Full List of Symbols

Paul D. Groves

This document presents the full list of symbols appearing in the book *Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems*, 2nd edition, and its appendices on CD. The symbols are divided into matrices, denoted by upper case bold; vectors, denoted by lower case bold; scalars, denoted by italics; subscripts and superscripts; and qualifiers. Subscripts and superscripts are only listed separately where they are used with more than one parent symbol; otherwise, the compound symbol is listed. Components of vectors and matrices are denoted by the equivalent scalar with subscript indices added. The magnitude of a vector is denoted by the equivalent scalar with no subscript index. Submatrices retain matrix notation, but have subscript indices added.

Matrices

$\mathbf{A}$	smoothing gain
$\mathbf{A}$	generic matrix
$\mathbf{B}$	covariance of the state vector difference
$\mathbf{B}$	generic matrix
$\mathbf{C}$	coordinate transformation matrix
$\mathbf{C}$	covariance matrix (general)
$\mathbf{C}$	generic matrix
$\mathbf{C}_{\delta\mathbf{z}}^{-}$	covariance of measurement innovations
$\mathbf{C}_{\delta\mathbf{z}}^{+}$	covariance of measurement residuals
$\mathbf{D}$	differencing matrix
$\mathbf{D}$	diagonal matrix
$\mathbf{E}$	matrix of eigenvectors
$\mathbf{F}$	system matrix
$\mathbf{G}$	continuous system noise distribution matrix
$\mathbf{G}_g$	gyro g-dependent errors
$\mathbf{H}$	measurement matrix
$\mathbf{H}$	homography matrix
$\mathbf{I}_n$	$n \times n$ identity matrix (diagonal elements = 1, off-diagonal elements = 0)
$\mathbf{J}$	measurement matrix for unestimated parameters
$\mathbf{K}$	Kalman gain
$\mathbf{K}$	Coefficients of the restoring force
$\mathbf{L}$	Coefficients of the damping force
$\mathbf{L}$	lower triangular matrix
$\mathbf{L}$	weighting matrix

$\mathbf{M}$	generic matrix
$\mathbf{M}$	scale-factor and cross-coupling errors
$\mathbf{O}$	observability matrix
$\mathbf{P}$	error covariance matrix
$\mathbf{Q}$	system noise covariance matrix
$\mathbf{Q}'$	approximated system noise covariance matrix
$\mathbf{Q}_U$	system noise covariance matrix for unestimated parameters
$\mathbf{R}$	measurement noise covariance matrix
$\mathbf{R}$	correlation matrix
$\mathbf{S}$	power spectral density matrix
$\mathbf{S}$	square root of error covariance matrix
$\mathbf{S}$	scaling matrix
$\mathbf{T}$	position change transformation matrix
$\mathbf{T}$	generic transformation matrix
$\mathbf{U}$	correlation matrix between states and unestimated parameters
$\mathbf{U}$	upper triangular matrix
$\mathbf{W}$	error covariance matrix for unestimated parameters
$\mathbf{Y}$	information matrix
$\mathbf{Z}$	decorrelation matrix
$\mathbf{\Gamma}$	discrete system noise distribution matrix
$\mathbf{\Pi}$	cofactor matrix
$\mathbf{\Phi}$	transition matrix
$\mathbf{\Phi}_U$	transition matrix for unestimated parameters
$\mathbf{\Psi}$	transition matrix linking states with unestimated parameters
$\mathbf{\Omega}$	skew-symmetric matrix of angular rate

Vectors

$\mathbf{a}$	acceleration
$\mathbf{a}$	generic vector
$\mathbf{b}$	bias errors
$\mathbf{b}$	generic vector
$\mathbf{c}$	vector of calibration coefficients
$\mathbf{c}$	generic vector
$\mathbf{c}_i$	i^{th} row of coordinate transformation matrix
$\mathbf{d}$	generic vector
$\mathbf{e}$	eigenvector and rotation axis unit vector
$\mathbf{f}$	specific force
$\mathbf{f}$	system function
$\mathbf{g}$	acceleration due to gravity
$\mathbf{g}$	generic function
$\mathbf{h}$	measurement function
$\mathbf{h}$	angular momentum
$\mathbf{k}$	Kalman gain for scalar measurement
$\mathbf{k}_n$	Runge-Kutta integration intermediate step result
$\mathbf{l}$	lever arm
$\mathbf{m}$	cross-coupling errors
$\mathbf{m}$	magnetic flux density
$\mathbf{m}$	quantities measured
$\mathbf{n}$	unit normal

$\mathbf{p}$	curvilinear position (geodetic latitude, longitude, and geodetic height)
$\mathbf{p}$	parity vector
$\mathbf{q}$	quaternion attitude
$\mathbf{r}$	Cartesian position
$\mathbf{s}$	scale-factor errors
$\mathbf{s}_{cg}$	satellite clock g-dependent error coefficients
$\mathbf{u}$	unit vector and line-of-sight unit vector
$\mathbf{u}$	control vector
$\mathbf{v}$	velocity
$\mathbf{w}$	vector of white noise sources
$\mathbf{w}_m$	measurement noise vector
$\mathbf{w}_s$	system noise vector
$\mathbf{x}$	generic vector or set of observations
$\mathbf{x}$	state vector
$\mathbf{y}^-$	normalized measurement innovation vector
$\mathbf{y}^+$	normalized measurement residual vector
$\mathbf{z}$	measurement vector
$\boldsymbol{\alpha}$	attitude increment
$\boldsymbol{\gamma}$	acceleration due to the gravitational force
$\Delta\mathbf{r}$	position displacement
$\delta\mathbf{x}$	state vector residual
$\delta\mathbf{z}^-$	measurement innovation
$\delta\mathbf{z}^+$	measurement residual
$\boldsymbol{\eta}$	flexure coefficients
$\boldsymbol{\mu}$	means
$\boldsymbol{\rho}$	rotation vector
$\boldsymbol{\tau}$	torque
$\boldsymbol{\upsilon}$	integrated specific force
$\boldsymbol{\psi}$	Euler attitude {roll, pitch, yaw} (no superscript)
$\boldsymbol{\psi}$	Small-angle attitude (superscript indicates resolving axes)
$\boldsymbol{\omega}$	angular rate

Scalars

A	area
A	constant proportional to signal amplitude
a	length of the semi-major axis
a	generic scalar
A_a	signal amplitude following amplification
a_f	satellite clock calibration coefficient
B	magnetic flux density
B	double-sided noise bandwidth
b	bias error
b	generic scalar
B_{GD}	broadcast group delay
B_{L_CA}	carrier-phase tracking-loop bandwidth
B_{L_CF}	carrier-frequency tracking-loop bandwidth
B_{L_CO}	code tracking-loop bandwidth
B_{PC}	double-sided precorrelation bandwidth
C	spreading code

C	orbital harmonic correction term
c	speed of light in free space or fiber-optic coil
c	calibration coefficient
c	connectivity
c	generic scalar
c	central moment
c_E	early count
c_L	late count
C/N_0	$10\log_{10}$ carrier power to noise density
c/n_0	carrier power to noise density
D	navigation data message
D	dilution of precision
D	code discriminator function
d	spacing of early and late correlation channels in code chips
d	depth
d	normal distance
d	generic scalar
d	day of year
$d_{\min}$	day of year when cycle starts
E	eccentric anomaly
e	eccentricity of the ellipsoid
e	subcarrier offset
e	water vapor pressure
e_o	eccentricity of the orbit
F	carrier-frequency discriminator function
F	focal length
F	cumulative distribution function
f	flattening of the ellipsoid
f	frequency
f	probability density function
f	generic function
f_a	ADC sampling frequency
G	antenna gain
g	generic function
H	orthometric height
H	hypothesis
h	geodetic height
h	scaling factor in measurement matrix
h_i	mean ionosphere height
I	in-phase sample
I	intensity
i	inclination angle
ISC	intersignal correction
J_2	Earth's second gravitational constant
K	loop gain
k	discriminator gain
k_N	renormalization coefficient
k_R, k_θ, k_ψ	sign dependent on error-state or total-state implementation
k_T	atmospheric temperature gradient
L	geodetic latitude

l	number of system-noise-vector components
l	number of filter hypotheses
l	number of matrix rows
M	mean anomaly
M	narrowband to wideband accumulation interval
M	number of elevation measurements
m	number of measurement-vector components
m	number of smoothing iterations
m	quantity measured
m	number of vector components or matrix rows/columns
m	number of transmitters
m	moment
N	geoid height
N	number of turns
N	normalization function
N	number of samples
N	sample from Gaussian distribution
N	integer ambiguity
n	number of state-vector components
n	number of vector components or matrix columns
n	number of observations
n	number of receivers
n	number of steps
n	number of degrees of freedom of chi-square distribution
n_D	number of successive discriminator measurements
n_k	number of measurement-vector hypotheses at iteration k
N_0	carrier phase tracking initialization integer
n_0	noise power spectral density (not root)
P	power
P	carrier-phase discriminator function
P	probability (of an event)
p	first component of angular-rate vector
p	pressure
p	hypothesis probability
p	probability (of taking a value)
p_{fa}	false alarm probability
p_{md}	missed detection probability
Q	quadraphase sample
q	second component of angular-rate vector
R	correlation function
R	gas constant
R	average Earth radius
R	reflection coefficient
R	cross-correlation or auto-correlation
r	(geometric) range
r	third component of angular-rate vector
r	iteration counter in summation
r	root mean square
r_D	radial distance RMS error
R_E	transverse radius of curvature

r_H	radius of radio horizon
R_N	meridian radius of curvature
R_P	polar Earth radius
R_0	equatorial Earth radius
S	subcarrier function
S	power spectral density or cross spectral density
S	signal strength
s	signal amplitude
s	root chi-square test statistic
s	scale factor error
s	geodesic
s	generic scalar
$s_{L\lambda}$	latitude and longitude scaling
T	temperature
T	track width
t	time
t	generic scalar
$T_{b\mu}$	innovation-bias threshold
T_{GD}	group delay correction
t_{oe}	reference time of ephemeris
t_{sa}	time of signal arrival
t_{st}	time of signal transmission
t'_{st}	code phase
T_V	vertical TEC
t_0	arrival time of pulse start
u	corrected argument of latitude
u	angle subtended by the geodesic
W	weighting factor
w	white noise source
w_r	pseudo-range rate tracking error
w_ρ	pseudo-range tracking error
w_Φ	carrier phase tracking error expressed as a range
x	generic process
x	first component of Cartesian position or a generic vector
x	code tracking error in code chips
x	generic argument of probability density
x	generic scalar
x_-, x_+	confidence limits
y	second component of Cartesian position or a generic vector
y	generic scalar
Z	statistical measurement or moment used for c/n_0 estimation
z	third component of Cartesian position or a generic vector
z	generic scalar
Z_{cc}	correlator-comparison measurement
α	relative amplitude of multipath component
α	magnetic declination angle/variation
α	right ascension
α	significance level
$\alpha_{0/1/2/3}$	Klobuchar model coefficients
β	magnitude of the projection of position onto the equatorial plane

β	operating characteristic
β	temperature lapse rate
$\beta_{0/1/2/3}$	Klobuchar model coefficients
Γ	gamma function
γ	magnetic inclination/dip angle
Δ	range lag of multipath component
δ	range lag of multipath component in code chips
δ	Kronecker delta function (equals one when indices match and zero otherwise)
δ	declination
Δf	Doppler frequency shift
Δ_{ij}	scalar product of i^{th} and j^{th} coordinate transformation matrix rows
Δn	mean motion difference from computed value
δN_s	refractive index perturbation
Δr	distance traveled
δr_L	linearization error
δt	time increment
δt_c	clock offset (time)
Δt_{NED}	time difference of transmission/ nominal emission delay
Δt_{rt}	round-trip time
δt_s	time synchronization error
Δt_{TD}	time difference of arrival
Δx	rise time of signal waveform in code chips
$\delta \rho_c$	range error due to clock offset
$\Delta \rho_{dc}$	differential correction
$\delta \rho_e$	range error due to ephemeris data
$\delta \rho_I$	ionosphere propagation error
$\delta \rho_{ie}$	Sagnac correction
$\delta \rho_L$	linearization error
$\delta \rho_M$	range error due to multipath
$\delta \rho_T$	troposphere propagation error
$\delta \dot{\rho}_r$	range rate error due to relativistic time dilation
$\delta \Phi_b$	phase bias, expressed as a range
$\delta \Phi_I$	phase error due to ionosphere propagation , expressed as a range
$\delta \Phi_p$	line-of-sight-dependent phase wind-up error, expressed as a range
θ	pitch or elevation angle
θ	generic angle
θ_{nu}	elevation angle of satellite line-of-sight vector
Λ	likelihood
λ	longitude
λ	eigenvalue
λ	water vapor lapse rate
λ_{ca}	carrier wavelength
λ_0	wavelength
μ	magnitude of angle or rotation
μ	Earth's gravitational constant
μ	mean innovation test statistic
μ	mean
ν	true anomaly
ρ	pseudo-range
ρ	density

ρ	correlation coefficient
σ	standard deviation or error standard deviation
σ_{IQ}	noise standard deviation of accumulated correlator outputs
τ	correlation time
τ	propagation time
τ_a	correlator accumulation interval
τ_i	inertial navigation integration interval
τ_o	odometer measurement interval
τ_P	PDR measurement interval
τ_p	pulse time constant
τ_r	base station response time
τ_s	system propagation time
Φ	geocentric latitude
Φ	argument of latitude
Φ	accumulated delta range
ϕ	roll or bank angle
ϕ	phase
ϕ_0	phase offset
φ	phase lag of multipath component
χ^2	chi-square statistic
ψ	yaw or heading angle
ψ_{bh}	boresight angle
Ψ_E	Earth-centered angle
ψ_{mu}	bearing of line-of-sight vector with respect to magnetic north
ψ_{nu}	azimuth angle of line-of-sight vector
Ω	longitude/ right ascension of the ascending node
ω	angular frequency
ω	argument of perigree

Subscripts and Superscripts

A	denotes local magnetic anomalies
A	denotes accelerometer indicated
A	denotes attitude-matching transfer alignment measurement
a	denotes a vibrating element
a	denotes accelerometer
a	denotes user antenna body coordinate frame
a	denotes user feature-matching sensor coordinate frame (in Section 16.3)
a	denotes at the antenna
a	denotes acceleration-dependent
ASF	denotes additional secondary factor
B	denotes barometric height measurement
b	denotes body or INS body coordinate frame
b	denotes backward filter
b	denotes barometric altimeter
bad	denotes accelerometer dynamic bias
bgd	denotes gyro dynamic bias
C	denotes receiver-generated carrier
C	denotes postcorrelation
C	denotes corrected

C	denotes PRN-code derived
c	denotes from the coil
c	denotes due to or of coning motion
c	denotes cosine term
c	denotes camera or feature-matching sensor body coordinate frame
c	denotes cardinal direction
c	denotes c/n_0-dependent
c	denotes cross-track component of velocity
ca	denotes carrier or carrier phase
cc	denotes correlator comparison
cf	denotes carrier frequency
cf	denotes clock frequency drift
co	denotes code
$c\phi$	denotes clock phase drift
D	denotes down component
D	denotes Doppler measurement
D	denotes database-indicated
d	denotes at the detector
d	denotes dynamic
d	denotes direction
d	denotes Doppler sensor body frame
d	denotes dry
DC	denotes differentially corrected
E	denotes east component
E	denotes early correlation channel
E	denotes Earth's geomagnetic field
e	denotes Earth-centered Earth-fixed coordinate frame
eb	denotes ECEF-frame-resolved velocity and position errors with body-frame-resolved attitude errors
ECD	denotes envelope-to-cycle difference
ew	denotes ECEF-frame-resolved measurement with wander-azimuth-resolved states
F	denotes feature-matching measurement
F	denotes float filter
f	denotes forward filter
f	denotes front-wheel coordinate frame
f	denotes fused solution
f	denotes of the specific force
f	denotes finish-point coordinate frame
f	denotes feature coordinate frame
G	denotes resultant position and time
G	denoted GNSS-derived
G	denotes Gaussian distribution
g	denotes gyro
GL	denotes GLONASS-GPS time offset
$GNSS$	denotes GNSS partition
H	denotes horizontal
h	denotes height
h	denotes hard-iron
HF	denotes high-frequency correlation channel

I	denotes ECI frame synchronized with ECEF at time of signal arrival
I	denotes in-phase
I	denotes INS-derived
I	denotes subionospheric
i	generic index
i	denotes Earth-centered inertial coordinate frame
i	filter bank hypothesis index
i	denotes applicable to the inclination angle
ib	denotes ECI-frame-resolved velocity and position errors with body-frame-resolved attitude errors
IC	denotes ionosphere-corrected
IF	denotes intermediate frequency
INS	denotes INS partition
is	denotes intersignal bias
j	generic index
j	satellite, signal, or tracking channel number
k	iteration index for Kalman filter or tracking loop
k	generic index
L	denotes late correlation channel
L	denotes latitude
L	denotes left (wheel)
L	denotes leveling measurement
L	denotes road link
L	denotes line-fix measurement
l	denotes local tangent plane coordinate frame
l	denotes a particular signal from a given satellite or other transmitter
lag	denotes lag induced tracking error
LF	denotes low-frequency correlation channel
LS	denotes lunisolar
Lλ	denotes latitude and longitude
M	denotes magnetic heading measurement or error states
M	denotes due to multipath
M	denotes geomagnetic
m	denotes Markov process
m	denotes magnetometer-measured flux density and frame thereof
m	denotes magnetometer
m	denotes coordinate frame of nearest point on a line
N	denotes north component
N	denotes narrowband
N	denotes noise channel
N	denotes ambiguity states
n	denotes local navigation coordinate frame
Nav	denotes navigation solution
nb	denotes local-navigation-frame-resolved velocity and position errors with body-frame-resolved attitude errors
nC	denotes local navigation frame with Cartesian position error
ND	denotes normalized code discriminator
NED	denotes nominal emission delay
NF	denotes normalized carrier-frequency discriminator
NΦ	denotes normalized carrier-phase discriminator

O	denotes odometry measurement
o	denotes orbital coordinate frame
o	denotes odometer
P	denotes position
P	denotes prompt correlation channel
P	denotes PDR measurement
P	denotes prefilter
p	denotes precession
p	denotes from the phase modulator
p	denotes planar coordinate frame
p	denotes curvilinear position
p	denotes proximity
PC	denotes phase code
Q	denotes quadraphase
Q	denotes quasi-stationary alignment measurement
q	denotes coordinate frame of intersection point on a line
R	denotes right (wheel)
R	denotes terrestrial radio navigation and measurement thereof
R	denotes feature-matching measurement (in Section 16.3)
R	denotes reference-navigation-system-indicated
R	denotes raw (i.e. uncorrected)
R	denotes on reflection
r	denotes applicable to the orbit radius
r	denotes pseudo-range rate
r	denotes rear-wheel coordinate frame
r	denotes receiver
r	denotes reference receiver or reference antenna body frame
r	denotes random walk process
r	denotes reference body coordinate frame
ra	denotes accelerometer random noise
Ref	denotes reference navigation system
rg	denotes gyro random noise
S	denotes a point on the Earth's ellipsoidal surface
S	denotes subcarrier-derived
s	denotes a point on the Earth's geoid surface or water surface
s	denotes static
s	denotes due to or of sculling motion
s	denotes of the Schuler oscillation
s	denotes subcarrier
s	denotes sine term
s	denotes satellite body coordinate frame
s	denotes soft-iron
s	denotes scattering-surface coordinate frame
s	denotes start-point coordinate frame
s	denotes sensor
Sensor	denotes sensor
si	denotes at the subionospheric point
T	denotes the transpose of a matrix
T	denotes time
T	denotes TRN measurement

t	denotes transmitter or transmit antenna body frame
t	denotes feature or feature body frame (in Section 16.3)
t	denotes terrain
TD	denotes time difference
u	denotes applicable to the argument of latitude
u	denotes user receiver
V	denotes velocity-matching transfer alignment measurement
V	denotes vertical
v	denotes oscillatory/vibratory
v	denotes velocity
VC	denotes velocity constraint measurement
VE	denotes very early correlation channel
VL	denotes very late correlation channel
W	denotes wideband
w	denotes wander-azimuth coordinate frame
w	denotes due to tracking noise
w	denotes wet
x	denotes first component of a vector or axis
y	denotes second component of a vector or axis
Z	denotes zenith
Z	denotes decorrelated ambiguities
z	denotes third component of a vector or axis
ZA	denotes ZARU measurement
ZV	denotes ZVU measurement
α	denotes a generic object frame
α	generic index
β	denotes a generic reference frame
β	generic index
γ	denotes a generic set of resolving axes or frame
γ	generic index
Δ	denotes differenced measurement or subcarrier
δ	denotes a generic coordinate frame
Δo	denotes differential odometer
δx	denotes difference in state estimates
δz	denotes measurement innovation/residual
$\Delta\Delta$	denotes double-delta discriminator
$\delta\psi$	denotes heading error
ε	denotes measurement residual of single-epoch navigation solution
θ	denotes elevation measurement
λ	denotes longitude
ρ	denotes pseudo-range
Σ	denotes a summation
σ	denotes amplitude of seasonal variation
Φ	denotes accumulated delta range
ψ	denotes attitude/ GNSS attitude/ heading/azimuth measurement
0	denotes value at the geoid
0	denotes initialization value
0	denotes a constant value
0	denotes at the reference time
0	denotes samples after carrier correlation and before code correlation

0	denotes average value
$-$	denotes after state propagation and before measurement update
$-$	denotes prediction
$+$	denotes after measurement update
$\perp$	denotes perpendicular
(i)	denotes i^{th} sigma point

See also the list of acronyms and abbreviations

Qualifiers

E()	expectation operator
δ	denotes a small increment or error
Δ	denotes an increment, change, or difference across transmitters
∇	denotes a difference across receivers
$'$	denotes alternative version
$''$	denotes alternative version
$\hat{}$	denotes estimate
$\breve{}$	denotes ambiguity-fixed estimate
$\tilde{}$	denotes a navigation system measurement
$\bar{}$	denotes average value
$(-)$, $-$	denotes at the beginning of the navigation processing cycle
$(+)$, $+$	denotes at the end of the navigation processing cycle