EXAMPLE 2.1(a)

Conversion of Euler attitude to Coordinate Transformation Matrices and Quaternions

INPUTS: Set of Euler angles describing rotation from local navigation frame to body frame

Roll	ϕ_{nb}	-30	degrees	-0.5236	radians
Pitch	θ_{nb}	30	degrees	0.523599	radians
Yaw	ψ_{nb}	45	degrees	0.785398	radians

Conversion to Coordinate Transformation Matrix

Local navigation frame to body frame coordinate transformation matrix:

From (2.22),

$$\mathbf{C}_n^b = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_{nb} & \sin\phi_{nb} \\ 0 & -\sin\phi_{nb} & \cos\phi_{nb} \end{pmatrix} \begin{pmatrix} \cos\theta_{nb} & 0 & -\sin\theta_{nb} \\ 0 & 1 & 0 \\ \sin\theta_{nb} & 0 & \cos\theta_{nb} \end{pmatrix} \begin{pmatrix} \cos\psi_{nb} & \sin\psi_{nb} & 0 \\ -\sin\psi_{nb} & \cos\psi_{nb} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\mathbf{C}_n^b =$

1	0	0	0.866025	0	-0.5	0.707107	0.707107	0
0	0.866025	-0.5	0	1	0	-0.70711	0.707107	0
0	0.5	0.866025	0.5	0	0.866025	0	0	1

=

0.612372	0.612372	-0.5
-0.78915	0.435596	-0.43301
-0.04737	0.65974	0.75

Alternatively,

From (2.22),

$$\mathbf{C}_n^b = \begin{bmatrix} \cos\theta_{nb}\cos\psi_{nb} & \cos\theta_{nb}\sin\psi_{nb} & -\sin\theta_{nb} \\ \begin{pmatrix} -\cos\phi_{nb}\sin\psi_{nb} \\ +\sin\phi_{nb}\sin\theta_{nb}\cos\psi_{nb} \end{pmatrix} & \begin{pmatrix} \cos\phi_{nb}\cos\psi_{nb} \\ +\sin\phi_{nb}\sin\theta_{nb}\sin\psi_{nb} \end{pmatrix} & \sin\phi_{nb}\cos\theta_{nb} \\ \begin{pmatrix} \sin\phi_{nb}\sin\psi_{nb} \\ +\cos\phi_{nb}\sin\theta_{nb}\cos\psi_{nb} \end{pmatrix} & \begin{pmatrix} -\sin\phi_{nb}\cos\psi_{nb} \\ +\cos\phi_{nb}\sin\theta_{nb}\sin\psi_{nb} \end{pmatrix} & \cos\phi_{nb}\cos\theta_{nb} \end{bmatrix}$$

$\mathbf{C}_n^b =$

0.612372	0.612372	-0.5
-0.78915	0.435596	-0.43301
-0.04737	0.65974	0.75

Body frame to local navigation frame coordinate transformation matrix:

From (2.24),

$$\mathbf{C}_b^n = \begin{bmatrix} \cos\theta_{nb}\cos\psi_{nb} & \begin{pmatrix} -\cos\phi_{nb}\sin\psi_{nb} \\ +\sin\phi_{nb}\sin\theta_{nb}\cos\psi_{nb} \end{pmatrix} & \begin{pmatrix} \sin\phi_{nb}\sin\psi_{nb} \\ +\cos\phi_{nb}\sin\theta_{nb}\cos\psi_{nb} \end{pmatrix} \\ \cos\theta_{nb}\sin\psi_{nb} & \begin{pmatrix} \cos\phi_{nb}\cos\psi_{nb} \\ +\sin\phi_{nb}\sin\theta_{nb}\sin\psi_{nb} \end{pmatrix} & \begin{pmatrix} -\sin\phi_{nb}\cos\psi_{nb} \\ +\cos\phi_{nb}\sin\theta_{nb}\sin\psi_{nb} \end{pmatrix} \\ -\sin\theta_{nb} & \sin\phi_{nb}\cos\theta_{nb} & \cos\phi_{nb}\cos\theta_{nb} \end{bmatrix}$$

$\mathbf{C}_b^n =$

0.612372	-0.78915	-0.04737
0.612372	0.435596	0.65974
-0.5	-0.43301	0.75

Conversion to Quaternions

ϕ_{nb} /2 =	-0.2618
θ_{nb} /2 =	0.261799
ψ_{nb} /2 =	0.392699

From (2.38),

$$q_{n0}^{b} = \cos\left(\frac{\phi_{nb}}{2}\right)\cos\left(\frac{\theta_{nb}}{2}\right)\cos\left(\frac{\psi_{nb}}{2}\right) + \sin\left(\frac{\phi_{nb}}{2}\right)\sin\left(\frac{\theta_{nb}}{2}\right)\sin\left(\frac{\psi_{nb}}{2}\right)$$

$$q_{n1}^{b} = \sin\left(\frac{\phi_{nb}}{2}\right)\cos\left(\frac{\theta_{nb}}{2}\right)\cos\left(\frac{\psi_{nb}}{2}\right) - \cos\left(\frac{\phi_{nb}}{2}\right)\sin\left(\frac{\theta_{nb}}{2}\right)\sin\left(\frac{\psi_{nb}}{2}\right)$$

$$q_{n2}^{b} = \cos\left(\frac{\phi_{nb}}{2}\right)\sin\left(\frac{\theta_{nb}}{2}\right)\cos\left(\frac{\psi_{nb}}{2}\right) + \sin\left(\frac{\phi_{nb}}{2}\right)\cos\left(\frac{\theta_{nb}}{2}\right)\sin\left(\frac{\psi_{nb}}{2}\right)$$

$$q_{n3}^{b} = \cos\left(\frac{\phi_{nb}}{2}\right)\cos\left(\frac{\theta_{nb}}{2}\right)\sin\left(\frac{\psi_{nb}}{2}\right) - \sin\left(\frac{\phi_{nb}}{2}\right)\sin\left(\frac{\theta_{nb}}{2}\right)\cos\left(\frac{\psi_{nb}}{2}\right)$$

$\mathbf{q}_{n}^{b} =$

0.836356
-0.32664
0.135299
0.418937

EXAMPLE 2.1(b)

Conversion of Coordinate Transformation Matrix to Euler attitude and Quaternions

INPUT: Coordinate transformation matrix from body frame to local navigation frame

$$\mathbf{C}_b^n = \begin{pmatrix} 0.612372 & -0.78915 & -0.04737 \\ 0.612372 & 0.435596 & 0.65974 \\ -0.5 & -0.43301 & 0.75 \end{pmatrix}$$

Check validity of matrix

From (2.17), $\mathbf{C}_b^n \mathbf{C}_n^b = \mathbf{I}_3$

$$\mathbf{C}_b^n \mathbf{C}_n^b = \begin{pmatrix} 1 & 7.98\text{E-}17 & 4.86\text{E-}17 \\ 7.98\text{E-}17 & 1 & 1.11\text{E-}16 \\ 4.86\text{E-}17 & 1.11\text{E-}16 & 1 \end{pmatrix}$$

Conversion to Euler angles

Set of Euler angles describing rotation from local navigation frame to body frame

From (2.25),

$$\phi_{nb} = \arctan_2\left(C_{b3,2}^n, C_{b3,3}^n\right)$$
$$\theta_{nb} = -\arcsin C_{b3,1}^n$$
$$\psi_{nb} = \arctan_2\left(C_{b2,1}^n, C_{b1,1}^n\right)$$

Note: The arguments of the Excel ATAN2 function are the opposite way round

Roll	ϕ_{nb}	-0.5236	radians	-30	degrees
Pitch	θ_{nb}	0.523599	radians	30	degrees
Yaw	ψ_{nb}	0.785398	radians	45	degrees

Conversion to Quaternions

Set of Quaternions describing rotation from local navigation frame to body frame

From (2.35)

$$q_{n0}^b = \tfrac{1}{2}\sqrt{1 + C_{b1,1}^n + C_{b2,2}^n + C_{b3,3}^n}$$
$$q_{n1}^b = \frac{C_{b3,2}^n - C_{b2,3}^n}{4q_{n0}^b}$$
$$q_{n2}^b = \frac{C_{b1,3}^n - C_{b3,1}^n}{4q_{n0}^b}$$
$$q_{n3}^b = \frac{C_{b2,1}^n - C_{b1,2}^n}{4q_{n0}^b}$$

$$\mathbf{q}_n^b = \begin{pmatrix} 0.836356 \\ -0.32664 \\ 0.135299 \\ 0.418937 \end{pmatrix}$$

EXAMPLE 2.1(c)

Conversion of Quaternions to Coordinate Transformation Matrix and Euler attitude

INPUTS: Quaterions describing rotation from local navigation frame to body frame

$\mathbf{q}_n^b =$

0.836356
-0.32664
0.135299
0.418937

Conversion to Coordinate Transformation Matrix

Local navigation frame to body frame coordinate transformation matrix:

From (2.34),

$$\mathbf{C}_n^b = \begin{pmatrix} {q_{n0}^b}^2 + {q_{n1}^b}^2 - {q_{n2}^b}^2 - {q_{n3}^b}^2 & 2(q_{n1}^b q_{n2}^b + q_{n3}^b q_{n0}^b) & 2(q_{n1}^b q_{n3}^b - q_{n2}^b q_{n0}^b) \\ 2(q_{n1}^b q_{n2}^b - q_{n3}^b q_{n0}^b) & {q_{n0}^b}^2 - {q_{n1}^b}^2 + {q_{n2}^b}^2 - {q_{n3}^b}^2 & 2(q_{n2}^b q_{n3}^b + q_{n1}^b q_{n0}^b) \\ 2(q_{n1}^b q_{n3}^b + q_{n2}^b q_{n0}^b) & 2(q_{n2}^\alpha q_{n3}^\alpha - q_{n1}^\alpha q_{n0}^\alpha) & {q_{n0}^\alpha}^2 - {q_{n1}^\alpha}^2 - {q_{n2}^\alpha}^2 + {q_{n3}^\alpha}^2 \end{pmatrix}$$

$\mathbf{C}_n^b =$

0.612372	0.612372	-0.5
-0.78915	0.435596	-0.43301
-0.04737	0.65974	0.75

Body frame to local navigation frame coordinate transformation matrix:

$\mathbf{C}_b^n =$

0.612372	-0.78915	-0.04737
0.612372	0.435596	0.65974
-0.5	-0.43301	0.75

Conversion to Euler angles

Set of Euler angles describing rotation from local navigation frame to body frame

From (2.37),

$$\phi_{nb} = \arctan_2\left[2\left(q_{n0}^b q_{n1}^b + q_{n2}^b q_{n3}^b\right), \left(1 - 2{q_{n1}^b}^2 - 2{q_{n2}^b}^2\right)\right]$$

$$\theta_{nb} = \arcsin\left[2\left(q_{n0}^b q_{n2}^b - q_{n1}^b q_{n3}^b\right)\right]$$

$$\psi_{nb} = \arctan_2\left[2\left(q_{n0}^b q_{n3}^b + q_{n1}^b q_{n2}^b\right), \left(1 - 2{q_{n2}^b}^2 - 2{q_{n3}^b}^2\right)\right]$$

Note: The arguments of the Excel ATAN2 function are the opposite way round

Roll	ϕ_{nb}	-0.5236	radians	-30	degrees
Pitch	θ_{nb}	0.523599	radians	30	degrees
Yaw	ψ_{nb}	0.785398	radians	45	degrees