EXAMPLE 2.5(a)

Transformation of resolving axes from local navigation frame to ECEF frame

INPUTS:

Velocity

$\mathbf{v}^n_{eb}$ = 5 m s^{-1}, 5 m s^{-1}, 5 m s^{-1}

Acceleration

$\mathbf{a}^n_{eb}$ = 5 m s^{-2}, 0 m s^{-2}, 0 m s^{-2}

Angular rate

$\boldsymbol{\omega}^n_{nb}$ = 0.001 rad s^{-1}, 0 rad s^{-1}, 0 rad s^{-1}

Latitude

L_b = 45 degrees 0.785398 rad

Height

h_b = 0 m

Longitude

λ_b = 45 degrees 0.785398 rad

NED to ECEFcoordinate transformation matrix

From (2.150),

$$\mathbf{C}^e_n = \begin{pmatrix} -\sin L_b \cos \lambda_b & -\sin \lambda_b & -\cos L_b \cos \lambda_b \\ -\sin L_b \sin \lambda_b & \cos \lambda_b & -\cos L_b \sin \lambda_b \\ \cos L_b & 0 & -\sin L_b \end{pmatrix}$$

$\mathbf{C}^e_n$ =

-0.5	-0.70711	-0.5
-0.5	0.707107	-0.5
0.707106781	0	-0.70711

Velocity transformation

From (2.152), $\mathbf{v}^e_{eb} = \mathbf{C}^e_n \mathbf{v}^n_{eb}$

$\mathbf{v}^e_{eb}$ = -8.53553391 m s^{-1}, -1.46446609 m s^{-1}, 4.44089E-16 m s^{-1}

Acceleration transformation

From (2.152), $\mathbf{a}^e_{eb} = \mathbf{C}^e_n \mathbf{a}^n_{eb}$

$\mathbf{a}^e_{eb}$ = -2.5 m s^{-2}, -2.5 m s^{-2}, 3.535533906 m s^{-2}

Angular rate transformation

From (2.153), $\boldsymbol{\omega}^e_{eb} = \mathbf{C}^e_n \left(\boldsymbol{\omega}^n_{nb} + \boldsymbol{\omega}^n_{en}\right)$

Transverse Radius of Curvature

From (2.106), $R_E(L_b) = \dfrac{R_0}{\sqrt{1 - e^2 \sin^2 L_b}}$

R_E = 6.39E+06

Meridian Radius of Curvature

$R_N(L_b) = \dfrac{R_0\left(1 - e^2\right)}{\left(1 - e^2 \sin^2 L_b\right)^{3/2}}$

R_N = 6.37E+06

From (5.44)

$$\boldsymbol{\omega}_{en}^{n} = \begin{pmatrix} v_{eb,E}^{n} \big/ \left(R_E(L_b) + h_b\right) \\ -v_{eb,N}^{n} \big/ \left(R_N(L_b) + h_b\right) \\ -v_{eb,E}^{n} \tan L_b \big/ \left(R_E(L_b) + h_b\right) \end{pmatrix}$$

$\boldsymbol{\omega}_{en}^{n} =$

7.83E-07	rad s^{-1}
-7.85E-07	rad s^{-1}
-7.83E-07	rad s^{-1}

$\boldsymbol{\omega}_{eb}^{e} =$

-0.00049944	rad s^{-1}
-0.00050056	rad s^{-1}
0.000708214	rad s^{-1}

EXAMPLE 2.5(b)

Transformation of resolving axes from ECEF frame to local navigation frame

INPUTS:

Velocity

$$\mathbf{v}^e_{eb} = \begin{pmatrix} -8.53553391 \\ -1.46446609 \\ 4.44089E-16 \end{pmatrix} \begin{matrix} \text{m s}^{-1} \\ \text{m s}^{-1} \\ \text{m s}^{-1} \end{matrix}$$

Acceleration

$$\mathbf{a}^e_{eb} = \begin{pmatrix} -2.5 \\ -2.5 \\ 3.535534 \end{pmatrix} \begin{matrix} \text{m s}^{-2} \\ \text{m s}^{-2} \\ \text{m s}^{-2} \end{matrix}$$

Angular rate

$$\boldsymbol{\omega}^e_{eb} = \begin{pmatrix} -0.0005 \\ -0.0005 \\ 0.000708 \end{pmatrix} \begin{matrix} \text{rad s}^{-1} \\ \text{rad s}^{-1} \\ \text{rad s}^{-1} \end{matrix}$$

Latitude

L_b = 45 degrees 0.785398 rad

Height

h_b = 0 m

Longitude

λ_b = 45 degrees 0.785398 rad

ECEFto NED coordinate transformation matrix

From (2.150),

$$\mathbf{C}^n_e = \begin{pmatrix} -\sin L_b \cos \lambda_b & -\sin L_b \sin \lambda_b & \cos L_b \\ -\sin \lambda_b & \cos \lambda_b & 0 \\ -\cos L_b \cos \lambda_b & -\cos L_b \sin \lambda_b & -\sin L_b \end{pmatrix}$$

$$\mathbf{C}^n_e = \begin{pmatrix} -0.5 & -0.5 & 0.707107 \\ -0.70710678 & 0.707107 & 0 \\ -0.5 & -0.5 & -0.70711 \end{pmatrix}$$

Velocity transformation

From (2.152), $\mathbf{v}^n_{eb} = \mathbf{C}^n_e \mathbf{v}^e_{eb}$

$$\mathbf{v}^n_{eb} = \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix} \begin{matrix} \text{m s}^{-1} \\ \text{m s}^{-1} \\ \text{m s}^{-1} \end{matrix}$$

Acceleration transformation

From (2.152), $\mathbf{a}^n_{eb} = \mathbf{C}^n_e \mathbf{a}^e_{eb}$

$$\mathbf{a}^n_{eb} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \begin{matrix} \text{m s}^{-2} \\ \text{m s}^{-2} \\ \text{m s}^{-2} \end{matrix}$$

Angular rate transformation

From (2.153), $\boldsymbol{\omega}^n_{nb} = \mathbf{C}^n_e \boldsymbol{\omega}^e_{eb} - \boldsymbol{\omega}^n_{en}$

Transverse Radius of Curvature

From (2.106), $R_E(L_b) = \dfrac{R_0}{\sqrt{1 - e^2 \sin^2 L_b}}$

R_E = 6.39E+06

Meridian Radius of Curvature

$R_N(L_b) = \dfrac{R_0(1 - e^2)}{(1 - e^2 \sin^2 L_b)^{3/2}}$

R_N = 6.37E+06

From (5.44)

$$\boldsymbol{\omega}_{en}^{n} = \begin{pmatrix} v_{eb,E}^{n} / \left(R_E(L_b) + h_b\right) \\ -v_{eb,N}^{n} / \left(R_N(L_b) + h_b\right) \\ -v_{eb,E}^{n} \tan L_b / \left(R_E(L_b) + h_b\right) \end{pmatrix}$$

$\boldsymbol{\omega}_{en}^{n} =$

7.83E-07	rad s^{-1}
-7.85E-07	rad s^{-1}
-7.83E-07	rad s^{-1}

$\boldsymbol{\omega}_{nb}^{n} =$

0.001	rad s^{-1}
5.72806E-20	rad s^{-1}
2.13876E-20	rad s^{-1}