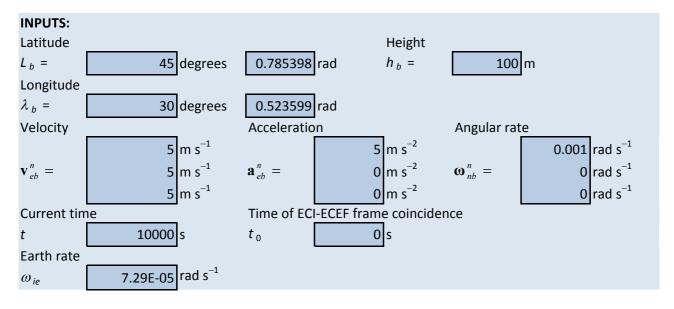
EXAMPLE 2.6(a)

Transformation of reference frame from ECEF frame to ECI frame and resolving axes from local navigation frame to ECI frame



Transverse Radius of Curvature

From (2.106),
$$R_E(L_b) = \frac{R_0}{\sqrt{1 - e^2 \sin^2 L_b}}$$

Meridian Radius of Curvature
$$P_{0}\left(1-a^{2}\right)$$

$$R_E(L_b) = \frac{R_0}{\sqrt{1 - e^2 \sin^2 L_b}} \qquad R_N(L_b) = \frac{R_0 (1 - e^2)}{(1 - e^2 \sin^2 L_b)^{3/2}}$$

$$R_E = 6.39E+06$$

$$R_N = 6.37E+06$$

Cartesian Position

From (2.112),
$$x_{eb}^{e} = \left(R_{E}(L_{b}) + h_{b}\right) \cos L_{b} \cos \lambda_{b}$$

$$y_{eb}^{e} = \left(R_{E}(L_{b}) + h_{b}\right) \cos L_{b} \sin \lambda_{b}$$

$$z_{eb}^{e} = \left[\left(1 - e^{2}\right)R_{E}(L_{b}) + h_{b}\right] \sin L_{b}$$

$$\mathbf{r}_{eb}^{e} = \begin{bmatrix} 3912409.702 & m \\ 2258830.795 & m \\ 4487419.12 & m \end{bmatrix}$$

ECEF to ECI Coordinate transformation matrix

From (2.145),
$$\mathbf{C}_{e}^{i} = \begin{pmatrix} \cos \omega_{ie} (t - t_{0}) & -\sin \omega_{ie} (t - t_{0}) & 0 \\ \sin \omega_{ie} (t - t_{0}) & \cos \omega_{ie} (t - t_{0}) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{C}_{e}^{i} = egin{array}{cccc} 0.745699997 & -0.66628 & 0 \\ 0.666281858 & 0.7457 & 0 \\ 0 & 0 & 1 \\ \end{array}$$

Cartesian position transformation

From (2.146),
$$\mathbf{r}_{ib}^{i} = \mathbf{C}_{e}^{i} \mathbf{r}_{eb}^{e}$$

$$\mathbf{r}_{ib}^{i} = \begin{bmatrix} 1412465.926 & m \\ 4291177.722 & m \\ 4487419.12 & m \end{bmatrix}$$

Skew symmetric matrix of Earth rate

,			
$\Omega_{ie}^{e} =$	0	-7.29E-05	0
$\mathbf{\Omega}_{ie}^{i} =$	7.29E-05	0	0
	0	0	0

NED to ECI coordinate transformation matrix From (2.154),

From (2.154),
$$\mathbf{C}_{n}^{i} = \begin{pmatrix} -\sin L_{b} \cos \left(\lambda_{b} + \omega_{ie}(t - t_{0})\right) & -\sin \left(\lambda_{b} + \omega_{ie}(t - t_{0})\right) & -\cos L_{b} \cos \left(\lambda_{b} + \omega_{ie}(t - t_{0})\right) \\ -\sin L_{b} \sin \left(\lambda_{b} + \omega_{ie}(t - t_{0})\right) & \cos \left(\lambda_{b} + \omega_{ie}(t - t_{0})\right) & -\cos L_{b} \sin \left(\lambda_{b} + \omega_{ie}(t - t_{0})\right) \\ \cos L_{b} & 0 & -\sin L_{b} \end{pmatrix}$$

$$\left(\lambda_{b} + \omega_{ie}(t - t_{0})\right) = \boxed{1.25281}$$

$$\begin{bmatrix} -0.22107991 & -0.94987 & -0.22108 \\ -0.67165741 & 0.312654 & -0.67166 \\ 0.707106781 & 0 & -0.70711 \end{bmatrix}$$

$$(\lambda_b + \omega_{ie}(t - t_0)) =$$
 1.25281
 $\mathbf{C}_n^i =$ -0.22107991 -0.94987 -0.22108
-0.67165741 0.312654 -0.67166
0.707106781 0 -0.70711

Velocity transformation

From (2.155),
$$\mathbf{v}_{ib}^i = \mathbf{C}_n^i \mathbf{v}_{eb}^n + \mathbf{C}_e^i \mathbf{\Omega}_{ie}^e \mathbf{r}_{eb}^e$$

$$\mathbf{v}_{ib}^{i} = \begin{bmatrix} -319.877749 & \text{m s}^{-1} \\ 97.84533668 & \text{m s}^{-1} \\ 4.44089E-16 & \text{m s}^{-1} \end{bmatrix}$$

Acceleration transformation

From (2.156),
$$\mathbf{a}_{ib}^{i} = \mathbf{C}_{n}^{i} \left(\mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{n} \mathbf{v}_{eb}^{n} \right) + \mathbf{C}_{e}^{i} \Omega_{ie}^{e} \Omega_{ie}^{e} \mathbf{r}_{eb}^{e}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{i} \mathbf{C}_{n}^{i} \mathbf{v}_{eb}^{n} + \mathbf{C}_{e}^{i} \Omega_{ie}^{e} \Omega_{ie}^{e} \mathbf{r}_{eb}^{e}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{i} \mathbf{C}_{n}^{i} \mathbf{v}_{eb}^{n} + \mathbf{C}_{e}^{i} \Omega_{ie}^{e} \Omega_{ie}^{e} \mathbf{r}_{eb}^{e}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{i} \mathbf{C}_{n}^{i} \mathbf{v}_{eb}^{n} + \mathbf{C}_{e}^{i} \Omega_{ie}^{e} \Omega_{ie}^{e} \mathbf{r}_{eb}^{e}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{i} \mathbf{C}_{n}^{i} \mathbf{v}_{eb}^{n} + \mathbf{C}_{e}^{i} \Omega_{ie}^{e} \Omega_{ie}^{e} \mathbf{r}_{eb}^{e}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{i} \mathbf{C}_{n}^{i} \mathbf{v}_{eb}^{n} + \mathbf{C}_{e}^{i} \Omega_{ie}^{e} \mathbf{n}_{eb}^{e}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{i} \mathbf{C}_{n}^{i} \mathbf{v}_{eb}^{n} + \mathbf{C}_{e}^{i} \Omega_{ie}^{e} \mathbf{n}_{eb}^{e}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{i} \mathbf{C}_{n}^{i} \mathbf{v}_{eb}^{n} + \mathbf{C}_{e}^{i} \Omega_{ie}^{e} \mathbf{n}_{eb}^{e}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{n} \mathbf{c}_{n}^{i} \mathbf{v}_{eb}^{n} + \mathbf{C}_{e}^{i} \Omega_{ie}^{e} \mathbf{n}_{eb}^{e}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{n} \mathbf{c}_{n}^{i} \mathbf{v}_{eb}^{n} + \mathbf{C}_{e}^{i} \Omega_{ie}^{e} \mathbf{n}_{eb}^{e}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{n} \mathbf{c}_{n}^{i} \mathbf{v}_{eb}^{n} + \mathbf{C}_{e}^{i} \Omega_{ie}^{e} \mathbf{n}_{eb}^{e}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{n} \mathbf{c}_{n}^{i} \mathbf{v}_{eb}^{n} + \mathbf{C}_{e}^{i} \Omega_{ie}^{e} \mathbf{n}_{eb}^{e}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{n} \mathbf{c}_{n}^{i} \mathbf{c}_{eb}^{n} \mathbf{c}_{eb}^{n}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{n} \mathbf{c}_{eb}^{n} \mathbf{c}_{eb}^{n} \mathbf{c}_{eb}^{n}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} + 2\Omega_{ie}^{n} \mathbf{c}_{eb}^{n} \mathbf{c}_{eb}^{n} \mathbf{c}_{eb}^{n}$$

$$= \mathbf{C}_{n}^{i} \mathbf{a}_{eb}^{n} \mathbf{c}_{eb}^{n} \mathbf{c}_{eb}^{n}$$

Angular rate transformation

From (2.157),
$$\mathbf{\omega}_{ib}^{i} = \mathbf{C}_{n}^{i} \left(\mathbf{\omega}_{nb}^{n} + \mathbf{\omega}_{en}^{n}\right) + \mathbf{\omega}_{ie}^{i}$$

From (5.44)
$$\mathbf{\omega}_{en}^{n} = \begin{pmatrix} v_{eb,E}^{n} / (R_{E}(L_{b}) + h_{b}) \\ -v_{eb,N}^{n} / (R_{N}(L_{b}) + h_{b}) \\ -v_{eb,E}^{n} \tan L_{b} / (R_{E}(L_{b}) + h_{b}) \end{pmatrix}$$

$$\mathbf{\omega}_{en}^{n} = \begin{bmatrix} 7.83\text{E-07} & \text{rad s}^{-1} \\ -7.85\text{E-07} & \text{rad s}^{-1} \\ -7.83\text{E-07} & \text{rad s}^{-1} \end{bmatrix}$$

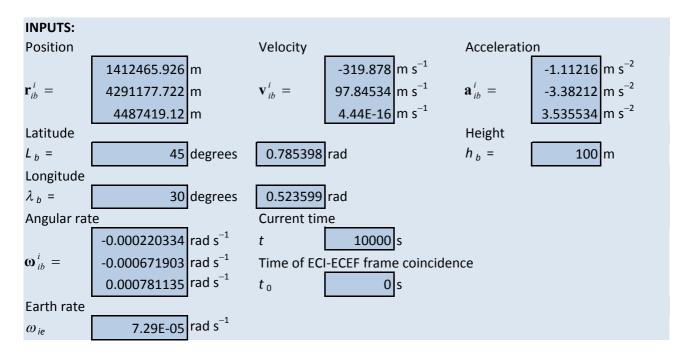
$$\mathbf{\omega}_{ie}^{i} = \begin{bmatrix}
0 & \text{rad s}^{-1} \\
0 & \text{rad s}^{-1} \\
7.29E-05 & \text{rad s}^{-1}
\end{bmatrix}$$

$$\mathbf{\omega}_{ib}^{i} = \begin{bmatrix} -0.00022033 & \text{rad s}^{-1} \\ -0.0006719 & \text{rad s}^{-1} \\ 0.000781135 & \text{rad s}^{-1} \end{bmatrix}$$

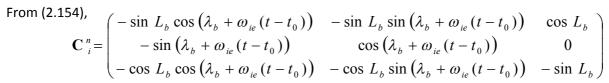
EXAMPLE 2.6(b)

Transformation of reference frame from ECI frame to ECEF frame and resolving axes from ECI frame to local navigation frame

See Example 2.2 for determination of Latitude, Longitude, and Height from Cartesian Position



Coordinate transformation matrix



$$(\lambda_b + \omega_{ie}(t - t_0)) =$$
 1.25E+00

$$\mathbf{C}_{i}^{n} = \begin{bmatrix} -0.221079914 & -0.67166 & 0.707107 \\ -0.949867014 & 0.312654 & 0 \\ -0.221079914 & -0.67166 & -0.70711 \end{bmatrix}$$

Skew symmetric matrix of Earth rate

$$\mathbf{\Omega}_{ie}^{i} = egin{pmatrix} 0 & -7.29 \text{E-}05 & 0 & 0 \ 7.29 \text{E-}05 & 0 & 0 \ 0 & 0 & 0 \end{pmatrix}$$

Velocity transformation

From (2.155),
$$\mathbf{v}_{eb}^{n} = \mathbf{C}_{i}^{n} \left(\mathbf{v}_{ib}^{i} - \mathbf{\Omega}_{ie}^{i} \mathbf{r}_{ib}^{i} \right)$$

$$\mathbf{v}_{eb}^{n} = \begin{bmatrix} 5 & \text{m s}^{-1} \\ 5 & \text{m s}^{-1} \\ 5 & \text{m s}^{-1} \end{bmatrix}$$

Acceleration transformation

From (2.156),
$$\mathbf{a}_{eb}^{n} = \mathbf{C}_{i}^{n} \left(\mathbf{a}_{ib}^{i} - 2\Omega_{ie}^{i} \mathbf{v}_{ib}^{i} + \Omega_{ie}^{i} \Omega_{ie}^{i} \mathbf{r}_{ib}^{i} \right)$$

$$\mathbf{a}_{eb}^{n} = \begin{bmatrix} 5 & \text{m s}^{-2} \\ -2.22045\text{E-}16 & \text{m s}^{-2} \\ 0 & \text{m s}^{-2} \end{bmatrix}$$

Angular rate transformation

From (2.157),
$$\mathbf{\omega}_{nb}^{n} = \mathbf{C}_{i}^{n} \left(\mathbf{\omega}_{ib}^{i} - \mathbf{\omega}_{ie}^{i}\right) - \mathbf{\omega}_{en}^{n}$$

Transverse Radius of Curvature

 $R_E =$

Transverse Radius of Curvature Meridian Radius of Curvature From (2.106),
$$R_E(L_b) = \frac{R_0}{\sqrt{1-e^2\sin^2 L_b}}$$
 $R_N(L_b) = \frac{R_0 \left(1-e^2\right)}{\left(1-e^2\sin^2 L_b\right)^{3/2}}$

Meridian Radius of Curvature

$$R_N(L_b) = \frac{R_0 (1 - e^2)}{(1 - e^2 \sin^2 L_b)^{3/2}}$$

$$R_N = 6.37E+06$$

From (5.44)
$$\mathbf{\omega}_{en}^{n} = \begin{pmatrix} v_{eb,E}^{n} / (R_{E}(L_{b}) + h_{b}) \\ -v_{eb,N}^{n} / (R_{N}(L_{b}) + h_{b}) \\ -v_{eb,E}^{n} \tan L_{b} / (R_{E}(L_{b}) + h_{b}) \end{pmatrix}$$

$$\mathbf{\omega}_{en}^{n} = \begin{bmatrix} 7.83\text{E-07} & \text{rad s}^{-1} \\ -7.85\text{E-07} & \text{rad s}^{-1} \\ -7.83\text{E-07} & \text{rad s}^{-1} \end{bmatrix}$$

$$\mathbf{\omega}_{ie}^{i} = \begin{bmatrix} 0 & \text{rad s}^{-1} \\ 0 & \text{rad s}^{-1} \\ 7.29\text{E-}05 & \text{rad s}^{-1} \end{bmatrix}$$

$$\mathbf{\omega}_{nb}^{n} = \begin{bmatrix} 0.001 & \text{rad s}^{-1} \\ -1.05879\text{E-}21 & \text{rad s}^{-1} \\ 4.93397\text{E-}20 & \text{rad s}^{-1} \end{bmatrix}$$