EXAMPLE 6.1
Magnetic Heading

INPUTS:

Attitude

True attitude (Eulers)	ϕ_{nb} =	4	deg	0.069813	rad
	θ_{nb} =	-6	deg	-0.10472	rad
	ψ_{nb} =	30	deg	0.523599	rad
Roll measurement error	$\delta\phi_{nb}$ =	0.1	deg	0.001745	rad
Pitch measurement error	$\delta\theta_{nb}$ =	-0.15	deg	-0.00262	rad

Earth's Magnetic Field

True magnitude	B_E =	45	μT		
True declination	α_{nE} =	10	deg	0.174533	rad
True dip	γ_{nE} =	40	deg	0.698132	rad
Database declination error	$\delta\alpha_{nE}$ =	0.3	deg	0.005236	rad

Other Sources of Magnetism

Local magnetic anomalies (N, E, D)		2	μT
	$\mathbf{m}_A^n$ =	-1.8	μT
		3	μT
Hard-iron magnetism		-1	μT
(body-frame axes)	$\mathbf{b}_m$ =	-0.5	μT
		1	μT

Soft-iron magnetism	0.012	-0.014	0.006
(body-frame axes) $\mathbf{M}_m$ =	-0.008	0.007	0.017
	0.003	-0.019	-0.011

Calculate magnetometer measurements

Local navigation frame to body frame coordinate transformation matrix:

From (2.22),

$$\mathbf{C}_n^b = \begin{bmatrix} \cos\theta_{nb}\cos\psi_{nb} & \cos\theta_{nb}\sin\psi_{nb} & -\sin\theta_{nb} \\ \begin{pmatrix} -\cos\phi_{nb}\sin\psi_{nb} \\ +\sin\phi_{nb}\sin\theta_{nb}\cos\psi_{nb} \end{pmatrix} & \begin{pmatrix} \cos\phi_{nb}\cos\psi_{nb} \\ +\sin\phi_{nb}\sin\theta_{nb}\sin\psi_{nb} \end{pmatrix} & \sin\phi_{nb}\cos\theta_{nb} \\ \begin{pmatrix} \sin\phi_{nb}\sin\psi_{nb} \\ +\cos\phi_{nb}\sin\theta_{nb}\cos\psi_{nb} \end{pmatrix} & \begin{pmatrix} -\sin\phi_{nb}\cos\psi_{nb} \\ +\cos\phi_{nb}\sin\theta_{nb}\sin\psi_{nb} \end{pmatrix} & \cos\phi_{nb}\cos\theta_{nb} \end{bmatrix}$$

$$\mathbf{C}_n^b = \begin{bmatrix} 0.861281226 & 0.497260948 & 0.104528 \\ -0.505096681 & 0.860270041 & 0.069374 \\ -0.055425555 & -0.112547797 & 0.992099 \end{bmatrix}$$

Flux density of Earth's magnetic field

From (6.1),

$$\mathbf{m}_E^n = \begin{pmatrix} \cos\alpha_{nE}\cos\gamma_{nE} \\ \sin\alpha_{nE}\cos\gamma_{nE} \\ \sin\gamma_{nE} \end{pmatrix} B_E$$

$\mathbf{m}_E^n =$ 33.9482928 μT, 5.98599997 μT, 28.92544244 μT

Total magnetic flux density

From (6.9) $\mathbf{m}_m^b = \mathbf{b}_m + (\mathbf{I}_3 + \mathbf{M}_m)\mathbf{C}_n^b(\mathbf{m}_E^n + \mathbf{m}_A^n)$

$\mathbf{m}_E^n + \mathbf{m}_A^n =$ 35.9482928 μT, 4.18599997 μT, 31.92544244 μT

$\mathbf{C}_n^b(\mathbf{m}_E^n + \mathbf{m}_A^n) =$ 36.38024145 μT, -12.34146652 μT, 29.20962962 μT

$(\mathbf{I}_3 + \mathbf{M}_m) =$

1.012	-0.014	0.006
-0.008	1.007	0.017
0.003	-0.019	0.989

$(\mathbf{I}_3 + \mathbf{M}_m)\mathbf{C}_n^b(\mathbf{m}_E^n + \mathbf{m}_A^n) =$ 37.16484265 μT, -12.22233501 μT, 29.23195228 μT

$\mathbf{m}_m^b =$ 36.16484265 μT, -12.72233501 μT, 30.23195228 μT

Calculate Magnetic Heading Measurement

Roll and pitch measurements:

$\tilde{\phi}_{nb} = \phi_{nb} + \delta\phi_{nb} =$ 0.071558499 rad

$\tilde{\theta}_{nb} = \theta_{nb} + \delta\theta_{nb} =$ -0.107337749 rad

From (6.6),

$$\tilde{\psi}_{mb} = \arctan_2\left(\begin{array}{l} -\tilde{m}_{m,y}^b \cos\hat{\phi}_{nb} + \tilde{m}_{m,z}^b \sin\hat{\phi}_{nb}, \\ \tilde{m}_{m,x}^b \cos\hat{\theta}_{nb} + \tilde{m}_{m,y}^b \sin\hat{\phi}_{nb} \sin\hat{\theta}_{nb} + \tilde{m}_{m,z}^b \cos\hat{\phi}_{nb} \sin\hat{\theta}_{nb} \end{array}\right)$$

$-\tilde{m}_{m,y}^b \cos\hat{\phi}_{nb} + \tilde{m}_{m,z}^b \sin\hat{\phi}_{nb} =$ 14.85128 μT

$\tilde{m}_{m,x}^b \cos\hat{\theta}_{nb} + \tilde{m}_{m,y}^b \sin\hat{\phi}_{nb} \sin\hat{\theta}_{nb} + \tilde{m}_{m,z}^b \cos\hat{\phi}_{nb} \sin\hat{\theta}_{nb} =$ 32.82364 μT

$\tilde{\psi}_{mb} =$ 0.424895188 rad, 24.34470099 deg

Note: The arguments of the Excel ATAN2 function are the opposite way round

Calculate True Heading

Database-indicated magnetic declination:

$\hat{\alpha}_{nE} = \alpha_{nE} + \delta\alpha_{nE} =$ 10.3 deg 0.179769 rad

True heading:

From (6.7),

$\tilde{\psi}_{nb} = \tilde{\psi}_{mb} + \tilde{\alpha}_{nE} =$ 34.64470099 deg 0.604664 rad

Heading Error

$\tilde{\psi}_{nb} - \psi_{nb} =$ 4.644700992 deg 0.081065 rad