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Shuttle Entry Guidance Revisited Using Nonlinear Geometric Methods

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The entry guidance law for the Space Shuttle Orbiter is revisited using nonlinear geometric methods. The Shuttle guidance concept is to track a reference drag trajectory that has been designed to lead to a specified range and velocity. The current guidance law provides exponential tracking locally. We show that the approach taken in the original derivation of the Shuttle entry guidance has much in common with the more recently developed feedback linearization method of differential geometric control. Using the feedback linearization method, however, we are led to an alternative, potentially superior, guidance law. To compare the two guidance laws, stability and performance domains in state space are defined, taking into account the nonlinear dynamics, a state constraint, and a control constraint. The stability and performance domains for the Shuttle law and the alternative law are constructed numerically. The effects of increasing the control capability and changing a parameter in the guidance laws are illustrated. The alternative guidance law achieves the desired performance over a larger domain of the state space; the stability domains for the two laws are similar. For the current operating domain of the Shuttle, the performance improvement offered by the alternative guidance law is probably not significant. With a larger operating domain for the Shuttle or some other entry vehicle, the alternative guidance law should be considered. A more comprehensive comparison taking into account important factors not considered here, such as robustness, would be necessary to decide whether or not the alternative guidance law is superior.

Nomenclature

A^u = domain of attraction for control law u
C = controllable set
C_D = drag coefficient
C_L = lift coefficient
D = specific drag
e = drag error, ΔD
g = specific gravity
g_a = apparent gravity
H = atmosphere scale height
K_d = derivative feedback gain
K_p = proportional feedback gain
L = specific lift
L_v = vertical component of $L_v = L\cos\sigma$
m = vehicle mass
P^u = performance domain for control law u
R = downrange
r = radial distance from Earth center
r_s = Earth radius
S_a = reference surface area
S^u = domain of *safe* attraction
T = period
t = time
U = class of piecewise continuous controls
u = control
V = velocity
γ = flight path angle
$\Delta(\cdot)$ = error, $(\cdot)-(\cdot)_r$
$\delta(\cdot)$ = linear part of $\Delta(\cdot)$
ζ = damping ratio
λ = eigenvalue
μ = gravitational constant
ρ = atmospheric density
ρ_s = atmospheric density at r_s
σ = bank angle
ω = characteristic frequency
$(\dot{\cdot})$ = first time derivative
$(\ddot{\cdot})$ = second time derivative
$(\cdot)_0$ = initial value
$(\cdot)_f$ = final value
$(\cdot)_r$ = reference value (at given V)
$\mathcal{O}(\cdot)$ = 'order of' operator

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Introduction

IN designing entry guidance logic for future aerospace vehicles—an aeroassisted orbital transfer vehicle, a Mars lander, an aerospace plane, a space station assured crew return vehicle—one is faced with a perhaps overwhelming number of potentially applicable approaches: neighboring optimal control, dynamic programming, neural networks, H_∞ control, μ-synthesis, regular and singular perturbation methods, adaptive control, quantitative feedback theory, differential geometric control, and, let us not forget, classical linear control. It is instructive in this situation to consider what approaches have been used in designing the guidance logic for current operational vehicles, why these approaches were chosen, and how effective they have been. Important references in this regard are Refs. 1 and 2, which cover operational vehicles through Apollo. The U.S. Space Shuttle is another obvious point of focus at present.

The entry guidance for the Shuttle, in particular the logic for guiding the vehicle from atmospheric entry to the range and velocity at which the terminal area energy management phase is initiated, is the topic of this paper. The Shuttle entry guidance logic has been presented by Harpold and Graves.[3] The guidance concept evolved from the Apollo entry guidance concept.[4,5] The Shuttle entry guidance has proven very effective operationally.[6]

In this paper, the Shuttle entry guidance is revisited. We show that the approach taken in the original derivation of the Shuttle entry guidance has much in common with the more recently developed feedback linearization method of differential geometric control.[7,8] Using the feedback linearization method, however, we are led to an alternative, potentially superior, guidance law. We then proceed to compare the stability and performance of the Shuttle and the alternative guidance laws. By taking a numerical approach to determining

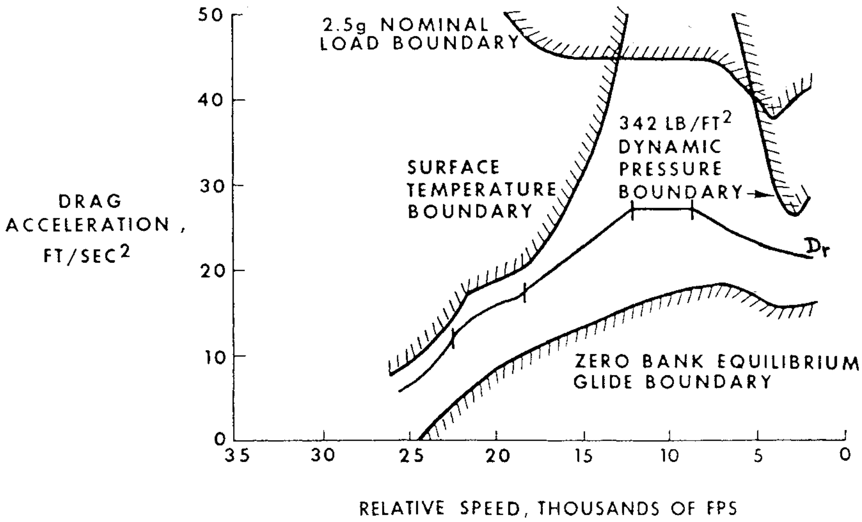

Fig. 1 Operational entry corridor and reference drag profile (taken from Ref. 3).

stability and performance domains in the state space, we are able to compare the two guidance laws, taking into account nonlinearity, a state constraint, and a control constraint. Stability boundaries for an aircraft control system subject to control constraints were determined by a related approach.[9] Our definitions of various state-space domains are in the spirit of definitions used in dynamical systems theory[10] and nonlinear control theory[7,8] but have been adapted to account for the presence of state and control constraints. The effects of increasing the control capability and changing a parameter in the guidance laws on the stability and performance domains are illustrated. The objective is to gain additional insight into why the Shuttle guidance law is successful, some perspective on its range of applicability, and an understanding of the conditions, if any, under which the alternative guidance law would be superior.

The notation and assumptions for the formulation of the entry guidance problem are consistent with those in Ref. 3, except that several additional simplifications are made in order not to clutter the presentation of the key points. Only the longitudinal translational motion of the vehicle is considered. The aerodynamic coefficients are assumed constant. The bank angle is considered to be the only control for the translational dynamics, the angle of attack being fixed at a value dictated by heating considerations. Furthermore, we restrict our attention to nominal stability and performance; the issue of robustness is not addressed.

Guidance Problem Formulation

We consider the following entry longitudinal guidance problem. Given the current translational state and equations of motion, determine the bank angle program required to achieve the prescribed range and speed without violating the state and control constraints.

Equations of Motion

The Earth-relative longitudinal translational state of the Shuttle is represented by the variables R, r, V, and γ. Neglecting the Coriolis and centrifugal terms due to Earth rotation and the effects of winds, the equations of unpowered flight are

$$\frac{\mathrm{d}R}{\mathrm{d}t} = V\cos\gamma \tag{1}$$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = V\sin\gamma \tag{2}$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -D - g\sin\gamma \tag{3}$$

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = \frac{1}{V}\left[L_v - \left(g - \frac{V^2}{r}\right)\cos\gamma\right] \tag{4}$$

The specific drag and the specific vertical lift are given by

$$D = \tfrac{1}{2}\rho\frac{S_a}{m}V^2C_D \tag{5}$$

$$L_v = \tfrac{1}{2}\rho\frac{S_a}{m}V^2C_L\cos\sigma \tag{6}$$

The aerodynamic coefficients C_D and C_L are assumed constant for our purposes here. An exponential atmospheric density model

$$\rho = \rho_s\exp\left(-\frac{r-r_s}{H}\right) \tag{7}$$

with constant density multipiier ρ_s and constant scale height H and a constant gravitational acceleration g are assumed for the guidance law development.

The translational state is controlled by adjusting the bank angle. Equivalently, the vertical lift-to-drag ratio L_v/D is taken to be the control in the following analysis.

State and Control Constraints

The entry corridor is defined by structural, thermal, and controllability constraints and can be represented in the drag-velocity plane as shown in Fig. 1. The state constraints are thus in the form of minimum and maximum drag limits as functions of velocity, specifically,

$$D_{\min}(V) \leq D \leq D_{\max}(V) \tag{8}$$

The control constraint is

$$|L_V/D| \leq (L_V/D)_{\max} \tag{9}$$

In general, the control bound is state dependent. Any value of L_V/D within and including the upper and lower bounds can be achieved by an appropriate value of the bank angle. We restrict our attention to the class U of piecewise continuous functions of time that satisfy at each time the above constraint.

Guidance Law Derivation

The Shuttle entry guidance concept[3] is to choose a flyable reference trajectory that leads to the desired range at a specified velocity and to track the reference trajectory based on feedback. Following a brief discussion of the reference trajectory, we focus our attention on the tracking problem for the remainder of the paper.

Reference Drag Trajectory

Since the destination for the entry phase under consideration is a desired range at a specified velocity, it is appropriate to look at how range changes with velocity. Combining Eqs. (1) and (3), separating variables, and integrating yields

$$R(V_f) = -\int_{V_0}^{V_f} \frac{V}{D(V)} \mathrm{d}V \tag{10}$$

assuming $R(V_0) = 0$ and $D \gg |g \sin\gamma|$. (If the latter assumption is invalid, energy can be used in place of velocity as the independent variable.) In this light, the range is seen to depend solely on the drag-vs-velocity profile. This is convenient because the entry corridor can be represented in the drag-velocity plane as described in the previous section. A reference trajectory $D_r(V)$ lying inside the entry corridor is selected such that the desired range is achieved at the specified velocity (see Fig. 1). The reference trajectory for the Shuttle entry is a continuous spline of two quadratic segments, a pseudo-equilibrium glide segment, a constant drag segment, and a linear energy segment. The reference trajectory is biased toward the higher drag region of the entry corridor to reduce the total heat load by reducing the flight time to the specified velocity.

Reference Trajectory Tracking

Once the reference-drag vs velocity trajectory is chosen, it remains to develop the bank-angle control law for tracking the reference trajectory. The drag is the focus of attention, so we think of it as the output of the third-order dynamic system given by Eqs. (2–4). (The range equation is neglected since the dynamic system that generates the drag does not depend on the range.) The approach to deriving the bank-angle control law begins with time differentiating the drag along a trajectory (i.e., taking the Lie derivative of the drag) until the first appearance of the control. The first and second derivatives are

$$\dot{D} = -\frac{DV}{H}\sin\gamma - \frac{2D^2}{V} \tag{11}$$

$$\ddot{D} = a(V, D, \dot{D}) + b(V, D, \dot{D})u \tag{12}$$

where

$$a = \dot{D}(\dot{D}/D - 3D/V) - 4D^3/V^2 + D/H(g - V^2/r)$$

$$b = -D^2/H \qquad u = L_v/D$$

Equations (11) and (12) have been derived with the approximations $D + g\sin\gamma \approx D$ and $\cos\gamma \approx 1$. The physically meaningful domain of the $(D, \dot{D}, V)$ space for atmospheric entry is defined by $V > 0$ and $D > 0$ and is singularity free. We further assume that r in the apparent gravity term $(g - V^2/r)$ is constant, $r = \bar{r}$, to simplify the subsequent developments.

We set as our goal to construct a control law that yields asymptotic tracking of the reference trajectory when the models given above are exact and the fedback state variables are measured without error. It is also necessary that the transient response is sufficiently fast. It is the reference-drag vs velocity trajectory that leads to the desired range at the specified velocity; if the control law does not cause quick enough recovery from perturbations off the reference trajectory, the desired final condition will not be achieved.

Let

$$\nu = a(V, D, \dot{D}) + b(V, D, \dot{D})u(V, D, \dot{D}) \tag{13}$$

and consider u to be a function of the arguments shown. Defining $\Delta a = a - a_r$, $\Delta b = b - b_r$, and $\Delta u = u - u_r$, where $a_r = a(V, D_r, \dot{D}_r)$ and similarly for b_r and u_r, Eq. (13) is rewritten as

$$\nu = (a_r + b_r u_r) + (\Delta a + \Delta b\, u_r + b\, \Delta u) = \nu_r + \Delta\nu \tag{14}$$

where ν_r and $\Delta\nu$ correspond to the first and second expressions in parentheses, respectively, and $\Delta\nu(V, D_r, \dot{D}_r) = 0$. Equation (12) becomes

$$\ddot{D} = \nu_r + \Delta\nu \tag{15}$$

Choosing

$$\nu_r = \ddot{D}_r \tag{16}$$

and defining $\Delta D = D - D_r$, the dynamics for the tracking error are given by

$$\Delta\ddot{D} = \Delta\nu \tag{17}$$

The feedforward control

$$u_r = (1/b_r)(-a_r + \ddot{D}_r) \tag{18}$$

obtained by solving Eq. (16) for u_r using the relation in Eq. (14) between ν_r and u_r thus produces perfect tracking beyond any value of V for which ΔD and $\Delta\dot{D}$ are both zero. (Note that $b_r \neq 0$ when $D > 0$.) However, if only the feedforward control is used (i.e., $\Delta u = 0$), the resulting time-varying error dynamics are

$$\Delta\ddot{D} = \Delta a + \Delta b\, u_r \tag{19}$$

which may not be acceptable. The simplest acceptable error dynamics are of the linear, time-invariant form

$$\Delta\ddot{D} + 2\zeta\omega\,\Delta\dot{D} + \omega^2\Delta D = 0 \tag{20}$$

with positive natural frequency ω and positive damping ratio ζ, since the error dynamics are then asymptotically (or, more precisely, exponentially) stable with respect to the origin. These dynamics shall be referred to as the *desired error dynamics*.

Local Asymptotic Tracking

The approach taken in the derivation of the Shuttle guidance was to specify a linear feedback control u such that the desired tracking error dynamics given in Eq. (20) are achieved locally. Thus consider Δu to be linear in D and $\dot{D}$,

$$\Delta u = -K_p(V)\Delta D - K_d(V)\Delta\dot{D} \tag{21}$$

where K_p and K_d are gains as yet unspecified, and consider the linear part of the expansion of $\Delta\nu$ about the reference drag trajectory for fixed V,

$$\delta\nu = \left(\frac{\partial a}{\partial D} + u_r\frac{\partial b}{\partial D}\right)\Delta D + \left(\frac{\partial a}{\partial \dot{D}} + u_r\frac{\partial b}{\partial \dot{D}}\right)\Delta\dot{D} + b_r\Delta u \tag{22}$$

where all the partial derivatives are evaluated on the reference trajectory. The closed-loop linear variational equation is

$$\Delta\ddot{D} + \left(b_r K_d - \frac{\partial a}{\partial \dot{D}} - u_r\frac{\partial b}{\partial \dot{D}}\right)\Delta\dot{D} + \left(b_r K_p - \frac{\partial a}{\partial D} - u_r\frac{\partial b}{\partial D}\right)\Delta D = 0 \tag{23}$$

The desired exponentially stable error dynamics [Eq. (20)] are achieved locally by specifying the gains

$$K_p(V) = \frac{1}{b_r}\left[\omega^2 + \left(\frac{\partial a}{\partial D}\right)_r + u_r\left(\frac{\partial b}{\partial D}\right)_r\right] \tag{24}$$

$$K_d(V) = \frac{1}{b_r}\left[2\zeta\omega + \left(\frac{\partial a}{\partial \dot{D}}\right)_r + u_r\left(\frac{\partial b}{\partial \dot{D}}\right)_r\right] \tag{25}$$

which are analytically scheduled on V (note that the partial derivatives vary with V). The complete *Shuttle entry guidance law* is

$$u_L = \frac{1}{b_r}(-a_r + \ddot{D}_r) - K_p(V)\Delta D - K_d(V)\Delta\dot{D} \tag{26}$$

The subscript L denotes that the law is linear in D and $\dot{D}$.

In the Shuttle entry guidance, there are additional features that are neglected here. Altitude rate is fed back in place of drag rate, since it can be determined with less error. For the usual reasons there is an integral feedback term in addition to the proportional and derivative feedback terms. And there is the capability of recomputing the reference drag trajectory during the entry, although it has not been important for the Shuttle flights to date. In the following, an alternative guidance law is derived and is subsequently compared to the Shuttle law. The features just mentioned could be incorporated into the alternative law, but the differences between the two laws are more clearly identified by neglecting these features in both.

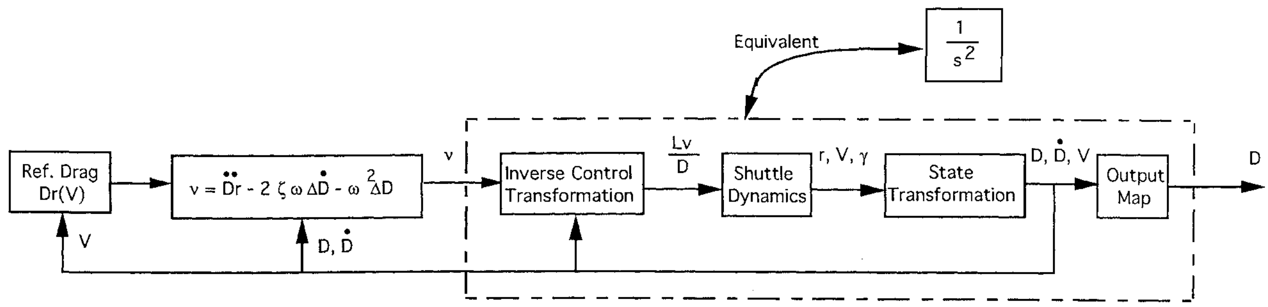

Fig. 2 Block diagram of feedback linearized system subject to feedforward and feedback control.

Feedback Linearization Method

In the formalism of the feedback linearization method,[7,8] we have used a nonlinear state transformation and a state-dependent control transformation to linearize the input-output dynamics. The state variables (r, γ, V) were transformed to $(D, \dot{D}, V)$ and the control was transformed according to Eq. (13). Using the notation $(z_1, z_2, z_3) = (D, \dot{D}, V)$, the transformed state equations are

$$\dot{z}_1 = z_2 \tag{27}$$

$$\dot{z}_2 = \nu \tag{28}$$

$$\dot{z}_3 = -z_1 + gH\frac{z_2 z_3 + 2z_1^2}{z_1 z_3^2} \tag{29}$$

$$y = z_1 \tag{30}$$

The dynamic system between input ν and output y is linear, time invariant, and in double-integrator form.

Note that by specifying

$$\nu = a + bu = \ddot{D}_r - \omega^2 \Delta D - 2\zeta\omega\Delta\dot{D} \tag{31}$$

the desired error dynamics given in Eq. (20) are achieved exactly. That is, by skipping the linearization step, used in deriving the Shuttle law, we are able to *match* the desired error dynamics exactly, not just approximately. The required actual control, obtained by solving Eq. (31) for u, is

$$u_N = \frac{1}{b}(-a + \ddot{D}_r - \omega^2\Delta D - 2\zeta\omega\Delta\dot{D}) \tag{32}$$

This asymptotic (exponential) tracking control law is *nonlinear*, as denoted by the subscript N, due to the dependence of a and b on D and $\dot{D}$. By expanding u_N about D_r and $\dot{D}_r$ at each value of V to obtain the form

$$u_N = u_r + \delta u + \mathcal{O}(\delta u^2) \tag{33}$$

where δu contains the linear terms in ΔD and $\Delta\dot{D}$, it can be shown that the Shuttle law in Eq. (26) is obtained by neglecting the second and higher order terms, i.e., $u_N = u_L + \mathcal{O}(\delta u^2)$. Figure 2 shows the block diagram for the closed-loop control of the feedback linearized system.

Since the control appears in the second derivative of the drag, the linear input-output system is second order. The original system [Eqs. (2–4)] is said to have degree 2 relative to the drag output. When the relative degree is less than the dimension of the internal state, 3 in our case, some of the behavior of the system is not observable in the output, and there is concern as to the nature of this behavior. For the tracking problem, an important variable is the tracking error $\Delta D = D - D_r = z_1 - D_r$. Perfect tracking implies that z_1 and z_2 are behaving as desired but implies nothing about the behavior of z_3. Thus one must examine the dynamics of z_3 when the tracking error is zero to determine if they are acceptable. Geometrically, the perfect tracking conditions $\Delta D \equiv 0$ and $\Delta\dot{D} \equiv 0$ define a one-dimensional manifold in the three-dimensional state space, and the z_3 dynamics, with $z_1 = D_r$ and $z_2 = \dot{D}_r$, dictate the motion on this manifold. For a gliding (unpowered), shallow entry, such as the reference drag trajectory shown in Fig. 1, the velocity z_3 is strictly decreasing and presents no cause for concern.

Remark. Since D_r is specified as a function of V and since the original dynamics [Eqs. (2–4)] do not depend explicitly on the time, V could have been treated as the independent variable. With V as the independent variable, the original system is second order and still has degree 2 relative to the drag output. Thus there would be no unobservable internal behavior in this formulation. We have retained t as the independent variable for consistency with the derivation of the Shuttle guidance in Ref. 3.

In general, in using the feedback linearization method, one must also be concerned that, although the behavior of the observable part of the transformed state is acceptable, the behavior of the original variables may not be. This is not a concern here because the transformed variables are the variables of interest; they are physical variables, the variables in which the important performance requirements and constraints are expressed.

Another important concern is control saturation. Both the linear (Shuttle) and nonlinear guidance laws are obtained by specifying desired closed-loop error dynamics and then obtaining the required control by an inversion process. If the L_v/D commanded by the tracking law is outside of the achievable range, there is saturation. In the case of control saturation, the linear and nonlinear laws may not yield asymptotic tracking; when they do, the performance may not conform to the desired error dynamics in Eq. (20). Control saturation is an important factor in comparing u_L and u_N and cannot be neglected. Hence, the guidance laws that will be compared are

$$u_S = \begin{cases} (L_V/D)_{\max} & \text{if} \quad u_L > (L_V/D)_{\max} \\ u_L & \text{if} \quad |u_L| \le (L_V/D)_{\max} \\ -(L_V/D)_{\max} & \text{if} \quad u_L < -(L_V/D)_{\max} \end{cases} \tag{34}$$

and

$$u_A = \begin{cases} (L_V/D)_{\max} & \text{if} \quad u_N > (L_V/D)_{\max} \\ u_N & \text{if} \quad |u_N| \le (L_V/D)_{\max} \\ -(L_V/D)_{\max} & \text{if} \quad u_N < -(L_V/D)_{\max} \end{cases} \tag{35}$$

Henceforth, we refer to u_S as the Shuttle guidance law and u_A as the alternative guidance law. Due to the saturation, both laws are nonlinear.

Stability and Performance

In this section, the closed-loop behavior is analyzed for the Shuttle and the alternative guidance laws. Given the extensive testing and operational success[6] of the Shuttle guidance law, the stability and performance of the Shuttle law will surely be adequate under Shuttle conditions. Our interest is in how the stability and performance of the Shuttle law compares to that of the alternative law and in the effects of changes in control and other parameters. The presence of state and control constraints complicates the stability and performance analysis.

Exact tracking $[e(t) \equiv 0]$ implies that the trajectory evolves along the V axis in the three-dimensional $(e, \dot{e}, V)$ space. For a particular control law, the error state $(e, \dot{e})$ is mapped from an initial value $(e_0, \dot{e}_0)$ to the value $(e(t), \dot{e}(t))$ during the time interval t. We denote the map, which in general depends on V_0 as well as on t and $u[0, t]$, by $\phi_t^{V_0,u}$ such that

$$(e(t), \dot{e}(t)) = \phi_t^{V_0,u}(e_0, \dot{e}_0) \tag{36}$$

A control law u is said to achieve asymptotic tracking from $(e_0, \dot{e}_0, V_0)$ if $\|\phi_t^{V_0,u}(e_0, \dot{e}_0)\| \to 0$ as $t \to \infty$, where $\|\cdot\|$ is the Euclidean norm. We define the *domain of attraction* for a control law u as the set of all points $(e, \dot{e}, V)$ for which asymptotic tracking is achieved, namely,

$$A^u = \left\{(e, \dot{e}, V) \colon \left\|\phi_t^{V,u}(e, \dot{e})\right\| \to 0 \text{ as } t \to \infty\right\} \tag{37}$$

Remark. In this and subsequent definitions, there is a mathematical idealization in considering asymptotic behavior for $t \to \infty$. Our model becomes invalid once the velocity has decreased to the point where the flight path is no longer small and the $g \sin\gamma$ term is no longer negligible compared to the drag. The idealization is reasonable provided that the significant transient behavior is short relative to the time interval over which the reference trajectory is traversed.

In Ref. 11, the domains of attraction for the linear and nonlinear control laws were constructed and compared. With a state constraint, however, the domain of attraction is not the most appropriate concept for assessing a control law. We should rather determine the set of points that are mapped to the origin *without violating the state constraint*.

In terms of the error, the state constraint Eq. (8) takes the form

$$e_{\min} \le e \le e_{\max} \tag{38}$$

with $e_{\min} < 0$ and $e_{\max} > 0$. For simplicity, we assume that both $e_{\min}$ and $e_{\max}$ are constant; in particular, we assume that they do not depend on V. The *domain of safe attraction* is defined as

$$S^u = \{(e, \dot{e}, V) \colon (e, \dot{e}, V) \in A^u \text{ and}$$
$$e_{\min} \le e(t) \le e_{\max} \text{ for all } t \text{ in } [0, \infty]\} \tag{39}$$

where $e(t)$ is the first component of $\phi_t^{V,u}(e, \dot{e})$. We use $S_2^u(V)$ to denote the two-dimensional constant V slice of the three-dimensional domain S^u for the value of V in the argument. Points that are in A^u but not in S^u are mapped to the origin (i.e., are driven back to the reference trajectory under the action of the control) but violate the drag constraint along the way, endangering the vehicle and crew.

We define the *controllable set* C as

$$C = \left\{(e, \dot{e}, V) \colon \exists u \in U \text{ and } t_1 < \infty \text{ such that } \phi_{t_1}^{V,u}(e, \dot{e}) = 0\right.$$
$$\left.\text{and } e_{\min} \le e(t) \le e_{\max} \text{ for all } t \text{ in } [0, t_1]\right\} \tag{40}$$

and use $C_2(V)$ to denote the constant V slice of C for the value of V in the argument. The controllable set is dictated by the vehicle capability, and it is control law independent. It establishes the largest possible domain of safe attraction.

The domains of safe attraction for the Shuttle and alternative guidance laws and the controllable set can be constructed numerically. For a given value of V, a point on the boundary of $C_2(V)$ is obtained by integrating the error differential equation backward in time, using the maximum negative or positive L_v/D as appropriate, from $(e, \dot{e}, V) = (e_{\min}, 0, V_0)$ and $(e, \dot{e}, V) = (e_{\max}, 0, V_0)$. This procedure is iterated on V_0, with $V_0 < V$, to construct the complete boundary. The domains of safe attraction are constructed similarly, except that the control is determined according to either the Shuttle or the alternative law.

Figure 3a shows the slices of the controllable set and the domains of safe attraction at $V = 4.0$ km/s for a constant reference drag $D_r(t) \equiv 9$ m/s^2. The bounds on e are taken to be $e_{\min} = -3$ m/s^2 and $e_{\max} = +3$ m/s^2. The maximum control capability $(L_v/D)_{\max} = 1.0$. The values $\zeta = 0.7$ and $T = 90$ s are used for both guidance laws. The domains of safe attraction for the two laws are not much different from each other and are almost as large as the controllable set. The upper boundaries of the controllable set and the two domains of safe attraction are almost identical, since they are all determined primarily with the control saturated.

The desired drag error dynamics are linear, second order with prescribed period T and damping ratio ζ. The mapping of error states that is consistent with the desired error dynamics is denoted by ϕ_t^D and is independent of V_0. We define the *performance set* (or *domain*) P^u, which depends on the control law, as the set

$$P^u = \left\{(e, \dot{e}, V) \colon (e, \dot{e}, V) \in S^u \text{ and}\right.$$
$$\left.\phi_t^{V,u}(e, \dot{e}) = \phi_t^D(e, \dot{e}) \text{ for all } t \text{ in } [0, \infty)\right\} \tag{41}$$

Here $P_2^u(V)$ denotes the constant V slice of P^u for the value of V in the argument. The set P^u can be constructed numerically. For the alternative guidance law, u_A, the desired error dynamics are achieved as long as the control is unsaturated. The maximum size of P^{u_A} is achieved when $(L_v/D)_{\max}$ is large enough that control saturation is not a factor. In the absence of saturation, $P^{u_A} = S^{u_A}$ and its size is dictated by the state constraint. The construction of a cross section of the performance domain for the alternative guidance law, $P_2^{u_A}(V = 4.0 \text{ km/s})$, is illustrated in Fig. 3a. First the control saturation boundaries are drawn for $V = 4.0$ km/s. As trajectories in the error plane evolve, V is changing and the control saturation boundaries, which depend on V, are moving. However, for the velocity range of importance, the saturation boundaries are approximately

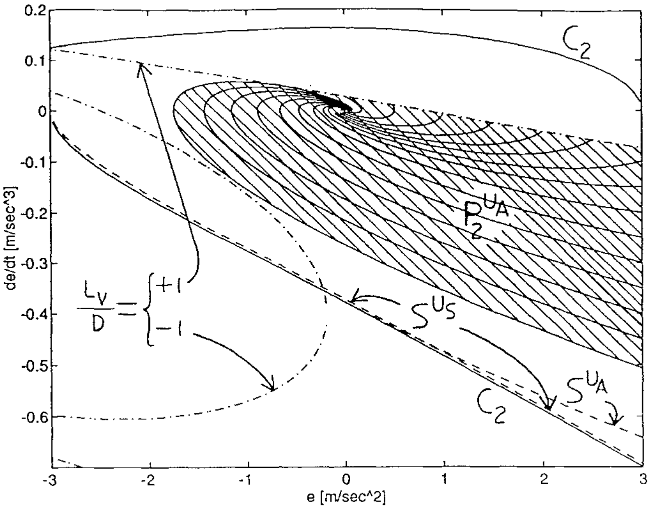

Fig. 3a Cross-section of controllable set C_2 in error state plane for $V = 4$ km/s, $D_r = 9$ m/s^2, $h = 60$ km, $e_{\min} = -3$ m/s^2, $e_{\max} = +3$ m/s^2, and $(L_v/D)_{\max} = 1.0$. Dashed lines are cross sections of domains of safe attraction, S^{u_S} and S^{u_A}, for same conditions and $\zeta = 0.7$ and $T = 90$ s (note that upper boundaries are confounded with C_2). Also cross section of performance domain for alternative guidance law P^{u_A} (shaded area, with superimposed second-order trajectories). Dot-dashed contours are control saturation boundaries at $V = 4$ km/s.

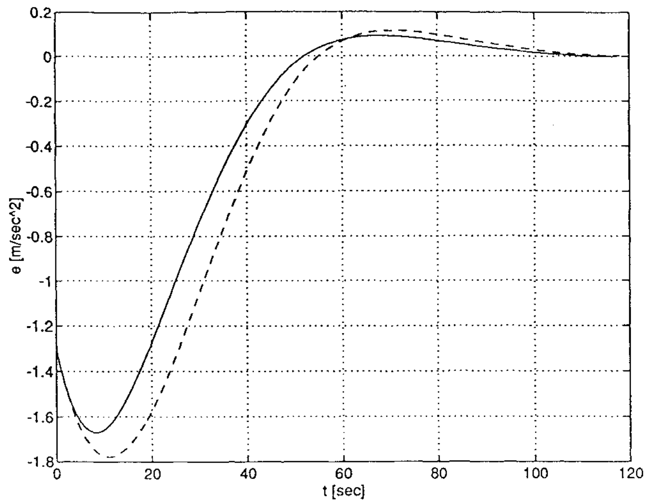

Fig. 3b Time histories showing corresponding performance of alternative guidance law (plain line) and of Shuttle guidance law (dashed line) for initial speed $V = 4$ km/s and initial perturbations $e = -1.3$ m/s^2 and $\dot{e} = -0.1$ m/s^3 (equivalent to $\Delta r = +1131$ m and $\Delta\gamma = +1.416$ deg).

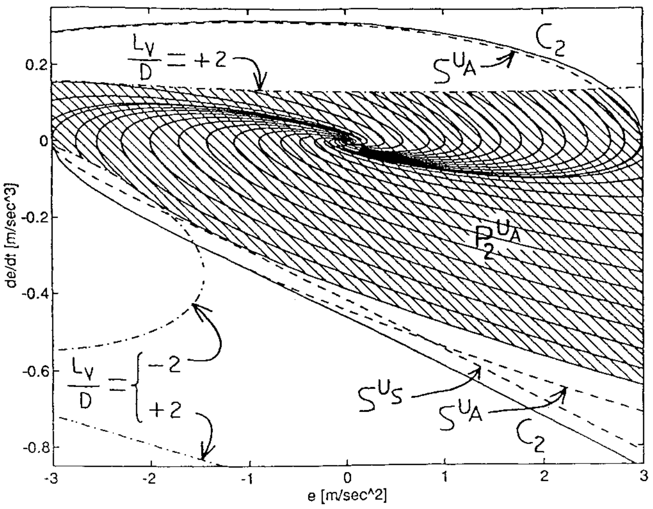

Fig. 4a Same as Fig. 3a, except for increased control capability $(L_v/D)_{\max} = 2.0$.

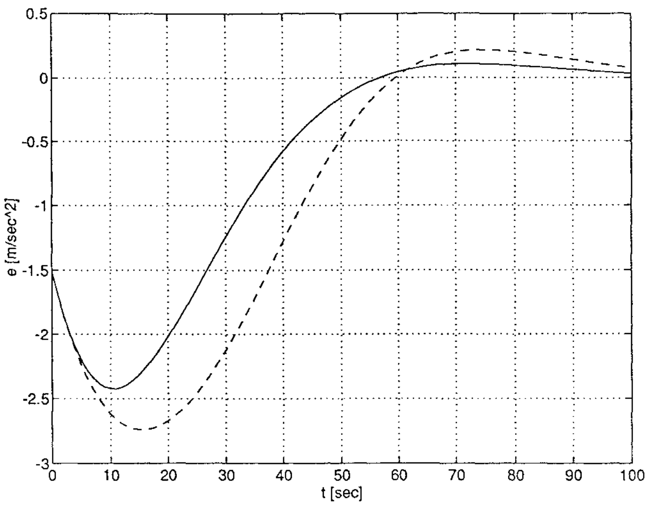

Fig. 4b Time histories for $(L_v/D)_{\max} = 2.0$ and initial perturbations $e = -1.5$ m/s^2 and $\dot{e} = -0.2$ m/s^3 (equivalent to $\Delta r = +1322$ m and $\Delta\gamma = +2.848°$).

stationary. The construction of $P_2^{u_A}$ is greatly simplified by assuming that the control saturation boundaries are fixed. The construction involves finding the family of trajectories that evolve according to the desired second-order dynamics and do not penetrate the regions where the control saturates. The second-order trajectories that are tangent to the control saturation boundaries are limiting cases in this respect. Depending on the particular conditions considered, one or more of these tangent trajectories will be part of the performance domain boundaries. It is worth noting that the second-order dynamics trajectories always evolve clockwise in the error plane and depend solely on T and ζ, not on V. Finally, the performance domain is drawn as the intersection of the region of second-order trajectories that do not violate the control constraints, with the domain of safe attraction S^{u_A}.

For the linear tracking law u_L, the desired closed-loop error dynamics will only be achieved in an infinitesimal neighborhood of the origin; outside that neighborhood the nonlinearities neglected in the derivation of the Shuttle law will alter the performance. The actual performance domain for the Shuttle law, P^{u_L}, has zero volume as we have defined it. One could construct an approximate performance domain for the Shuttle law by establishing a limit on the allowable degradation of performance. Instead, we compare the performances of the Shuttle law and the alternative law by simulation for particular initial conditions. Figure 3b compares the performances of the two laws for an initial condition just inside of P^{u_A}. Since the initial condition is in P^{u_A}, the alternative law achieves

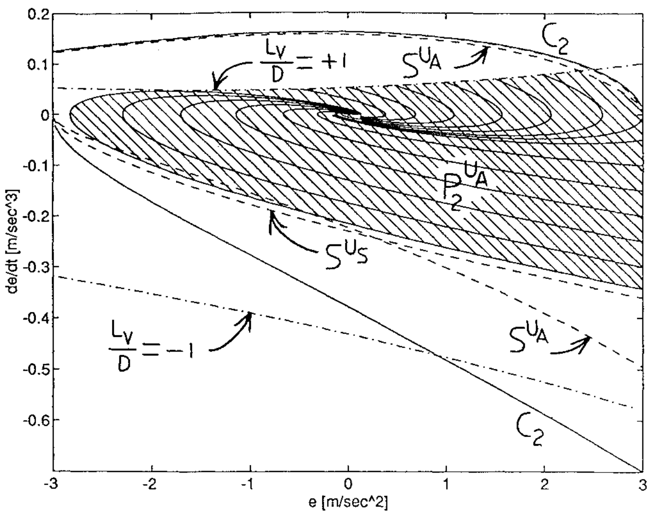

Fig. 5 Same as Fig. 3a, except for increased control parameter $T = 180$ s.

exactly the desired performance. The performance of the Shuttle law deviates from that desired.

Using the conditions for Fig. 3 as a baseline, we now illustrate how changes in individual parameters affect the stability and performance of the two laws. Figure 4 presents the same information as Fig. 3 under the same conditions except that $(L_v/D)_{\max} = 2.0$. As expected, all the sets are larger with greater control capability. There is again little difference between the domains of safe attraction for the two laws. In some directions, the domain for the Shuttle law extends farther; in other directions, the domain for the alternative law extends farther. The control saturated region in the lower left portion of $S_2^{u_A}(V)$ is relatively smaller, allowing the performance domain for the alternative law to extend closer to the lower boundary of $S_2^{u_A}(V)$. Figure 4b compares the performances of the two laws for an initial error state just inside of P^{u_A}. The difference in performance is more substantial for this case of higher control capability.

Figure 5 illustrates how an increase in the control parameter T affects the stability and performance of the two laws. All other conditions are unchanged from the baseline. The value of T is 180 s, twice the baseline value. In physical terms, a larger value of T reduces the control magnitude commanded for a given error. Observe that the lower boundaries of the domains of safe attraction are substantially inside of the controllable set, and thus these domains are reduced by the increase in T. The domains are not much different for the two laws, and neither one is uniformly preferable. The upper boundaries of the domains of safe attraction are determined primarily with saturated controls and thus do not differ much from the controllable domain. Here $P_2^{u_A}$ (4.0) is somewhat altered from the baseline case due to the change in the location and extent of the saturated control regions.

Remark. The preceding stability and performance analysis considered only constant drag reference trajectory tracking. The actual reference trajectory tracked by the Shuttle, shown in Fig. 1, varies with V. However, since the rate of change of the actual D_r is relatively slow (estimated to be less than 0.02 m/s^3), it is expected that the results we have obtained would not be significantly different if the variation in D_r were taken into account. However, this does not imply that taking into account the rate of change of D_r is not of quantitative importance in stability and performance.

Assessment

The Shuttle guidance law has feedback terms that only ensure locally the desired asymptotic tracking of the reference drag trajectory. This is because the terms are derived using only the linear part of the Taylor series expansion of the drag dynamics about the reference trajectory. The feedback linearization method allows a more global linearization of the drag dynamics, limited by singularities and control saturation rather than truncation, and thus it allows the prospect for achieving the desired asymptotic tracking over a larger domain. Both the truncated Taylor series and feedback linearization approaches lead to tracking terms that are analytically scheduled on the velocity.

Using feedback linearization, we were led to a linear double-integrator system with a transformed control as input and the drag as output. Following the derivation of the Shuttle guidance law as closely as possible, we chose the transformed control such that the drag error dynamics were linear, second order with a particular damping ratio and natural frequency. For any guidance law u, the domain P^u in the $(e, \dot{e}, V)$ space in which the desired performance is actually achieved without violating the drag constraint is contained in the controllable set C. The alternative guidance law we have derived yields the *largest performance domain*, i.e., $P^u \subseteq P^{u_A}$, $\forall u \in U$.

What are the implications of our results for the entry guidance of the Shuttle and other aerospace vehicles? For a given vehicle and a given reference drag trajectory, we define the *operating domain* as the set of points in the $(e, \dot{e}, V)$ space at which the vehicle may find itself. It is reasonable to expect that the current operating domain of the Shuttle is well within P^{u_A}. The alternative law achieves exactly the desired performance within P^{u_A}; the Shuttle law does not, strictly speaking, except in an infinitesimal neighborhood of the origin of the error plane. However, the smaller the operating domain, the less likely the performance difference between the two laws is to be appreciable. For larger operating domains that encompass most of P^{u_A}, the improved performance offered by the alternative law may be significant. A more definite conclusion regarding the superiority of the alternative guidance law for entry of the Shuttle or some other vehicle would require a comprehensive comparison with attention to robustness and other important factors that have not been addressed here.

It is also interesting to consider the case where the operating domain extends beyond P^{u_A}. The operating domain should, however, be contained within C; otherwise, there is no control law that can bring the vehicle safely back to the reference trajectory. For points $(e, \dot{e}, V)$ outside the performance domain P^{u_A} but inside the controllable set, there exists a control that will null the drag error without violating the drag constraint. In the cases investigated, neither the Shuttle nor the alternative guidance laws uniformly generate such a control within C, since neither S^{u_S} nor S^{u_A} encompass C. In fact, neither control law is clearly better in this regard. Outside of P^{u_A}, it may be more appropriate to use a control law derived to meet some other objective, such as driving the state back to the performance domain. In cases of nonuniqueness, the minimum-time control could be used. Some relevant discussion can be found in Ref. 12.

In keeping with the Shuttle guidance concept, our objective in developing the alternative guidance law was uniform performance; i.e., the guidance law was designed such that the closed-loop error dynamics *matched* the desired exponentially stable error dynamics. The feedback linearization method allowed this model-matching objective to be achieved exactly. In general, uniformity in performance is achieved at the expense of nonuniformity in control effort. A compromise may be more appropriate. Although we used model matching, any linear control design method could be used to design the transformed control for the feedback linearized system. A difficulty in designing a reduced control effort control law is that the actual system dynamics and the actual control are buried in the state and control transformations employed for feedback linearization and are not easily taken into account in designing the transformed control.

Conclusions

The Shuttle entry guidance concept is to track a reference drag trajectory that leads to the specified range and velocity for the initiation of the terminal energy management phase. The current guidance law provides exponential tracking locally. Using the feedback linearization method, an alternative guidance law yielding an expanded region of exponential tracking has been derived. To compare the two guidance laws, stability and performance domains in state space were defined, taking into account the nonlinear dynamics, a state constraint, and a control constraint. The stability and performance domains for the Shuttle law and the alternative law were constructed numerically. The effects of increasing the control capability and changing a parameter in the guidance laws on the stability and performance domains were illustrated. The alternative guidance law achieves a larger performance domain; the stability domains for the two laws are similar. For the current operating domain of the Shuttle, the performance improvement offered by the alternative guidance law is probably not significant. With a larger operating domain for the Shuttle or some other entry vehicle, the alternative guidance law should be considered. A more comprehensive comparison taking into account important factors not considered here, such as robustness, would be necessary to decide whether or not the alternative guidance law is superior.

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