EXAMPLE 3.2 (Chapter 3 Example B)
Kalman filter estimating 2D position

INPUTS:

Initial position (x)		1	m
Initial position (y)		0	m
Initial position uncertainty (x)		0.5	m
Initial position uncertainty (y)		0.5	m
Initialx-y position covariance		0.1	m^2
Velocity PSD (x)		1.8	$m^2\ s^{-1}$
Velocity PSD (y)		2.2	$m^2\ s^{-1}$
Time between epochs	$\tau_s =$	0.5	s
Position measurement (x, inc. noise)		2	m
		-2	m
Measurement noise SD (x)		1	m
Measurement noise SD (y)		1	m
Meas. noise x-y covariance		0.1	m^2

INITIALIZATION
State vector estimate
From (3.13),

$$\mathbf{x}_B = \begin{pmatrix} r^i_{ib,x} \\ r^i_{ib,y} \end{pmatrix}$$

Thus,

$$\hat{\mathbf{x}}_0^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ m}$$

Error covariance matrix

$$\mathbf{P}_0^+ = \begin{pmatrix} 0.25 & 0.1 \\ 0.1 & 0.25 \end{pmatrix}$$

SYSTEM PROPAGATION PHASE
Step 1: Calculate transition matrix
From (3.13),

$$\Phi_B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Thus,

$$\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Step 2: Calculate system noise covariance matrix
From (3.49),

$$\mathbf{Q}_B = \begin{pmatrix} S_{vx}\tau_s & 0 \\ 0 & S_{vy}\tau_s \end{pmatrix}$$

Thus,

$$\mathbf{Q}_0 = \begin{pmatrix} 0.9 & 0 \\ 0 & 1.1 \end{pmatrix}$$

Step 3: State vector time propagation

From (3.14), $\hat{\mathbf{x}}_k^- = \mathbf{\Phi}_{k-1}\hat{\mathbf{x}}_{k-1}^+$

Thus,

$\hat{\mathbf{x}}_1^- =$

1	m
0	m

$\mathbf{x}_B = \begin{pmatrix} r_{ib,x}^i \\ r_{ib,y}^i \end{pmatrix}$

Step 4: Error covariance matrix time propagation

From (3.15), $\mathbf{P}_k^- = \mathbf{\Phi}_{k-1}\mathbf{P}_{k-1}^+\mathbf{\Phi}_{k-1}^{\mathrm{T}} + \mathbf{Q}_{k-1}$

Thus,

$\mathbf{\Phi}_0\mathbf{P}_0^+\mathbf{\Phi}_0^{\mathrm{T}} =$

0.25	0.1
0.1	0.25

$\mathbf{P}_1^- =$

1.15	0.1
0.1	1.35

MEASUREMENT UPDATE PHASE

Step 5: Calculate Measurement Matrix

From (3.18), $\mathbf{H}_B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Thus,

$\mathbf{H}_1 =$

1	0
0	1

Step 6: Calculate Measurement Noise Covariance Matrix

Diagonal elements are the squares of the measurement noise SD:

Off-diagonals are the covariance of the noise on the two measurements

$\mathbf{R}_1 =$

1	0.1
0.1	1

Step 7: Calculate Kalman Gain Matrix

From (3.21), $\mathbf{K}_k = \mathbf{P}_k^-\mathbf{H}_k^{\mathrm{T}}\left(\mathbf{H}_k\mathbf{P}_k^-\mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k\right)^{-1}$

$\mathbf{P}_1^-\mathbf{H}_1^{\mathrm{T}} =$

1.15	0.1
0.1	1.35

$\mathbf{H}_1\mathbf{P}_1^-\mathbf{H}_1^{\mathrm{T}} =$

1.15	0.1
0.1	1.35

$\mathbf{H}_1\mathbf{P}_1^-\mathbf{H}_1^{\mathrm{T}} + \mathbf{R}_1 =$

2.15	0.2
0.2	2.35

$\left(\mathbf{H}_1\mathbf{P}_1^-\mathbf{H}_1^{\mathrm{T}} + \mathbf{R}_1\right)^{-1} =$

0.46882793	-0.0399
-0.03990025	0.428928

$\mathbf{K}_1 =$

0.535162095	-0.00299
-0.00698254	0.575062

Step 8: Formulate Measurement

From (3.18), $$\mathbf{z}_B = \begin{pmatrix} r^i_{ib,x} + w_{m,x} \\ r^i_{ib,y} + w_{m,y} \end{pmatrix}$$

$\mathbf{z}_1 =$

2	m
-2	m

Step 9: Update State Vector

From (3.24), $$\hat{\mathbf{x}}^+_k = \hat{\mathbf{x}}^-_k + \mathbf{K}_k\left(\mathbf{z}_k - \mathbf{H}_k\hat{\mathbf{x}}^-_k\right)$$

$\mathbf{z}_1 - \mathbf{H}_1\hat{\mathbf{x}}^-_1 =$

1
-2

$\mathbf{K}_1\left(\mathbf{z}_1 - \mathbf{H}_1\hat{\mathbf{x}}^-_1\right) =$

0.541147132
-1.15710723

$\hat{\mathbf{x}}^+_1 =$

1.541147132	m
-1.157107232	m

$$\mathbf{x}_B = \begin{pmatrix} r^i_{ib,x} \\ r^i_{ib,y} \end{pmatrix}$$

Step 10: Update Error Covariance Matrix

From (3.25), $$\mathbf{P}^+_k = \left(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k\right)\mathbf{P}^-_k$$

$\mathbf{K}_1\mathbf{H}_1 =$

0.535162095	-0.00299
-0.00698254	0.575062

$\left(\mathbf{I} - \mathbf{K}_1\mathbf{H}_1\right) =$

0.464837905	0.002993
0.006982544	0.424938

$\mathbf{P}^+_1 =$

0.534862843	0.050523691
0.050523691	0.57436409