gram-Schmidt procedure to construct an orthonormal basis for $\boldsymbol{R}^4$. Do the same for the four vectors $\mathbf{y}_1$, $\mathbf{y}_2$, $\mathbf{y}_3$, and $\mathbf{y}_4$ in **Problem 04 (b)**.

(a) Use the Gram-Schmidt procedure to construct an orthonormal basis for $\boldsymbol{R}^4$ starting with the vectors $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, and $\mathbf{x}_4$ below.

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \qquad \mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \qquad \mathbf{x}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \qquad \mathbf{x}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

(b) Use the Gram-Schmidt procedure to construct an orthonormal basis for $\boldsymbol{R}^4$ starting with the vectors $\mathbf{y}_1$, $\mathbf{y}_2$, $\mathbf{y}_3$, and $\mathbf{y}_4$ given below.

$$\mathbf{y}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \qquad \mathbf{y}_2 = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \qquad \mathbf{y}_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \qquad \mathbf{y}_4 = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

(c) Check your work using MATLAB by writing your own Gram-Schmidt algorithm and upload you MATLAB file, which should be named,

yourusername_Gram_Schmidt.m

or

yourusername_Modified_Gram_Schmidt.m

where your UCD email address is

yourusername@ucdavis.edu.

HINT: There is MATLAB code in the Fifth Edition of Strang for the Modified Gram Schmidt algorithm

Problem 05 (This is also a review problem.) Consider the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 5 & 1 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

(a) View this matrix as a linear transformation, T, between two vector spaces. What are the *domain* and the *codomain* of T?

(b) What is the column space, $C(A)$, or *image*, of the linear transformation T.

(c) Find a basis for the column space, $C(A)$ of A.

(d) What is the nullspace, $N(A)$, which is also known as the kernel of T.

(e) Find a basis for the nullspace $N(A)$ of A.

(f) What is the row space of A?

(g) Find a basis for the row space, $C(A^T)$ of A.

(h) What is the left null space, $N(A^T)$, of A.

(i) Find a basis for the left nullspace $N(A^T)$.

(j) What is the rank of the matrix A?