EXAMPLE 2.6(a)
Transformation of reference frame from ECEF frame to ECI frame and resolving axes from local navigation frame to ECI frame

INPUTS:

Latitude
L_b = 45 degrees, 0.785398 rad

Height
h_b = 100 m

Longitude
λ_b = 30 degrees, 0.523599 rad

Velocity

$\mathbf{v}^n_{eb}$ =

5	m s^{-1}
5	m s^{-1}
5	m s^{-1}

Acceleration

$\mathbf{a}^n_{eb}$ =

5	m s^{-2}
0	m s^{-2}
0	m s^{-2}

Angular rate

$\boldsymbol{\omega}^n_{nb}$ =

0.001	rad s^{-1}
0	rad s^{-1}
0	rad s^{-1}

Current time
t = 10000 s

Time of ECI-ECEF frame coincidence
t_0 = 0 s

Earth rate
ω_{ie} = 7.29E-05 rad s^{-1}

Transverse Radius of Curvature
From (2.106), $$R_E(L_b) = \frac{R_0}{\sqrt{1 - e^2 \sin^2 L_b}}$$

R_E = 6.39E+06

Meridian Radius of Curvature
$$R_N(L_b) = \frac{R_0\left(1 - e^2\right)}{\left(1 - e^2 \sin^2 L_b\right)^{3/2}}$$

R_N = 6.37E+06

Cartesian Position
From (2.112),
$$\begin{aligned} x^e_{eb} &= \left(R_E(L_b) + h_b\right)\cos L_b \cos \lambda_b \\ y^e_{eb} &= \left(R_E(L_b) + h_b\right)\cos L_b \sin \lambda_b \\ z^e_{eb} &= \left[\left(1 - e^2\right)R_E(L_b) + h_b\right]\sin L_b \end{aligned}$$

$\mathbf{r}^e_{eb}$ =

3912409.702	m
2258830.795	m
4487419.12	m

ECEF to ECI Coordinate transformation matrix
From (2.145),
$$\mathbf{C}^i_e = \begin{pmatrix} \cos \omega_{ie}(t - t_0) & -\sin \omega_{ie}(t - t_0) & 0 \\ \sin \omega_{ie}(t - t_0) & \cos \omega_{ie}(t - t_0) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\mathbf{C}^i_e$ =

0.745699997	-0.66628	0
0.666281858	0.7457	0
0	0	1

Cartesian position transformation
From (2.146), $\mathbf{r}^i_{ib} = \mathbf{C}^i_e \mathbf{r}^e_{eb}$

$\mathbf{r}^i_{ib}$ =

1412465.926	m
4291177.722	m
4487419.12	m

Skew symmetric matrix of Earth rate

$\mathbf{\Omega}_{ie}^{e} =$

$\mathbf{\Omega}_{ie}^{i} =$

0	-7.29E-05	0
7.29E-05	0	0
0	0	0

NED to ECI coordinate transformation matrix

From (2.154),

$$\mathbf{C}_n^i = \begin{pmatrix} -\sin L_b \cos\left(\lambda_b + \omega_{ie}(t-t_0)\right) & -\sin\left(\lambda_b + \omega_{ie}(t-t_0)\right) & -\cos L_b \cos\left(\lambda_b + \omega_{ie}(t-t_0)\right) \\ -\sin L_b \sin\left(\lambda_b + \omega_{ie}(t-t_0)\right) & \cos\left(\lambda_b + \omega_{ie}(t-t_0)\right) & -\cos L_b \sin\left(\lambda_b + \omega_{ie}(t-t_0)\right) \\ \cos L_b & 0 & -\sin L_b \end{pmatrix}$$

$\left(\lambda_b + \omega_{ie}(t-t_0)\right) =$ 1.25281

$\mathbf{C}_n^i =$

-0.22107991	-0.94987	-0.22108
-0.67165741	0.312654	-0.67166
0.707106781	0	-0.70711

Velocity transformation

From (2.155), $\mathbf{v}_{ib}^i = \mathbf{C}_n^i \mathbf{v}_{eb}^n + \mathbf{C}_e^i \mathbf{\Omega}_{ie}^e \mathbf{r}_{eb}^e$

$\mathbf{v}_{ib}^i =$

-319.877749	m s^{-1}
97.84533668	m s^{-1}
4.44089E-16	m s^{-1}

Acceleration transformation

From (2.156),

$$\begin{aligned} \mathbf{a}_{ib}^i &= \mathbf{C}_n^i \left(\mathbf{a}_{eb}^n + 2\mathbf{\Omega}_{ie}^n \mathbf{v}_{eb}^n\right) + \mathbf{C}_e^i \mathbf{\Omega}_{ie}^e \mathbf{\Omega}_{ie}^e \mathbf{r}_{eb}^e \\ &= \mathbf{C}_n^i \mathbf{a}_{eb}^n + 2\mathbf{\Omega}_{ie}^i \mathbf{C}_n^i \mathbf{v}_{eb}^n + \mathbf{C}_e^i \mathbf{\Omega}_{ie}^e \mathbf{\Omega}_{ie}^e \mathbf{r}_{eb}^e \end{aligned}$$

$\mathbf{a}_{ib}^i =$

-1.11215878	m s^{-2}
-3.38212043	m s^{-2}
3.535533906	m s^{-2}

Angular rate transformation

From (2.157), $\boldsymbol{\omega}_{ib}^i = \mathbf{C}_n^i \left(\boldsymbol{\omega}_{nb}^n + \boldsymbol{\omega}_{en}^n\right) + \boldsymbol{\omega}_{ie}^i$

From (5.44)

$$\boldsymbol{\omega}_{en}^n = \begin{pmatrix} v_{eb,E}^n / \left(R_E(L_b) + h_b\right) \\ -v_{eb,N}^n / \left(R_N(L_b) + h_b\right) \\ -v_{eb,E}^n \tan L_b / \left(R_E(L_b) + h_b\right) \end{pmatrix}$$

$\boldsymbol{\omega}_{en}^n =$

7.83E-07	rad s^{-1}
-7.85E-07	rad s^{-1}
-7.83E-07	rad s^{-1}

$\boldsymbol{\omega}_{ie}^i =$

0	rad s^{-1}
0	rad s^{-1}
7.29E-05	rad s^{-1}

$\boldsymbol{\omega}_{ib}^i =$

-0.00022033	rad s^{-1}
-0.0006719	rad s^{-1}
0.000781135	rad s^{-1}

EXAMPLE 2.6(b)
Transformation of reference frame from ECI frame to ECEF frame
and resolving axes from ECI frame to local navigation frame

See Example 2.2 for determination of Latitude, Longitude, and Height from Cartesian Position

INPUTS:

Position

$\mathbf{r}^i_{ib}$ =

1412465.926	m
4291177.722	m
4487419.12	m

Velocity

$\mathbf{v}^i_{ib}$ =

-319.878	m s^{-1}
97.84534	m s^{-1}
4.44E-16	m s^{-1}

Acceleration

$\mathbf{a}^i_{ib}$ =

-1.11216	m s^{-2}
-3.38212	m s^{-2}
3.535534	m s^{-2}

Latitude

L_b = 45 degrees, 0.785398 rad

Height

h_b = 100 m

Longitude

λ_b = 30 degrees, 0.523599 rad

Angular rate

$\boldsymbol{\omega}^i_{ib}$ =

-0.000220334	rad s^{-1}
-0.000671903	rad s^{-1}
0.000781135	rad s^{-1}

Current time

t 10000 s

Time of ECI-ECEF frame coincidence

t_0 0 s

Earth rate

ω_{ie} 7.29E-05 rad s^{-1}

Coordinate transformation matrix

From (2.154),

$$\mathbf{C}^n_i = \begin{pmatrix} -\sin L_b \cos\left(\lambda_b + \omega_{ie}(t - t_0)\right) & -\sin L_b \sin\left(\lambda_b + \omega_{ie}(t - t_0)\right) & \cos L_b \\ -\sin\left(\lambda_b + \omega_{ie}(t - t_0)\right) & \cos\left(\lambda_b + \omega_{ie}(t - t_0)\right) & 0 \\ -\cos L_b \cos\left(\lambda_b + \omega_{ie}(t - t_0)\right) & -\cos L_b \sin\left(\lambda_b + \omega_{ie}(t - t_0)\right) & -\sin L_b \end{pmatrix}$$

$\left(\lambda_b + \omega_{ie}(t - t_0)\right)$ = 1.25E+00

$\mathbf{C}^n_i$ =

-0.221079914	-0.67166	0.707107
-0.949867014	0.312654	0
-0.221079914	-0.67166	-0.70711

Skew symmetric matrix of Earth rate

$\boldsymbol{\Omega}^i_{ie}$ =

0	-7.29E-05	0
7.29E-05	0	0
0	0	0

Velocity transformation

From (2.155), $\mathbf{v}^n_{eb} = \mathbf{C}^n_i\left(\mathbf{v}^i_{ib} - \boldsymbol{\Omega}^i_{ie}\mathbf{r}^i_{ib}\right)$

$\mathbf{v}^n_{eb}$ =

5	m s^{-1}
5	m s^{-1}
5	m s^{-1}

Acceleration transformation

From (2.156), $\mathbf{a}_{eb}^{n} = \mathbf{C}_{i}^{n}\left(\mathbf{a}_{ib}^{i} - 2\mathbf{\Omega}_{ie}^{i}\mathbf{v}_{ib}^{i} + \mathbf{\Omega}_{ie}^{i}\mathbf{\Omega}_{ie}^{i}\mathbf{r}_{ib}^{i}\right)$

$\mathbf{a}_{eb}^{n} =$

5	m s^{-2}
-2.22045E-16	m s^{-2}
0	m s^{-2}

Angular rate transformation

From (2.157), $\boldsymbol{\omega}_{nb}^{n} = \mathbf{C}_{i}^{n}\left(\boldsymbol{\omega}_{ib}^{i} - \boldsymbol{\omega}_{ie}^{i}\right) - \boldsymbol{\omega}_{en}^{n}$

Transverse Radius of Curvature

From (2.106), $R_E(L_b) = \dfrac{R_0}{\sqrt{1 - e^2 \sin^2 L_b}}$

$R_E =$ 6.39E+06

Meridian Radius of Curvature

$R_N(L_b) = \dfrac{R_0\left(1 - e^2\right)}{\left(1 - e^2 \sin^2 L_b\right)^{3/2}}$

$R_N =$ 6.37E+06

From (5.44)

$$\boldsymbol{\omega}_{en}^{n} = \begin{pmatrix} v_{eb,E}^{n} \big/ \left(R_E(L_b) + h_b\right) \\ -v_{eb,N}^{n} \big/ \left(R_N(L_b) + h_b\right) \\ -v_{eb,E}^{n} \tan L_b \big/ \left(R_E(L_b) + h_b\right) \end{pmatrix}$$

$\boldsymbol{\omega}_{en}^{n} =$

7.83E-07	rad s^{-1}
-7.85E-07	rad s^{-1}
-7.83E-07	rad s^{-1}

$\boldsymbol{\omega}_{ie}^{i} =$

0	rad s^{-1}
0	rad s^{-1}
7.29E-05	rad s^{-1}

$\boldsymbol{\omega}_{nb}^{n} =$

0.001	rad s^{-1}
-1.05879E-21	rad s^{-1}
4.93397E-20	rad s^{-1}