EXAMPLE 2.4(a)

Transformation of reference frame and resolving axes from ECEF frame to ECI frame

INPUTS:

Position

$\mathbf{r}_{eb}^{e} =$ 4000000 m, 2900000 m, 4000000 m

Velocity

$\mathbf{v}_{eb}^{e} =$ 5 m s^{-1}, 5 m s^{-1}, 5 m s^{-1}

Acceleration

$\mathbf{a}_{eb}^{e} =$ 5 m s^{-2}, 0 m s^{-2}, 0 m s^{-2}

Angular rate

$\boldsymbol{\omega}_{eb}^{e} =$ 0.001 rad s^{-1}, 0 rad s^{-1}, 0 rad s^{-1}

Current time

t = 10000 s

Time of ECI-ECEF frame coincidence

t_0 = 0 s

Earth rate

ω_{ie} = 7.29E-05 rad s^{-1}

Coordinate transformation matrix

From (2.145),

$$\mathbf{C}_{e}^{i} = \begin{pmatrix} \cos \omega_{ie}(t - t_0) & -\sin \omega_{ie}(t - t_0) & 0 \\ \sin \omega_{ie}(t - t_0) & \cos \omega_{ie}(t - t_0) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{C}_{e}^{i} = \begin{pmatrix} 0.745699997 & -0.66628 & 0 \\ 0.666281858 & 0.7457 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Skew symmetric matrix of Earth rate

$$\boldsymbol{\Omega}_{ie}^{e} = \begin{pmatrix} 0 & -7.29\text{E-}05 & 0 \\ 7.29\text{E-}05 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Position transformation

From (2.146), $\mathbf{r}_{ib}^{i} = \mathbf{C}_{e}^{i} \mathbf{r}_{eb}^{e}$

$\mathbf{r}_{ib}^{i} =$ 1050582.602 m, 4827657.423 m, 4000000 m

Velocity transformation

From (2.147), $\mathbf{v}_{ib}^{i} = \mathbf{C}_{e}^{i} \left(\mathbf{v}_{eb}^{e} + \boldsymbol{\Omega}_{ie}^{e} \mathbf{r}_{eb}^{e} \right)$

$\mathbf{v}_{ib}^{i} =$ -351.64124 m s^{-1}, 83.66960078 m s^{-1}, 5 m s^{-1}

Acceleration transformation

From (2.148), $\mathbf{a}_{ib}^{i} = \mathbf{C}_{e}^{i}\left(\mathbf{a}_{eb}^{e} + 2\boldsymbol{\Omega}_{ie}^{e}\mathbf{v}_{eb}^{e} + \boldsymbol{\Omega}_{ie}^{e}\boldsymbol{\Omega}_{ie}^{e}\mathbf{r}_{eb}^{e}\right)$

$$\mathbf{a}_{ib}^{i} = \begin{bmatrix} 3.721883887 \\ 3.305796161 \\ 0 \end{bmatrix} \text{ m s}^{-2}$$

Angular rate transformation

From (2.149),

$$\boldsymbol{\omega}_{ib}^{i} = \mathbf{C}_{e}^{i}\left(\boldsymbol{\omega}_{eb}^{e} + \begin{pmatrix} 0 \\ 0 \\ \omega_{ie} \end{pmatrix}\right)$$

$$\boldsymbol{\omega}_{eb}^{e} + \begin{pmatrix} 0 \\ 0 \\ \omega_{ie} \end{pmatrix} = \begin{bmatrix} 0.001 \\ 0 \\ 7.29\text{E-}05 \end{bmatrix} \text{ rad s}^{-1}$$

$$\boldsymbol{\omega}_{ib}^{i} = \begin{bmatrix} 0.0007457 \\ 0.000666282 \\ 7.29212\text{E-}05 \end{bmatrix} \text{ rad s}^{-1}$$

EXAMPLE 2.4(b)
Transformation of reference frame and resolving axes from ECI frame to ECEF frame

INPUTS:

Position

$\mathbf{r}_{ib}^{i} =$ 1050583 m; 4827657 m; 4000000 m

Velocity

$\mathbf{v}_{ib}^{i} =$ -351.641 m s^{-1}; 83.6696 m s^{-1}; 5 m s^{-1}

Acceleration

$\mathbf{a}_{ib}^{i} =$ 3.721884 m s^{-2}; 3.305796 m s^{-2}; 0 m s^{-2}

Angular rate

$\boldsymbol{\omega}_{ib}^{i} =$ 0.000746 rad s^{-1}; 0.000666 rad s^{-1}; 7.29E-05 rad s^{-1}

Current time

t 10000 s

Time of ECI-ECEF frame coincidence

t_0 0 s

Earth rate

ω_{ie} 7.29E-05 rad s^{-1}

Coordinate transformation matrix

From (2.145),

$$\mathbf{C}_{i}^{e} = \begin{pmatrix} \cos\omega_{ie}(t-t_0) & \sin\omega_{ie}(t-t_0) & 0 \\ -\sin\omega_{ie}(t-t_0) & \cos\omega_{ie}(t-t_0) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{C}_{i}^{e} = \begin{pmatrix} 0.7457 & 0.666282 & 0 \\ -0.66628 & 0.7457 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Skew symmetric matrix of Earth rate

$$\boldsymbol{\Omega}_{ie}^{i} = \begin{pmatrix} 0 & -7.29E\text{-}05 & 0 \\ 7.29E\text{-}05 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Position transformation

From (2.146), $\mathbf{r}_{eb}^{e} = \mathbf{C}_{i}^{e}\mathbf{r}_{ib}^{i}$

$\mathbf{r}_{eb}^{e} =$ 4000000 m; 2900000 m; 4000000 m

Velocity transformation

From (2.147), $\mathbf{v}_{eb}^{e} = \mathbf{C}_{i}^{e}\left(\mathbf{v}_{ib}^{i} - \boldsymbol{\Omega}_{ie}^{i}\mathbf{r}_{ib}^{i}\right)$

$\mathbf{v}_{eb}^{e} =$ 5 m s^{-1}; 5 m s^{-1}; 5 m s^{-1}

Acceleration transformation

From (2.148), $\mathbf{a}_{eb}^{e} = \mathbf{C}_{i}^{e}\left(\mathbf{a}_{ib}^{i} - 2\boldsymbol{\Omega}_{ie}^{i}\mathbf{v}_{ib}^{i} + \boldsymbol{\Omega}_{ie}^{i}\boldsymbol{\Omega}_{ie}^{i}\mathbf{r}_{ib}^{i}\right)$

$$\mathbf{a}_{eb}^{e} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \text{ m s}^{-2}$$

Angular rate transformation

From (2.149),

$$\boldsymbol{\omega}_{eb}^{e} = \mathbf{C}_{i}^{e}\left(\boldsymbol{\omega}_{ib}^{i} - \begin{pmatrix} 0 \\ 0 \\ \omega_{ie} \end{pmatrix}\right)$$

$$\boldsymbol{\omega}_{ib}^{i} - \begin{pmatrix} 0 \\ 0 \\ \omega_{ie} \end{pmatrix} = \begin{bmatrix} 0.000746 \\ 0.000666 \\ 0.00\text{E+}00 \end{bmatrix} \text{ rad s}^{-1}$$

$$\boldsymbol{\omega}_{eb}^{e} = \begin{bmatrix} 0.001 \\ 0 \\ 0 \end{bmatrix} \text{ rad s}^{-1}$$