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A Predictor–corrector Entry Guidance Method Using Gaussian Integration

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Abstract: To solve the reentry guidance problem of lift vehicle, a predictor–corrector entry guidance method using Gaussian integration is proposed. First, a simplified dynamic differential model is established. Second, when the angle-of-attack command for the guidance control variable is fixed as a function of velocity, the range corresponding to the bank angle profile is predicted by Gauss integral, instead of numerical integration, and the bank of angle command for another guidance control variable matched with the range to go can be solved by the Newton–Raphson iteration. In order to prevent altitude jump, the feedback of altitude change rate is added to the command profile of the bank angle. Finally, lateral guidance is realized by limiting the heading angle error in the corridor, and the simulation is carried out for the given condition. The results show that the method performs well: the calculation time of the proposed method to complete a predictor–corrector calculation is less than 20ms, and the method has a high accuracy for terminal altitude, terminal velocity, and terminal position.

Key Words: Gaussian Integration, Quasi-equilibrium Glide, Predictor–corrector

1 Introduction

The lift vehicle represented by the space shuttle can be used repeatedly to go back and forth between the ground and space [1]. In the reentry phase, the lift vehicle flies in the atmosphere for a long time with high speed, and there are many constraints that need to be tolerated in the long range [2]. Because the various situations need to be adapted, the guidance for reentry is difficult.

Due to the uncertainty of the atmospheric density in the atmosphere and the inaccuracy of the aerodynamic model of the lift vehicle, the reentry flight is greatly disturbed and it is difficult to improve the guidance accuracy. A numerical predictor–corrector entry guidance method (NPCG) was proposed by Lu in [3]. In the method, the angle of attack is fixed as a function of energy, and then the glide terminal impact point is predicted by numerical integration of the dynamic differential model. The bank angle command profile is corrected by the impact point deviation, and the reentry guidance problem is transformed into a single parameter correction problem. High guidance accuracy is obtained for the reentry guidance. However, due to the long calculation time of numerical integration, it is difficult for the existing hardware to meet the requirements of real-time guidance. Some virtual target points are selected and the reentry segment is divided into several segments in [4], the calculation time to complete a predictor–corrector calculation is reduced. In [5], the range is calculated by the drag acceleration, and the drag acceleration profile spliced by several lines which meets the requirements of the range to go, can be obtained by iteration, so the guidance commands can be calculated by tracking the drag acceleration.

A predictor–corrector entry guidance method using Gaussian integration (GPCG) is proposed in this paper, and it overcomes the low computational efficiency, and inherits the high accuracy of the NPCG. First, the angle of attack is constructed by a function of the Mach, and the bank angle is fixed as the first-order function with the energy as the independent variable, where the terminal is constant. Second the bank angle profile is transformed into the drag acceleration profile with the quasi-equilibrium glide condition (QEGC), and the range of the drag acceleration profile can be calculated by Gaussian integration to correct the bank angle profile, so the bank angle profile corresponding to the range-to-go can be obtained. Third, the feedback of altitude change rate is added to the command profile of the bank angle to prevent altitude jump, and the sign of bank angle is changed to ensure that heading angle error is within the corridor. Finally, a simulation of the predictor–corrector entry guidance method using Gaussian integration is carried out for CAV-H model [6].

2 Dynamic Model

Due to the complexity of reentry process, in order to simplify the guidance process, the following assumptions are made:

- The vehicle is rigid and inelastic.
- The earth is a uniform sphere.

The three-dimensional dynamic model of a lift vehicle over a spherical, rotating Earth in terms of non-dimensional variables is

$$\dot{r} = V\sin\gamma \tag{1}$$

$$\dot{\theta} = \frac{V\cos\gamma\sin\psi}{r\cos\phi} \tag{2}$$

$$\dot{\phi} = \frac{V\cos\gamma\cos\psi}{r} \tag{3}$$

$$\begin{aligned}\dot{V} = -D & - \frac{\sin\gamma}{r^2} \\ & +\Omega^2 r\cos\phi\left(\sin\gamma\cos\phi-\cos\gamma\sin\phi\cos\psi\right)\end{aligned} \tag{4}$$

$$\dot{\gamma} = \frac{1}{V}[L\ \cos\sigma + (V^2 - \frac{1}{r})\frac{\cos\gamma}{r}$$
$$+2\Omega V\cos\phi\sin\psi \quad (5)$$
$$+\Omega^2 r\cos\phi\left(\cos\gamma\cos\phi + \sin\gamma\sin\phi\cos\psi\right)]$$

$$\dot{\psi} = \frac{1}{V}[\frac{L\ \sin\sigma}{\cos\gamma} + \frac{V^2\cos\gamma\sin\psi\tan\phi}{r}$$
$$-2\Omega V\left(\tan\gamma\cos\psi\cos\phi - \sin\phi\right) \quad (6)$$
$$+\frac{\Omega^2 r}{\cos\gamma}\sin\psi\sin\phi\cos\phi]$$

where the time is normalized by $\tau = t / t_{scale}$ ($t_{\text{scale}} = \sqrt{R_0 / g_0}$), r normalized by the equatorial radius of the Earth R_0, is the radial distance from the Earth center to the vehicle. θ is the longitude and ϕ is the latitude. V is the Earth-relative velocity, normalized by $V = V_{\text{d}} / V_{\text{scale}}$ (where $V_{\text{scale}} = \sqrt{g_0 \cdot R_0}$), γ is flight-path angle of the Earth-relative velocity vector, ψ is the heading angle of the same velocity vector, measured clockwise in the local horizontal plane from the north, Ω is the Earth self-rotation rate. The bank angle σ is the roll angle of the vehicle about the relative velocity vector. The terms L and D are the non-dimensional aerodynamic lift and drag acceleration (in g_0), respectively.

3 Predictor–corrector Method using Gaussian Integration

In predictor–corrector method using Gaussian integration, the angle of attack α is fixed as a function of Mach numbers (the corresponding angle-of-attack profile at different Mach numbers is plotted in Fig.1), and the bank angle σ is fixed as a function of the energylike variable e

$$|\sigma| = \sigma_0 + \frac{e - e_0}{e_{\text{f}} - e_0}(\sigma_{\text{f}} - \sigma_0) \quad (7)$$

where

$$e = \frac{1}{r} - \frac{V^2}{2} \quad (8)$$

In the method, the range is calculated by Gaussian integration, and then the bank angle profile can be obtained by the predictor–corrector method.

3.1 Predictor for range using Gaussian Integration

In the process of reentry, the formula of range calculation under the quasi-equilibrium glide condition ($\dot{\gamma} \approx 0, \gamma \approx 0$), can be transformed into:

$$\frac{\mathrm{d}s}{\mathrm{d}e} = \frac{1}{D} \quad (9)$$

To calculate the range, the drag acceleration corresponding to e needs to be determined. Considering the quasi-equilibrium glide condition:

$$L\cos\sigma + \left(V^2 - \frac{1}{r}\right)\frac{\cos\gamma}{r} = 0 \quad (10)$$

where, the flight altitude in the reentry process is very small compared with the radius of the earth, so $r \approx 1$. And under the quasi-equilibrium glide condition ($\dot{\gamma} \approx 0, \gamma \approx 0$), Eq. (10) can be rewritten as:

$$L\cos\sigma + \left(V^2 - 1\right) = 0 \quad (11)$$

In Eq. (10), the bank angle σ can be obtained by Eq. (7) using the energylike e. And due to $r \approx 1$, the energylike e can be rewritten as:

$$e = 1 - \frac{V^2}{2} \quad (12)$$

The Earth-relative velocity V can also be obtained using the energylike e in Eq. (10). Therefore, under the quasi-equilibrium glide condition, the lift acceleration corresponding to the e can be calculated by:

$$L(e) = \frac{1 - \left[V(e)\right]^2}{\cos(\sigma(e))} \quad (13)$$

In reentry model, aerodynamic coefficient is related to velocity and the angle of attack. The Earth-relative velocity V can be calculated using the e, and the angle of attack can be calculated using the Mach numbers which is related to the Earth-relative velocity V, so the lift and drag aerodynamic coefficients can be calculated by the e. The drag acceleration corresponding to the energylike e can be calculated by:

$$D(e) = L(e) \cdot \frac{C_D(e)}{C_L(e)} \quad (14)$$

The range corresponding to the bank-angle profile can be predicted using Eq. (9) and Eq. (10) using numerical integration. But the cost time of the numerical integration is long. It is difficult to fast calculate the guidance command profile online.

Considering that Gaussian integration that has high integral accuracy is a kind of interpolation integration, it needs to determine the integrand value of Gauss points. In the Eq. (9), the integral independent variable is energylike variable e that can be the variable determined by the velocity and altitude. The integral variable is the reciprocal of the drag acceleration that is the variable determined by Eq. (14). So the range in Eq. (9) can be calculated by Gaussian integration.

In addition to the range requirement, terminal velocity and altitude requirements should also be included in the reentry process. Considering that the collocation points can contain the terminal point in Legendre-Gauss-Lobatto method. In this method, the Legendre polynomial is

$$P_K(\tau) = \frac{1}{2^K K!}\frac{\mathrm{d}^K}{\mathrm{d}\tau^K}\left[\left(\tau^2 - 1\right)^K\right] \quad (15)$$

In Eq. (15), the integral collocation points consist of the roots of $P_K(\tau)$ in Eq. (15) and the terminal e determined by the terminal velocity and altitude requirements. The Gaussian integration can be calculated by

$$\int_{-1}^{1} f(\tau) d\tau \approx \sum_{k=1}^{K+1} A_k f(\tau_k) \quad (16)$$

In Eq. (16), the Gauss integral coefficients are $A_k = \int_{-1}^{+1} L_k(\tau)\mathrm{d}\tau,\ k = 1,2...K+1$, $L_k(\tau)$ is a Lagrange interpolation function of order k.

The integral collocation points of order K and integral coefficients A_k can be calculated offline in Eq. (15) and Eq. (16). In order to calculate the range online, the integral intervals only needs to be linearly transformed to $[-1, +1]$.

$$\begin{aligned} S_{togo} &= R_0 \int_{e_0}^{e_f} \frac{1}{D(e)} de \\ &= R_0 \cdot \frac{e_f - e_0}{2} \cdot \int_{-1}^{+1} \frac{1}{D(\tau)} d\tau \\ &= R_0 \cdot \frac{e_f - e_0}{2} \cdot \sum_{k=1}^{K+1} A_k \frac{1}{D(\tau_k)} \end{aligned} \tag{17}$$

3.2 Correction and Compensation of The Bank Angle

In reentry guidance, the angle attack is fixed as a function of Mach number, and the bank angle command needs to be determined.

To fixed the terminal bank angle σ_{f}, the range $S(e_f)$ can be calculated by Eq. (17), for each initial value of the bank angle σ_0 in Eq. (7).

The terminal range deviation is defined as:

$$z(\sigma_0) = S(e_f) - S_f^* \tag{18}$$

where, S_f^* is the range-to-go of the vehicle.

In the guidance cycle, for each initial value of the bank angle σ_0, the unique corresponding terminal range deviation can be obtained. Then the $z(\sigma_0)$ can be regarded as a function of σ_0. Since the terminal range deviation requirement is 0, the guidance problem is transformed into a root-seeking problem of $z(\sigma_0)$.

In each iteration cycle, at the k th iteration, the $z(\sigma_0^k)$ can be calculated according to σ_0^k. If the condition is not met, the bank angle can be updated by the Gauss–Newton method:

$$\sigma_0^{(k+1)} = \sigma_0^{(k)} - \frac{z(\sigma_0^{(k)})}{z(\sigma_0^{(k)}) - z(\sigma_0^{(k-1)})} \left(\sigma_0^{(k)} - \sigma_0^{(k-1)}\right) \tag{19}$$

When the vehicle has sufficient energy, the $\sigma_0^{(k+1)}$ must be obtained, due to the strong convergence of the Eq. (19). The convergence condition is:

$$\left|\sigma_0^{(k+1)} - \sigma_0^{(k)}\right| \le \varepsilon \tag{20}$$

where a preselected small $\varepsilon > 0$.

Due to the deviation of the actual aerodynamic coefficient and the model aerodynamic coefficient, the bank angle command profile obtained by correcting terminal range deviation cannot make the vehicle meet the equilibrium glide condition. Therefore, the bank angle command needs to be compensated by:

$$L\cos(\sigma_{cx}) = L\cos(\sigma_{base}) - k\dot{h} \tag{21}$$

In Eq. (21), σ_{base} is the bank angle obtained by correcting the terminal range deviation, $\dot{h}$ is the altitude change rate, and k is the compensation coefficient. σ_{cx} is the bank angle command after compensation that is a unsigned positive value.

3.3 Lateral Guidance Method

The compensated bank angle command needs to determine the sign in reentry guidance. This paper reverses the bank angle based on the heading angle deviation in the lateral guidance.

The heading angle deviation corridor is set as a function of velocity:

$$\Delta\psi_{up} = \begin{cases} 18°, & V_{\mathrm{d}} > 5000 \\ (18 - \dfrac{V_d - 5000}{5000 - 3000} \square 8)°, & 3000 < V_{\mathrm{d}} \le 5000 \\ 8°, & V_{\mathrm{d}} \le 3000 \end{cases} \tag{22}$$

When the heading angle deviation exceeds the corridor, the bank angle is reversed. Fig. 1 shows the predictor–corrector method process using Gaussian Integration:

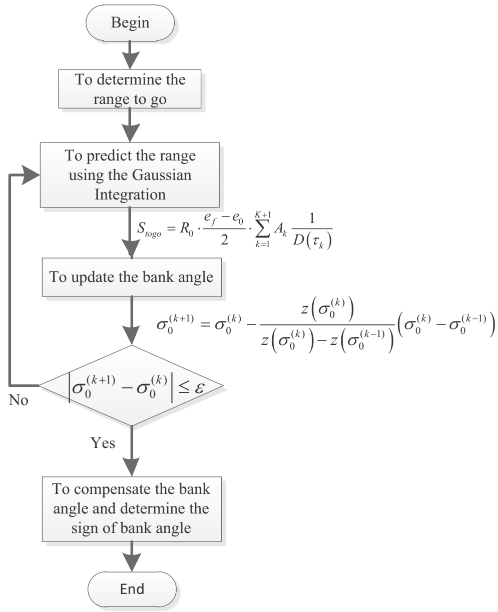

Fig. 1: Predictor–corrector process

4 Simulation Results

For the CAV-H model, a simulation with three degrees of freedom is carried using the proposed method， and the results of numerical predictor–corrector guidance method is used to compare with the predictor–corrector guidance method using Gaussian integration. The initial conditions and terminal requirements (Task1, Task2, and Task3) are shown in Table 1.

Table 1: Simulation Conditions

Parameter Names	Initial conditions	Task1	Task2	Task3
Altitude（km）	80	20	20	20
Longitude（°）	10	85	90	80
Latitude（°）	-20	25	25	25
Velocity（m/s）	7100	1800	1800	1800
The flight path angle	-1	/	/	/

(°)				
The heading angle (°)	61	/	/	/

In Fig. 2, the angle of attack corresponding to the Mach number is shown. The change rate of bank angle is limited in 30°/s, and the compensation coefficient of bank angle is:

$$k_0 = \begin{cases} 20, V_d > 5000 \\ 20 + \dfrac{V_d - 5000}{2000} \cdot 20, \ 3000 \le V_d \le 5000 \\ 0, \quad V_d < 3000 \end{cases} \quad (23)$$

Table 2: Simulation Results Compared with NPCG

Parameter Names	Terminal Requirement	Terminal Results (GPCG)	Terminal Results (NPCG)
Altitude (km)	20	20.661	18.387
Longitude (°)	85	85.0120	85.0211
Latitude (°)	25	24.9813	24.9852
Velocity (m/s)	1800	1795.9	1807.7
The range to go (km)	0	2.406	2.689

Table 3: Simulation Results of Different Tasks

Parameter Names	Task1	Task2	Task3
Altitude (km)	20.661	20.378	20.742
Longitude (°)	85.0120	90.0031	80.0257
Latitude (°)	24.9813	24.9749	24.9933
Velocity (m/s)	1795.9	1796.8	1795.7
The range to go (km)	2.406	2.807	2.697

Table 2 shows the terminal simulation results compared with NPCG: the terminal altitude error is 0.661km, the terminal velocity error is 4.1m/s, and the terminal position error is 2.406km in the proposed method. Compared with the numerical predictor–corrector guidance method, the proposed method has no loss of accuracy. The cost time to complete a predictor–corrector process is less 20ms using proposed method, and the cost time is more than 1s using NPCG in the same hardware environment. The proposed method has a high calculation speed and can be calculated online.

Table 3 shows the terminal simulation results of different tasks. The proposed method can adapt to different tasks.

Fig. 3~Fig. 6 show the altitude, velocity, bank of angle, and flight path angle of the vehicle with respect to time. The red and blue curves in the figures are the simulation results of the GPCG and the NPCG respectively. And Fig. 7 shows the latitude with respect to longitude. The results show that there is no jump in the altitude due to the compensation of bank angle, the bank angle is changed to reduce the lateral deviation, and the vehicle finally reaches target point. Due to approaching the target point, the heading angle need should be considered, so the sign of bank angle changes frequently. And the flight path angle changes dramatically in Fig. 5~Fig. 6.

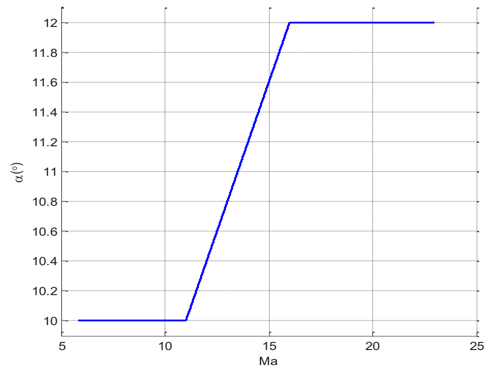

Fig. 2: Angle of attack with respect to Mach

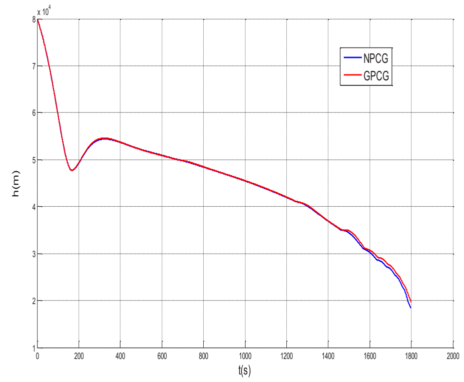

Fig. 3: Vehicle altitude with respect to time

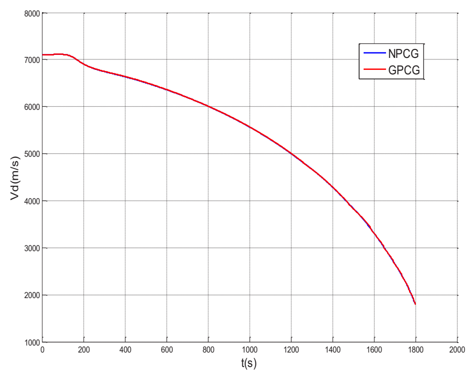

Fig. 4: Vehicle velocity with respect to time

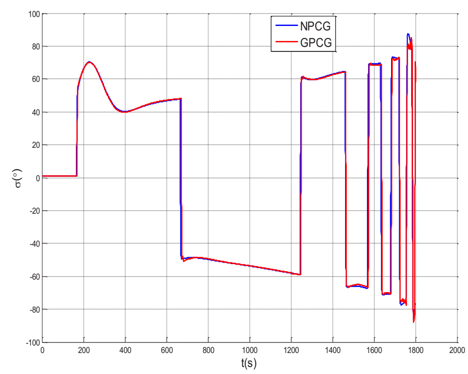

Fig. 5: The bank of angle with respect to time

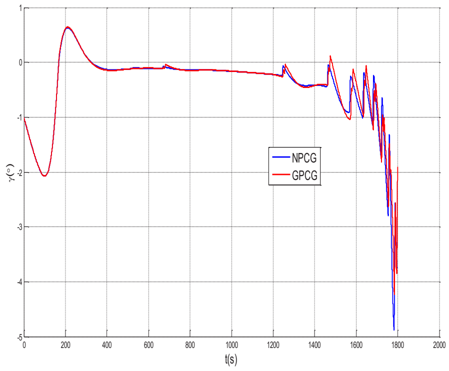

Fig. 6: Vehicle flight path angle with respect to time

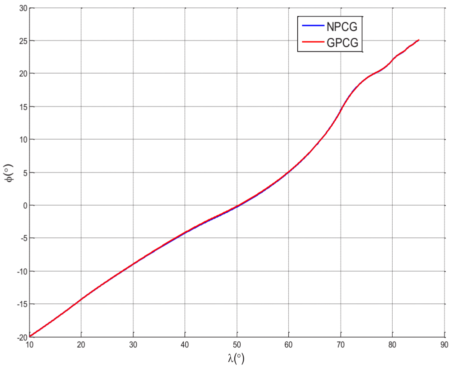

Fig. 7: Latitude with respect to longitude

5 Conclusions

A predictor–corrector entry guidance method using Gaussian integration is used in reentry guidance. The range is predicted by Gaussian integration, and the bank angle is corrected by Gauss–Newton method. The proposed method has high accuracy and fast calculation speed.

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