EXAMPLE 5.2
ECI-Frame Inertial Navigation Update Cycle using First-order Approximations

INPUTS:
Initial navigation solution

Initial position		0	m	Note: The user is at the	
	$\mathbf{r}_{ib}^{i}(-) =$	0	m	north pole	
		6356760	m		
Initial velocity		0	m s^{-1}		
	$\mathbf{v}_{ib}^{i}(-) =$	0	m s^{-1}		
		0	m s^{-1}		
Initial attitude (Eulers)	$\phi_{ib}(-) =$	0	deg	0	rad
	$\theta_{ib}(-) =$	0	deg	0	rad
	$\psi_{ib}(-) =$	30	deg	0.523599	rad

Measurements

Time interval	$\tau =$	0.01	s
Angular rate		0	rad s^{-1}
	$\boldsymbol{\omega}_{ib}^{b} =$	0	rad s^{-1}
		7.29E-05	rad s^{-1}
Body-frame-resolved		2	m s^{-2}
specific force	$\mathbf{f}_{ib}^{b} =$	0	m s^{-2}
		9.832	m s^{-2}

Body frame to ECI frame coordinate transformation matrix:
From (2.24),

$$\mathbf{C}_b^i = \begin{bmatrix} \cos\theta_{ib}\cos\psi_{ib} & \begin{pmatrix} -\cos\phi_{ib}\sin\psi_{ib} \\ +\sin\phi_{ib}\sin\theta_{ib}\cos\psi_{ib} \end{pmatrix} & \begin{pmatrix} \sin\phi_{ib}\sin\psi_{ib} \\ +\cos\phi_{ib}\sin\theta_{ib}\cos\psi_{ib} \end{pmatrix} \\ \cos\theta_{ib}\sin\psi_{ib} & \begin{pmatrix} \cos\phi_{ib}\cos\psi_{ib} \\ +\sin\phi_{ib}\sin\theta_{ib}\sin\psi_{ib} \end{pmatrix} & \begin{pmatrix} -\sin\phi_{ib}\cos\psi_{ib} \\ +\cos\phi_{ib}\sin\theta_{ib}\sin\psi_{ib} \end{pmatrix} \\ -\sin\theta_{ib} & \sin\phi_{ib}\cos\theta_{ib} & \cos\phi_{ib}\cos\theta_{ib} \end{bmatrix}$$

$$\mathbf{C}_b^i(-) = \begin{bmatrix} 0.866025404 & -0.5 & 0 \\ 0.5 & 0.866025404 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Update Attitude

From (5.13), $\mathbf{C}_b^i(+) \approx \mathbf{C}_b^i(-)\left(\mathbf{I}_3 + \boldsymbol{\Omega}_{ib}^{b}\tau_i\right)$ Assuming a constant angular rate

$$\boldsymbol{\Omega}_{ib}^{b}\tau_i = \begin{bmatrix} 0 & -7.29212E\text{-}07 & 0 \\ 7.29212E\text{-}07 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ rad}$$

$$\mathbf{I}_3 + \boldsymbol{\Omega}_{ib}^{b}\tau_i = \begin{bmatrix} 1 & -7.3E\text{-}07 & 0 \\ 7.29212E\text{-}07 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\mathbf{C}_b^i(+) =$

0.866025039	-0.500000632	0
0.500000632	0.866025039	0
0	0	1

Set of Euler angles describing rotation from ECI frame to body frame

From (2.25), $\phi_{ib} = \arctan_2\left(C^i_{b3,2}, C^i_{b3,3}\right)$

$\theta_{ib} = -\arcsin C^i_{b3,1}$

$\psi_{ib} = \arctan_2\left(C^i_{b2,1}, C^i_{b1,1}\right)$

Note: The arguments of the Excel ATAN2 function are the opposite way round

Roll	ϕ_{ib}	0	radians	0.00000	degrees
Pitch	θ_{ib}	0	radians	0.00000	degrees
Yaw	ψ_{ib}	0.523599505	radians	30.00004	degrees

Specific Force Frame Transforation

From (5.16), $\mathbf{f}^i_{ib} \approx \frac{1}{2}\left(\mathbf{C}^i_b(-) + \mathbf{C}^i_b(+)\right)\mathbf{f}^b_{ib}$

$\mathbf{f}^i_{ib} =$

1.732050443	m s^{-2}
1.000000632	m s^{-2}
9.832	m s^{-2}

Update Velocity

From (5.18), $\mathbf{a}^i_{ib} = \mathbf{f}^i_{ib} + \boldsymbol{\gamma}^i_{ib}\left(\mathbf{r}^i_{ib}\right)$

$\boldsymbol{\gamma}^i_{ib}\left(\mathbf{r}^i_{ib}\right) =$

0	m s^{-2}
0	m s^{-2}
-9.832	m s^{-2}

$\mathbf{a}^i_{ib} =$

1.73205	m s^{-2}
1.000001	m s^{-2}
0	m s^{-2}

From (5.20), $\mathbf{v}^i_{ib}(+) = \mathbf{v}^i_{ib}(-) + \mathbf{a}^i_{ib}\tau_i$

$\mathbf{v}^i_{ib}(+) =$

0.017320504	m s^{-1}
0.010000006	m s^{-1}
0	m s^{-1}

Update Position

From (5.23), $\mathbf{r}^i_{ib}(+) = \mathbf{r}^i_{ib}(-) + \left(\mathbf{v}^i_{ib}(-) + \mathbf{v}^i_{ib}(+)\right)\frac{\tau_i}{2}$

$\mathbf{r}^i_{ib}(+) =$

8.66025E-05	m
5E-05	m
6356760	m