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Improved MPSP Method-based Cooperative Re-entry Guidance for Hypersonic Gliding Vehicles

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Abstract: A computationally sufficient technique is used to solve the 3-D cooperative re-entry guidance problem for hypersonic gliding vehicles. Due to the poor surrounding adaptive ability of the traditional cooperative guidance methods, a novel methodology, named as model predictive static programming (MPSP), is used to solve a class of finite-horizon optimal control problems with hard terminal constraints. The main feature of this guidance law is that it is capable of hitting the target with high accuracy for each one of the cooperative vehicles at the same time. In addition, it accurately satisfies variable constraints. Performance of the proposed MPSP-based guidance is demonstrated in 3-D nonlinear dynamics scenario. The numerical simulation results show that the proposed cooperative re-entry guidance methodology has the advantage of computational efficiency and better robustness against the perturbations.

1 Introduction

Hypersonic vehicles have been drawing a lot of attention in recent decades. In order to improve the survivability and fighting strength, it is significant to consider the multiple vehicles as a cooperative group and design an optimal trajectory for each of them. It can be foreseen that cooperative hypersonic vehicles will replace single ones for more complicated flight missions. What's more, the core task of the multi-vehicle cooperative guidance is to avoid the threat and finally reach the target to complete the task. It can be achieved in two ways. One is to make the vehicle select more curved trajectory to increase flight time, waiting for the later arrival vehicle. Another is to require the later arrival vehicle to accelerate, and thus catch up. But for the hypersonic gliding vehicles, the re-entry process has no power, so it is difficult to achieve the normal acceleration process. Therefore, the first way is desirable.

Vehicles cooperative operations can complete many tasks that a single vehicle cannot complete, such as tactical stealth, the ability of electronic countermeasures and so on [1]. Vehicles not only need to achieve their precise guidance, but also need to cooperate with other vehicles to complete a variety of operational tasks, so the overall effectiveness of one's own operations can be maximized.

Many researchers have done a lot of works on the cooperative guidance. A flight controllable guidance law, Impact-Time-Control Guidance (ITCG) is given in [2]. And in [3], a two-level cooperative guidance architecture for multi-missile attack is proposed. The architecture is composed

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with the guidance law for each missile on the lower level, and the upper level guidance law used the coordination strategy. Zhao Shiyu proposed a cooperative guidance scheme based on distributed coordination [4].

In this paper, a nonlinear suboptimal guidance scheme is developed for the re-entry phase of the reusable launch vehicles here. It is based on a quite straightforward methodology, named as MPSP [5-6], which combines the philosophies of nonlinear model predictive control (MPC) [7] and approximate dynamic programming (ADP) [8]. The method provides a finite time nonlinear suboptimal guidance law which leads to a rapid solution of the update. To the best of authors' knowledge, this method have not been applied to the cooperative guidance for hypersonic gliding vehicles.

The rest of this paper is organized as follows. The problem formulation and preliminaries are presented in the next section. The improved MPSP theory used in the cooperative guidance is given in Sec 3. The simulation results and the analysis are then presented in Sec 4. Finally, the appropriate conclusions are drawn in Sec 5.

2 Mathematic model for hypersonic gliding vehicles

2.1 System dynamics model description of gliding vehicles

The nonlinear equations assuming a spherical, rotating Earth can be described as:

$$
\begin{aligned}
&\dot{r} = V\sin\gamma \\
&\dot{\theta} = -\frac{V\cos\gamma\sin\psi}{r\cos\phi} \\
&\dot{\phi} = \frac{V\cos\gamma\cos\psi}{r} \\
&\dot{V} = -D - g\sin\gamma + \omega^2 r\cos\phi(\sin\gamma\cos\phi - \cos\gamma\sin\phi\cos\psi) \\
&\dot{\gamma} = \frac{1}{V}[\mathrm{L}\cos\sigma + (\frac{V^2}{r} - g)\cos\gamma] + 2\omega\cos\phi\sin\psi + \frac{\omega^2 r}{V}\cos\phi(\cos\gamma\cos\phi - \sin\gamma\sin\phi\cos\psi) \\
&\dot{\psi} = -\frac{\mathrm{L}\sin\sigma}{V\cos\gamma} + \frac{V}{r}\cos\gamma\sin\psi\tan\phi + 2\omega(\sin\phi - \tan\gamma\cos\phi\cos\psi) + \frac{\omega^2 r}{V}\cos\gamma\cos\psi\sin\phi\cos\phi
\end{aligned}
\tag{1}
$$

2.2 Standard predictive equations

Consider that the cooperative strategy is deceleration and waiting. The terminal speed is unconstraint in the process of cooperative control. The state variables are $X = [\mathrm{r}, \theta, \phi, V, \gamma, \psi]^T$, and the control variables are $U = [\alpha, \sigma]^T$. The nonlinear function can be described as $\dot{X} = F(\mathrm{X}, \mathrm{U})$ and discretization handled by Euler method as the discretization systems model. The state and output are given by

$$X_{k+1} = F_k(\mathrm{X}_k + \mathrm{U}_k) \qquad \mathrm{Y}_k = X_k X_{k+1} = F_k(\mathrm{X}_k + \mathrm{U}_k) \tag{2}$$

3 Improved MPSP design: mathematical details

3.1 Solution of the sensitivity matrix

The primary principle of MPSP guidance is to update the control variable and obtain a suitable control history U_k, $k = 1,2,\cdots,N-1$, so that the output at the final time step Y_N goes to a desired value Y_N^d, i.e. $Y_N \to Y_N^d$. Expanding Y_N about Y_N^d using Taylor series expansion and then we can write the error in the output as

$$\Delta Y_N \cong dY_N = \left[\frac{\partial Y_N}{\partial X_N}\right] dX_N \tag{3}$$

From (1), the error in state at time step (k+1) can be expressed as

$$dX_{k+1} = \left[\frac{\partial F_k}{\partial X_k}\right] dX_k + \left[\frac{\partial F_k}{\partial U_k}\right] dU_k \tag{4}$$

where dX_k is the error of state at time step (k+1), and dU_k is the error of state at time step (k+1). According to the function (5), we can write the error in output at time step (N-1) as

$$dY_N = \left[\frac{\partial Y_N}{\partial X_N}\right]\left(\left[\frac{\partial F_{N-1}}{\partial X_{N-1}}\right] dX_{N-1} + \left[\frac{\partial F_{N-1}}{\partial U_{N-1}}\right] dU_{N-1}\right) \tag{5}$$

And since the initial condition is specified, there is no error in state at the first step. It means

$$dY_N = B_1 dU_1 + B_2 dU_2 + \cdots + B_{N-1} dU_{N-1} = \sum_{k=1}^{N-1} B_k dU_k \tag{6}$$

Note that it is quite difficult to evaluate each of the B_k, $k = 1,2,\cdots,N-1$. Especially when N is high, it will be the main impediment to computational efficiency.

3.2 Update the control

If the terminal error cannot meet the requirement, the optimal control theory will be used in the MPSP method to correct the control variable. For equation (6), each control variable is independent from others. And the cost function is described as

$$J = \frac{1}{2}\sum_{k=1}^{N-1}\left(dU_k\right)^T R_k \left(dU_k\right) \tag{7}$$

After deducing, we can get

$$dU_k = -R_k^{-1} B_k^T A_\lambda^{-1} dY_N \tag{8}$$

In each guidance cycle, the control at the k th time step is updated to $U_k = U_k^0 - dU_k$, where U_k^0 is the control history of the previous update

4 Numerical results and Discussions

4.1 Time and accuracy

To start the MPSP algorithm, the guess value of the guidance command history makes full use of prior knowledge and improves the accuracy of prediction-correction. The trajectory along with lower and upper bounds of constraints can be seen in Figure 1. All the path constraints and terminal constraints are satisfied.

The guidance cycle (for which the guidance command is updated) is assumed to be 1.8 s. The total time taken by the Vehicle A and Vehicle B to reach the desired end point conditions are around 2830s and 2780s respectively. This means that vehicle B takes off 50s more lately than the vehicle A.

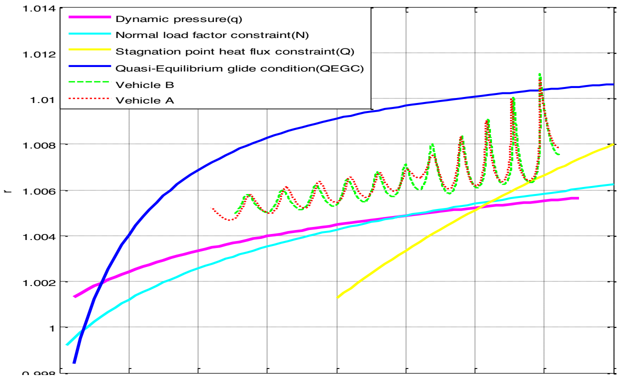
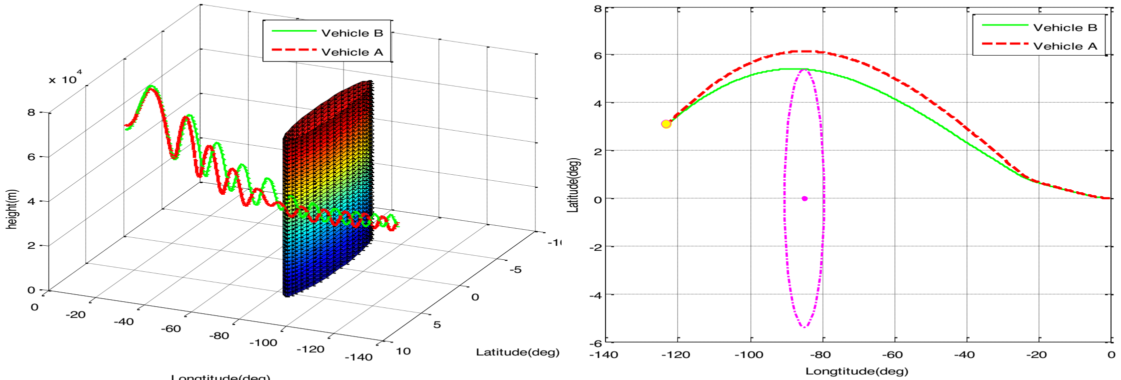


Figure 1. Velocity vs. geocentric distance

The improved MPSP prediction and correction algorithm requires only 7 iterations per iteration cycle for Vehicle B and only 3 iterations per iteration cycle for Vehicle A. It does not consume the computational time. Thus, the MPSP method-based cooperative re-entry guidance has the possibility of online implementation.



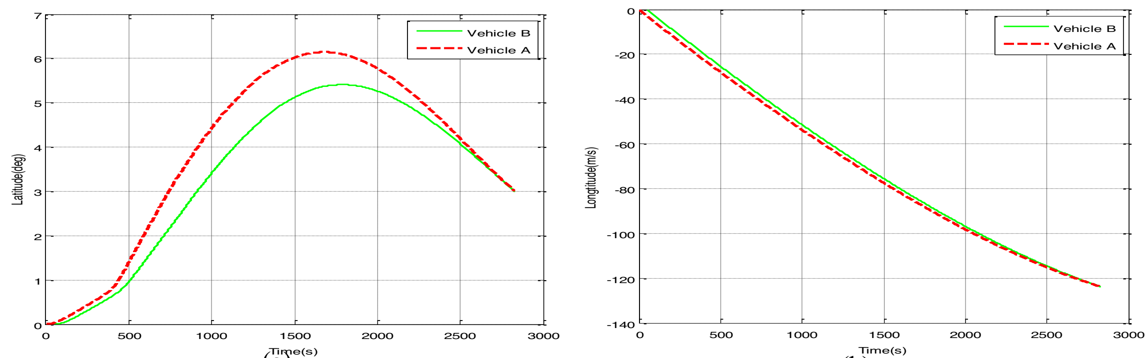
(a) (b)

Figure 2. (a) 3-D figure of reentry trajectory (b) Flat trajectory

The no-fly zones can be described as the cylinder. Figure 2(a) and 2(b) shows that the 3-D trajectory and the flat trajectory of MPSP method-based cooperative re-entry guidance for hypersonic gliding vehicles respectively. In order to arrive at the target at the same time， Vehicle A chose to glide in a more curved trajectory. Furthermore, both Vehicle A and Vehicle B avoid the no-fly zone successfully.

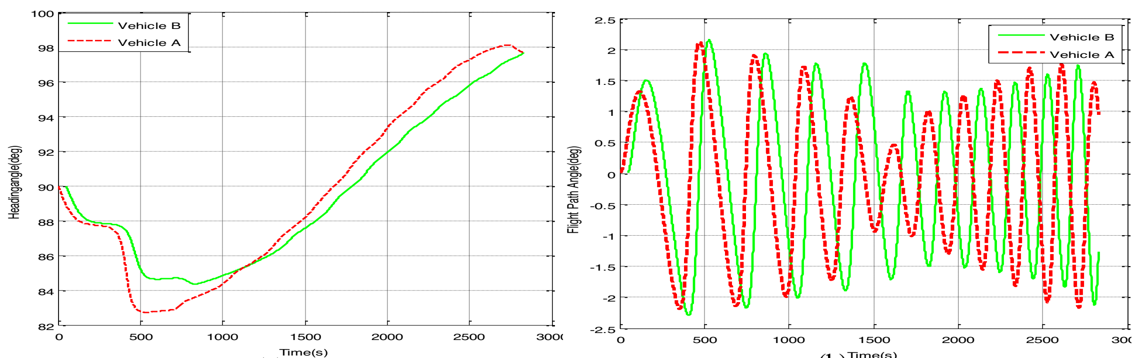
4.2 The analysis of MPSP method-based cooperative re-entry guidance process

Figure 3 and 4 show the latitude and longitude, the heading angle and flight path angle. Without the MPSP method-based cooperative guidance, the trajectories of the two vehicles are very close. This means that they cannot hit the same target.



(a) (b)

Figure 3. (a) Latitude of the cooperative vehicles (b) Longitude of the cooperative vehicles



(a) (b)

Figure 4. (a) Heading angle (b) Flight path angle

4.3 Aerodynamic coefficient perturbation

Considering the aerodynamic coefficient and the perturbation effect, take both the deviation of 20%, and get the simulation results as shown below.

Table 1. The Terminal states of different case

	Vehicle	C_L C_D	L/km	E_Y	r/km	dL/km
Case 1	*Vehicle A*	0	*13771.7154814*	*0.0000258*	*6411.4*	*0.4223065*
	Vehicle B			*0.0001689*	*6408.96*	*2.9496947*
Case 2	*Vehicle A*	20%	*13771.7154814*	*0.0001547*	*6411.5*	*2.2496153*
	Vehicle B			*0.0001085*	*6409.9*	*1.4978738*

The entry guidance algorithm is proved to be successful. In table 1, the aerodynamic perturbation is simulated with 20% aerodynamic perturbation in Case 2. As expected, it is seen that the MPSP method-based cooperative re-entry guidance can enhance the flexibility of the cooperative guidance law. Obviously, the 3-D cooperative guidance law has better robustness and improves the guidance accuracy of the hypersonic gliding vehicles.

5 Conclusions

There were few researches in developing the algorithms and methods for hypersonic gliding vehicles in the cooperative re-entry guidance. In this paper, a computationally sufficient MPSP technique has been used to solve the 3-D cooperative re-entry guidance problem for hypersonic gliding vehicles. The direct adaptive technique undermines the need to update the control quickly and uses for the cooperative re-entry vehicles online. Numerical simulation results validate the high performance about computational efficiency and accuracy and also show great robustness against perturbations in the aerodynamic conditions.

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