Problems and Exercises

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Problems and exercises are grouped according to chapter with the answers listed at the end. The worked examples in Microsoft Excel format may be of use in some cases. In addition, some of the exercises involve using and modifying the MATLAB simulation software that accompanies the book.

1 Introduction

1.1 Classify the following applications into real-time or postprocessed, static or mobile, and self- or remote positioning:

a) Navigation of a car from one city to another.
b) Testing of a gun-launched munition on a firing range.
c) Mapping of a road.
d) Tracking of parts in a car assembly plant.
e) Setting out markers on a construction site.

1.2 Which of the following is *not* a property of dead-reckoning?

a) Initialization at a known position is required.
b) Known landmarks are required.
c) Operation is continuous.
d) The position error grows with time (on average).

1.3 Which of the following sensors may be used to *directly* measure speed?

a) Magnetometer
b) Accelerometer
c) Radar

d) Odometer
e) Sonar

1.4 a) Which position-fixing methods derive LOPs or SOPs from the measurements?
b) What is the minimum number of planar SOPs required to determine a 3D position solution?
c) What is the minimum number of spherical SOPs required to determine a 3D position solution?

1.5) Which of the following signals are better suited to indoor positioning than outdoor positioning?
a) GNSS
b) Infrared
c) Mobile phone signals
d) RFID
e) Sonar

1.6 Which of the following environmental features may be used as position-fixing landmarks?
a) Stars
b) Buildings
c) Cars
d) Terrain height
e) Gravity variations

1.7 a) List three different operating contexts applicable to a car navigation system.
b) What are the two main contexts of submarine navigation?

1.8 a) How might GNSS be used to aid odometry?
b) How might Wi-Fi be used to aid *and* assist GNSS?
c) How might inertial navigation be used to aid TRN?

2 Coordinate Frames, Kinematics, and the Earth

2.1 Consider the ECI, ECEF, local navigation, and local tangent-plane coordinate frames. Which of these
a) Remain fixed with respect to the Earth?
b) Rotates with respect to the Earth due to user motion?
c) Rotates with respect to the Earth independently of user motion?
d) Share a common origin with each other?
e) Shares a common origin with the body frame?

2.2 An aircraft undergoes a rotation of 30° roll, –30° pitch, and 30° yaw. This is followed by a second rotation of –30° roll, 30° pitch, and –30° yaw.
a) Convert each rotation to the corresponding coordinate transformation matrix.
b) Obtain the combined rotation by multiplying the coordinate transformation matrices.
c) Express the result as a set of Euler angles.

2.3 During an aerobatic maneuver, an aircraft reports a bank of 45°, an elevation of 90°, and a heading of 45°.
a) Obtain the body- frame to local-navigation-frame coordinate transformation matrix.
b) Convert this to a set of quaternions.
c) Convert it back to Euler angles and comment on the result.

2.4 The angular rate of a body with respect to inertial space, resolved about east, north, and up (ENU) is $(0.1\ \ 0.2\ \ 0.3)^T$ rad s^{-1}

a) Convert the resolving axes to north, east, and down (NED) by applying the appropriate coordinate transformation matrix.
b) Determine the skew-symmetric matrices of both versions of the angular rate.
c) Obtain the NED-resolved form of the skew-symmetric matrix directly from the ENU version and verify that your result agrees with part b.

2.5 Assuming a spherical Earth of radius 6,400 km,
a) What precision of latitude corresponds to a Cartesian position precision of one meter?
b) What precisions of longitude corresponds to a Cartesian position precision of one meter at the equator and at a latitude of 60°?
Give your answers in both radians and degrees.

2.6 The Cartesian position with respect to and resolved about the axes of an ECEF frame is $\mathbf{r}^e_{eb} = (1548 \quad -4497 \quad 4284)^\mathrm{T}$ km, the velocity is $\mathbf{v}^e_{eb} = (9.46 \quad 3.26 \quad 0)^\mathrm{T}$ m s^{-1}, and the acceleration is $\mathbf{a}^e_{eb} = (-0.43 \quad 1.27 \quad 1.49)^\mathrm{T}$ m s^{-2}.
a) Determine the Cartesian position, velocity, and acceleration with respect to and resolved along the axes of an ECI frame that was aligned with the ECEF frame 6 hours earlier (assume the WGS84 Earth rotation rate).
b) Obtain the geodetic latitude, longitude, and geodetic height assuming the WGS84 ellipsoid. Verify your result by converting it back to Cartesian position.
c) Resolve the Earth-referenced velocity and acceleration about north, east, and down.

2.7 a) Using the Somigliana model, estimate the acceleration due to gravity at the ellipsoid at latitude 51° longitude 0.
b) If the geodetic height is 1km, determine the north and down components of the acceleration due to gravity using (2.139) and (2.140).
c) Compute the centrifugal acceleration resolved along north, east, and down (assuming the WGS84 ellipsoid and Earth rotation rate).
d) Deduce the acceleration due to the gravitational force.

2.8 The left rear wheel of a car is travelling north with respect to the Earth at a constant speed of 10 meters per second. At the same time, the car is turning right at a constant rate of 10 degrees per second.
a) If the car's center of mass is 2m in front of and 0.8m to the right of the left rear wheel, what is its velocity?
b) What is the acceleration of the left rear wheel? (Hint: the speed is constant, but the direction is changing)
c) What is the acceleration of the center of mass?
(Answers should be expressed as north, east, and vertical components. The rotation of these axes with respect to the Earth as the car moves may be neglected.)

3 Kalman Filter-Based Estimation

3.1 Which of the following are assumed in an ideal Kalman filter?
a) The system noise is white.
b) The measurement noise is time correlated.
c) The measurements are linear functions of the states.
d) The errors in the estimates of different states are mutually independent.
e) All states are time varying.
f) All state estimation errors have Gaussian distributions.
g) All measurement errors have Gaussian distributions.

3.2 Which of the following matrices are symmetrical about the diagonal?
a) State estimation covariance, $\mathbf{P}$.
b) Transition matrix, $\mathbf{\Phi}$.
c) System noise covariance, $\mathbf{Q}$.
d) Measurement matrix, $\mathbf{H}$.
e) Measurement noise covariance, $\mathbf{R}$.
f) Kalman gain, $\mathbf{K}$.

3.3 A Kalman filter estimates six states using a local-tangent-plane coordinate system. These are north position, east position, north velocity, east velocity, north acceleration, and east acceleration.
a) Determine the exact transition matrix as a function of the state propagation interval, τ_s.
b) If the propagation interval is 0.5s and the horizontal acceleration is 4 m s^{-2}, what position error arises from using the first-order approximation of the transition matrix as a function of the system matrix?

3.4 Consider Example A with an acceleration PSD of 1 m^2 s^{-3} and propagation intervals of 0.2s, 1s, and 5s.
a) Calculate the exact system noise covariance matrix using (3.47).
b) Calculate the approximation, $\mathbf{Q}'_{k-1}$, neglecting the propagation of the system noise, using (3.48).
c) Calculate the first-order approximation $\frac{1}{2}\mathbf{\Phi}_{k-1}\mathbf{Q}'_{k-1}\mathbf{\Phi}_{k-1}^{\mathrm{T}} + \frac{1}{2}\mathbf{Q}'_{k-1}$.
d) Discuss the appopriateness of the two approximations when the initial position uncertainty is 3m, the velocity uncertainty is 1 m s^{-1}, and the position-velocity covariance is 0.4 m^2 s^{-1}.

3.5 Build a four-state Kalman filter that estimates 2D position and velocity, and inputs 2D position measurements at 1Hz. Supposing:
- The initial position and velocity estimates are zero.
- The initial position and velocity uncertainties are 100m and 10 m s^{-1} per axis, respectively, and all covariances between states are initially zero.
- The acceleration PSD is 5 m^2 s^{-3} per axis.
- The first four position measurements are (–89.9, 210.2), (–96.1, 204.9), (–103.2, 201.3), and (–111.0, 197.9) m.
- The measurement noise uncertainty is 0.5m per axis with a covariance between x and y components of 0.1m^2.

What are the estimated position and velocity and their associated error covariance after 4s?

3.6 Which of the following are possible reasons for modeling a measurement noise covariance larger than the truth in a Kalman filter?
a) Approximations in the system model.
b) Timing errors.
c) The correlation time of the measurement noise exceeding the measurement update interval.
d) Unknown levels of vibration.

3.7 Consider a Kalman filter with 10 states, 10 measurements, and 6 system noise sources. What is the computational load of:
a) The system propagation phase?
b) A vector measurement update?
c) A sequential measurement update (assuming diagonal $\mathbf{R}$)?

3.8 What type of estimation algorithm is most suited to each of the following problems? Consider a basic Kalman filter, EKF, UKF, multiple-hypothesis Kalman filter, and particle filter.

a) Position determination using time-of-flight measurements from nearby radio transmitters. (Hint: Error sources may be assumed to have Gaussian distributions, but the relationship between range and position is nonlinear. Furthermore, the ranging errors are large compared to the distances from the transmitters.)

b) Real-time calibration of INS errors using the position solution of a sonar ranging system. (Hint: The relationship between the INS errors and sonar position may be assumed to be linear and the error distributions assumed to be Gaussian.)

c) Terrain referenced navigation, which determines position by comparing terrain height measurements with a database. (Hint: The relationship between measurement error and position error is irregular and there can be multiple candidate matches between measurements and database.)

4 Inertial Sensors

4.1 IMUs may be classified into aviation, consumer, intermediate, marine, and tactical grades.

a) List these in order of cost and performance, lowest to highest.

b) What grade is an IMU that produces a horizontal position error of 1km after an hour of inertial navigation?

c) What grade is an IMU with accelerometer biases of order 2mg and gyro biases of order $10\ ^\circ\ \mathrm{hr}^{-1}$?

d) Name three applications of marine-grade IMUs.

4.2 Which of these technologies are used for accelerometers and which are used for gyros?

a) Ring laser.

b) Force-feedback pendulum.

c) Vibrating cylinder.

d) Vibrating beam.

e) Hemispherical resonator.

4.3 The displacement of the *x*-axis accelerometer from the IMU reference point is 5cm, 1cm, and 2cm in the *x*, *y*, and *z* directions, respectively. The following are known:

- The *x*-axis accelerometer measures a specific force of 180mg;
- The angular rate is (0.4, 0.6, 0.8) rad s^{-1};
- The angular acceleration is (0.1, 0.2, 0.3) rad s^{-2};
- The *x*-axis accelerometer bias is 160mg.

Estimate the *x*-axis specific force at the IMU reference point in m s^{-2}.

4.4 An IMU is placed on a perfectly flat surface with its *z*-axis pointing downwards. The IMU is stationary with respect to the Earth and the local acceleration due to gravity is 9.80 m s^{-2}.

a) If the IMU is perfect, what is the magnitude of the measured angular rate?

b) If the IMU is perfect, what are the three accelerometer measurements?

c) If the sensitive axis of the *x*-axis accelerometer is tilted upwards by 0.02°, what value does it measure?

4.5 Which of the following errors may be compensated by an IMU's processor?

a) Gyro in-run bias variation.

b) Accelerometer temperature-dependent scale-factor error.

c) Gyro mounting misalignment.

d) Accelerometer size effect.

e) Gyro random noise.

4.6 Which of the following statements are true?

a) Accelerometer biases vary by more than five orders of magnitude over the different grades of inertial sensor.
b) Gyro noise PSD may be expressed in units of degrees squared per hour.
c) RLGs typically exhibit lower scale-factor errors than other types of inertial sensor.
d) The VRE is directly proportional to the amplitude driving the vibration.
e) Accelerometers exhibit angular-rate-dependent errors.

4.7 Consider an IMU that outputs measurements at a rate of 200Hz.

a) If the root-PSD of the accelerometer random noise is 100μg/√Hz, what is the noise standard deviation of the individual specific-force measurements in m s^{-2}?
b) If the root-PSD of the gyro random noise is 0.1°/√hr, what is the noise standard deviation of the individual angular-rate measurements in rad s^{-1}?
c) If the IMU outputs "delta-v"s and "delta-θ"s instead, what are their noise standard deviations?

(You can assume that the noise is white.)

5 Inertial Navigation

5.1 What are the four stages of inertial navigation processing and what constraints on their order apply?

5.2 Consider inertial navigation in two dimensions. Given the following information, determine the position, velocity, and heading after 2s.

- The initial position is (100, 100) m, the initial velocity is (10, –10) m s^{-1}, and the initial heading is –45°.
- Acceleration and angular rate measurements are produced at 5Hz.
- The next 12 acceleration and angular rate measurements are:

Body x-axis accel. (m s^{-2})	0	0	0	0	1	2.5	2.5	1	0	0	0	0
Body y-axis accel. (m s^{-2})	0	–2	–4	–4	–2	0	0	2.2	4.4	4.4	2.2	0
Angular rate (mrad s^{-1})	0	–140	–280	–280	–140	0	0	140	280	280	140	0

5.3 To which implementations of the inertial navigation equations do each of the following statements apply? Consider the ECI-frame, ECEF-frame, local-navigation-frame, and wander-azimuth-frame implementations and note that some statements apply to more than one implementation.

a) Coriolis forces must be accounted for.
b) Singularities occur at the north and south poles.
c) A coordinate transformation/conversion must be applied to the position solution in order to obtain geodetic height.
d) Gravitational acceleration must be calculated (either directly or indirectly).

5.4 a) If the velocity, $\mathbf{v}^e_{eb}$, is (10, 10, 10) m s^{-1}, and remains constant, what position error arises after two minutes if the Coriolis term in the ECEF-frame velocity update equation is neglected?

b) In the local-navigation-frame navigation equations, what attitude and velocity errors arise from neglecting the transport-rate term for 100s when the velocity is 250 m s^{-1} due north? (You may assume that $R_N + h_b$ = 6,400 km.)

5.5 A gyro triad measures an attitude increment, $\boldsymbol{\alpha}^b_{ib}$, of (0.2, 0.3, 0.4) rad.

a) Calculate the first-order, fourth-order, and precise versions of the attitude update matrix, $\mathbf{C}^{b-}_{b+}$.

b) Calculate the first-order and fourth-order attitude errors, expressed as Euler rotations from the true to estimated body frame after the rotation.

5.6 Consider the coordinate transformation matrix:

$$\begin{pmatrix} 0.86 & 0.49 & 0.06 \\ -0.50 & 0.86 & 0.04 \\ -0.03 & -0.06 & 0.99 \end{pmatrix}.$$

a) Show that this matrix is not orthonormal.
b) Re-orthogonalize this matrix.
c) Re-normalize the answer to part b.

5.7 An IMU is subject to angular vibration of (0.1sin(200t), 0.2cos(200t),0)° and a linear vibration of (0, 0, 0.2sin(200t)) cm, both expressed in the body frame. Calculate the magnitudes of the coning and sculling errors for IMU update rates of:
a) 100 Hz.
b) 250 Hz.
c) 1 kHz.

5.8 A stationary IMU produces the measurements $\mathbf{f}_{ib}^{b} = (0.41, \quad 0.52, \quad -9.78)\,\mathrm{m\,s^{-2}}$ and $\boldsymbol{\omega}_{ib}^{b} = (3.22, \quad 5.63, \quad 5.44) \times 10^{-5}\ \mathrm{rad\,s^{-1}}$.
a) Calculate its attitude with respect to north, east, and down, expressed as a set of Euler angles.
b) Estimate the latitude. (Hint: Consider the rotation of the Earth resolved about north, east, and down.)
c) If the accelerometers are accurate to 200 μg and the gyros to 0.02 ° hr^{-1}, determine the uncertainty of the attitude solution. (Hint: Take the root sum of squares of the contributions from each error source.)

5.9 An INS is told that it is level and given a heading solution from a magnetic compass. This results in pitch, roll, and heading initialization errors of 0.2°, –0.1°, and 2°, respectively.
a) What is the position error after 60s if the INS remains stationary? (Assume an acceleration due to gravity of 9.8 m s^{-2}.)
b) How does the position error change if the INS spends the first 10s accelerating forward at 1.5 m s^{-2}?
c) In each case, what accelerometer errors (in mg) would result in the position error being zero at 60s?

5.10 Using the MATLAB inertial navigation simulation software, modifying the demonstration scripts as necessary, investigate the following:
a) How each error source affects each component of the position, velocity, and attitude errors.
b) How the maneuvers in the different motion profiles affect the propagation of the errors.

6 Dead Reckoning, Attitude, and Height Measurement

6.1 A strapdown magnetic compass produces magnetometer readings of (25, 20, 35) μT and accelerometer readings of (0.35, 0.45, –9.78) m s^{-2} when stationary.
a) If the declination of the Earth's magnetic field is 10°, what is the heading with respect to true north?
b) If the magnetic compass is subject to a 1 m s^{-2} acceleration along its x-axis, what is the resulting heading error?

c) If a piece of equipment that produces a magnetic field of (5, 10, 5) μT (in compass axes) is placed next to the compass, what is the resulting heading error?
d) If the assumed declination of the Earth's magnetic field is in error by −1°, what is the resulting heading error?

6.2 Which of the following measure absolute attitude (as opposed to changes in attitude)?
a) Interferometric GNSS.
b) Electrolytic tilt sensor.
c) Differential odometry.
d) Gyrocompass.
e) Strapdown gyroscope.
f) Horizon sensing.

6.3 An AHRS with a fixed-gain smoothing filter attributes ten times the weight to changes in attitude measured by a gyro triad than to absolute attitude measured by magnetometer and accelerometer triads. Given the following information, determine the AHRS attitude solution after three epochs. (Earth rotation and velocity-dependent reference-frame rotation may be neglected).

- The initial attitude solution is $\phi_{nb} = 2°$, $\theta_{nb} = 5°$, $\psi_{nb} = 90°$.
- The following attitude solutions are obtained from the magnetometers and accelerometers (all are Euler angles):
  (1.2°, 4.5°, 90.1°); (0.5°, 4.8°, 92.3°); (−0.3°, 4.7°, 93.9°).
- The following attitude increments are measured by the gyros between epochs of the above measurements (all resolved about body-frame axes):
  (−13.4, −8.3, 0.1) mrad; (−13.3, 4.1, 39.8) mrad; (−13.2, −2.0, 31.2) mrad.

6.4 a) If the surface pressure and temperature are 103.5kPa and 294K, respectively, and the orthometric height is 6 km, what pressure will a barometric altimeter measure?
b) What height measurement is obtained from this pressure using a standard atmospheric model?

6.5 A car begins its journey at the origin of a local-tangent-plane coordinate frame, facing due north. The origin of the car body coordinate frame is the mid point of the rear axle and the rear wheels are 1.6m apart. The following distances are recorded using the wheel speed sensors each second:

Left rear (m)	3.0	7.0	10.0	14.8	14.9	12.0	9.0	8.9	9.0	8.9
Right rear (m)	3.0	7.0	10.0	14.7	14.8	11.9	8.9	8.5	8.6	8.5

a) Assuming the road is flat, calculate the car's position after 10s.
b) What would be the position error if both WSS had a scale-factor error of 1%?
c) What would be the position error if the left WSS had a +1% scale-factor error and the right WSS a −1% scale-factor error?
d) What would be the position error if the road was sloping upward at an angle of 10°?

6.6 Which of the following statements about odometry are true?
a) If all of the scale-factor errors are calibrated to an accuracy of 0.1%, differential odometry will always provide yaw rate to an accuracy of better than 1 ° s^{-1}, equating to a low-grade gyroscope.
b) The relationship between the front-wheel speeds and the vehicle speed depends on the steering angle.
c) Quantization errors affect speed measurement much less than distance measurement.
d) Speed measurement is not affected by road banking.

6.7 Which of the following can affect the motion detected by inertial sensors during walking?
a) Sensor location on the body.
b) Individual walking style.
c) The nature of the terrain (slope, ground texture, obstacles, etc).
d) Turning.
e) Sensor quality.

6.8 A remotely operated underwater vehicle is equipped with a Doppler sonar system operating at 400 kHz. The beam elevations are −60° and the azimuths of beams 1 to 4 are, respectively, 45°, 135°, 225°, and 315°.
a) If the Doppler shifts measured by each beam are 1868, −1675, −1694, and 1847 Hz, determine the velocity of the vehicle, resolved in the axes of its body frame. (Assume the speed of sound is 1,500 m s^{-1}.)
b) What velocity error will arise if beam 1 is misaligned from its nominal direction by +2° in azimuth and −3° in elevation?
c) What velocity error will arise if the real speed of sound is 1,475 m s^{-1}?
d) What is the principal advantage of using four beams instead of three?

6.9 Extend the MATLAB simulation software suite to model a 3-axis magnetic compass and wheel-speed sensors. Examples 6.1 and 6.2 will help you structure the software.

7 Principles of Radio Positioning

7.1 Which of the following are characteristic of self positioning using radio signals and which of remote positioning?
a) The transmitters are at known locations.
b) There is always a limit to the number of mobile users that may be positioned simultaneously.
c) Signals must contain information that enables the transmitter to be identified.
d) Position is calculated at a master processing station.

7.2 The measured time of flight between a transmitter and a receiver is 35μs.
a) If the transmitter clock is running 25μs ahead of system time and the receiver clock is running 40μs behind system time, what is the distance between the transmitter and the receiver? (Assume that $c = 3\times10^8$ m s^{-1}.)
b) If a signal were to be transmitted in the other direction, what would the pseudo-range be?
c) What would the round-trip time of a two-way ranging measurement be if the response time interval is 50μs?
d) If a reference station with a clock running 10μs behind system time is located 21 km away from the transmitter, what is the measured TDOA?

7.3 The positions of two transmitters, expressed using a local-tangent-plane coordinate frame with axes aligned with north, east, and down are $\mathbf{r}^l_{l1} = (100, \quad 200, \quad -10)\,\mathrm{m}$ and $\mathbf{r}^l_{l2} = (-50, \quad 50, \quad -10)\,\mathrm{m}$.
a) If a user measures azimuths of 30° and 240° to transmitters 1 and 2, respectively, what is that user's horizontal position?
b) If an elevation to transmitter 1 of 3.52° is measured, what is the user's vertical position?
c) What is the elevation to transmitter 2?

7.4 The 2D positions of transmitters 1 and 2 are (3.2, 20.6) km and (14.3, 7.2) km.
a) If the measured ranges to transmitters 1 and 2 are 13.9 km and 11.0 km, respectively, what are the two possible user positions?

b) If a range of 10.49 km is measured to a third transmitter at (5.4, –3.6) km, what is the user position?
c) If all three ranges are measured to an accuracy of 10m (1σ), what is the position accuracy?

7.5 Which of the following characteristics apply to LF and MF signals and which apply to VHF and UHF signals?
a) The signal bandwidths are limited to no more than 20 kHz (double-sided).
b) The signals are suitable for both remote positioning and self positioning.
c) The maximum range of the signal depends on the transmit and receive antenna heights.
d) The maximum range of the signal for terrestrial propagation depends on the wavelength, transmission power, and terrain characteristics.
e) Signal reception can be blocked by buildings, which can also reflect the signals.
f) Signal propagation can be affected by the atmosphere.

7.6 An 800-MHz signal is transmitted from an antenna 50m above the ground.
a) Estimate the maximum range of the signal where the receive antenna is close to the ground.
b) By how much must the transmit antenna height be increased to double this range?
c) What height must the receiver's antenna be in order to pick up a signal 15 km beyond the radio horizon?

7.7 Which of the five radio positioning methods can exhibit position errors as a result of the following error sources?
a) Transmitter position error.
b) Transmitter timing error.
c) Time-varying multipath interference.
d) Atmospheric propagation errors.
e) Signal attenuation.

8 GNSS: Fundamentals, Signals, and Satellites

8.1 Which of the GNSS segments undertakes the following functions?
a) Calculates the user position solution.
b) Predicts the satellite orbits.
c) Transmits the navigation data message.
d) Receives signals from other segments.

8.2 Which of the following can contribute significantly to the GNSS position error?
a) Ionospheric refraction.
b) Reflection of signals by buildings.
c) Drift of the receiver clock.
d) Thermal and RF noise.
e) Variations in the transmission power.
f) Errors in the satellite ephemeris data.

8.3 To which GNSS systems do each of the following statements apply?
a) Most signals use frequency division multiple access.
b) Satellites transmit on up to three frequencies, known as L1, L2, and L5.
c) The system was implemented regionally prior to global deployment.
d) Two of the ten signals are reserved for a possible subscription-funded service.
e) The system is undergoing a substantial modernization process.

8.4 Which of the following capabilities are currently offered by one or more SBAS system?
a) GPS integrity alerts.
b) Troposphere propagation delay corrections.
c) Ranging.
d) Galileo satellite clock corrections.

8.5 Features of GNSS signal designs include low or high chipping rates, long or short code lengths, BPSK or BOC modulation, and presence or absence of navigation data. To what design features do the following characteristics apply?
a) It facilitates low-cost receiver design.
b) It minimizes the time and/or processing load for signal acquisition.
c) It minimizes vulnerability to multipath interference.
d) It minimizes interference to and from BPSK signals on the same carrier frequency.
e) It maximizes general interference resistance.

8.6 Let all of the ephemeris parameters listed in Table 8.17 be zero except for the semi-major axis and the inclination angle.
a) Calculate the ECI-frame positions of a GPS satellite, a GLONASS satellite, a Galileo satellite, and a Compass satellite at the reference time and one hour later.
b) Calculate the corresponding ECEF-frame positions.
c) Calculate the velocities with respect to both frames.

8.7 A user at latitude 50°, longitude zero, and height 100m is stationary with respect to the Earth and has a clock offset of 1.7ms and a clock drift of one part in 10^6. A satellite has a position of $\mathbf{r}_{es}^{e} = \begin{bmatrix} 1.535 & 1.535 & 1.535 \end{bmatrix}^{\mathrm{T}} \times 10^7\,\mathrm{m}$, a velocity of $\mathbf{v}_{es}^{e} = \begin{bmatrix} 2300 & -4555 & 2255 \end{bmatrix}^{\mathrm{T}}\,\mathrm{m\,s^{-1}}$, a clock offset of 63μs, and a clock drift of one part in 10^9.
a) Calculate the range, range rate, line-of-sight vector, azimuth, and elevation.
b) If the ionosphere propagation delay is 5m and increasing at 1 cm s^{-1}, the troposphere propagation delay is fixed at 4m, the pseudo-range tracking error due to noise and multipath interference is –3m, and the pseudo-range rate tracking error is – 3 cm s^{-1}, what are the raw pseudo-range and pseudo-range rate measurements?
c) If the navigation data message reports a satellite clock offset of 62μs and the model-predicted ionosphere and troposphere propagation delays are 8m and 3.6m, respectively, what is the corrected pseudo-range?

9 GNSS: User Equipment Processing and Errors

9.1 Which of the following functions are performed by the front-end of a GNSS receiver?
a) Downconversion to the intermediate frequency or baseband.
b) Signal correlation.
c) Digital sampling of the signal.
d) Bandpass filtering.
e) Amplification.
f) Navigation data message demodulation.

9.2 A TCXO in a GNSS receiver has a frequency bias of one part in $2{\times}10^5$.
a) By how much does the receiver clock drift in one second?
b) What is the receiver clock contribution to the pseudo-range rate?
c) How long does it take for the clock to drift by 1ms?
d) If the actual frequency bias is one part in 10^8 higher than the predicted frequency bias, what is the difference between the true and predicted pseudo-ranges after 1s?

9.3 Calculate the expected values of the early, prompt, and late in-phase and quadraphase accumulated correlator outputs within a GNSS receiver given the following information:

- The carrier-power-to-noise-density ratio is 43 dB-Hz.
- The accumulation interval is 10 ms.
- The code tracking error is 0.01 chips, the carrier frequency tracking error is 5 Hz, and the carrier phase tracking error is 0.1 rad.
- The early-late correlator spacing is 0.5 chips and the effects of precorrelation bandlimiting may be neglected.
- The noise standard deviation is 10.
- The navigation data bit is positive.

9.4 Which of the following are required before an unambiguous GNSS pseudo-range measurement can be produced?

a) Acquisition and tracking of the PRN code.
b) Acquisition and tracking of the carrier frequency.
c) Acquisition and tracking of the carrier phase.
d) Navigation data message bit synchronization.
e) Downloading (or otherwise obtaining) the ephemeris data.

9.5 Calculate the number of acquisition cells required for a 1023-chip length PRN code on a 1575.42 MHz carrier frequency under the following conditions.

a) The user equipment and satellite position, velocity, and clock parameters are unkown; the maximum range rate is 1,200 m s^{-1}; the maximum receiver oscillator frequency error is one part in 2×10^5; and the coherent integration interval is 1 ms.
b) The satellite positions and velocities are known; the user equipment position and velocity are known to within 200m and 1 m s^{-1}, respectively, the receiver clock offset to within 1 ms, and the receiver clock drift to within 50 Hz; and the coherent integration interval is 16 ms.

9.6 One second ago, the carrier-smoothed pseudo-range was 23,456,789.00m. If the unsmoothed pseudo-range measurement is 23,455,432,1m, determine the current carrier-smoothed pseudo-range, given that:

a) The delta-range measurement is −1357.63m and the smoothing time constant is 100s.
b) The Doppler shift measurement is 7127.2 Hz, the carrier frequency is 1575.42 MHz, and the smoothing time constant is 60s.

9.7
a) Consider a code tracking loop with a bandwidth of 1 Hz using an early-minus-late power discriminator with a 20ms coherent integration interval and a 0.5-chip early-late correlator spacing. If the code chipping rate is 1 Mchip s^{-1}, estimate the tracking noise standard deviation in meters for $C/N_0 = 43$ dB-Hz and $C/N_0 = 17$ dB-Hz.
b) Consider a third-order carrier phase tracking loop with a bandwidth of 18 Hz using a Costas discriminator with a 20ms coherent integration interval. Estimate the maximum tolerable jerk at a carrier frequency of 1575.42 MHz and the tracking noise standard deviation for $C/N_0 = 43$ dB-Hz and $C/N_0 = 27$ dB-Hz.

9.8 Which of the following can cause significant multipath interference?

a) Reception via a still pond and by the direct path.
b) Reception via a passing bus and by the direct path.
c) Reception only via a glass and steel building.
d) Ionospheric scintillation.
e) Reception through trees.
f) Reception via diffraction by a stone building and reflection by a metal building.

9.9 a) Determine the ECEF-frame Cartesian user position and receiver clock offset from the following measurement data:

Satellite	Pseudo-range measurement (m)	Satellite position x^e_{es} (m)	y^e_{es} (m)	z^e_{es} (m)
1	21495331.60	-60085.77	-21194220.57	15059927.1
2	20125530.93	17684740.50	-4800817.31	18444568.48
3	22928956.20	6729295.17	14404839.62	20572243.00
4	22075058.44	-9110309.47	-13030160.38	20572243.00
5	19841135.00	13000000.00	-12915025.89	18444568.48
6	22508958.72	18324690.50	10649146.09	15059927.10

b) What is the position error if the Sagnac correction is neglected?
c) Calculate the HDOP, VDOP, PDOP, and GDOP.

9.10 GNSS_Demo_2 in MATLAB uses a Kalman filter with system noise tuned for road vehicle motion while GNSS_Demo_3 uses a Kalman filter tuned for a stationary application.
a) What happens if the vehicle tuning is used with the stationary motion profile?
b) What happens if the stationary tuning is used with the vehicle motion profile?

9.11 Extend the MATLAB simulation software to incorporate some or all of the following features:
a) Simulation of a second GNSS constellation with a different orbital radius, inclination angle, and timebase.
b) Multiconstellation positioning algorithms that estimate the interconstellation timing bias.
c) Positioning algorithms based on measuring the TDOA across satellites.

10 GNSS: Advanced Techniques

10.1 Consider a GNSS system using carrier-smoothed pseudo-range measurements from both a user and a reference receiver, giving a noise standard deviation of 0.1m per receiver. The reference station is located where multipath interference is negligible. The spatial decorrelations of the ionosphere and troposphere errors are 0.3m per 100 km and 0.15m per 100 km, respectively, while that of the ephemeris errors is negligible. If the HDOP is 0.8 and the VDOP is 1.2, determine the UERE and the horizontal and vertical position error SDs under the following conditions.
a) The user and reference antennas are 1 km apart and multipath interference at the user is negligible.
b) The user and reference antennas are 1 km apart and the range error SD due to multipath interference at the user is 1.5m.
c) The user and reference antennas are 300 km apart and multipath interference at the user is negligible.
c) The user and reference antennas are 300 km apart, multipath interference at the user is negligible, and a weather front has decorrelated the troposphere error such that the SD of its variation is 1.2m for separations greater than 100 km.

10.2 Consider the local-area, regional-area, and wide-area implementations of DGNSS. To which of them do the following statements apply?
a) Multiple reference stations are used.
b) Troposphere propagation errors are corrected for.
c) Interpolation between different reference stations is performed within the user equipment.
d) Satellite clock, ephemeris, and ionosphere corrections are transmitted separately.
e) Accuracy depends on the distance between the rover and reference.
f) Differential corrections may be transmitted to users via the Internet.

10.3 Estimate the double-differenced integer wavelength ambiguity for the case where the user equipment is initialized at known locations. The initial positions of the user antenna, denoted a, reference antenna r, and satellites, s and t, in meters, are

$$\mathbf{r}_{ia}^{i} = \begin{pmatrix} 4{,}236{,}346.84 \\ -2{,}442{,}899.36 \\ 4{,}081{,}708.55 \end{pmatrix}, \quad \mathbf{r}_{ir}^{i} = \begin{pmatrix} 4{,}237{,}540.78 \\ -2{,}446{,}545.31 \\ 4{,}078{,}306.97 \end{pmatrix}, \quad \mathbf{r}_{is}^{i} = \begin{pmatrix} 21{,}630{,}742.37 \\ -7{,}872{,}946.37 \\ 13{,}290{,}000.00 \end{pmatrix}, \quad \mathbf{r}_{it}^{i} = \begin{pmatrix} 9{,}799{,}722.43 \\ -11{,}678{,}854.41 \\ 21{,}773{,}061.34 \end{pmatrix}.$$

The ADR measurement is 1,646.413m and the GPS L1 frequency is used.

10.4 Consider an ensemble of three GNSS antennas, a, b, and c, mounted on the roof of a vehicle. The position of antennas a and c with respect to antenna b, resolved in the axes of the vehicle body frame, are (1, 0, 0) m and (0, 1, 0) m, respectively. The following baselines have been measured using differential carrier-phase GNSS in the local navigation frame:

$$\tilde{\mathbf{r}}_{ba}^{n} = \begin{pmatrix} 765.6 \\ 642.4 \\ 34.9 \end{pmatrix} \text{mm}, \quad \tilde{\mathbf{r}}_{bc}^{n} = \begin{pmatrix} -643.2 \\ 765.5 \\ 17.4 \end{pmatrix} \text{mm}.$$

Using these measurements, determine the attitude of the vehicle body frame with respect to north, east, and down, expressed as a set of Euler angles. It may be assumed that the integer wavelength ambiguity has already been resolved.

10.5 Which of the following techniques may be used to mitigate interference and which may be used to aid reception of weak signals?

a) A null-steering CRPA system.
b) Extended coherent integration.
c) Spectral filtering.
d) Narrow correlator spacing.
e) Vector tracking.

10.6 Which of the following techniques may be used to mitigate the effects of multipath interference and which may be used to mitigate the effects of NLOS signal reception?

a) Methods using additional correlators.
b) Beam-forming CRPA system.
c) Extended-range tracking.
d) Dual-polarization antenna.
e) Comparing measurements on different frequencies.
f) Comparing signals from different satellites for consistency.

10.7 A GNSS user is located in an urban street which is oriented east-west, has a width of 20m, and has buildings 20m high on both sides. Signals were received from satellites with azimuth 45°, elevation 45° and azimuth 280°, elevation 15°, but were not received from satellites with azimuth 350°, elevation 40° and azimuth 215°, elevation 80°. Given this information, which side of the street is the user on?

11 Long- and Medium-range Radio Navigation

11.1 DME measurements are received from the following transmitters:

Transmitter	Latitude (°)	Longitude (°)	Height (m)	Slant range measurement (m)
1	51	0	30	45,264.3
2	52	-1	40	101,404.8
3	52	1	60	91,348.3

a) Calculate the user latitude and longitude assuming a user height of 5,000m.

b) If the true user height is 4,700m, what are the north and east position errors in meters due to this height error?
c) If the errors in the three slant-range measurements are, respectively, –40m, 70m, and –60m, what are the ensuing north and east position errors?

11.2 Which of the following statements apply to ELoran signal propagation over sea paths compared to land paths
a) The signal waveform is distorted less, enabling more precise ranging measurements to be made.
b) The propagation speed is more predictable, enabling better calibration of the ranging biases.
c) The signal is attenuated less with distance, enabling it to propagate further.
d) There is less interference from sky waves, particularly at night.

11.3 An ELoran transmitter is at latitude 54.92°, longitude –3.28°, while a receiver is at latitude 51.94°, longitude 1.27°.
a) Calculate the geodesic.
b) If the ASF is 0.5% of the distance traveled by the signal, the receiver clock offset is equivalent to a +6,000m range error, and the measurement error due to noise is –10m, what is the raw pseudo-range measurement?

11.4 Consider the following signals of opportunity: AM radio broadcasts, digital television, Iridium satellite communications, and mobile phone signals.
a) Arrange them in order of coverage radius, largest to smallest.
b) Which of these exhibits signal propagation closest to that of marine radio beacons?
c) Which of these may be used for Doppler positioning?
d) Which of these may be used to aid GNSS signal acquisition?

12 Short-range Positioning

12.1 Which positioning method (proximity, ranging, angular, pattern matching, or Doppler) is most suitable for obtaining a position accurate to within 5m from the following types of signal?
a) Sonar.
b) Passive RFID.
c) ISM-band pseudolite.
d) Visible light communications.
e) WLAN.
f) Standard Bluetooth.

12.2 Consider a temporary UWB positioning system deployed to locate firefighters within a building. Base stations are located outside at coordinates (0, 0, 1), (9, 1, 1.5), (–1, 10, 1.3), and (3, 1, 3) meters in a local-tangent-plane coordinate system with arbitrarily aligned horizontal axes and the *z*-axis pointing upward.
a) If the signal bandwidth is 0.6 GHz, what is the resolution with which multipath components may be separated?
b) If signal propagation through walls introduces an uncertainty of 0.75m and signals may be timed to a precision of 1 ns, what is the ranging error standard deviation?
c) Assuming passive ranging, what is the position uncertainty in each direction if the mobile user is located at (13, 12, 3) m?
d) What are the position uncertainties if two-way ranging is used and the range measurement error is the same?
e) How is the positioning accuracy affected if the height of the fourth base station is increased from 3m to 5m?

12.3 Which of the following apply to WLAN, WPAN, and RFID?
a) An approximate position solution may be determined using the existing infrastructure in many places.
b) It can operate using base stations without their own power supplies.
c) The 2.4-GHz ISM band is most commonly used.
d) Ranges of tens of meters can be achieved using the appropriate version of the technology.

12.4 To which sonar positioning techniques do the following apply?
a) The position solution is more accurate when the mobile user is closer to their support vessel.
b) Angular positioning is used in addition to ranging.
c) At least three transponders at known locations are required to determine position.
d) Vehicle motion during the time it takes to measure the range must be accounted for to obtain the best positioning accuracy.

13 Environmental Feature Matching

13.1 A car has just passed a fork in the road and could be on two possible road links Using a local-tangent-plane frame with axes aligned with north and east, road link 1 starts at (0, 0) and finishes at (20, 40) meters, while road link 2 also starts at (0, 0), but finishes at (14, 40) meters. It may be assumed that the road link position is accurate to within 2 meters per axis and the heading of the road links to within 3°.
a) If the user position solution is (10, 20) m with an uncertainty of 6m per axis, which road link is most likely to be correct?
b) If, in addition, the user heading is known to be $67.5 \pm 2.5°$, which road link is more likely to be correct?
c) If the position uncertainty is 3m per axis and the heading uncertainty 5°, which is the more likely road link?

13.2 Which of the following are characteristic of road, rail, and pedestrian map matching?
a) The user is always on a link of a line-based map.
b) The user is usually on a link of a line-based map, but can sometimes go off the map.
c) The user is constrained by obstacles, but can otherwise go anywhere.
d) Turns through approximately 90° are common.

13.3 What types of sensor are suited to the following potential applications of terrain referenced navigation?
a) An aircraft flying 1,000m above the terrain.
b) Hydrographic surveying on a river estuary.
c) Road navigation.
d) Aircraft precision approach and landing.

13.4 A camera facing due north with a focal length of 10 cm, observes objects in its image plane at coordinates of (–0.04, 0.013) and (0.017, 0.015) meters.
a) Determine the azimuth and elevation of each object.
b) If the (north, east, down) local-tangent-plane coordinates of the two objects are (1,500, –500, –10) m and (900, 600, –57.8) m, what is the user position?
c) If the second object is incorrectly identified by the feature comparison algorithm and assumed to be an object located at 500m north, 750m east, what is the ensuing position error?

13.5 Which of the following statements apply to feature matching using line features and which apply to small-feature descriptors?
a) Features can be matched independently of the sensor viewpoint.
b) Features may be described using the coordinates of just two points.

c) Repetition of similar features at different locations can hinder matching.
d) A position solution may sometimes be determined from a single feature.

13.6 Consider the following feature-matching techniques: gravity gradiometry, magnetic anomaly matching, map matching, radar image matching, stellar navigation. Which of these techniques is most suited to each of the following?
a) Submarine.
b) Indoor pedestrian.
c) High-altitude aircraft.
d) Train.
e) Mid-ocean ship.

14 INS/GNSS Integration

14.1 Which of the following statements apply to loosely coupled INS/GNSS integration and which apply to tightly coupled integration?
a) The integration algorithm must calibrate the GNSS receiver clock.
b) A stand-alone GNSS position solution is required.
c) State estimates may be applied as corrections within the INS to constrain the error growth.
d) GNSS integration is performed in the range domain using pseudo-range and pseudo-range rate measurements.
e) It is compatible with all types of GNSS user equipment.
f) The GNSS accumulated correlator outputs are input to the integration algorithm.

14.2 An aircraft is flying due east at 200 m s^{-1} as measured by the INS. It is level, but pitching up at an angular rate of 0.5 rad s^{-1}.
a) If the GNSS antenna is 10m forward of the INS and 5m above it, what is its velocity?
b) If the aircraft position is L_b = 29.9791°, λ_b = 31.1342°, h_b = 2,000m, what is the antenna velocity resolved about ECEF-frame axes?
c) A satellite has a position of $\mathbf{r}_{es}^{e} = \begin{bmatrix}1.535 & 1.535 & 1.535\end{bmatrix}^{\mathrm{T}} \times 10^{7}\,\mathrm{m}$ and a velocity of $\mathbf{v}_{es}^{e} = \begin{bmatrix}2300 & -4555 & 2255\end{bmatrix}^{\mathrm{T}}\,\mathrm{m\,s^{-1}}$. The range-rate errors due to the receiver and satellite clock errors are –330 m s^{-1} and 3 m s^{-1}, respectively. Predict the pseudo-range rate.
d) If the predicted pseudo-range rate is used to control the carrier NCO, how accurate must it be to maintain signal coherence over a 100-ms accumulation interval (assuming a data-free GNSS signal is used)?

14.3 Which of the following affect GNSS signal tracking in a deeply coupled INS/GNSS?
a) IMU quality.
b) Oscillator quality.
c) Prefilter design (including coherent versus noncoherent).
d) C/N_0 or signal amplitude measurement accuracy.

14.4 List the following INS error sources in decreasing order of observability:
- Gyro bias about the forward (x) axis;
- Heading error;
- Pitch error;
- Transverse (y-axis) accelerometer bias;
- Velocity error;
- Vertical (z) accelerometer bias;
- Vertical (z-axis) gyro bias.

14.5 Which of the following should be modeled as system noise in tightly coupled INS/GNSS integration?
a) Acceleration.
b) Receiver oscillator random walk.
c) Gyro bias variation.
d) GNSS signal tracking noise.
e) Accelerometer random walk.
f) Variation in ionosphere propagation delay.

14.6 Consider missile guidance, mobile mapping, and pedestrian navigation. Which application is most suited to each of the following advanced INS/GNSS integration techniques and why?
a) Carrier-phase positioning and GNSS attitude determination combined with an intermediate-grade IMU.
b) A tactical-grade IMU integrated with GPS using noncoherent deeply coupled integration.
c) A MEMS IMU integrated with GNSS using an adaptive unscented Kalman filter.

14.7 Using the MATLAB INS/GNSS simulation software, investigate the effect of varying the Kalman filter tuning on the performance of an INS/GNSS system. Consider the following:
- Whether the estimation errors are consistent with the state uncertainties (see out_KF_SD, out_IMU_bias_est, and out_clock in the MATLAB workspace);
- How the IMU and oscillator specifications impact on the tuning;
- The effects of different host vehicle maneuvers;
- The effect of varying GNSS error sources to model different reception environments.

14.8 Extend the MATLAB INS/GNSS simulation software to incorporate some of the following features:
a) Scale-factor and cross-coupling error estimation.
b) Elevation-dependent measurement noise covariance.
c) Unscented Kalman filter.
d) Adaptive Kalman filter.

15 INS Alignment, Zero Updates, and Motion Constraints

15.1 Which of the following statements apply to transfer alignment, but not to INS/GNSS integration?
a) Heading changes enhance the observability of the attitude and accelerometer errors.
b) Lever arm flexure is the main contributor to the measurement noise.
c) The duration of the process is limited.
d) Attitude measurements may be added.

15.2 Which of the following INS calibration algorithms may be applied underwater to an ROV launched from a ship?
a) Transfer alignment.
b) Quasi-stationary alignment.
c) Sideways motion constraint.
d) Zero angular-rate update.
e) INS/GNSS integration.

15.3 Which of the following statements apply to pedestrians and which apply to land vehicles?
a) Does not turn when stationary.
b) Can be assumed to be stationary when the acceleration is below a certain threshold.
c) Can be assumed to be stationary when the velocity is below a certain threshold.
d) The height above the ground is approximately constant.
e) Can move sideways.

15.4 Extend the MATLAB simulation software to incorporate some of the following features:
a) A partial IMU with dummy outputs in place of some of the sensors.
b) The addition of land vehicle motion constraint measurements to a version of the INS/GNSS integration Kalman filter.

16 Multisensor Integrated Navigation

16.1 An integration architecture may be single-epoch or filtered, cascaded or centralized, total-state or error-state, and may also be federated, hybrid, or modal. To which architectures do the following statements apply?
a) One navigation system produces a reference navigation solution that is corrected using the measurements from the other systems.
b) Estimates of the systematic errors of the various sensors and/or signals are maintained and used to correct future measurements.
c) Information from different navigation and positioning technologies is combined in the position domain.
d) A reference navigation system is separately integrated with each of the other systems.
e) The integration architecture varies according to the environment, user behavior, or signal availability.

16.2 Which of the following combinations of sensors may be integrated in an error-state architecture and which may only be integrated in a total-state architecture. In the cases where error-state integration is possible, which should be the reference system?
a) Inertial navigation and GNSS.
b) GNSS, ELoran, and WLAN.
c) Odometer, magnetic compass, yaw-axis gyro, and GNSS.
d) LBL sonar ranging, Doppler sonar, and inertial navigation.
e) Image-based feature recognition, RFID, and Wi-Fi.
f) Orbital force modeling, limited-availability GNSS, and stellar attitude determination.

16.3 Consider the integration of two fictional 2D navigation systems. Alpha is a very-short-range proximity positioning system with small gaps in its coverage; position errors may be treated as random. Beta continuously measures the direction of travel and exhibits a slowly varying bias.
a) Explain why a total-state integration architecture must be used.
b) Explain why a filtered integration architecture is required.
c) Which states should be estimated by the integration filter?
d) What additional information must Beta measure if it is to be used as the reference within an error-state integration architecture?

16.4 Which states should be estimated in order to integrate the following combination of measurements? Assume a filtered integration architecture and consider whether the positioning is 2D or 3D.
a) Speed from WSSs, yaw rate from a single gyro, and 3D position from GNSS.
b) Position, velocity, and attitude from a tactical-grade INS and position from TRN and the radalt height measurement.
c) Pseudo-ranges from GNSS and mobile-phone base stations, which are equipped with high-quality oscillators, but are not synchronized to UTC.
d) Position, velocity, and attitude from an aviation-grade INS, height from a barometric altimeter, and ranges from multiple DME transponders.

16.5 Which of the following positioning systems could benefit from the use of a multiple-hypothesis Kalman filter or a particle filter to integrate it with other sensors and, where so, why?
a) ELoran.
b) Pattern matching using WLAN and RFID signal strength.

c) WSSs and yaw-axis gyro.
d) UWB
e) Aerial image-based line-feature matching.

16.6 Extend the MATLAB simulation software to incorporate some of the following features:
a) Integration of odometry, magnetic compass, and GNSS.
b) Integration of odometry, yaw-axis gyro, and GNSS.
c) Addition of odometry and magnetic compass measurements and states to the INS/GNSS integration algorithms.

17 Fault Detection, Integrity Monitoring, and Testing

17.1 Is each of the following cases an example of fault detection only, FDR, FDI, or FDE?
a) A chi-square test statistic based on the least-squares residuals of an eight-satellite single-epoch GPS position solution exceeds the detection threshold. However, the corresponding test statistic for one of the seven-satellite subsets is within the threshold.
b) One of the accelerometers in a standard six-sensor IMU is producing measurements outside the specified range of the sensor.
c) An examination of a Kalman filter's innovation sequence identifies a bias in one of the measurement streams. To compensate, a covariance reset is performed and further measurements from that stream are rejected.
d) A GPS pseudo-range measurement is rejected because it has undergone a sudden change between epochs. A single-epoch solution is computed using signals from the four remaining satellites.

17.2 Which of the following might result in a range-check failure?
a) Three identical measurements in a row.
b) A ship position solution changing by 1 km in one second.
c) A set of GNSS measurements that is mutually inconsistent.
d) A car-mounted IMU measuring an instantaneous specific force of 60 m s^{-2}.
e) A foot-mounted IMU measuring an instantaneous specific force of 60 m s^{-2}.
f) An estimated accelerometer bias of six times the manufacturer's specification.
g) An estimated gyro bias of three times the manufacturer's specification.

17.3 A GNSS pseudo-range measurement is 23,457,836.23m. A measurement noise standard deviation of 3m is assumed. The EKF-predicted range is 21,392,465.17 ± 2.34m and the estimated receiver clock offset is 2,065,363.85 ± 1.63m.
a) Calculate the measurement innovation.
b) Determine the normalized measurement innovation.
c) Should this measurement be accepted by the EKF?

17.4 The last-known user position is $\mathbf{r}_{is}^{i} = \left[2{,}642{,}093 \quad -2{,}225{,}401 \quad 5{,}343{,}941\right]^{\mathrm{T}}$ m and the following GNSS measurements are available:

Satellite	Pseudo-range measurement (m)	Satellite position x_{is}^{i} (m)	y_{is}^{i} (m)	z_{is}^{i} (m)
1	22420685.25	14502482.77	-21194220.57	4059927.09
2	27740633.53	-24966153.63	-4800817.31	5444568.48
3	22928969.66	6729295.17	14404839.62	20572243.00
4	22075114.59	-9110309.47	-13030160.38	20572243.00
5	19841131.65	13000000.00	-12915025.89	18444568.48
6	23138185.74	20982700.02	10649146.09	11059927.10

Determine if any of these measurements are faulty and, if so, which one.

17.5 Consider a navigation system incorporating a certified integrity monitoring system. How would the following changes affect the position solution availability?
a) Reducing the permitted probability that the position error falls outside the alert limits.
b) Increasing the vertical alert limit.
c) Increasing the time interval over which continuity is required.
d) Increasing the number of signals used to determine the position solution.

17.6 Navigation systems may be tested using field trials, recorded data testing, laboratory testing, software simulation, or a combination of these. What is the most suitable testing method to determine each of the following?
a) How a GNSS receiver responds to jamming.
b) The optimum tuning of an EKF.
c) How an IMU responds to a high-vibration environment in a guided weapon.
d) Whether a RAIM algorithm is working correctly.
e) Whether an INS/GNSS integration algorithm is suited to a motorsport application and, if not, what changes should be made.
f) Whether a new product meets a set of requirements specified by the customer.

17.7 Extend the MATLAB simulation software to incorporate some of the following features:
a) Measurement innovation filtering in the Kalman filters (and EKFs).
b) RAIM.

18 Applications

18.1 Classify the following requirements into performance, environmental constraints, engineering constraints, and economic constraints.
a) Service or product.
b) Maximum acceleration.
c) Continuity.
d) Stealth.
e) Vibration level.
f) Size.
g) Availability.
h) Delivery schedule.

18.2 What are the key GNSS capability gaps that lead to the use of multisensor navigation systems for each of the following applications?
a) Rail navigation.
b) Military aviation.
c) Pedestrian navigation.
d) Civil aviation.

18.3 Arrange the following applications in order of accuracy requirement, most accurate first.
- General pedestrian navigation.
- Oceanic aircraft navigation.
- Road navigation in rural areas.
- Ship navigation in ports.
- Structural deformation monitoring.

18.4 Consider DME, Doppler radar, ELoran, foot-mounted inertial navigation, GNSS, gravity gradiometry, map matching, odometry, precision inertial navigation, UWB, and WLAN. Which of these technologies are suited to each of the following navigation applications?
a) Firefighters inside buildings.
b) A long-range submarine.
c) A ship about 100 km from the coast.

d) A helicopter over land.
e) A private plane over land.

18.5 Which of the following statements are true?
a) GPS augmented by SBAS is certified for category I landing of aircraft.
b) Short-range missiles can use inertial navigation alone to meet their positioning requirements.
c) Only MEMS-based inertial sensors are suitable for UAVs.
d) Road-user charging requires a positioning accuracy of 10m.
e) For moving-block rail signaling, the continuity and integrity of the train position solution are more important than the accuracy.
f) The accuracy requirement for ship navigation in ports is 2.5m.
g) Doppler radar is commonly used by autonomous underwater vehicles.
h) Solar radiation pressure modeling is used within spacecraft navigation.
i) The most effective technologies for pedestrian navigation have yet to be determined.
j) Inertial sensors may be used for time synchronization.

Answers

1 Introduction

1.1 a) Real-time mobile self-positioning; b) Postprocessed mobile remote positioning; c) Postprocessed static remote positioning; d) Real-time mobile remote positioning; e) Real-time static self-positioning.

1.2 b.

1.3 c, d, and e.

1.4 a) Ranging, angular positioning, and Doppler positioning; b) 3; c) 4.

1.5 b and d.

1.6 All except c.

1.7 a) Stationary, urban, and open road; b) Submerged and surfaced or periscope raised (GNSS and other radio signals are not receivable underwater).

1.8 a) Initialization of the position and calibration of sensor errors; b) Provision of approximate user position and time, and satellite position information; c) Provision of velocity and approximate position.

2 Coordinate Frames, Kinematics, and the Earth

2.1 a) ECEF and local-tangent plane; b) Local navigation; c) ECI; d) ECEF and ECI; e) Local navigation.

2.2 a) $\begin{pmatrix} 0.75 & 0.43301 & 0.5 \\ -0.64952 & 0.625 & 0.43301 \\ -0.125 & -0.64952 & 0.75 \end{pmatrix}$, $\begin{pmatrix} 0.75 & -0.43301 & -0.5 \\ 0.21651 & 0.875 & -0.43301 \\ 0.625 & 0.21651 & 0.75 \end{pmatrix}$;

b) $\begin{pmatrix} 0.90625 & 0.37889 & -0.1875 \\ -0.35182 & 0.92188 & 0.16238 \\ 0.23438 & -0.08119 & .96875 \end{pmatrix}$; c) 9.5153° roll, 10.8069° pitch, 22.6889° yaw.

2.3 a) $\mathbf{C}_b^n = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$; b) $\mathbf{q}_b^n = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}^{\mathrm{T}}$; c) The elevation is 90°, but the bank and heading are undefined because of division by zero. This occurs because Euler angles exhibit a singularity when the pitch is ±90°.

2.4) a) $\boldsymbol{\omega}_{ib}^n = \begin{pmatrix} 0.2 & 0.1 & -0.3 \end{pmatrix}^{\mathrm{T}}$;

b) $\boldsymbol{\Omega}_{ib}^{ENU} = \begin{pmatrix} 0 & -0.3 & 0.2 \\ 0.3 & 0 & -0.1 \\ -0.2 & 0.1 & 0 \end{pmatrix}$ $\boldsymbol{\Omega}_{ib}^n = \begin{pmatrix} 0 & 0.3 & 0.1 \\ -0.3 & 0 & -0.2 \\ -0.1 & 0.2 & 0 \end{pmatrix}$.

2.5) a) 1.56×10^{-7} rad = 8.95×10^{-6}°; b) 1.56×10^{-7} rad = 8.95×10^{-6}° at the equator; 3.12×10^{-7} rad = 1.79×10^{-5}° at 60° latitude.

2.6) a) $\mathbf{r}_{ib}^i = \begin{pmatrix} 4490 & 1567 & 4284 \end{pmatrix}^{\mathrm{T}}$ km, $\mathbf{v}_{ib}^i = \begin{pmatrix} -117.6 & 336.9 & 0 \end{pmatrix}^{\mathrm{T}}$ m s^{-1}, $\mathbf{a}_{ib}^i = \begin{pmatrix} -1.29 & -0.44 & 1.49 \end{pmatrix}^{\mathrm{T}}$ m s^{-2}; b) L_b = 0.7363288 rad = 42.18851°; λ_b =–1.2392715 rad = –71.00503°; h_b = 31060m; c) $\mathbf{v}_{eb}^n = \begin{pmatrix} 0.002 & 10.006 & 0.003 \end{pmatrix}^{\mathrm{T}}$ m s^{-1}, $\mathbf{a}_{eb}^n = \begin{pmatrix} 2.004 & 0.007 & -0.007 \end{pmatrix}^{\mathrm{T}}$ m s^{-2}.

2.7) a) 9.8116 m s^{-2}; b) Down: 9.8085 m s^{-2}, North: -7.9×10^{-6} m s^{-2}; c) North: 0.0258 m s^{-2}, East: 0; Down: 0.0209 m s^{-2}; d) North: –0.0258 m s^{-2}, East: 0; Down: 9.7877 m s^{-2}.

2.8) a) $\mathbf{v}_{eb}^n = \begin{pmatrix} 9.860 & 0.349 & 0 \end{pmatrix}^{\mathrm{T}}$ m s^{-1}; b) 1.745 m s^{-2} East;

c) $\mathbf{a}_{eb}^n = \begin{pmatrix} -0.061 & 1.721 & 0 \end{pmatrix}^{\mathrm{T}}$ m s^{-2}

3 Kalman Filter-Based Estimation

3.1) a, c, f, and g.

3.2) a, c, and e.

3.3) a) $\mathbf{\Phi} = \begin{pmatrix} 1 & 0 & \tau_s & 0 & 0.5\tau_s^2 & 0 \\ 0 & 1 & 0 & \tau_s & 0 & 0.5\tau_s^2 \\ 0 & 0 & 1 & 0 & \tau_s & 0 \\ 0 & 0 & 0 & 1 & 0 & \tau_s \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$; b) 0.5m.

3.4) a) $\begin{pmatrix} 0.00266 & 0.02 \\ 0.02 & 0.2 \end{pmatrix}$ for 0.2s, $\begin{pmatrix} 0.333 & 0.5 \\ 0.5 & 1 \end{pmatrix}$ for 1s, $\begin{pmatrix} 41.7 & 12.5 \\ 12.5 & 5 \end{pmatrix}$ for 5s to 3 s.f.;

b) $\begin{pmatrix} 0 & 0 \\ 0 & 0.2 \end{pmatrix}$ for 0.2s, $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ for 1s, $\begin{pmatrix} 0 & 0 \\ 0 & 5 \end{pmatrix}$ for 5s;

c) $\begin{pmatrix} 0.004 & 0.02 \\ 0.02 & 0.2 \end{pmatrix}$ for 0.2s, $\begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 1 \end{pmatrix}$ for 1s, $\begin{pmatrix} 62.5 & 12.5 \\ 12.5 & 5 \end{pmatrix}$ for 5s;

d) $\mathbf{\Phi}_{k-1}\mathbf{P}_{k-1}^{+}\mathbf{\Phi}_{k-1}^{\mathrm{T}} = \begin{pmatrix} 9.2 & 0.6 \\ 0.6 & 1 \end{pmatrix}$ after 0.2s, $\begin{pmatrix} 10.8 & 1.4 \\ 1.4 & 1 \end{pmatrix}$ after 1s, and $\begin{pmatrix} 38 & 5.4 \\ 5.4 & 5 \end{pmatrix}$ after 5s.

Therefore, comparing these with the system noise covariance, both approximations are reasonable for the 0.2s propagation interval; approximation c, but not b, is reasonable for the 1s interval; and neither approximation is reasonable for the 5s interval.

3.5) a) $\hat{\mathbf{x}}_4^{+} = \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{v}_x \\ \hat{v}_y \end{pmatrix}_4^{+} = \begin{pmatrix} -110.97 & \mathrm{m} \\ 197.90 & \mathrm{m} \\ -7.85 & \mathrm{m\,s^{-1}} \\ -3.29 & \mathrm{m\,s^{-1}} \end{pmatrix}$; b) $\mathbf{P}_4^{+} = \begin{pmatrix} 0.236 & 0.091 & 0.251 & 0.087 \\ 0.091 & 0.236 & 0.087 & 0.251 \\ 0.251 & 0.087 & 2.133 & 0.205 \\ 0.087 & 0.251 & 0.205 & 2.133 \end{pmatrix}$.

3.6) b, c, and d.

3.7) a) 2760 multiplications and 2490 additions; b) 7695 multiplications and ~5000 additions; c) 4400 multiplications and 3010 additions.

3.8) a) UKF; b) Basic Kalman filter; c) A particle filter is best in theory, but a multiple-hypothesis Kalman filter may be more practical.

4 Inertial Sensors

4.1) a) Consumer, tactical, intermediate, aviation, marine; b) Aviation grade; c) Tactical grade; d) Military ships, submarines, inter-continental ballistic missiles, spacecraft.

4.2) Accelerometers: b and d; Gyros: a, c, d, and e.

4.3) $1.765 + 0.05 - (0.0025 + 0.0064) + (0.004 - 0.003) - 1.569064 = 0.2383\ \mathrm{m\,s^{-2}}$.

4.4) a) 7.292115×10^{-5} rad s^{-1}; b) 0, 0, and –9.80 m s^{-2}; c) + 0.00342 m s^{-2}.

4.5) b, c, and d.

4.6) b and c. (Note that VRE is typically a nonlinear function of vibration amplitude.)

4.7) a) 1.387×10^{-2} m s^{-2}; b) 4.114×10^{-4} rad s^{-1}; c) 6.934×10^{-5} m s^{-1}, 2.057×10^{-6} rad.

5 Inertial Navigation

5.1) Attitude update, specific-force frame transformation, velocity update, and position update. Velocity must be updated before position and the specific-force frame transformed before the velocity update. The ordering of the attitude update depends on the implementation.

5.2) (118.55, 77.26) m; (10.68, –11.29) m s^{-1}; –46.60°.

5.3) a) ECEF, local-navigation, and wander azimuth; b) local-navigation; c) ECI and ECEF; d) all implementations, directly for ECI.

5.4) a) (–10.5, 10.5, 0) m; b) attitude error of –3.9 mrad or –0.22° about east and velocity error of 0.977 m s^{-1} downward.

5.5) a) 1st order: $\begin{pmatrix} 1 & -0.4 & 0.3 \\ 0.4 & 1 & -0.2 \\ -0.3 & 0.2 & 1 \end{pmatrix}$, 4th order: $\begin{pmatrix} 0.8780 & -0.3514 & 0.3245 \\ 0.4099 & 0.9024 & -0.1318 \\ -0.2465 & 0.2489 & 0.9366 \end{pmatrix}$,

precise: $\begin{pmatrix} 0.8780 & -0.3517 & 0.3248 \\ 0.4102 & 0.9024 & -0.1319 \\ -0.2467 & 0.2490 & 0.9366 \end{pmatrix}$; b) 1st order: (–4.02°, 0.51°, –3.35°), 4th order: (–0.0033°, –0.0134°, –0.0125°).

5.6) a) $\mathbf{C}\mathbf{C}^{\mathrm{T}} = \begin{pmatrix} 0.983 & -0.006 & 0.004 \\ -0.006 & 0.991 & 0.003 \\ 0.004 & 0.003 & 0.984 \end{pmatrix} \neq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$;

b) $\begin{pmatrix} 0.8585 & 0.4928 & 0.0580 \\ -0.4973 & 0.8616 & 0.0387 \\ -0.0311 & -0.0623 & 0.9898 \end{pmatrix}$; c) $\begin{pmatrix} 0.8658 & 0.4970 & 0.0585 \\ -0.4995 & 0.8654 & 0.0389 \\ -0.0313 & -0.0628 & 0.9975 \end{pmatrix}$.

5.7) a) Coning: 6.64×10^{-4} rad s^{-1}, Sculling 3.88×10^{-2} m s^{-2}; b) Coning: 1.26×10^{-4} rad s^{-1}, Sculling 7.2×10^{-3} m s^{-2}; c) Coning: 8.11×10^{-6} rad s^{-1}, Sculling 4.6×10^{-4} m s^{-2}.

5.8) a) (–3.0°, 2.4°, –59.9°); b) –43.2°; c) (0.011°, 0.011°, 0.105°).

5.9) a) (30.79, 61.58, 0) m; b) Additional error: (0, –28.79, –1.44) m, Total error: (30.79, 32.79, –1.44) m; c) Stationary case: (–1.74, –3.49, 0) mg, Accelerating case: (–1.74, –1.86, 0.082) mg.

6 Dead Reckoning, Attitude, and Height Measurement

6.1) a) –29.5°; b) 3.2°; c) –5.9°; d) –1.0°.

6.2) a, b, d, and f.

6.3) Roll: –0.0°, pitch: 4.6°, heading: 94.1°.

6.4) a) 53.785 kPa; b) 5,678m.

6.5) a) (88.6, 24.7) m; b) (0.73, 0.45) m; c) (–41.3, 37.8) m; d) (1.14, 0.68) m.

6.6) b and d.

6.7) a, b, c, and d.

6.8) a) (9.392, 0.053, 0.187) m s^{-1}; b) (–0.289, –0.289, –0.118) m s^{-1}; c) (–0.156, –0.001, –0.003) m s^{-1}; d) Fault detection.

7 Principles of Radio Positioning

7.1) a) Self; b) Remote; c) Both; d) Remote.

7.2) a) 30 km; b) 49.5 km; c) 250μs; d) zero.

7.3) a) (4.9, 145.1) m; b) $z^{l}_{la} = -3.24\text{m}$; c) 3.52°.

7.4) a) (3.311, 6.700) km and (16.835, 17.904) km; b) (3.314, 6.691) km; c) x-axis: 9.9 m, y-axis: 7.2m, radial: 12.2m.

7.5) a) LF/MF; b) VHF/UHF; c) VHF/UHF; d) LF/MF; e) VHF/UHF; f) All.

7.6) a) 29 km; b) by 150m to 200m; c) 13.2m.

7.7) a) Proximity, ranging, angular positioning, and Doppler positioning; b) Ranging and Doppler positioning if the timing error is changing; c) Ranging, angular positioning, pattern matching, and Doppler positioning; d) Ranging with a small effect on angular and Doppler positioning; e) Pattern matching.

8 GNSS: Fundamentals, Signals, and Satellites

8.1) a) User; b) Control; c) Space; d) All.

8.2) a, b, d, and f.

8.3) a) GLONASS; b) GPS; c) Compass; d) Galileo; e) GPS and GLONASS.
8.4) a and c.
8.5) a) BPSK modulation with a low chipping rate; b) A short code length; c) A high chipping rate and/or BOC modulation; d) BOC modulation; e) Omitting the navigation data.
8.6) See the table below:

	At the reference time				After one hour				
	GPS	GLONASS	Galileo	Compass	GPS	GLONASS	Galileo	Compass	
$\mathbf{r}_{ib}^{i}$	2.6580	2.5500	2.9620	2.7840	2.3007	2.1630	2.6724	2.4573	$\times 10^7$ m
	0	0	0	0	0.7635	0.5750	0.7143	0.7505	$\times 10^7$ m
	0	0	0	0	1.0903	1.2220	1.0589	1.0718	$\times 10^7$ m
$\mathbf{r}_{eb}^{e}$	2.6580	2.5500	2.9620	2.7840	2.4200	2.2381	2.7662	2.5679	$\times 10^7$ m
	0	0	0	0	0.1403	-0.0060	-0.0037	0.0871	$\times 10^7$ m
	0	0	0	0	1.0903	1.2220	1.0589	1.0718	$\times 10^7$ m
$\mathbf{v}_{ib}^{i}$	0.0	0.0	0.0	0.0	-1939.2	-2094.0	-1581.9	-1778.4	m s^{-1}
	2221.2	1683.4	2051.3	2170.3	1922.6	1427.9	1850.8	1915.7	m s^{-1}
	3172.2	3577.4	3041.2	3099.6	2745.8	3034.4	2743.9	2735.9	m s^{-1}
$\mathbf{v}_{eb}^{e}$	0.0	0.0	0.0	0.0	-1271.6	-1656.0	-1050.1	-1156.8	m s^{-1}
	282.9	-176.1	-108.6	140.2	595.3	290.3	180.7	439.0	m s^{-1}
	3172.2	3577.4	3041.2	3099.6	2745.8	3034.4	2743.9	2735.9	m s^{-1}

8.7) a) Range = 21,718,391.2m, range rate = –939.95 m s^{-1}, LOS (ECEF axes) = (0.5174, 0.7066, 0.4827), azimuth = 96.95), elevation = 44.62°; b) Pseudo-range = 22,228,395.4m, pseudo-range rate = –640.27 m s^{-1}; c) Corrected pseudo-range = 22,228,385.6m.

9 GNSS: User Equipment Processing and Errors

9.1) a, c, d, and e.
9.2) a) 5μs; b) 1,500 m s^{-1}; c) 200s; d) 3m.
9.3) I_E = 150.4; I_P = 196.0; I_L = 146.5; Q_E = 15.09; Q_P = 19.66; Q_L = 14.70 (Note: use equation (9.22).
9.4) a, b, and d.
9.5) a) $2046 \times 58 = 118668$; b) $2046 \times 5 = 10230$.
9.6) a) 23,455,431.38m; b) 23,455,431.81m.
9.7) a) 1.06m, 32.40m; b) 480 m s^{-3}, 0.03 rad, 0.19 rad.
9.8) a, b, d, and f.
9.9) a) $\tilde{x}_{ea}^{e} = 2{,}642{,}112.08$m, $\tilde{y}_{ea}^{e} = -2{,}225{,}376.80$m, $\tilde{z}_{ea}^{e} = 5{,}343{,}931.42$m, $\delta\tilde{\rho}_{c}^{a} = 12{,}339.57$m; b) $\Delta\tilde{x}_{ea}^{e} = -11.47$m, $\Delta\tilde{y}_{ea}^{e} = -14.46$m, $\Delta\tilde{z}_{ea}^{e} = 1.73$m, $\Delta\delta\tilde{\rho}_{c}^{a} = 1.29$m; c) HDOP = 2.250, VDOP = 3.421, PDOP = 4.094, GDOP = 4.752
9.10) a) The position solution is noisier because the Kalman filter is more receptive to new measurement data. b) The velocity errors are bigger because the Kalman filter is insufficiently receptive to new measurement data during maneuvers.

10 GNSS: Advanced Techniques

10.1) a) $\sigma_\rho = 0.141$m, $\sigma_H = 0.113$m, $\sigma_V = 0.170$m; b) $\sigma_\rho = 1.51$m, $\sigma_H = 1.21$m, $\sigma_V = 1.81$m; c) $\sigma_\rho = 1.02$m, $\sigma_H = 0.813$m, $\sigma_V = 1.22$m; d) $\sigma_\rho = 1.51$m, $\sigma_H = 1.21$m, $\sigma_V = 1.81$m.
10.2) a) Regional- and wide-area; b) local- and regional-area; c) regional-area; d) wide-area; e) local-area; f) all three.
10.3) 1234.
10.4) $\phi_{nb} = 1°$, $\theta_{nb} = -2°$, $\psi_{nb} = 40°$.
10.5) Interference: a, b, c, and e; Weak signal: b and e.
10.6) Multipath: a, b, e, and f; NLOS: d and f.
10.7) The south side.

11 Long- and Medium-range Radio Navigation

11.1) a) L_b = 51.39971°, λ_b = 0.10119°; b) –3.84m north, –12.02m east; c) –28.4m north, 225.1m east.
11.2) b and c.
11.3) a) 448,659.8m, b) 456,893.1m.
11.4) a) Iridium, AM, TV, phones; b) AM radio; c) Iridium; d) Potentially all of them.

12 Short-range Positioning

12.1) a) Ranging; b) proximity; c) ranging; d) angular; e) pattern matching; f) none.
12.2) a) 0.5m; b) 0.95m; c) x: 7.0m, y: 5.6m, z: 11.8m; d) x: 1.2m, y: 1.1m, z: 8.5m; e) The vertical accuracy is improved by a factor of ~2 with a small improvement to the horizontal accuracy; the 1-way accuracy is x: 6.7m, y: 5.4m, z: 5.2m; and the 2-way accuracy is x: 1.0m, y: 1.0m, z: 4.3m.
12.3) a) WLAN; b) RFID; c) WLAN and WPAN; d) all three.
12.4) a) SBL and USBL; b) USBL and homing; c) LBL and SBL; d) all methods.

13 Environmental Feature Matching

13.1) a) Link 1 (likelihood 1); b) Link 2 (likelihood 0.645); c) Link 1 (likelihood 0.784).
13.2) a) Rail, b) road, c) pedestrian; d) road and pedestrian.
13.3) a) Radar altimeter; b) multibeam sonar; c) barometric altimeter; d) laser scanner.
13.4) a) The azimuths are –21.8° and 9.6°, and the elevations are –6.8° and –8.4°; b) –608.8m north, 343.5m east, and –284.1m down; c) –382.5m north and 153.0m east.
13.5) a) Small-feature descriptors; b) line features; c) both; d) line feature.
13.6) a) Gravity gradiometry; b) magnetic anomaly matching; c) radar image matching; d) map matching; e) stellar navigation.

14 INS/GNSS Integration

14.1) a) Tightly coupled; b) loosely coupled; c) both; d) tightly coupled; e) loosely coupled; f) neither.
14.2) a) 197.5 m s^{-1} east and 5 m s^{-1} upward; b) (–98.409, 171.291, 2.498) m s^{-1}; c) –633.387 m s^{-1}; d) 0.8 m s^{-1}.
14.3) All of them.
14.4) Velocity error; Vertical (z) accelerometer bias; Gyro bias about the forward (x) axis; Pitch error; Transverse (y-axis) accelerometer bias; Heading error; Vertical (z-axis) gyro bias.
14.5) Should be modeled: b, c, and e; Occasionally modeled: f.
14.6) a) Mobile mapping, because the accuracy of both position and attitude are important.
b) Missile guidance, because this is vulnerable to jamming and spoofing.
c) Pedestrian navigation, because the sensors must be small and light, a UKF handles larger heading errors, and adaptive filtering responds better to noise levels which can be variable and difficult to predict.

15 INS Alignment, Zero Updates, and Motion Constraints

15.1) b and c.
15.2) a, b, and d.
15.3) a) Vehicle; b) pedestrian; c) vehicle; d) vehicle; e) pedestrian.

16 Multisensor Integrated Navigation

16.1) a) Error state; b) filtered; c) cascaded; d) federated; e) modal.
16.2) a) With the INS as the reference; c) with the odometer and gyro as the reference; d) with the INS as the reference; and f) integrating GNSS and force modeling only with the force modeling as the reference (the stellar attitude determination is separate).
16.3) a) Neither system provides a complete continuous position solution which could be used as the reference; b) To enable a continuous integrated position solution to be maintained

using state prediction and to calibrate the bias of Beta; c) 2D position, 2D velocity, and the Beta directional bias; d) speed.

16.4) a) 2D position error, 2D velocity error, heading error, gyro bias, and odometry scale-factor error (note that the height information is only provided by GNSS so is not integrated); b) INS 3D position, velocity and attitude errors, and accelerometer and gyro biases (note that a TRN database bias is not observable unless position fixes from another technology, such as GNSS, are added); c) 3D position and velocity, receiver clock offset and drift, clock offset of each phone base station; d) INS 3D position and velocity errors, baro bias and scale-factor error.

16.5) b) Because a good match may be obtained at more than one candidate position; c) because a standard KF, EKF, or UKF cannot represent the probability distribution in cases where the heading is completely unknown; e) because line features are difficult to identify unambiguously.

17 Fault Detection, Integrity Monitoring, and Testing

17.1) a) FDE; b) fault detection only; c) FDR; d) FDI.

17.2) a, b, d, and f.

17.3) a) 7.21m; b) 1.74; c) yes.

17.4) There is a fault on measurement 4.

17.5) a) Decreases it; b) increases it; c) decreases it; d) increases it.

17.6) a) Laboratory trials; b) recorded data testing and software simulation; c) field trials; d) software simulation; e) recorded data testing; f) potentially all four methods.

18 Applications

18.1) a) Economic; b) environmental; c) performance; d) engineering; e) environmental; f) engineering; g) performance; h) economic.

18.2) a) Signal availability; b) vulnerability to jamming; c) signal availability, particularly indoors; d) integrity and continuity.

18.3) Structural deformation monitoring; ship navigation in ports; general pedestrian navigation; road navigation in rural areas; oceanic aircraft navigation.

18.4) a) UWB and foot-mounted inertial navigation; b) precision inertial navigation and gravity gradiometry; c) GNSS and ELoran; d) GNSS and Doppler radar; e) GNSS and DME.

18.5) b, e, h, and i. (Notes: a) GPS augmented by GBAS; c) micro air vehicles; f) this is the proposed GNSS alert limit.)