EXAMPLE 5.1

Inertial Navigation in Two Dimensions

INPUTS:

Initial navigation solution

Initial position

 $x_{pb}^{p} = y_{pb}^{p} = v_{pb,x}^{p} = v_{pb,x}^{p} = v_{pb,y}^{p} = v_{pb,y}$

Initial velocity

 $\psi_{pb} =$

0 m s⁻¹ 0 m s⁻¹

0.785398 rad

Epoch 1 Measurements

Initial heading

Time interval Angular rate

 $\omega_{pb,z}^{b} = a_{pb,x}^{b} = a_{pb,y}^{b} =$ Body x-axis acceleration Body y-axis acceleration

 $0.5 \, \text{rad s}^{-1}$ 2 m s^{-2}

Note: The time interval would normally be much shorter.

Update Heading

Assuming a constant angular rate

59.32394 deg

Transform acceleration to p frame

 $\begin{pmatrix} a_{pb,x}^{p} \\ a_{pb,y}^{p} \end{pmatrix} = \begin{pmatrix} \cos \psi_{pb} & -\sin \psi_{pb} \\ \sin \psi_{pb} & \cos \psi_{pb} \end{pmatrix} \begin{pmatrix} a_{pb,x}^{b} \\ a_{pb,y}^{b} \end{pmatrix}$ From (5.6),

 $\begin{pmatrix} \cos \psi_{pb} & -\sin \psi_{pb} \\ \sin \psi_{pb} & \cos \psi_{pb} \end{pmatrix} = \begin{array}{c} \text{Before update} \\ \hline 0.707106781 & -0.70711 \\ 0.707106781 & 0.707107 \\ \hline \end{array}$

After update 0.510184 -0.86007 0.860066 0.510184

0.783586171 0.608645

Update Velocity

 $\begin{pmatrix} v_{pb,x}^{p}(t+\tau) \\ v_{pb,y}^{p}(t+\tau) \end{pmatrix} = \begin{pmatrix} v_{pb,x}^{p}(t) \\ v_{pb,y}^{p}(t) \end{pmatrix} + \begin{pmatrix} a_{pb,x}^{p} \\ a_{pb,y}^{p} \end{pmatrix} \tau$ From (5.3),

$$v_{pb,x}^{p} = 0.569465845 \text{ m s}^{-1}$$
 $v_{pb,y}^{p} = 0.814018429 \text{ m s}^{-1}$

Update Position

 $\begin{pmatrix} x_{pb}^{p}(t+\tau) \\ y_{pb}^{p}(t+\tau) \end{pmatrix} = \begin{pmatrix} x_{pb}^{p}(t) \\ y_{pb}^{p}(t) \end{pmatrix} + \left[\begin{pmatrix} v_{pb,x}^{p}(t) \\ v_{pb,y}^{p}(t) \end{pmatrix} + \begin{pmatrix} v_{pb,x}^{p}(t+\tau) \\ v_{pb,y}^{p}(t+\tau) \end{pmatrix} \right] \frac{\tau}{2}$ From (5.4),

$$x_{pb}^{p} = 0.142366461 \text{ m}$$

 $y_{pb}^{p} = 0.203504607 \text{ m}$

Epoch 2 Measurements

Time interval $ au$ =	0.5	S
Angular rate $\omega^{b}_{pb,z}=$	0.2	rad s ⁻¹
Body x-axis acceleration $a_{pb,x}^b =$	5	ms^{-2}
Body y-axis acceleration $a_{pb,y}^b =$	0.1	$m s^{-2}$

Update Heading

From (5.5),
$$\psi_{pb}(t+\tau) = \psi_{pb}(t) + \omega_{pb,z}^b \tau$$
 Assuming a constant angular rate $\psi_{pb} = \frac{1.135398163}{1.135398163}$ rad $\frac{65.05352}{1.135398163}$

Transform acceleration to p frame

From (5.6),
$$\begin{pmatrix} a_{pb,x}^{p} \\ a_{pb,y}^{p} \end{pmatrix} = \begin{pmatrix} \cos \psi_{pb} & -\sin \psi_{pb} \\ \sin \psi_{pb} & \cos \psi_{pb} \end{pmatrix} \begin{pmatrix} a_{pb,x}^{b} \\ a_{pb,y}^{b} \end{pmatrix}$$

$$\begin{pmatrix} \cos \psi_{pb} & -\sin \psi_{pb} \\ \sin \psi_{pb} & \cos \psi_{pb} \end{pmatrix} = \begin{pmatrix} 0.510183526 & -0.86007 \\ 0.860065561 & 0.510184 \end{pmatrix}$$

$$Average$$

$$\begin{pmatrix} 0.465977488 & -0.88338 \\ 0.883383871 & 0.465977 \end{pmatrix}$$

$$\begin{pmatrix} a_{pb,x}^{p} \\ a_{pb,y}^{p} \end{pmatrix} = \begin{pmatrix} 2.241549055 \\ 4.463517102 \end{pmatrix} \text{m s}^{-2}$$

$$M \text{m s}^{-2}$$

After update

0.421771	-0.9067
0.906702	0.421771

Update Velocity

From (5.3),
$$\begin{pmatrix} v_{pb,x}^{p}(t+\tau) \\ v_{pb,y}^{p}(t+\tau) \end{pmatrix} = \begin{pmatrix} v_{pb,x}^{p}(t) \\ v_{pb,y}^{p}(t) \end{pmatrix} + \begin{pmatrix} a_{pb,x}^{p} \\ a_{pb,y}^{p} \end{pmatrix} \tau$$

$$v_{pb,x}^{p} =$$

$$v_{pb,y}^{p} =$$

$$1.690240373 \text{ m s}^{-1}$$

$$v_{pb,y}^{p} =$$

$$3.04577698 \text{ m s}^{-1}$$

Update Position

From (5.4),
$$\begin{pmatrix} x_{pb}^{p}(t+\tau) \\ y_{pb}^{p}(t+\tau) \end{pmatrix} = \begin{pmatrix} x_{pb}^{p}(t) \\ y_{pb}^{p}(t) \end{pmatrix} + \begin{bmatrix} v_{pb,x}^{p}(t) \\ v_{pb,y}^{p}(t) \end{pmatrix} + \begin{pmatrix} v_{pb,x}^{p}(t+\tau) \\ v_{pb,y}^{p}(t+\tau) \end{pmatrix} \frac{\tau}{2}$$

$$x_{pb}^{p} = \begin{bmatrix} 0.707293016 \\ y_{pb}^{p} = \end{bmatrix}$$

$$x_{pb}^{p} = \begin{bmatrix} 0.707293016 \\ 1.168453459 \end{bmatrix}$$
m
$$x_{pb}^{p} = \begin{bmatrix} 0.707293016 \\ 1.168453459 \end{bmatrix}$$
m

Epoch 3 Measurements

Time interval	τ =	0.5	s	
Angular rate	$\omega_{pb,z}^{b} =$	0.1	rad s ⁻¹	
Body x-axis acceleration	$a_{pb,x}^{b} =$		ms^{-2}	
Body y-axis acceleration	$a_{pb,y}^{b} =$	-0.05	$\mathrm{m\ s}^{-2}$	

Update Heading

From (5.5),
$$\psi_{pb}(t+\tau) = \psi_{pb}(t) + \omega_{pb,z}^b \tau$$
 Assuming a constant angular rate $\psi_{pb} = 1.185398163$ rad 67.91831 deg

After update

0.375928 -0.92665 0.926649 0.375928

Transform acceleration to p frame

From (5.6),
$$\begin{pmatrix} a_{pb,x}^{p} \\ a_{pb,y}^{p} \end{pmatrix} = \begin{pmatrix} \cos \psi_{pb} & -\sin \psi_{pb} \\ \sin \psi_{pb} & \cos \psi_{pb} \end{pmatrix} \begin{pmatrix} a_{pb,x}^{b} \\ a_{pb,y}^{b} \end{pmatrix}$$

$$\begin{pmatrix} \cos \psi_{\ pb} & -\sin \psi_{\ pb} \\ \sin \psi_{\ pb} & \cos \psi_{\ pb} \end{pmatrix} = \begin{array}{c} \text{Before update} \\ \hline 0.42177145 & -0.9067 \\ 0.90670218 & 0.421771 \\ \hline \end{array}$$

Average 0.398849787 -0.91668 0.916675503 0.39885

 $\begin{pmatrix} a_{pb,x}^{p} \\ a_{pb,y}^{p} \end{pmatrix} = \begin{pmatrix} 0.84353335 & \text{m s}^{-2} \\ 1.813408516 & \text{m s}^{-2} \end{pmatrix}$

Update Velocity

From (5.3),
$$\begin{pmatrix} v_{pb,x}^{p}(t+\tau) \\ v_{pb,y}^{p}(t+\tau) \end{pmatrix} = \begin{pmatrix} v_{pb,x}^{p}(t) \\ v_{pb,y}^{p}(t) \end{pmatrix} + \begin{pmatrix} a_{pb,x}^{p} \\ a_{pb,y}^{p} \end{pmatrix} \tau$$

$$v_{pb,x}^{p} =$$
 2.112007048 m s⁻¹
 $v_{pb,y}^{p} =$ 3.952481238 m s⁻¹

Update Position

From (5.4),
$$\begin{pmatrix} x_{pb}^{p}(t+\tau) \\ y_{pb}^{p}(t+\tau) \end{pmatrix} = \begin{pmatrix} x_{pb}^{p}(t) \\ y_{pb}^{p}(t) \end{pmatrix} + \begin{bmatrix} v_{pb,x}^{p}(t) \\ v_{pb,y}^{p}(t) \end{pmatrix} + \begin{pmatrix} v_{pb,x}^{p}(t+\tau) \\ v_{pb,y}^{p}(t+\tau) \end{pmatrix} \frac{\tau}{2}$$

$$x_{pb}^{p} = \begin{cases} 1.657854871 \\ v_{pb}^{p}(t+\tau) \end{cases}$$

$$x_{pb}^{p} = \begin{cases} 2.918018014 \\ 0.918018014 \end{cases}$$

Epoch 4 Measurements

Time interval	τ =	0.5	S
Angular rate	$\omega_{pb,z}^b =$	-0.1	rad s ⁻¹
Body x-axis acceleration	$a_{pb,x}^{b} =$	0	ms^{-2}
Body y-axis acceleration	$a_{pb,v}^b =$	0	$\mathrm{m\ s}^{-2}$

Update Heading

From (5.5),
$$\psi_{pb}(t+\tau) = \psi_{pb}(t) + \omega_{pb,z}^b \tau$$
 Assuming a constant angular rate $\psi_{pb} = 1.135398163$ rad 65.05352 deg

Transform acceleration to p frame

From (5.6),
$$\begin{pmatrix} a_{pb,x}^{p} \\ a_{pb,y}^{p} \end{pmatrix} = \begin{pmatrix} \cos \psi_{pb} & -\sin \psi_{pb} \\ \sin \psi_{pb} & \cos \psi_{pb} \end{pmatrix} \begin{pmatrix} a_{pb,x}^{b} \\ a_{pb,y}^{b} \end{pmatrix}$$

$$\begin{pmatrix} \cos \psi_{pb} & -\sin \psi_{pb} \\ \sin \psi_{pb} & \cos \psi_{pb} \end{pmatrix} = \begin{bmatrix} \text{Before update} \\ 0.375928124 & -0.92665 \\ 0.926648825 & 0.375928 \end{bmatrix}$$

$$After update$$

$$0.421771 & -0.9067 \\ 0.906702 & 0.421771 \\ Average$$

-0.91668

0.39885

0.398849787

0.916675503

Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems, 2nd Edition, by Paul D. Groves

$$\begin{pmatrix} a_{pb,x}^{p} \\ a_{pb,y}^{p} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ m s^{-2} \end{pmatrix}$$

Update Velocity

From (5.3),
$$\begin{pmatrix} v_{pb,x}^{p}(t+\tau) \\ v_{pb,y}^{p}(t+\tau) \end{pmatrix} = \begin{pmatrix} v_{pb,x}^{p}(t) \\ v_{pb,y}^{p}(t) \end{pmatrix} + \begin{pmatrix} a_{pb,x}^{p} \\ a_{pb,y}^{p} \end{pmatrix} \tau$$

$$v_{pb,x}^{p} =$$

$$v_{pb,y}^{p} =$$

$$2.112007048 \text{ m s}^{-1}$$

$$3.952481238 \text{ m s}^{-1}$$

Update Position

From (5.4),
$$\begin{pmatrix} x_{pb}^{p}(t+\tau) \\ y_{pb}^{p}(t+\tau) \end{pmatrix} = \begin{pmatrix} x_{pb}^{p}(t) \\ y_{pb}^{p}(t) \end{pmatrix} + \left[\begin{pmatrix} v_{pb,x}^{p}(t) \\ v_{pb,y}^{p}(t) \end{pmatrix} + \begin{pmatrix} v_{pb,x}^{p}(t+\tau) \\ v_{pb,y}^{p}(t+\tau) \end{pmatrix} \right] \frac{\tau}{2}$$

$$x_{pb}^{p} = \begin{cases} 2.713858395 \\ y_{pb}^{p} = \end{cases}$$

$$4.894258633$$
m