EXAMPLE 5.3
Leveling and Direct Gyrocompassing

INPUTS:

Latitude	L_b =	45	deg	0.785398	rad
True attitude (Eulers)	ϕ_{nb} =	2	deg	0.034907	rad
	θ_{nb} =	-3	deg	-0.05236	rad
	ψ_{nb} =	30	deg	0.523599	rad
Acceleration due to gravity	$g^n_{b,D}$ =	9.8	m s^{-2}	(Exact value not important)	
Earth rotation rate	ω_{ie} =	7.29E-05	rad s^{-1}		
Accelerometer errors		0.011	m s^{-2}		
	$\delta\mathbf{f}^b_{ib}$ =	-0.007	m s^{-2}		
		0.009	m s^{-2}		
Gyro errors		1.20E-06	rad s^{-1}		
	$\delta\boldsymbol{\omega}^b_{ib}$ =	-1.80E-06	rad s^{-1}		
		2.50E-06	rad s^{-1}		

Accelerometer measurements

True specific force

From (5.100), $\begin{pmatrix} f^b_{ib,x} \\ f^b_{ib,y} \\ f^b_{ib,z} \end{pmatrix} = \begin{pmatrix} \sin\theta_{nb} \\ -\cos\theta_{nb}\sin\phi_{nb} \\ -\cos\theta_{nb}\cos\phi_{nb} \end{pmatrix} g^n_{b,D}$

$$\begin{pmatrix} f^b_{ib,x} \\ f^b_{ib,y} \\ f^b_{ib,z} \end{pmatrix} = \begin{matrix} -0.512892371 \\ -0.341546348 \\ -9.780607727 \end{matrix} \text{ m s}^{-2}$$

Measured specific force

From (4.17) $\tilde{\mathbf{f}}^b_{ib} = \mathbf{f}^b_{ib} + \delta\mathbf{f}^b_{ib}$

$$\tilde{\mathbf{f}}^b_{ib} = \begin{matrix} -0.501892371 \\ -0.348546348 \\ -9.771607727 \end{matrix} \text{ m s}^{-2}$$

Gyro measurements

Local navigation frame to body frame coordinate transformation matrix:

From (2.22),

$$\mathbf{C}^b_n = \begin{bmatrix} \cos\theta_{nb}\cos\psi_{nb} & \cos\theta_{nb}\sin\psi_{nb} & -\sin\theta_{nb} \\ \begin{pmatrix} -\cos\phi_{nb}\sin\psi_{nb} \\ +\sin\phi_{nb}\sin\theta_{nb}\cos\psi_{nb} \end{pmatrix} & \begin{pmatrix} \cos\phi_{nb}\cos\psi_{nb} \\ +\sin\phi_{nb}\sin\theta_{nb}\sin\psi_{nb} \end{pmatrix} & \sin\phi_{nb}\cos\theta_{nb} \\ \begin{pmatrix} \sin\phi_{nb}\sin\psi_{nb} \\ +\cos\phi_{nb}\sin\theta_{nb}\cos\psi_{nb} \end{pmatrix} & \begin{pmatrix} -\sin\phi_{nb}\cos\psi_{nb} \\ +\cos\phi_{nb}\sin\theta_{nb}\sin\psi_{nb} \end{pmatrix} & \cos\phi_{nb}\cos\theta_{nb} \end{bmatrix}$$

$$\mathbf{C}_n^b = \begin{bmatrix} 0.864838546 & 0.499314767 & 0.052336 \\ -0.501277208 & 0.864584595 & 0.034852 \\ -0.027846909 & -0.056375888 & 0.998021 \end{bmatrix}$$

True angular rate

From (5.104),

$$\boldsymbol{\omega}_{ib}^b = \mathbf{C}_n^b \begin{pmatrix} \cos L_b \omega_{ie} \\ 0 \\ -\sin L_b \omega_{ie} \end{pmatrix}$$

$$\begin{pmatrix} \cos L_b \omega_{ie} \\ 0 \\ -\sin L_b \omega_{ie} \end{pmatrix} = \begin{bmatrix} 5.16\text{E-}05 \\ 0 \\ -5.16\text{E-}05 \end{bmatrix} \text{rad s}^{-1}$$

$$\boldsymbol{\omega}_{ib}^b = \begin{bmatrix} 4.18951\text{E-}05 \\ -2.76444\text{E-}05 \\ -5.28969\text{E-}05 \end{bmatrix} \text{rad s}^{-1}$$

Measured angular rate

From (4.17), $\tilde{\boldsymbol{\omega}}_{ib}^b = \boldsymbol{\omega}_{ib}^b + \delta\boldsymbol{\omega}_{ib}^b$

$$\tilde{\boldsymbol{\omega}}_{ib}^b = \begin{bmatrix} 4.31\text{E-}05 \\ -2.94\text{E-}05 \\ -5.04\text{E-}05 \end{bmatrix} \text{rad s}^{-1}$$

Levelling

From (5.101),

$$\tilde{\theta}_{nb} = \arctan\left(\frac{\tilde{f}_{ib,x}^b}{\sqrt{\tilde{f}_{ib,y}^{b\ 2} + \tilde{f}_{ib,z}^{b\ 2}}} \right), \qquad \tilde{\phi}_{nb} = \arctan_2\left(-\tilde{f}_{ib,y}^b, -\tilde{f}_{ib,z}^b \right)$$

$\tilde{\phi}_{nb} =$ 0.035654178 rad 2.042834 deg

Note: The arguments of the Excel ATAN2 function are the opposite way round

$\tilde{\theta}_{nb} =$ -0.051284661 rad -2.93839 deg

Gyrocompassing

From (5.105),

$$\sin\tilde{\psi}_{nb} = -\tilde{\omega}_{ib,y}^b \cos\tilde{\phi}_{nb} + \tilde{\omega}_{ib,z}^b \sin\tilde{\phi}_{nb}$$

$$\cos\tilde{\psi}_{nb} = \tilde{\omega}_{ib,x}^b \cos\tilde{\theta}_{nb} + \tilde{\omega}_{ib,y}^b \sin\tilde{\phi}_{nb} \sin\tilde{\theta}_{nb} + \tilde{\omega}_{ib,z}^b \cos\tilde{\phi}_{nb} \sin\tilde{\theta}_{nb}$$

$$\tilde{\psi}_{nb} = \arctan_2\left(\sin\tilde{\psi}_{nb}, \cos\tilde{\psi}_{nb}\right)$$

$\sin\tilde{\psi}_{nb} =$ 2.76E-05

$\cos\tilde{\psi}_{nb} =$ 4.57E-05

$\tilde{\psi}_{nb} =$ 0.544030747 rad 31.17067 deg

Note: The arguments of the Excel ATAN2 function are the opposite way round