EXAMPLE 2.2(a)
Conversion of Curvilinear Position to Cartesian ECEF Position

INPUTS:

Equatorial radius	R_0 =	6378137	m		
Ellipsoid eccentricity	e =	0.081819191			
Latitude	L_b =	45	degrees	0.785398	rad
Longitude	λ_b =	30	degrees	0.523599	rad
Height	h_b =	1000	m		

Transverse Radius of Curvature
From (2.106),

$$R_E(L_b) = \frac{R_0}{\sqrt{1 - e^2 \sin^2 L_b}}$$

R_E = 6.39E+06

Cartesian Position (ECEF-frame resolevd and referenced
From (2.112),

$$x_{eb}^e = \left(R_E(L_b) + h_b\right)\cos L_b \cos \lambda_b$$
$$y_{eb}^e = \left(R_E(L_b) + h_b\right)\cos L_b \sin \lambda_b$$
$$z_{eb}^e = \left[\left(1 - e^2\right)R_E(L_b) + h_b\right]\sin L_b$$

$\mathbf{r}_{eb}^e$ =

3912960.837	m
2259148.993	m
4488055.516	m

EXAMPLE 2.2(b)
Conversion of Cartesian ECEF Position to Curvilinear Position
Iterative Method 1

INPUTS:

Equatorial radius	$R_0 =$	6378137	m
Ellipsoid eccentricity	$e =$	0.08181919	
Cartesian position	$x_{eb}^{e} =$	3912960.837	m
	$y_{eb}^{e} =$	2259148.993	m
	$z_{eb}^{e} =$	4488055.516	m

e^2	0.00669438	
$1-e^2$	0.99330562	
$\beta_{eb}^{e} = \sqrt{x_{eb}^{e\,2} + y_{eb}^{e\,2}} =$	4518297.986	m

Longitude

From (2.113), $\lambda_b = \arctan_2\left(y_{eb}^{e}, x_{eb}^{e}\right)$

Note: The arguments of the Excel ATAN2 function are the opposite way round

$\lambda_b =$ 0.523598776 rad   30 degrees

Initialize latitude with geocentric latitude

From (2.114),

$$L_{b,0} = \Phi_b = \arctan\left(\frac{z_{eb}^{e}}{\sqrt{x_{eb}^{e\,2} + y_{eb}^{e\,2}}}\right)$$

$L_{b,0} =$ 0.782040272 rad

Calculate transverse radius of curvature

From (C.3), $$R_E(L_{b,k-1}) = \frac{R_0}{\sqrt{1 - e^2 \sin^2 L_{b,k-1}}}$$

$R_E =$ 6388766.243 m

Calculate height

From (C.4), $$h_{b,k-1} = \frac{\sqrt{x_{eb}^{e\,2} + y_{eb}^{e\,2}}}{\cos L_{b,k-1}} - R_E(L_{b,k-1})$$

$h_{b,0} =$ -20276.7051 m

Calculate geodetic latitude

From (C.5), $$L_{b,k} = \arctan\left(\frac{z_{eb}^{e}\left[R_E(L_{b,k-1}) + h_{b,k-1}\right]}{\sqrt{x_{eb}^{e\,2} + y_{eb}^{e\,2}}\left[\left(1 - e^2\right)R_E(L_{b,k-1}) + h_{b,k-1}\right]}\right)$$

$L_{b,1} =$ 0.78540942 rad $L_{b,1} - L_{b,0} =$ 0.003369 rad

Iterate:

$R_E =$ 6388838.532 m

$h_{b,1} =$ 1071.686617 m

$L_{b,2} =$ 0.785398126 rad $L_{b,2} - L_{b,1} =$ -1.1E-05 rad

R_E =	6388838.289	m				
$h_{b,2}$ =	999.7592861	m				
$L_{b,3}$ =	0.785398164	rad		$L_{b,3} - L_{b,2}$ =	3.79E-08	rad
R_E =	6388838.29	m				
$h_{b,3}$ =	1000.000808	m				
$L_{b,4}$ =	0.785398163	rad		$L_{b,4} - L_{b,3}$ =	-1.3E-10	rad
R_E =	6388838.29	m				
$h_{b,4}$ =	999.9999973	m				
$L_{b,5}$ =	0.785398163	rad		$L_{b,5} - L_{b,4}$ =	4.28E-13	rad
	45	degrees				
R_E =	6388838.29	m				
$h_{b,5}$ =	1000	m				

EXAMPLE 2.2(c)
Conversion of Cartesian ECEF Position to Curvilinear Position
Iterative Method 2

INPUTS:

Equatorial radius	$R_0 =$	6378137	m
Ellipsoid eccentricity	$e =$	0.08181919	
Cartesian	$x^e_{eb} =$	3912960.837	m
position	$y^e_{eb} =$	2259148.993	m
	$z^e_{eb} =$	4488055.516	m

e^2 0.00669438

$1-e^2$ 0.99330562

$$\beta^e_{eb} = \sqrt{{x^e_{eb}}^2 + {y^e_{eb}}^2} =$$ 4518297.986 m

Longitude

From (2.113), $\lambda_b = \arctan_2\left(y^e_{eb}, x^e_{eb}\right)$

Note: The arguments of the Excel ATAN2 function are the opposite way round

$\lambda_b =$ 0.523598776 rad 30 degrees

Initialize reduced latitude

From (C.10),

$$\zeta_{b,0} = \arctan\left(\frac{z^e_{eb}}{\sqrt{1-e^2}\sqrt{{x^e_{eb}}^2 + {y^e_{eb}}^2}}\right)$$

$\zeta_{b,0} =$ 0.783719472 rad

Calculate latitude

From (C.12),

$$L_{b,k} = \arctan\left[\frac{z^e_{eb}\sqrt{1-e^2} + e^2 R_0 \sin^3 \zeta_{b,k-1}}{\sqrt{1-e^2}\left(\sqrt{{x^e_{eb}}^2 + {y^e_{eb}}^2} - e^2 R_0 \cos^3 \zeta_{b,k-1}\right)}\right]$$

$L_{b,1} =$ 0.785398163 rad

Calculate reduced latitude

From (C.11), $\zeta_{b,k-1} = \arctan\left(\sqrt{1-e^2}\tan L_{b,k-1}\right)$

$\zeta_{b,1} =$ 0.783718945 rad (rad)

Iterate:

$L_{b,2} =$	0.785398163	rad	$L_{b,2} - L_{b,1} =$	-1.44E-15	rad
$\zeta_{b,2} =$	0.783718945	rad			
$L_{b,3} =$	0.785398163	rad	$L_{b,3} - L_{b,2} =$	0.00E+00	rad
	45	degrees			

Calculate height

From (C.8),

$$h_b = \frac{\sqrt{x_{eb}^{e\,2} + y_{eb}^{e\,2}}}{\cos L_b} - \frac{R_0}{\sqrt{1 - e^2 \sin^2 L_b}}$$

h_b =	1000	m

EXAMPLE 2.2(d)
Conversion of Cartesian ECEF Position to Curvilinear Position
Iterative Method 3

INPUTS:

Equatorial radius	R_0 =	6378137	m
Ellipsoid eccentricity	e =	0.08181919	
Cartesian	$x_{eb}^e =$	3912960.837	m
position	$y_{eb}^e =$	2259148.993	m
	$z_{eb}^e =$	4488055.516	m

e^2	0.00669438
$1-e^2$	0.99330562

$$\beta_{eb}^e = \sqrt{x_{eb}^{e\,2} + y_{eb}^{e\,2}} =$$ 4518297.986 m

Longitude

From (2.113), $\lambda_b = \arctan_2\left(y_{eb}^e, x_{eb}^e\right)$

Note: The arguments of the Excel ATAN2 function are the opposite way round

λ_b = 0.523598776 rad 30 degrees

Initialize latitude

From (C.13),
$$L_{b,0} = \arctan\left(\frac{z_{eb}^e}{\sqrt{1-e^2}\sqrt{x_{eb}^{e\,2} + y_{eb}^{e\,2}}}\right)$$

$L_{b,0}$ = 0.783719472

Calculate transverse radius of curvature

From (C.3),
$$R_E(L_{b,k-1}) = \frac{R_0}{\sqrt{1-e^2\sin^2 L_{b,k-1}}}$$

R_E = 6388802.272

Calculate latitude

From (C.14),
$$L_{b,k} = \arctan\left(\frac{z_{eb}^e + e^2 R_E\left(L_{b,k-1}\right)\sin L_{b,k-1}}{\sqrt{x_{eb}^{e\,2} + y_{eb}^{e\,2}}}\right)$$

$L_{b,1}$ = 0.785392522 rad $L_{b,1} - L_{b,0}$ = 0.00167305

Iterate:

R_E =	6388838.169			
$L_{b,2}$ =	0.785398144	rad	$L_{b,2} - L_{b,1}$ =	5.62266E-06
R_E =	6388838.29			
$L_{b,3}$ =	0.785398163	rad	$L_{b,3} - L_{b,2}$ =	1.88804E-08
R_E =	6388838.29			
$L_{b,4}$ =	0.785398163	rad	$L_{b,4} - L_{b,3}$ =	6.33986E-11
R_E =	6388838.29			

$L_{b,5}$ =	0.785398163	rad	$L_{b,5} - L_{b,4}$ =	2.1283E-13
	45	degrees		

Calculate height

From (C.8),

$$h_b = \frac{\sqrt{x_{eb}^{e\ 2} + y_{eb}^{e\ 2}}}{\cos L_b} - \frac{R_0}{\sqrt{1 - e^2 \sin^2 L_b}}$$

h_b = 1000 m

EXAMPLE 2.2(e)
Conversion of Cartesian ECEF Position to Curvilinear Position
Iterative Method 4

INPUTS:

Equatorial radius	R_0 =	6378137	m
Ellipsoid eccentricity	e =	0.081819191	
Cartesian position	x_{eb}^e =	3912960.837	m
	y_{eb}^e =	2259148.993	m
	z_{eb}^e =	4488055.516	m

e^2 0.00669438
$1-e^2$ 0.99330562

$\beta_{eb}^e = \sqrt{x_{eb}^{e\;2} + y_{eb}^{e\;2}} =$ 4518297.986 m

Longitude

From (2.113), $\lambda_b = \arctan_2\left(y_{eb}^e, x_{eb}^e\right)$

Note: The arguments of the Excel ATAN2 function are the opposite way round

λ_b = 0.523598776 rad 30 degrees

Calculate coefficient A

From (C.16),

$$A = \arctan\left(\frac{z_{eb}^e\sqrt{1-e^2}}{\sqrt{x_{eb}^{e\;2} + y_{eb}^{e\;2}}}\right)$$

A 0.78036111 rad

Calculate coefficient B

From (C.16),

$$B = \frac{e^2 R_0}{\sqrt{x_{eb}^{e\;2} + y_{eb}^{e\;2} + \left(1-e^2\right) z_{eb}^{e\;2}}}$$

B 0.006715694

Initialize reduced latitude

From (C.10),

$$\zeta_{b,0} = \arctan\left(\frac{z_{eb}^e}{\sqrt{1-e^2}\sqrt{x_{eb}^{e\;2} + y_{eb}^{e\;2}}}\right)$$

$\zeta_{b,0}$ = 0.783719472 rad

Calculate reduced latitude

From (C.15),

$$\zeta_{b,k} = \zeta_{b,k-1} - \frac{2\sin\left(\zeta_{b,k-1} - A\right) - B\sin 2\zeta_{b,k-1}}{2\cos\left(\zeta_{b,k-1} - A\right) - B\cos 2\zeta_{b,k-1}}$$

$\zeta_{b,1}$ = 0.783718945 rad $\zeta_{b,1} - \zeta_{b,0}$ = -5.2735E-07 rad

Iterate:

$\zeta_{b,2}$ =	0.783718945	rad	$\zeta_{b,2} - \zeta_{b,1}$ =	-5.9466E-12	rad
$\zeta_{b,3}$ =	0.783718945	rad	$\zeta_{b,3} - \zeta_{b,2}$ =	0	rad

Calculate latitude

From (C.17), $$L_b = \arctan\left(\frac{\tan \zeta_b}{\sqrt{1-e^2}}\right)$$

L_b =	0.785398163	rad
	45	deg

Calculate height

From (C.8), $$h_b = \frac{\sqrt{x_{eb}^{e\,2} + y_{eb}^{e\,2}}}{\cos L_b} - \frac{R_0}{\sqrt{1-e^2 \sin^2 L_b}}$$

h_b =	1000

EXAMPLE 2.2(f)
Conversion of Cartesian ECEF Position to Curvilinear Position
Heikkinen Closed-Form Exact Solution

INPUTS:

Equatorial radius	R_0 =	6378137	m
Ellipsoid eccentricity	e =	0.08181919	
Cartesian position	x_{eb}^{e} =	3912960.837	m
	y_{eb}^{e} =	2259148.993	m
	z_{eb}^{e} =	4488055.516	m

e^2	0.00669438	
$1-e^2$	0.99330562	
$\beta_{eb}^{e} = \sqrt{x_{eb}^{e\;2} + y_{eb}^{e\;2}}$ =	4518297.986	m

Longitude

From (2.113), $\lambda_b = \arctan_2\left(y_{eb}^{e}, x_{eb}^{e}\right)$

Note: The arguments of the Excel ATAN2 function are the opposite way round

λ_b = 0.523598776 rad | 30 degrees

From (C.19),
$$F = 54\left(1-e^2\right)R_0^2 z_{eb}^{e\;2}$$
F = 4.39522E+28 m^4

From (C.20),
$$G = \beta_{eb}^{e\;2} + \left(1-e^2\right)z_{eb}^{e\;2} - e^4 R_0^2$$
G = 4.0421E+13 m^2

From (C.21),
$$C = \frac{e^4 F \beta_{eb}^{e\;2}}{G^3}$$
C = 0.000608878

From (C.22),
$$S = \left(1 + C + \sqrt{C^2 + 2C}\right)^{1/3}$$
S = 1.01169944

From (C.23),
$$P = \frac{F}{3\left(S + \frac{1}{S} + 1\right)^2 G^2}$$
P = 0.996239781

From (C.24),
$$Q = \sqrt{1 + 2e^4 P}$$
Q = 1.000044645

From (C.25), $$T = \sqrt{\frac{R_0^2}{2}\left(1+\frac{1}{Q}\right) - \frac{P\left(1-e^2\right)z_{eb}^{e\ 2}}{Q\left(1+Q\right)} - \frac{P\beta_{eb}^{e\ 2}}{2} - \frac{Pe^2\beta_{eb}^e}{1+Q}}$$

$\frac{R_0^2}{2}\left(1+\frac{1}{Q}\right) =$ 4.06797E+13

$\frac{P\left(1-e^2\right)z_{eb}^{e\ 2}}{Q\left(1+Q\right)} =$ 9.96562E+12

$\frac{P\beta_{eb}^{e\ 2}}{2} =$ 1.01691E+13

$\frac{Pe^2\beta_{eb}^e}{1+Q} =$ 15066.39744

$T =$ 4517590.879

From (C.26), $$V = \sqrt{\left(\beta_{eb}^e - e^2T\right)^2 + \left(1-e^2\right)z_{eb}^{e\ 2}}$$

$V =$ 6336437.652

From (C.27), $$L_b = \arctan\left[\left(1+\frac{e^2R_0}{V}\right)\frac{z_{eb}^e}{\beta_{eb}^e}\right]$$

$L_b =$	0.785398163	rad
	45	degrees

Calculate height

From (C.28), $$h_b = \left[1-\frac{\left(1-e^2\right)R_0}{V}\right]\sqrt{\left(\beta_{eb}^e - e^2T\right)^2 + z_{eb}^{e\ 2}}$$

$h_b =$ 1000 m

EXAMPLE 2.2(g)
Conversion of Cartesian ECEF Position to Curvilinear Position
Borkowski Closed-Form Exact Solution

INPUTS:

Equatorial radius	R_0 =	6378137	m
Ellipsoid eccentricity	e =	0.08181919	
Cartesian position	$x_{eb}^e =$	3912961	m
	$y_{eb}^e =$	2259149	m
	$z_{eb}^e =$	4488056	m

e^2	0.00669438
$1-e^2$	0.99330562

$\beta_{eb}^e = \sqrt{x_{eb}^{e\,2} + y_{eb}^{e\,2}} =$ 4518298 m

Longitude

From (2.113), $\lambda_b = \arctan_2\left(y_{eb}^e, x_{eb}^e\right)$

Note: The arguments of the Excel ATAN2 function are the opposite way round

λ_b = 0.523598776 rad 30 degrees

From (C.29), $$E = \frac{\sqrt{1-e^2}\left|z_{eb}^b\right| - e^2 R_0}{\beta_{eb}^b}$$

E = 0.980526352

From (C.30), $$F = \frac{\sqrt{1-e^2}\left|z_{eb}^b\right| + e^2 R_0}{\beta_{eb}^b}$$

F = 0.999426245

From (C.31), $P = \frac{4}{3}(EF + 1)$

P = 2.639951693

From (C.32), $Q = 2\left(E^2 - F^2\right)$

Q = -0.07484178

From (C.33), $D = P^3 + Q^2$

D = 18.40433528

From (C.34), $$V = \left(D^{1/2} - Q\right)^{1/3} - \left(D^{1/2} + Q\right)^{1/3}$$

V = 0.018898933

From (C.35), $$G = \frac{1}{2}\left(\sqrt{E^2 + V} + E\right)$$

G = 0.985321471

From (C.36), $$T = \sqrt{G^2 + \frac{F - VG}{2G - E}} - G$$

T =	0.415197568

From (C.37), $$L_b = \operatorname{sign}(z^e_{eb}) \arctan\left(\frac{1 - T^2}{2T\sqrt{1 - e^2}}\right)$$

L_b =	0.785398163	rad
	45	degrees

Calculate height

From (C.38), $$h_b = \left(\beta^e_{eb} - R_0 T\right) \cos L_b + \left(z^e_{eb} - \operatorname{sign}(z^e_{eb}) R_0 \sqrt{1 - e^2}\right) \sin L_b$$

h_b =	1000	m