PREDICTIVE ENTRY GUIDANCE FOR AN APOLLO-TYPE VEHICLE

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ABSTRACT

Entry into the earth's atmosphere at the high speed of a lunar return poses a difficult guidance problem for a low L/D, roll-controlled vehicle such as the Apollo Command Module. This paper describes an approach to its solution based on fast numerical prediction of the future trajectory.

Previous predictive programs proposed for this task were too slow and cumbersome to be used in the Apollo Guidance Computer. The program described here, however, is small and fast enough to fit the on-board computer. Small size was attained by simplifying all elements of the guidance as much as satisfactory performance would permit. The necessary speed was guaranteed by using a small, fixed number of integration steps in the prediction; thus these steps had to be of variable length, and carefully chosen.

INTRODUCTION

The operational program developed by MIT[1] to guide the Apollo spacecraft during atmospheric entry uses reference trajectories. An alternative approach, conceptually offering greater flexibility (in ability to deal with a wider range of initial conditions than are encountered operationally in Apollo missions) would be to use prediction of the future trajectory. However, until now predictive programs have been too large and slow for use in the Apollo Guidance Computer. The present program does not have that drawback, and has at least comparable performance to the MIT program.

The supposed slowness of predictive schemes (due to the need to simulate a complete trajectory every cycle), has limited the amount of work done on them. Bailey and Duersch[2] considered guidance of a high-lift vehicle at subcircular entry velocities. Dow, Fields, and Scammell[3] covered supercircular entry, but for a variable L/D vehicle. A fixed L/D, roll-controlled, vehicle like Apollo was studied by Bryant and Frank,[4] but their program is too slow for operational use. The L/D of their vehicle, 0.5, is also somewhat higher than that of Apollo (0.25 to 0.35).

Our program is simpler and smaller than these previous predictive schemes; some of the simplifications will be mentioned later. Compared to MIT's, our program is smaller and slower, which is reasonable as reference trajectories would be expected to require more data storage and predictions more repetitive operations. Thus, though there are presently no plans to use our program on Apollo missions, it could be of service if computer space requirements became more pressing.

PROBLEMS AND REQUIREMENTS

Supercircular atmospheric entry provides one of the more difficult guidance tasks in the Apollo mission. This is due to the great sensitivity of supercircular trajectories, to the need to meet several performance requirements at once, and to the properties of the equipment: low spacecraft L/D, relatively sluggish roll dynamics, and limited on-board computing capacity.

Requirements on a guidance program, then, fall into three categories: size, speed, and performance. The size requirement is determined by the computer, the speed requirement by the computer and the frequency with which new guidance commands are needed. Performance requirements subdivide into the four categories (in order of importance) of skipout prevention, control of maximum acceleration, landing point control, and reasonable fuel use. The last is roughly equivalent to requiring a fairly smooth roll history, and so a comfortable ride for the crew.

No heating control requirements are placed on the guidance system. For an ablatively-cooled vehicle like Apollo, total heat load is the important heating quantity, and this is mainly influenced by the trajectory's initial conditions, rather than by guidance system operation.

RESULTS

The size requirement is more than satisfied. In fact, our program is only about 60% the size of the operational program. As to speed, our program will provide a new guidance command every four seconds when used in the Apollo Guidance Computer, and this is adequate for accurate guidance up to about 45,000 ft/sec entry velocity, which is well above the Apollo maximum of 38,000 ft/sec. This speed is about half that of the operational program, which provides a command every two seconds.

All performance requirements are satisfied. Up to 40,000 ft/sec skipout is prevented, and acceleration held below 10g, whenever the spacecraft has enough lift to do either. Control is possible over a narrower corridor up to 45,000 ft/sec, and with a faster computer (2 second guidance cycle) up to 50,000 ft/sec.

The landing point is controlled to within 2 miles over a footprint which extends from 900 to at least 3000 nautical miles for midcorridor entry (-6.3°) at lunar return speeds (37,000 ft/sec). Normal Apollo range is about 1300 n.m.

Fuel use is almost the same as MIT's - about 25 pounds per flight with one ring of thrusters firing. Roll histories are satisfactorily smooth. An example is shown in Figure 7.

Figures 1 and 2 sum up the spacecraft and guidance capability for L/D's at the ends of the Apollo range - .35 and .25. Added capability at the higher L/D is clearly visible. The shaded shallow skipout and 10g boundaries are the usual spacecraft corridor boundaries. The steep skipout boundary, which only occurs at high speeds, is due to the guidance system and could probably be designed out if desired. Within these shaded boundaries, target miss is less than 2 n.m., subject to the range limitations. Maximum range curves are spacecraft determined, being attained by holding full lift up for the whole flight. The minimum range curve is also affected by the guidance system, due to the need to avoid excessive acceleration. Minimum range is determined by the guidance system's ability to switch between lift full up, to reduce peak acceleration, and lift full down, to reduce the range.

SYSTEM DESCRIPTION

The system block diagram is shown in Figure 3. As in any predictive scheme, the three components are a profile generator, a predictor, and a processing block. The profile generator produces a future roll history which is used by the predictor, together with the current state, to determine a future trajectory by numerical integration. The maximum acceleration and landing point of this trajectory are processed to produce a command roll angle to the autopilot. This sequence is repeated every four-second guidance cycle. Only one future trajectory is predicted per guidance cycle. This is required for fast operation, but it does deprive the system of information about rate of change of miss with roll angle which would be given by two predictions. The two-prediction scheme is used by Bryant and Frank.[4] Our single prediction requires more of the processing block.

Whilst all three blocks influence the program's size, its speed is principally controlled by the predictor and profile. These are the most time-consuming activities because of the need to integrate complete trajectories step by step. The 12 integration steps used account for three-quarters of the program's running time.

PROFILE

The prediction integration is two-dimensional, crossrange not being considered. Thus all that is required from the profile generator is a vertical component of L/D, which is equivalent to a roll angle magnitude. Conversion between these requires knowledge of the spacecraft L/D, but tests have shown that this does not have to be exact. Thus monitoring the spacecraft's varying L/D is unnecessary.

The roll profile for each prediction is a constant roll angle, except on the first prediction step. This use of a constant greatly simplifies the profile calculation. References 3 and 4 use variable roll angles in their predictions, needing extra calculations to determine their values.

On the first step, terms linear in the command and actual roll angles are added to the vertical component of L/D, to take account of the current roll situation. As shown in Figure 3, this results in two inner feedback loops through the profile block, which stabilize the system and improve its accuracy. The gains in these loops depend on the state.

The constant part of the profile is normally set at 55° roll angle,* a value which gives about equal forward and back ranging capability on second entry of a trajectory which leaves the atmosphere. That is, the target is centered in the footprint. Early in the flight, however, this angle is increased to a maximum of 75°. This biases the flight trajectory upwards, giving a lower initial acceleration peak.

PREDICTOR

The predictor uses second-order predictor-corrector integration operating on the following two-dimensional, relative-coordinate equations:

$$\dot{V} = -D - g \sin \gamma$$

$$\dot{R} = V \sin \gamma$$

$$\dot{\theta} = \frac{V \cos \gamma}{R}$$

$$(\dot{\sin \gamma}) = [\dot{\theta} + (L/D_v \cdot D - g \cos \gamma + a_{cor})/V\} \cos \gamma$$

where

V = earth-relative velocity

γ = earth-relative flight path angle

R = distance from center of earth

θ = downrange angle from current position

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*0° roll angle is full lift up; 180°, full down; ±90°, sideways left or right. 55° is a magnitude only.

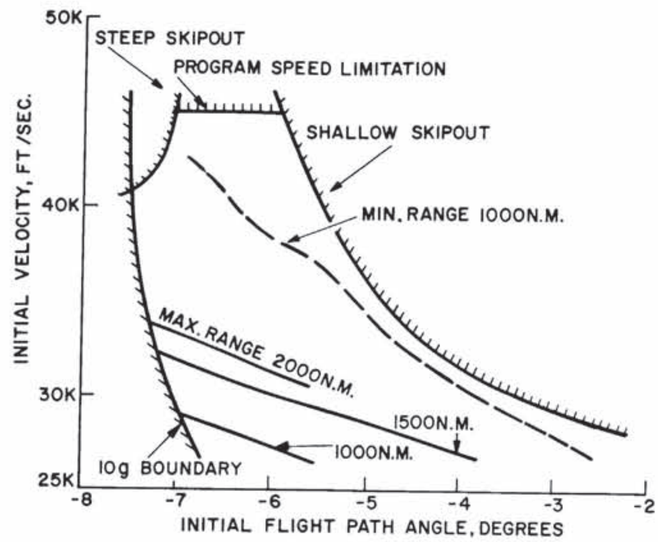

FIGURE 1. GUIDANCE CAPABILITY, L/D .35

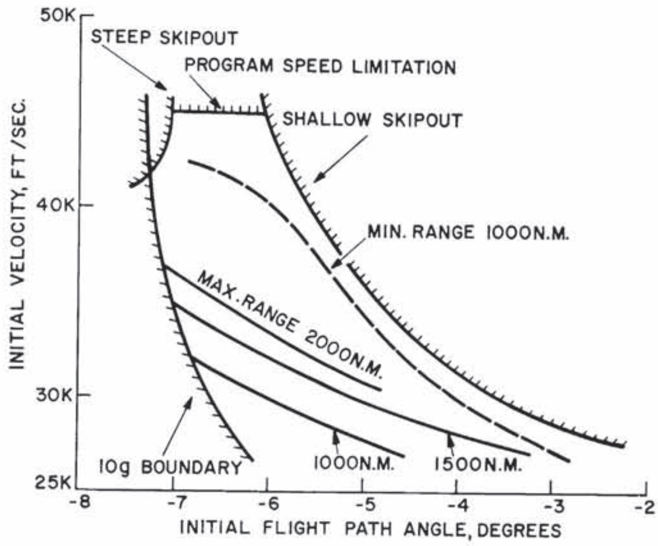

FIGURE 2. GUIDANCE CAPABILITY, L/D .25

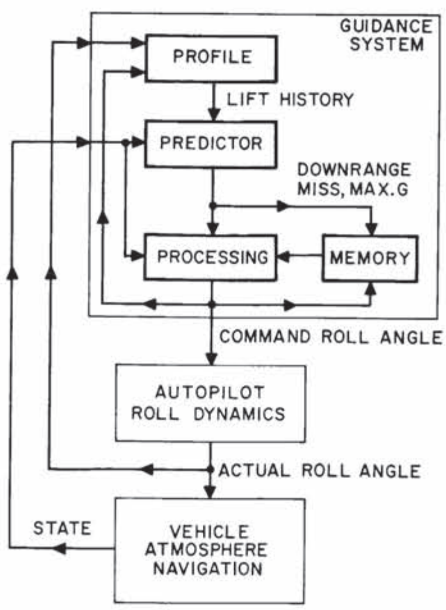

FIGURE 3. SYSTEM BLOCK DIAGRAM

and

D = drag acceleration

g = gravity acceleration

a_{cor} = vertical Coriolis acceleration

L/D_V = vertical component of lift to drag.

L/D_V comes from the profile generator, and D, g, a_{cor} are found by subsidiary formulas, which are kept simple to help program size and speed. The formula for D uses an exponential atmosphere with constant scale height, and that for g an inverse square law; a_{cor} is only calculated once at the beginning of each prediction. The formula for D is based on the current flight drag D_o, thus avoiding need to know the atmosphere base density and spacecraft ballistic coefficient. Thus

$$D = D_o \cdot \frac{V^2}{V_o^2} \cdot e^{-(R-R_o)/h_s}$$

where

D_o, V_o, R_o = current drag, velocity, radius

h_s = atmosphere scale height

It will be noted that time is used as independent variable. This allows the same predictor to integrate through exoatmospheric parts of trajectories, which would not be possible with velocity as independent variable (velocity is at first the obvious choice as constant velocity steps give an accurate integration). Removal of the need for separate treatment of exoatmospheric flight (by a conic formula) reduces the program's size significantly.

To ensure that the prediction will always be finished in the guidance cycle the number of steps used by the predictor is fixed at 12. This small number is made possible by a carefully-chosen variable step length. Approximately constant velocity increments are used, except for low drag parts of the trajectory, when the step length is limited to a maximum time increment. To convert the constant velocity steps to time steps, a knowledge of the future drag, which relates the two, is required. It was found that by considering only the altitude-dependent component of the drag, a simple conversion formula using the vertical velocity could be obtained. That is, if we assume D at time t is given by

$$D = D_o e^{-\dot{R}_o t/h_s}$$

where

D_o, $\dot{R}_o$ = current drag, vertical velocity,

h_s = atmosphere scale height,

we may integrate

$$\Delta V = \int_o^{\Delta t} D \, dt$$

to obtain

$$\Delta t = -\frac{h_s}{\dot{R}_o} \ln \left(1 - \frac{\Delta V}{D_o} \cdot \frac{\dot{R}_o}{h_s}\right),$$

which is well approximated by

$$\Delta t = \frac{\Delta V/D_o}{1 - c \cdot \frac{\Delta V}{D_o} \cdot \frac{\dot{R}_o}{h_s}}$$

This formula, in which the constant c may be chosen for most accurate results, converts the velocity step ΔV into a time step Δt.

PROCESSING

The processing block is required to provide a command roll angle that satisfies the hierarachical performance requirements of skipout prevention, acceleration control, range control, and smooth roll history. It was found possible to include skipout prevention under range control. This change in precedence had no effect up to 40,000 ft/sec entry velocity. Above this speed it resulted in the steep skipouts of Figures 1 and 2.

Crossrange is controlled by the sign of a command roll angle whose magnitude is determined to satisfy the other requirements (that is, by left-right choice of the lateral lift). Crossrange error* is confined to a shrinking band whose width is proportional to velocity squared. When the error exceeds this band, the lateral lift is reversed. If the required vertical lift places the lift vector close to full up or down, extra lateral lift is provided with almost no loss of vertical lift by deflecting the lift vector 15 degrees from the vertical. This crossrange control scheme is very similar to that used by MIT.

Normally, the vertical lift, or command roll angle magnitude, is determined to satisfy downrange control and smoothness requirements. However, whenever the predicted maximum drag rises above a certain

*Crossrange error is the distance of the target from the plane containing the current position, relative velocity vector, and earth center.

level, downrange control is overridden and lift rolled full up for acceleration control. Downrange control is returned to if either of two conditions is satisfied: if the predicted maximum drag falls below another level, or if the drag at the end of the first prediction step is decreasing. The second method often gives a quicker return, resulting in ability to hit shorter ranges; also, at high velocities, it alleviates the steep skipout problem. To prevent unduly early dropouts of this type, the first prediction step is limited while in g-override.

This simple scheme provides effective acceleration control, given a sufficiently accurate prediction and suitable choice of the cut-in and dropout levels. Maximum acceleration is generally kept below 9g, and acceleration control is not entered unnecessarily.

The roll angle magnitude to provide downrange control with a smooth history is basically obtained by multiplying a quadratic function of the predicted downrange miss by a state-dependent gain function, involving the factors velocity, drag, and range to go. Specifically,

$$\varphi_r = \varphi_n + f\ (D,V,\theta_t)\cdot\Delta\theta\cdot(c_1 + c_2\ |\Delta\theta\ |)$$

where

φ_r = magnitude of desired roll angle for range control

φ_n = nominal roll angle

θ_t = downrange to target

$\Delta\theta$ = predicted downrange miss

The constants c_1, c_2, and the gain function f, are chosen to provide overcontrol, so that the desired roll angle is driven fairly quickly to the nominal angle φ_n. This angle (55° except early in the flight) is the same constant angle used in the predictor. The overcontrol thus forces the predicted trajectory to hit the target, and so maintains the target roughly in the middle of the footprint, specifically at second entry of an exiting trajectory, but also more generally.

The gain function f is based on the sensitivity of miss to roll angle, but empirical approximations had to be made, as this sensitivity function varies considerably for different trajectories. The quadratic function of predicted miss prevents excessive response to small misses.

The basic angle φ_r is smoothed by simple logic using values of predicted miss and command roll angle retained from previous guidance cycles, as noted by the memory block in Figure 3. This smoothed angle is the new command roll angle magnitude, unless overridden by the need to roll full up for acceleration control. Addition of a sign determined by the crossrange logic gives the complete command angle.

AN EXAMPLE

To show the interaction of downrange, crossrange, and acceleration control, a sample trajectory is shown in Figures 4 - 8. The trajectory has an initial velocity of 37,000 ft/sec, flight path angle of -6.6°, and range of 1500 n.m. Spacecraft L/D is .3.

Figure 4 shows that the trajectory comes down into the atmosphere and then rises again to achieve the desired range. The first drag peak of Figure 5 is near the pullout point. Also shown on Figure 5 is the predicted future maximum drag. To ensure that the system responds only to imminent drag problems, and so minimize the effect of g-override on range control, this maximum drag is examined only for 100 seconds from the current time. This, together with the fact that drag is only calculated at the ends of prediction steps, accounts for the jumps in predicted drag. Initially, the predicted drag is high enough to trigger g-override, so for the first 76 seconds from entry the lift is rolled full up (0° roll angle).* The resulting drag peak is lower than that initially predicted because the prediction uses less than full lift (55° - 75° roll angle). The second predicted drag peak is not high enough to trigger g-override. Range control is started when the predicted drag drops low enough (first dropout method described earlier). During the period of g-override the predicted miss (Figure 6) changes from an undershoot of some 700 miles to an overshoot of 1000 miles, because full up lift is held too long. When range control starts, the lift is rolled down to reduce this overshoot, as shown in Figure 7. The predicted overshoot is rapidly removed, and from about 150 seconds on, the predicted trajectory hits close to the target, and the roll angle magnitude remains close to the nominal 55°.

At 100 seconds the crossrange error goes outside its band (Figure 8) and triggers a lateral reversal, which may be seen in Figure 7. Subsequently, there are three more such reversals.

CONCLUSIONS

The entry program described is small, fast, and has good performance over a wide range of initial conditions and ranges. The predictive concept's inherent simplicity should also make extension to an even wider range of conditions straightforward.

Most significant about the program is its demonstration of how small and fast a predictive entry program (and, by extension, any predictive program) can be, even though the trajectories involved are quite varied and complex. Ideas of technical significance contributing to the program's success are use of a fixed number of steps in the prediction; use of one integration scheme with time as independent variable for the whole trajectory; and development of a gain function which is applicable to a wide variety of trajectories and is at the same time fairly simple.

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*The small pip to 15° is due to extra lateral lift being provided for crossrange control.

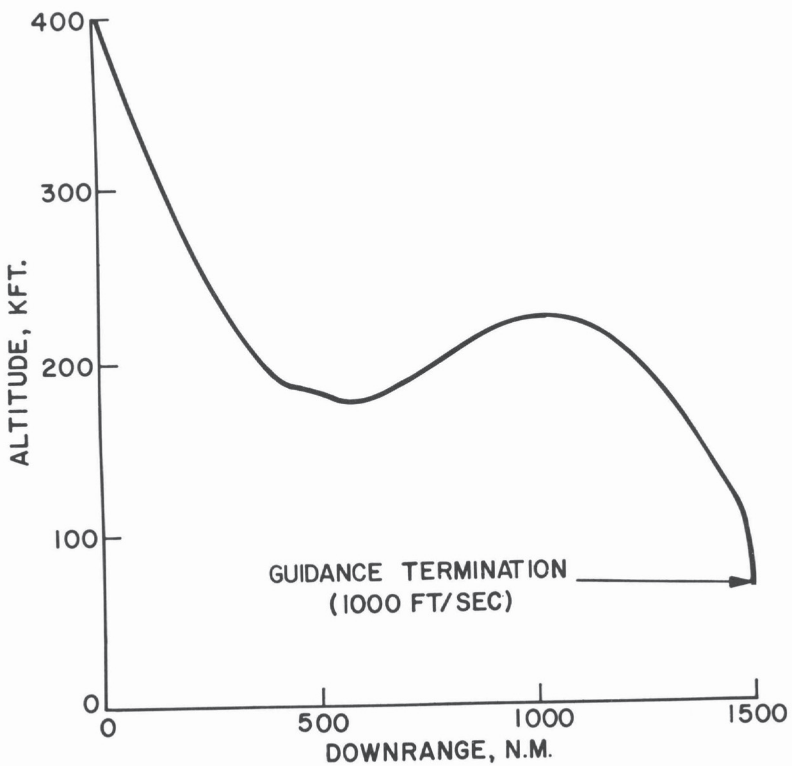

FIGURE 4. TRAJECTORY SHAPE, 1500N.M. FLIGHT

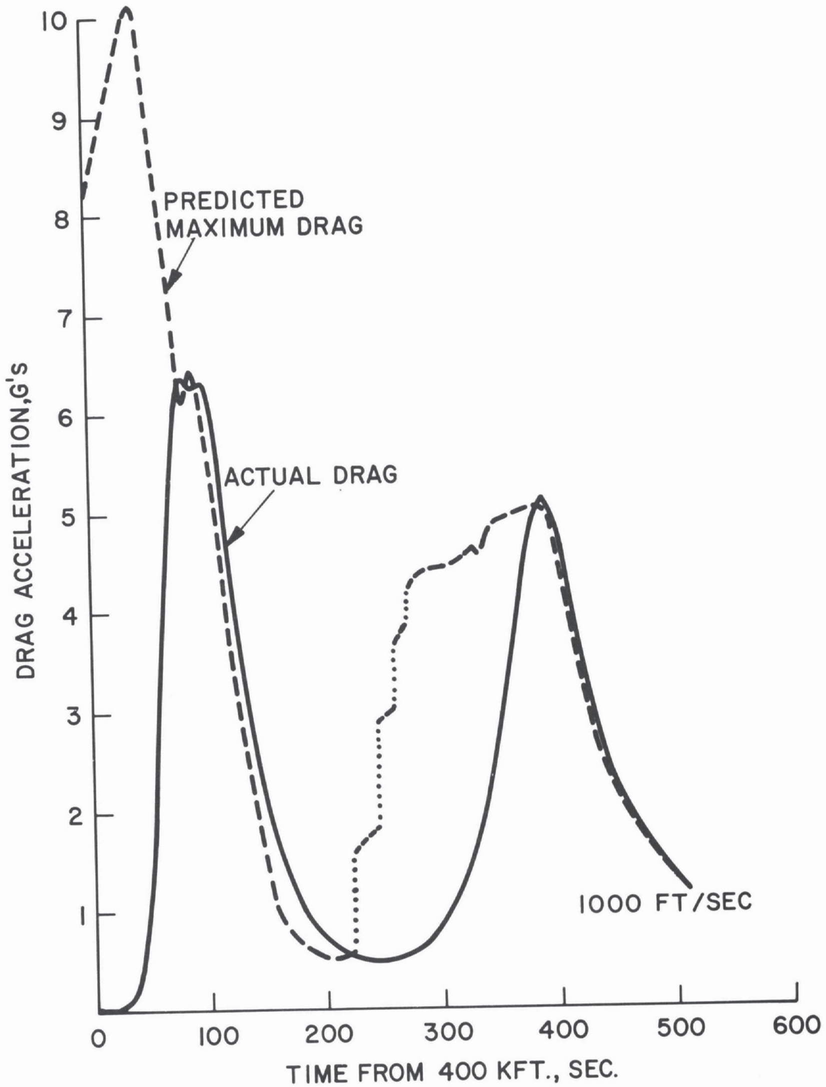

FIGURE 5. DRAG HISTORY, 1500 N.M. FLIGHT

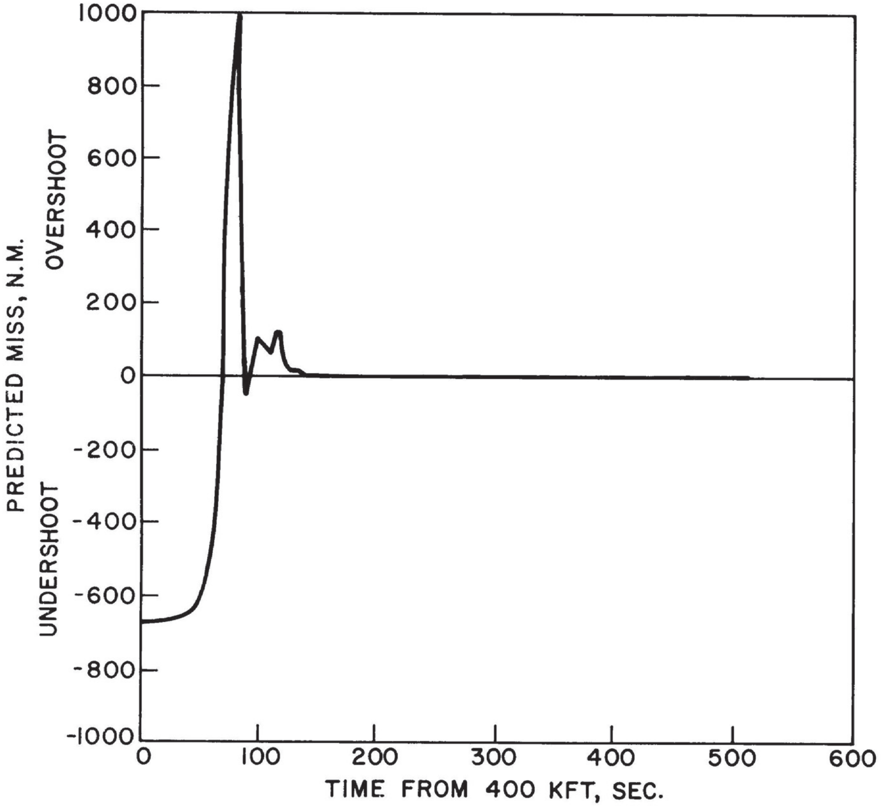

FIGURE 6. PREDICTED DOWNRANGE MISS

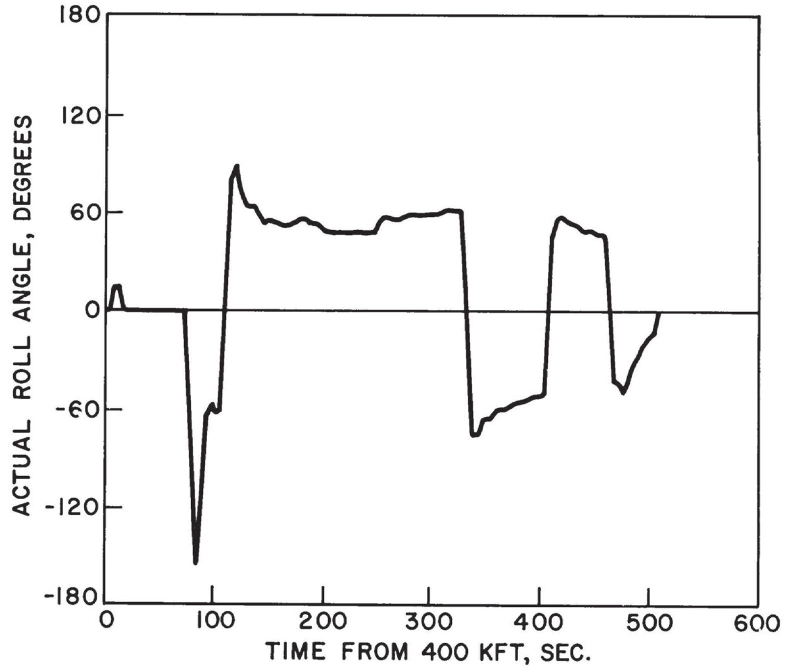

FIGURE 7. ROLL HISTORY, 1500N.M. FLIGHT

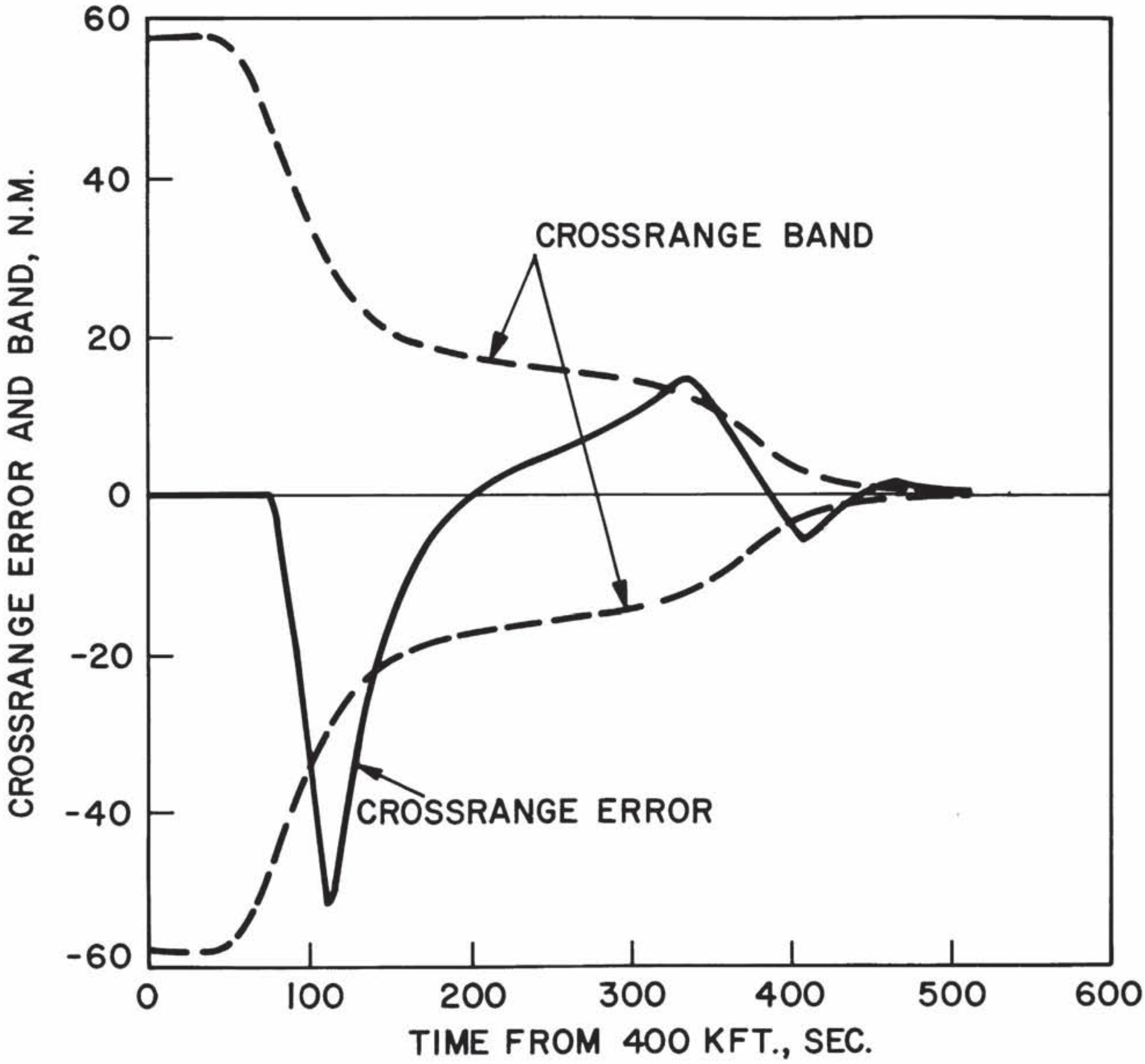

FIGURE 8. CROSSRANGE, 1500 N.M. FLIGHT

KEY WORDS

Apollo, guidance, prediction, reentry

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