APPENDIX K

Historical Navigation Systems

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There are three reasons for adding a historical appendix to a book on contemporary navigation systems. The first is simply completeness, the second is to put contemporary systems in context, helping to draw parallels between the different technologies, while the final reason is that some of the techniques used in past systems might be applicable in future systems. The focus of this appendix is limited to electronic navigation systems. For older navigation techniques, the reader is directed to [1].

Section K.1 discusses the general evolution of navigation technology over the decades since World War II from aids to a human navigator to systems providing a complete navigation solution. The remaining sections then describe a selection of obsolete, obsolescent, and historical experimental navigation technologies. Section K.2 covers very low frequency

(VLF) navigation systems, including Omega. Section K.3 describes the Decca Navigator System, while Section K.4 covers the legacy versions of Loran. Section K.5 describes the original satellite navigation systems, Transit and Tsikada. Section K.6 then summarizes the main features of a number of other radio navigation systems. Finally, Section K.7 discusses early celestial navigation, dead reckoning, and inertial attitude determination techniques. The histories of inertial navigation and GNSS are described, respectively, in Section E.4 of Appendix E and Section G.2 of Appendix G, both also on CD.

Some of the systems described were still operational at the time of writing, but with a small and declining user base. Some other systems were only ever deployed experimentally.

K.1 From navigation aids to navigation systems

Like any technology, the performance of electronic navigation systems has improved over the decades, while the cost, size, mass, and power consumption have dropped. Improvements have been both evolutionary, retaining backwards compatibility, and revolutionary, requiring new equipment to replace old. However, a key trend has been once of increasing automation.

Navigation of a ship or aircraft was originally performed by a human, known as a navigator. He determined the host vehicle’s position from landmarks, by observing the stars and using dead-reckoning sensors, such as a ship’s log or airspeed indicator, combined with a magnetic compass [1].

Radio signals were first used simply as electronic landmarks. The navigator determined the direction of the transmitter by rotating the receiver’s antenna (either physically or electronically) and identified it by its frequency and/or a Morse call sign or program content. These transmitters were known as *navigation aids* or *aids to navigation* as they helped the human navigator to determine position.

During World War II, a host of new navigation technologies were developed by a range of countries. As they needed to be brought into operation quickly, they were crude and often tailored to particular applications. Use of these systems required listening out for Morse codes or observing traces on oscilloscopes.

Following the war and through the 1950s, these technologies were improved and standardized across countries. A suite of systems: Decca, DME, Loran, and VOR, some of which are still in operation, took over. These provided navigators with range, bearing, or range difference measurements directly. For the hyperbolic systems, users were supplied with maps on which LOPs were printed, enabling them to determine position relatively easily from the measurements output by the user equipment.

The development of transistor-based digital computing through the 1960s enabled the computation of the position solution to be automated. Decca even produced equipment that displayed position on a map. At about the same time, inertial navigation, Omega, and Transit were introduced, providing global electronic navigation for the first time and finally rendering astronomical navigation largely obsolete. The navigators role was reduced to initializing the systems and selecting transmitters. Arguably, this marked the point at which navigation aids became navigation systems.

Through the 1970s and 1980s, the technology remained largely stable, with only evolutionary improvements. However, each navigation technology produced its own position solution and it was left to the human navigator to combine the information and decide which systems were reliable.

By the 1990s (much earlier for guided weapons), the final stage of processing, integration and integrity monitoring of the different navigation sensors had become automated. At the same time, GPS rendered older navigation systems, such as Decca, Omega, and Transit obsolete, while DME, VOR, and Loran-C were demoted to back-ups. By the end of the 20th century, the role of aircraft navigator had become obsolete. Figure K.1 summarizes the processing stages of an integrated navigation system.

In the first decade of the 21st century, the advent of cheap fully-automated navigation systems has massively widened the user base to include land vehicle and personal navigation. However, as GNSS performance in and around buildings is poor, new terrestrial radio navigation technologies are being developed to fill the gap, heralding the era of ubiquitous navigation and positioning.

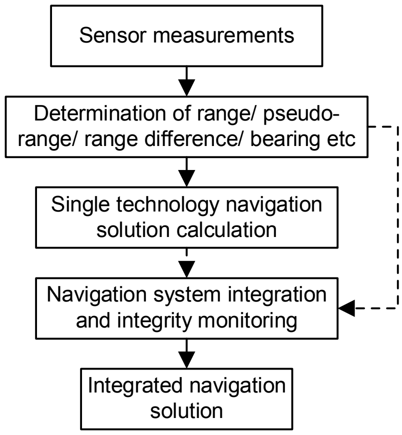

Figure K.1 Integrated navigation system processing stages.

K.2 Very Low Frequency Navigation

The very low frequency region of the radio spectrum ranges from 3 to 30 kHz in frequency and 10 to 100 km in wavelength. The Earth's surface and the lowest region of the ionosphere, known as the D layer, act as the boundaries of a waveguide. This enables VLF signals to propagate all of the way around the Earth. Thus, a VLF navigation or communication system can achieve global coverage with very few transmitters. The skin depth of sea water at 10 kHz is about 3m. Therefore, VLF signals may be received by submarines without the need to surface.

Propagation of VLF signals is complex [2–4]. As with Loran signals (see Section 11.2), the propagation speed over land varies with location and seasonally. Variation in the effective height of the ionosphere's D layer from about 60 km during the day to 90 km at night also introduces a time-varying phase shift, known as the diurnal effect. Further distortions in signal propagation arise due to the Earth's geomagnetic field and during solar storms. Close to a transmitter, interference from higher-order waveguide modes can occur, while at its antipode, signals traveling via different paths around the Earth will interfere with each other. The signal propagation can also be distorted by large structures and power-line harmonics near the receiver. Thus, to get the best position accuracy from VLF signals, those signals should be selected carefully and the various propagation effects accounted for.

Section K.2.1 describes the US-led Omega system, Section K.2.2 describes the Russian RSDN-20/ Alpha system, Section K.2.3 discusses the use of VLF communications signals and Section K.2.4 comprises a brief note on Decca Long-Range Area Cover (DELRAC).

K.2.1 Omega

Omega was the first navigation system to provide continuous world-wide coverage. It was operated by the United States in partnership with six other nations. Its origins lay in the DELRAC (Section K.2.4) and Radux (Section K.6.7) systems proposed in the 1950s. A nascent Omega system, serving North America and the North Atlantic, was implemented in the 1960s. This used four transmitters in New York state, Trinidad, Hawaii and Norway [3]. Global coverage using eight stations was achieved in the early 1970s with additional transmitters in Liberia, La Reunion (Indian Ocean), Argentina and Japan, while the mainland United States transmitter was moved from New York to North Dakota. In 1982, a new transmitter in Australia replaced Trinidad [2, 4, 5].

Using propagation models and databases, Omega was typically accurate to 2–4 km (1σ), though errors could exceed 10 km during ionospheric storms. It was used by aircraft in mid ocean, where more accurate positioning systems were out of range, and by submarines. Some ships also used it, though many favored Transit integrated with dead reckoning. Omega was closed down on 30 September 1997 after most of its user base had switched to GPS.

The original Omega signal format comprised unmodulated continuous-wave signals on three frequencies, 10.2, $11\frac{1}{3}$, and 13.6 kHz, which were transmitted by each of the eight stations in turn. The transmission cycle was divided into eight slots, or segments, of between 0.9 and 1.2s, separated by 0.2s intervals, with each station transmitting on one frequency at a time. The transmission slots were of different lengths to enable user equipment to distinguish between stations. The intervals were provided to allow time for the transmission on that frequency to disperse and for the transmitter antenna circuits to be retuned to the next frequency. All transmitters were synchronized to UTC with an error standard deviation of 2 μs (equivalent to 600m in range).

With only carrier-phase positioning available, the range ambiguity was 29.4 km at 10.2 kHz, 26.5 km at $11\frac{1}{3}$ kHz, and 22.1 km at 13.6 kHz. For basic user equipment using only the 10.2 kHz signals, the position solution ambiguity could be as low as 14.6 km, while for 2 or 3 frequency operation, the minimum position solution ambiguity was 131.4 km. This ambiguity was resolved by initializing at a known position.

The signal format was upgraded in the late 1970s with a fourth common frequency of 11.05 kHz added. This improved the ambiguity by a factor of 4, with a range ambiguity of 1059 km and minimum position ambiguity of 529 km. Furthermore, a set of transmitter-unique frequencies was added for use during the remaining four transmission slots of each station. With each station transmitting in all of the slots, the signal-to-noise performance and dynamics response of the user equipment was improved. Figure K.2 shows the transmission cycle.

	0.9 s	1.0 s	1.1 s	1.2 s	1.1 s	0.9 s	1.2 s	1.0 s
A. Norway	10.2	13.6	11 1/3	12.1	12.1	11.05	12.1	12.1
B. Liberia	12.0	10.2	13.6	11 1/3	12.0	12.0	11.05	12.0
C. Hawaii	11.55	11.55	10.2	13.6	11 1/3	11.55	11.55	11.05
D. N Dakota	11.05	13.1	13.1	10.2	13.6	11 1/3	13.1	13.1
E. La Reunion	12.3	11.05	12.3	12.3	10.2	13.6	11 1/3	12.3
F. Argentina	12.9	12.9	11.05	12.9	12.9	10.2	13.6	11 1/3
G. Australia	11 1/3	13.0	13.0	11.05	13.0	13.0	10.2	13.6
H. Japan	13.6	11 1/3	12.8	12.8	11.05	12.8	12.8	10.2

Above frequencies are in kHz; intervals between slots are each 0.2s.

Figure K.2 Final Omega transmission cycle.

Early designs of user equipment tended to measure relative phase differences between signals from the different transmitters (using an internally-generated signal as a “flywheel”) and used hyperbolic positioning (Section 7.1.4.2) to determine a navigation solution. Later user equipment constructed a pseudo-range measurement for each transmitter from the phase measurements on the various frequencies and then used passive ranging methods (Section 7.1.4.1) to determine a user position and receiver clock solution [4].

The accuracy of Omega could be improved substantially using differential techniques. By the 1990s, 30 differential Omega stations were in operation around Europe, NW Africa, Eastern Canada, the Caribbean, India and Indonesia. Corrections were transmitted using marine radio beacons, operating between 285 and 415 kHz. Beacon ranges were between 400 km and 1000 km and the differential Omega position error varied from about 200m (1σ) 200 km from the beacon to 1 km at maximum range [2, 4].

K.2.2 RSDN-20/ Alpha

The Russian version of Omega was known as RSDN-20 or Alpha [6]. RSDN is the Russian acronym for Normal Long-range Navigation System and was shared with Chayka chains known as RSDN-3, 4 and 5. The original system, developed in the 1960s, used three transmitters at Krasnodar, Novosibirsk, and Komsomolsk, all in Russia; these were collectively known as Sigma. Interestingly, the signal geometry was too poor to obtain a navigation solution within most of Russia, but was excellent over the Arctic, North America, China and Japan.

Three common frequencies were used, 11.905, 12.649, and 14.881 kHz, with the system occasionally using 12.500, 13.281, and 15.625 kHz instead. The range ambiguity using the 3 main frequencies was 403 km. The transmission cycle was 3.6s, divided into six 0.4s transmission slots, interspersed by 0.2s intervals, with each transmitter using different frequencies in different slots. Each frequency and transmitter was silent for a different part of the transmission cycle to allow the stations to be distinguished.

RSDN-20 was upgraded in 1991 with two new transmitters in Revda and Seyda, while the Komsomolsk transmitter was moved to Khaborovsk. This extended coverage to Europe and Russia itself. Revda and Seyda broadcast slots at 12.091 and 12.044 kHz, respectively, as well as using the main frequencies. The Revda and Seyda transmitters closed in 2010, together with the additional frequencies. However, the Krasnodar, Novosibirsk, and Khaborovsk transmitters were still operating on 11.905, 12.649, and 14.881 kHz at the time of writing.

K.2.3 VLF Communications

Many Omega receivers also made use of VLF communications stations operating between 15 and 25 kHz [2]. VLF communications stations do not frequency hop, though the signal modulation can be used to resolve the range ambiguity inherent in carrier phase measurements. However, these stations were not synchronized to UTC, so they were used only for range-rate or delta range measurements. Most VLF communications stations have been closed since Omega was decommissioned in 1997 as aircraft and ships have switched to satellite systems. However, a few remain operational for submarine use at depths down to about 15m. For deep submarines, the US and Russia operate communication systems at the much lower frequencies of 76 and 82 Hz, respectively.

K.2.4 DELRAC

DELRAC was a long-range VLF navigation system proposed by the UK’s Decca Navigator Company in 1954 [7, 8]. DELRAC was never built, being dropped in favor of DECTRA (Section K.3.3). However, its signal format formed the basis for Omega and RSDN-20 and the Decca company successfully sued the US government for patent infringement in the 1970s [9].

K.3 Decca Navigator System

The Decca Navigator System, usually referred to simply as Decca, was a medium to long-range low-frequency (LF) radio navigation system, which operated between 1946 and 2001. Transmitters were grouped into chains of four, comprising the master, red slave, green slave, and purple slave. After 1973, a few chains operated with only three transmitters, usually omitting purple. Each slave transmitter was phase-synchronized to the relevant master, but there was no synchronization between chains. All measurements made by the receiver compared a slave with its master. Hyperbolic positioning (Section 7.1.4.2) was then used to determine a latitude and longitude solution. Most Decca user equipment could only receive one chain at a time. However, multi-chain receivers were produced. Coverage of each chain was up to about 400 km from the master station [2, 7, 10–12].

Altogether, 55 Decca chains were operational over the history of the system. Figure K.3 shows the number of chains operating each year [13]. The first chain, opened in 1946, was the English chain, reflecting the development of Decca within the UK. This was followed by chains in Denmark, Germany, France, and other parts of the UK. In 1957, a series of chains were opened in Eastern Canada, with a New York chain opening the following year. Decca coverage peaked in 1980, by which time all of North and West Europe, Japan and South Africa had been added, together with California, Bangladesh, the United Arab Emirates, Nigeria, and parts of India and Australia.

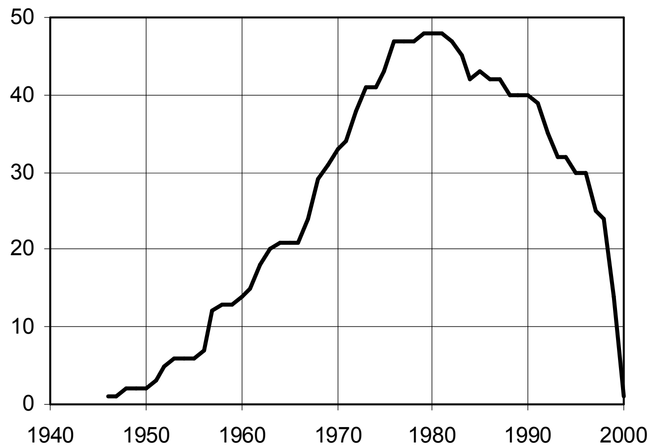

Figure K.3 Decca coverage, in chains, over the history of the system.

The decline of Decca began in the early 1980s when the North American chains were switched off, following the adoption of Loran-C. Further countries followed, but in others, such as Decca's native UK, the system remained popular until the completion of the GPS constellation in the 1990s. A coordinated shutdown of the remaining chains in Europe and South Africa occurred at the end of 1999 and spring 2000. The final chain, in Japan, closed in March 2001.

Decca was used primarily by the commercial shipping and yachting communities in coastal areas, but was also used in helicopters and light aircraft. Section K.3.1 describes the Decca signals, while Section K.3.2 discusses the system accuracy. Section K.3.3 describes Decca Track (DECTRA).

K.3.1 Signals

Decca used five LF frequency bands: 70.087–70.583, 84.105–85.900, 112.140–114.533, 114.943–117.397, and 126.157–128.850 kHz. Unlike Loran, Decca was a FDMA system. Each chain used one frequency within each band, based on a fundamental frequency, f, such that the chain's transmission frequencies were $5f$, $6f$, $8f$, $8.2f$, and $9f$. 63 different sets of frequencies, known as codes, were used.

For most of the Decca duty cycle, each transmitter broadcast an unmodulated carrier on one frequency, with the master and the red, green, and purple slaves using the $6f$, $8f$, $9f$, and $5f$ frequencies, respectively. LOPs were determined by comparing the slave carrier phase with that of the master, noting that the measurement was referred to by the slave color. However, with each transmitter on a different frequency, the ambiguity was at the wavelength corresponding to the lowest common multiple of each pair of frequencies. Consequently, the minimum separation of candidate LOPs, known as the lanewidth, was 351m for purple measurements, 439m for red measurements, and 585m for green measurements. The original Decca design relied on a known starting position for ambiguity resolution, then known as lane identification. However, the system was vulnerable to cycle slips, particularly for air users. Therefore, the signal was modified.

The first modified signal was known as V mode and involved the interruption of the regular Decca signals for three ~half-second intervals each minute, during which the master transmitted on the $5f$ and $6f$ frequencies and one of the slaves on the $8f$ and $9f$ frequencies. This enabled the minimum LOP ambiguity to be increased to 10.525 km. The separation of these LOPs was known as a zone.

V mode was superseded by multipulse, whereby the main signals were interrupted for four 0.75s intervals every 20s to enable each station in turn to transmit on all five frequencies for 0.45s. This multipulse increased the minimum LOP ambiguity to 52.625 km and was also known as zone identification. During the remainder of the duty cycle, transmissions on 8.2*f*, known as the orange frequency, were distributed amongst the four transmitters, with some time slots used for data transmission.

As Decca's main transmission mode occupied a much larger portion of the duty cycle, the resolution was much finer than for the multipulse or V mode transmission. Thus, the two types of Decca signal were often known as fine and coarse.

K.3.2 Accuracy

Decca signals were subject to the same propagation errors as Loran signals as described in Section 11.2.3. However, the continuous wave nature of the Decca signals, compared to the pulsed nature of Loran signals, brought two consequences. The resolution of Decca was better at around 5m. However, the sky wave component of the signal could not be separated from the ground wave in the receiver. Consequently, Decca performance at long ranges and at night was much poorer than that of Loran-C.

The specified Decca position error thus varied from less than 200m during the day within about 100 km of the master station to more than 2.5 km at maximum range during a winter night [2].

K.3.3 DECTRA

DECTRA was a long-range enhancement of Decca, making use of the master and purple slave transmissions on the 5*f* frequency from two distant Decca chains [7, 14]. One chain was phase-synchronized to the other. Accuracy was relatively poor at around 6 km along the track between the two chains and 15 km across the track. Only one DECTRA chain was ever implemented, linking the Newfoundland Decca chain in Canada with the Scottish chain in the UK. It was commissioned in 1957 and operated for about 10 years before being rendered obsolete by civil aviation's adoption of inertial navigation.

K.4 Loran

This section describes the main features of the Loran systems that preceded the current ELoran standard. Lorans A, B, C, and D are described in turn, along with Cytac and Pulse 8, followed by skywave-synchronized (SS) Loran.

K.4.1 Loran-A

The first version of Loran was developed by the US during World War II, with the first full-scale trial in 1942. It was originally known simply as Loran, before becoming standard Loran and then Loran-A as subsequent versions became established. By 1945, Loran-A transmitters at 72 sites were in operation, with only a few added subsequently. Most of the coastal areas of the North Atlantic, North Pacific and Caribbean, extending about 1000 km out to sea, were covered. Land coverage was much more limited. Closure of the Loran-A stations started in the 1970s as Loran-C took over. The last Atlantic transmitters were switched off in 1980, with a few Pacific transmitters surviving until the early 1990s [9, 11].

Loran-A transmitters were grouped in synchronized pairs, rather than chains, though most sites comprised transmitters for two pairs. Transmissions comprised a single 45 μs pulse, with the master station pulse triggering transmission of the slave station pulse [7, 9, 11, 15]. Hyperbolic LOPs were determined by measuring the difference in time of arrival of these two pulses and subtracting the difference in time of transmission. The pulses were short enough to enable the ground and sky wave components of the signal to be distinguished as in later versions of Loran. Most signals were transmitted on 1850 or 1950 kHz, though 1750 and 1900 kHz were also used. The pulse repetition interval varied between 29.3 and 50 ms. The position accuracy of Loran-A was typically 2–3 km, dominated in later years by the accuracy of the charts then used to determine position from the time difference measurements. The timing resolution was about 1 μs, corresponding to 300m of range, for both the transmitter to receiver and master to slave links.

K.4.2 Loran-B

Loran-B operated between 1948 and 1955 as an experimental system only. Based on Loran-A and using the same frequencies, it phase synchronized the pulses to the carrier to enable a much higher timing precision. Loran-B also enabled the transmitters to operate in chains rather than pairs. It was superseded by Loran-C [9, 13].

K.4.3 Loran-C and Cytac

Loran-C was developed in the late 1950s, evolving out of the Cytac system, which replaced Cyclan (Section K.6.2). Cytac operated on 100 kHz with a mixture of phase comparison and pulsing. It was tested in 1955 [9, 11].

The first Loran-C chains became operational on January 1st, 1958. The single pulses of Lorans A and B were replaced with groups of eight or nine, improving the signal-to-noise. However, the largest change was the switch to the low frequency of 100 kHz, increasing the coverage radius by about 50% over sea and more than a factor of ten over land.

Loran-C was developed primarily as a military system, covering the US coast, North Atlantic and North Pacific rims, the Mediterranean, and the Mid-Pacific around Hawaii. In 1974, it was made the primary civil navigation system for US coastal waters and extended to cover the Great Lakes. In 1981, it was certified for civil aviation use in the US and coverage was extended to the whole conterminous US. In the 1980s and early 1990s, chains were also installed by China, France, India and Saudi Arabia [16].

In the 1990s, military use of Loran-C was largely superseded by GPS. Consequently, the US ceased operating Loran-C chains abroad at the end of 1994. This resulted in the closure of the Greenland, Mediterranean, and Mid Pacific chains. However, the chains in, Japan, Korea, and North West Europe were taken over by their host countries, with the latter expanded. From the late 1990s onwards, Loran-C has been undergoing a gradual upgrade process to the new ELoran standard. However, the North American transmissions were terminated in 2010. At the time of writing, the ELoran standard remained backward compatible, enabling legacy Loran-C user equipment to continue operation. However, there is very little legacy equipment in use.

ELoran is described in detail in Section 11.2. Therefore, the discussion here focuses on the differences between Loran-C and ELoran. There are three main differences in the signal format. Firstly, Loran-C has no data link, so the user must have prior knowledge of the locations, GRIs, and NEDs of the transmitters. Changes to the GRIs of a number of European transmitters at the end of the 1990s (to reduce susceptibility to adjacent channel interference) therefore rendered a lot of legacy user equipment unusable.

The second difference is that there was no time synchronization between transmitters in different chains. Instead, chain timing was controlled by a system area monitor (SAM) that controlled the transmitters such that the time differences between signals received from the relevant chain at the SAM were constant. This meant that at least two transmitters from a given chain had to be received in order to obtain any positioning information.

The final difference was that a fault was indicated by a process known as blinking, whereby the first two pulses were omitted for 3.75 of every 4s, instead of ceasing transmission.

Loran-C user equipment operated using hyperbolic positioning (Section 7.1.4.2) instead of passive ranging. In early equipment, time differences were measured manually using an oscilloscope, hence the use of an additional pulse (as well as a different pulse code) to identify the master station. Automatic TD measurement was available from the 1960s, followed in the 1970s by multi-chain receivers. However, positioning was still performed manually. Operators typically used maps and charts with hyperbolic lines of position for certain TDs marked on them, interpolating by eye to determine position. The LOPs accounted for typical ASFs, but it was left to the operator's judgement and experience to account for seasonal variations in those ASFs. Later, printed tables became available. The first user equipment to calculate position was introduced in the late 1970s [16].

Loran-C receivers used the third zero crossing of the signal waveform within each pulse as a timing reference, rather than correlating the whole waveform. This was chosen because the initial part of each pulse is uncontaminated by sky-wave interference from the originating transmitter. However, as Figure 11.7 shows, this point is well below the maximum amplitude of the pulse. Consequently, effective coverage areas with Loran-C receivers were significantly less than with modern Loran receivers.

The accuracy specification for Loran-C was a 95% probability of a radial error within 463 m over water [2, 4, 5, 16].

The Russian Chayka system was very similar to Loran-C and remained fully operational at the time of writing, with plans proposed to upgrade it to an EChayka system similar to ELoran (see Section 11.2).

K.4.4 Loran-D and Pulse-8

Loran-D and Pulse-8 were shorter-range higher-accuracy versions of Loran-C, operating on the same frequency and using much more closely spaced transmitters. Loran-D, a military system, used transportable transmitters, and broadcast 16 pulses per group, instead of 8. The extra pulses were interleaved between the regular pulses and modulated so that a standard Loran-C receiver would ignore them. It was used for NATO military exercises, including those within Europe, in the 1960s and 1970s [9, 13].

Pulse-8 was a privately operated system, run by the Decca Navigator Company. It was used in the 1970s and 1980s for positioning in the North Sea and other offshore areas. It shared the same signal format as Loran-C, but was able to achieve an accuracy of 25m (1σ), largely by using only signals propagated over water so that the propagation speed was well known [13, 17].

K.4.5 Skywave-synchronized Loran

Skywave-synchronized (SS) Loran was a version of Loran-A with pairs of transmitters separated by a few thousand kilometers and the slaves synchronized to the masters using skywave signals instead of the normal ground wave. The system worked only at night, but provided extensive coverage over land as well as sea. Accuracy was similar to standard Loran-A. SS Loran was used towards the end of World War II for bombing over Europe [9].

K.5 Transit and Tsikada

The US Transit and Russian Tsikada (meaning cricket) were the first satellite navigation systems and the first radio navigation technologies with global coverage, albeit not continuous. The two systems were very similar in design. They were used for marine navigation, including by submarines (with an antenna at the sea surface), and for surveying [2, 18].

Both systems used satellites in low polar orbits, such that only one satellite was generally visible at a time. All satellites broadcast on the same frequencies, approximately 150 and 400 MHz, and signals comprised carrier and navigation data, but no ranging codes. Thus, where more than one satellite was visible at a time, they would interfere with each other. For marine navigation, inertial navigation or another dead-reckoning technique was used to determine position between Transit/Tsikada fixes.

Positioning in the horizontal plane was accurate to about 25m on a single pass where the velocity and height were well known and a dual frequency receiver was used. Where these conditions were not met, the system still worked, but performance was degraded. For surveying using multiple passes and differential techniques, an accuracy of around 5m could be achieved.

Transit was also known as the Navy Navigation Satellite System. Development started in 1958 as a US Navy system for submarine use. The first experimental satellite was launched in 1961 and the system became operational at the beginning of 1964. It was opened to civil use from 1967 and decommissioned in 1996 after being superseded by GPS. Between four and seven satellites were operational at a given time. Orbits were circular with a radius of 7,440 km, 1,075 km above the Earth's surface, and a period of 107 minutes. Transit satellites were visible for 10–18 minutes per pass [2]. Navigation data was modulated at ~50 symbol s^{-1} by phase shifting the carrier. Only some of this data was available to civil users.

Tsikada comprised three separate systems, two military and one civil, each using different satellites. The experimental military system was known as Tsiklon (meaning cyclone) with the first satellite launched in 1967. The permanent military system, variously known as Tsikada-M, Tsiklon-B, and Parus (meaning sail), became fully operational in 1974 and continues in use today, with the last satellite launch in April 2010. The Parus system is also used for communications. The constellation originally comprised eight satellites, but now comprises four. The four-satellite civil system, known simply as Tsikada, operated from 1978 to November 2003.

The orbital radius was 7,165 km, 800 km above the Earth's surface, and the period was 105 minutes. Navigation data was modulated at 50 symbol s^{-1} by varying the frequency of an amplitude-modulated tone.

Both Transit and Tsikada used the Doppler positioning method (Section 7.1.7). A position was obtained using signals from a single satellite. This required measurements to be taken over a period of time long enough for the signal geometry to change sufficiently for the different components of position to be separated. The Doppler frequency shift of the signal was measured and then integrated to give a measurement of delta range, the change in pseudo-range over time. Assuming the satellite position and velocity was known from the navigation data message and the user velocity was known from an INS or dead-reckoning system, four delta range measurements across a satellite pass were required to solve for a full position solution (at a given point in time during the pass) and the user–satellite relative clock drift. Figure K.4 illustrates this.

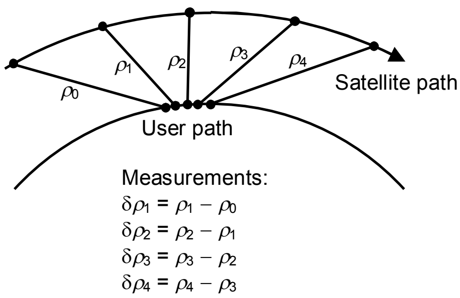

Figure K.4 Single-satellite Doppler positioning geometry.

Transit and Tsikada satellite orbits were polar with the ground tracks inclined at about 4° to a meridian due to the Earth's rotation. Consequently, the dilution of precision for determining latitude over the course of a satellite pass was much better than that for longitude and height. However, the height was often known for marine navigation. Therefore, by constraining this within the navigation solution, the accuracy of the longitude solution could be improved. If the satellite passed directly overhead, longitude could not be determined at all, so these satellites were not generally used.

Best signal-to-noise performance was obtained by tracking the signals continuously during a satellite pass. The measurements could be processed as four delta range measurements, each taken over several minutes, or as multiple delta range measurements, taken over shorter intervals. The latter approach would make position determination more complex, but enable variations in the relative clock drift over the course of a pass to be modeled.

Errors in the user velocity solution used to aid position computation could significantly degrade position accuracy. For Transit, a velocity error of 1 m s^{-1} led to a position error of around 700m [18]. Therefore, effective Doppler positioning with Transit or Tsikada on a moving vehicle required integration with a high-grade INS or other dead-reckoning system, in contrast to the relatively poor sensors often integrated with GNSS today.

There are two Doppler positioning systems currently in use: the Argos satellite tracking system and the DORIS satellite orbit determination system. They are described in Sections F.4.3 and F.4.4 of Appendix F, also on CD.

K.6 Other Radio Navigation Systems

This section briefly summarizes a range of other radio navigation systems, including experimental systems. Consol, Cyclan, Gee, Navaglobe and Navarho, Radio Mailles, Radio-range and Two-course Beacons, Radux, Rebecca-Eureka, Shoran, and Starfix are all described. Systems are listed in alphabetical order.

Other historical systems, not described here, include Condar, Electra, Navar, Post Office Position Indicator (POPI), and Visual Aural Range (VAR). A number of hydrographic surveying systems that operated on similar principles to radio navigation are described in [17]. There were also proposals to combine navigation and radar technology under programs such as Black Maria, Lanac, and Teleran [19].

K.6.1 Consol

Consol was a bearing-indicating beacon system. The range was about 1,500 km. Each station transmitted two directional signals with antenna patterns which varied with time over a 40–60s duty cycle, beginning with an omnidirectional synchronization signal. Both directional signals were on the same frequency and were distinguished by a Morse dash modulation on one signal and a dot on the other. By counting the number of dots or dashes between the synchronization signal and the point at which the two signals were equal, the bearing could be determined to an accuracy of 0.6° with an ambiguity of 7.5° [7, 11, 20].

Consol was developed by Germany as Sonne (meaning sun) and was commissioned in 1940 with two transmitters in Spain and one in Norway. The system was retained after World War II and renamed Consol. Transmitters in France and Northern Ireland were added. Between 1958 and 1960, the USA and USSR opened two transmitters each. The American version was known as Consolan. Consol broadcast between 257 and 363 kHz, while Consolan used 192 and 194 kHz. These signals could be received on a consumer AM radio equipped with a long-wave band, so the system continued to be used by pleasure craft, fishing vessels, and light aircraft after ships and larger aircraft had moved onto more sophisticated systems. Most Consol stations were switched off by 1980 [9].

K.6.2 Cyclan

Cyclan was an experimental phase-comparison hyperbolic navigation system, demonstrated in 1946 by the Sperry company in the USA. It used two frequencies in the LF band, 180 and 200 kHz. Stations transmitted in turn with two frequencies used to aid ambiguity resolution. This, in turn, was replaced by Cytac, which evolved into Loran-C (Section K.4.3) [9, 11].

K.6.3 Gee

Gee was developed in the UK during World War II and continued in operation until 1970. Transmitters were grouped into chains of 3 or 4 transmitters, like Loran-C and Decca, and positioning was hyperbolic. There were four chains in the UK, two in France and one in Germany [9].

Gee used VHF frequencies between 22 and 85 MHz, so coverage was line of sight. Its range varied from about 300 km at sea level to 650 km at high altitude. It was used mainly by aircraft. Signals comprised 6-μs pulses, repeated on a 4-ms cycle. Station A, the master, transmitter every cycle, stations B and C transmitted on alternate cycles, while station D broadcast double pulses. Station A broadcast additional pulses every 4th cycle to enable the 'B' and 'C' cycles to be distinguished. The slave stations transmitted their pulses at a fixed interval after receipt of pulses from the master station. Gee was accurate to a few hundred meters [7].

Gee was cast aside in favor of VOR/DME and TACAN because, prior to the advent of digital computers, it was easier to navigate in terms of range and bearing to a series of waypoints that to determine a hyperbolic position solution.

K.6.4 Navaglobe and Navarho

Navaglobe was a bearing-only system operating in the 70–150 kHz band. Development started in 1946 in the USA with a demonstration in 1954. Each beacon switched between four transmission patterns over a 1s duty cycle: an omnidirectional pattern and three bi-directional patterns, offset from each other by 60° in bearing. Signals were continuous wave. Users determined their bearings from the beacon by comparing the received signal strengths of the four transmission patterns, noting that there was a 180° ambiguity. The range was about 4000 km and the accuracy was 4° from individual measurements and 1° with time averaging [21].

Navarho superseded Navaglobe, combining it with a separately-developed range measurement technique known as Facon. It was first tested in 1956. Range determination was by passive ranging using carrier phase measurements. Ambiguity resolution and synchronization of the receiver clock to the transmitter was performed by initializing the user equipment at a known location. However, to maintain the proposed ranging accuracy of around 10 km over the course of an aircraft flight of up to 12 hours, a receiver clock oscillator accurate to 1 part in 10^9 was required [7, 21]. This is more accurate that the vast majority of today's GNSS receiver oscillators and would have required Navarho receivers to incorporate atomic clocks. Consequently, Navarho never reached operational status.

Hyperbolic versions of Navarho, known as Navarho-H and Navarho-HH were also proposed [22]. These synchronized the transmitters, avoiding the need for a precision receiver clock. Navarho-H combined a two station hyperbolic LOP with a bearing to obtain user position, while Navarho-HH used three stations to provide two hyperbolic LOPs (like Loran A-D and Decca). Neither system was adopted.

K.6.5 Radio Mailles

Radio Mailles (which translates as Radio Mesh or Radio Web) was an experimental navigation technique developed in France in the 1950s [7, 23]. A demonstration system in the Paris area operated on frequencies around 2 MHz. Four synchronized transmitters, each operating simultaneously on a different frequency, were located in the corners of the coverage area. The signals were modulated at 384 or 385 Hz, with the diagonally opposite transmitters carrying the same modulation. At a given location, the phase difference in the modulation received from neighboring transmitters varied over a 1s cycle. The time at which the phase difference was zero, known as an isophase, was measured for each of the four pairs of transmitters. These were then subtracted to produce two time differences, from which two near-perpendicular LOPs were obtained.

The baseline between transmitters for the Paris system was 100 km. With a 1 Hz modulation difference, this could have been increased to 390 km without ambiguity. For larger baselines without ambiguity, a smaller modulation difference could be used. However, this would be at the expense of a longer duty cycle and consequently slower update rate, though multiple modulation frequencies might have resolved this. The coverage area could also be extended with additional transmitters. A transmitter could serve multiple coverage areas, so a double coverage area would require 6 transmitters and a 2×2 quadruple coverage area 9 and so forth.

K.6.6 Radio-range and Two-course Beacons

From the 1920s and 1930s, airways in the USA and some other countries were delineated by MF radio beacons, known as Radio-range or four-course beacons. These transmitted directional signal patterns, such that users along four directions, each approximately 3° wide, would receive a continuous tone on an AM radio, whereas other users would receive either a Morse A or N, depending on which sector they were in. Aircraft flying directly towards or away from a beacon could thus maintain course by keeping within the continuous tone regions, with the signal received elsewhere indicating whether they should steer to the left or right. The range over land was 40–500 km, depending on the beacon design [7].

In the UK, two-course beacons were used. These operated on VHF frequencies in the 30–40 and 100–124 MHz bands. Continuous tones were received in only two directions with a Morse E or T received in the remaining sectors [7]. Both systems were superseded by VOR in the 1950s.

K.6.7 Radux

Radux was an experimental long-range hyperbolic navigation system, developed in the USA in the late 1940s [3, 8]. Ranging was performed using a 200 Hz signal modulated onto LF carriers operating around 40 kHz. The carrier frequency differed between transmitters, while the 200 Hz modulation was synchronized. The accuracy was around 4 km (1σ).

Using carrier-phase positioning to improve precision was considered. However, the modulation could not be measured to sufficient accuracy to resolve the ambiguity on the carrier measurements without long averaging periods (the same problem occurs with GNSS). Therefore, around 1950, it was decided to add an additional 10 kHz continuous wave signal to improve accuracy, with each station transmitting in turn for 1s. The combined system was known as Radux-Omega. As the VLF signals carried further than the 40 kHz signals, the Radux component was dropped later in the 1950s and the Omega component evolved into the system described in Section K.2.1.

K.6.8 Rebecca-Eureka and Shoran

Rebecca-Eureka was a two-way ranging system, developed during World War II by the UK and used in aircraft. Rebecca was the name given to the user equipment, while Eureka was the beacons [7]. It operated in the 190–240 MHz band. Rebecca equipment transmitted about 300 45 μs pulses per second with the beacons responding at the same repetition rate on a different frequency. Eureka beacons provided no bearing information, but Rebecca receivers used twin directional aerials on either side of the aircraft, enabling users to home in on the beacons using direction finding. The range of a beacon was about 400 km at high altitude, while the accuracy was in the 0.5–5 km range, depending on distance. User equipment for the shorter range Blind-approach beacon system (BABS) was often integrated into Rebecca units. The system was replaced by DME in the 1950s, except for Australia, where its use continued into the 1970s.

Short-range navigation (Shoran) was a two-way ranging system for aircraft developed during World War II by the USA. Aircraft user equipment determined position by transmitting pulses to two ground stations and timing the reponses. The system operated in the 200-300 MHz band and had a precision of a few meters. Despite the very high precision for its time, it was superseded by VOR/DME and TACAN for aircraft navigation. However, Shoran and its successor, Hiran, continued to be used for surveying [17, 24].

K.6.9 Starfix

Starfix was a satellite navigation system serving the oil industry in the Gulf of Mexico from 1986 until sometime in the 1990s [2]. It used three geostationary satellites, transmitting GNSS-like signals at around 4 GHz. The chipping rate was 2.4576 Mchip s^{-1}, with the code length matched to the data message rate of 150 symbol s^{-1}. Differential correction information was included in the data message. Positioning was horizontal only with an accuracy of about 5m where the user height was known. Starfix was superseded by the Omnistar DGPS system.

K.7 Celestial Navigation, Dead Reckoning, and Attitude Determination

This section discusses 20th century celestial navigation, dead-reckoning and attitude-determination systems. Traditional celestial navigation, using manual observations, is described first, followed by a discussion of the gimbaled star tracker and stellar-inertial navigation. Modern celestial, or stellar, navigation techniques are described in Section 13.4.1.

The mechanical vertical reference attitude determination system is described next. This used inertial sensors with axes aligned with the local horizontal and vertical. Note that gyrocompasses and directional gyros using mechanical sensors are still available and are described in Section 6.1.2 and Section E.2.3 of Appendix E, respectively. Finally, historical ship's logs are discussed.

K.7.1 Traditional Celestial Navigation

Celestial navigation has been used for latitude determination for millennia and for longitude determination since the 18th century when sufficiently accurate clocks were developed [1, 25]. It remained the primary position fixing method for ships in mid ocean until the 1960s and 1970s when Transit, Tsikada, and Omega became available. It was also used for transoceanic aircraft navigation until the 1970s when inertial navigation and Omega took over. Long-range aircraft used to be equipped with an astrodome to enable the crew to observe the stars.

Latitude and longitude may be determined from measurements of the elevation of two known stars with respect to the local horizontal plane. For best accuracy, the geometry should be optimal. This is achieved by selecting stars with azimuths that differ by around 90°. The normal vector, $\mathbf{n}^e_{eb}$, defined in Section C.3 of Appendix C on CD, is a unit vector defining the direction of the outward normal to the ellipsoid with respect to an ECEF frame. It may be determined from the two elevation measurements using

$$\begin{aligned} \mathbf{n}^{e\,\mathrm{T}}_{eb}\mathbf{C}^e_i\mathbf{u}^i_{i1} &= \cos(\pi/2-\theta_{n1}) = \sin\theta_{n1} \\ \mathbf{n}^{e\,\mathrm{T}}_{eb}\mathbf{C}^e_i\mathbf{u}^i_{i2} &= \cos(\pi/2-\theta_{n2}) = \sin\theta_{n2} \end{aligned}, \tag{K.1}$$

where θ_{n1} and θ_{n2} are the elevation angles of the two stars, denoted by the body frames 1 and 2, and $\mathbf{u}^i_{i1}$ and $\mathbf{u}^i_{i2}$ are the CIRS-inertial-frame line-of-sight vectors, computed from the right ascension and declination angles using (13.30). Note also that, as the normal vector is a unit vector,

$$\mathbf{n}^{e\,\mathrm{T}}_{eb}\mathbf{n}^e_{eb} = 1 \tag{K.2}$$

Solving (K.1) and (K.2) gives [26]

$$\mathbf{n}^e_{eb} = \frac{\mathbf{C}^e_i\left(\mathbf{g}\wedge\mathbf{h}\pm\mathbf{g}\sqrt{\mathbf{g}^\mathrm{T}\mathbf{g}-\mathbf{h}^\mathrm{T}\mathbf{h}}\right)}{\mathbf{g}^\mathrm{T}\mathbf{g}}, \tag{K.3}$$

where

$$\begin{aligned} \mathbf{g} &= \mathbf{u}^i_{i1}\wedge\mathbf{u}^i_{i2} \\ \mathbf{h} &= \sin\theta_{n2}\mathbf{u}^i_{i1} - \sin\theta_{n1}\mathbf{u}^i_{i2} \end{aligned}, \tag{K.4}$$

The latitude and longitude are then obtained using

$$L_b = \arctan\left(\frac{n^e_{eb,3}}{\sqrt{n^{e\,2}_{eb,1}+n^{e\,2}_{eb,2}}}\right) \qquad \lambda_b = \arctan_2\left(n^e_{eb,2}, n^e_{eb,1}\right). \tag{K.5}$$

Historically, the position calculation was formulated differently to facilitate manual computation assisted by an extensive set of look-up tables.

Measurements of the elevation angle were made using a sextant. A marine sextant, shown in Figure K.5, uses a manually rotated mirror to align the image of the star with an image of the horizon, all magnified using a telescope [26]. The angle of the mirror then determines the elevation angle; precise readings may be obtained using verniers or a micrometer drum. Star identification is performed manually by the operator.

For aircraft applications, the horizon cannot easily be used as a horizontal reference as it lies below the horizontal plane by an amount that depends on the aircraft altitude. Instead, bubble sextants were typically used, whereby the star image was aligned with the image of a gas bubble within a liquid (as in a spirit level). Bubble sextants were also used on land where the horizon is usually obstructed by the terrain. A pendulum can also be used to define the vertical.

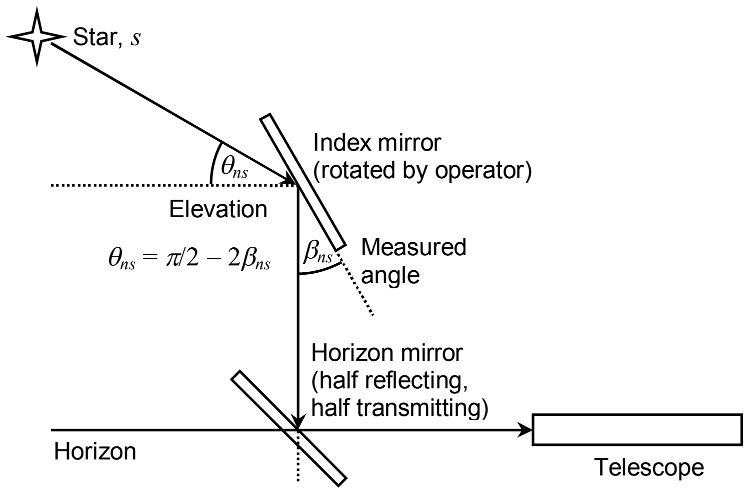

Figure K.5 Marine sextant.

K.7.2 Gimbaled Star Tracker

A gimbaled star tracker comprises a telescope with a light detector mounted on gimbals [27]. It is pointed in the expected direction of a known star and then scanned to maximize the light detector signal. Thus, only one star can be measured at a time, with measurements of three stars typically obtainable over the course of a minute. Rotary encoders measure the orientation of the telescope with respect to the instrument casing. The attitude of the casing must be determined, typically using an INS or AHRS, in order to determine the expected direction of the stars and account for any rotation of the instrument between successive star measurements. A gimbaled star tracker was often combined with a platform INS. It can be aimed over a much wider field of view than a strapdown star tracker so the star database was typically smaller, storing details of only the 50–100 brightest stars. Attitude and position can be determined in the same way as for a strapdown star tracker (see Section 13.3.7) provided corrections are applied for the rotation of the instrument case between measurements.

K.7.3 Stellar-Inertial Navigation

Prior to the advent of GNSS, stellar navigation was the only way of constraining the position error of an aircraft navigation system to less than 1 km at any location. Transit and Tsikada accuracy was severely degraded on high speed vehicles, Omega was not accurate enough, and other radio navigation signals were not available globally. Thus stellar-inertial navigation systems found application in military reconnaissance aircraft and long-range bombers that remained in the air for many hours. Early systems combined a gimbaled star tracker with a platform INS, such that the star tracker azimuth and elevation gimbals were mounted on the same platform as the inertial sensors. Strapdown stellar imagers only replaced gimbaled star trackers comparatively recently.

The star tracker constrained the INS attitude error, preventing the gyro biases from producing unbounded error growth. The effect of the remaining INS errors on horizontal positioning was then constrained by the Schuler feedback [27].

K.7.4 Vertical Reference

An averaging vertical reference was used for aircraft pitch and roll determination prior to the advent of the INS and low-cost AHRS [28]. It operated on a similar principal to the pitch and roll functions of an AHRS, but using gimbaled mechanical inertial sensors instead of today's strapdown, sensors. Pendulous accelerometers and spinning mass gyros are respectively described in Section 4.1.1 and Section E.2 of Appendix E, also on CD.

A spinning mass gyro with vertical spin axis was mounted in a pair of gimbals. When the host vehicle tilted, the gyro spin axis remained vertical and the roll and pitch were obtained from the gimbal readouts.

The spin axis moved relative to the vertical due to instrument errors and the rotation of the horizontal plane with respect to inertial space. This was detected by an orthogonal pair of horizontal accelerometers mounted on the gyro platform. They were used to control torquers that maintained the gyro spin axis in the vertical direction. Accelerometers will sense vehicle acceleration as well as platform tilt, which could cause erroneous platform torquing. This was mitigated by time-smoothing the accelerometer signals, i.e. using a relatively low gain in the torquer control loops. The time constant was determined by how quickly the gyro drifted out of alignment. With poor quality sensors this could be as low as 30 seconds, which can be shorter than the time taken for the aircraft to maneuver. Also, as the accelerometer axes within the horizontal plane were aligned with the aircraft body, the accelerations sensed would not cancel over a maneuver, particularly in the across-track direction. Therefore, an upper limit on the signals from the accelerometers was often applied to limit the effects of vehicle acceleration on platform alignment.

K.7.5 Ship's Speed Log

A ship's speed log measures the speed of a ship with respect to the water. Early designs used an actual wooden log, floating in the water. Speed was measured either by throwing the log off the front of the ship and timing how long it took the back of the ship to reach the log or by spooling out a knotted rope attached to a log in the water and timing the interval between knots. This is how the knot speed unit earned its name.

The first electronic logs were impellers, or turbines. These are now obsolete for large vessels, but are still used on yachts due to their simplicity and low cost. Their operation is described in Section 6.6.3 alongside other contemporary designs.

Pitot-static, or pitometer, speed logs were also widely used in the 20th century, but have now been mostly superseded. They operate on the same principle as pitot-static airspeed measurement (Section 6.6.2). The water speed is proportional to the square of the difference between a forward-facing pitot tube and a side-facing static tube, both hull- mounted. The accuracy is around 0.3 m s^{-1}, assuming calibration for the effects of water flow around the hull [29, 30]. Calibration is necessary because the water in the immediate vicinity of the hull can move relative to the main water mass and depends on the location of the speed log sensor(s) on the hull.

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