**EXAMPLE 5.1**

**Inertial Navigation in Two Dimensions**

**INPUTS:**

**Initial navigation solution**

| | | | | | |
|---|---|---|---|---|---|
| Initial position | $x^p_{pb} =$ | 0 | m | | |
| | $y^p_{pb} =$ | 0 | m | | |
| Initial velocity | $v^p_{pb,x} =$ | 0 | m s$^{-1}$ | | |
| | $v^p_{pb,y} =$ | 0 | m s$^{-1}$ | | |
| Initial heading | $\psi_{pb} =$ | 45 | deg | 0.785398 | rad |

**Epoch 1 Measurements**

| | | | |
|---|---|---|---|
| Time interval | $\tau =$ | 0.5 | s |
| Angular rate | $\omega^b_{pb,z} =$ | 0.5 | rad s$^{-1}$ |
| Body x-axis acceleration | $a^b_{pb,x} =$ | 2 | m s$^{-2}$ |
| Body y-axis acceleration | $a^b_{pb,y} =$ | 0.1 | m s$^{-2}$ |

Note: The time interval would normally be much shorter.

**Update Heading**

From (5.5), $\psi_{pb}(t+\tau) = \psi_{pb}(t) + \omega^b_{pb,z}\tau$ Assuming a constant angular rate

$\psi_{pb} =$ 1.035398163 rad 59.32394 deg

**Transform acceleration to *p* frame**

From (5.6),
$$\begin{pmatrix} a^p_{pb,x} \\ a^p_{pb,y} \end{pmatrix} = \begin{pmatrix} \cos\psi_{pb} & -\sin\psi_{pb} \\ \sin\psi_{pb} & \cos\psi_{pb} \end{pmatrix} \begin{pmatrix} a^b_{pb,x} \\ a^b_{pb,y} \end{pmatrix}$$

$$\begin{pmatrix} \cos\psi_{pb} & -\sin\psi_{pb} \\ \sin\psi_{pb} & \cos\psi_{pb} \end{pmatrix} =$$

Before update

| | |
|---|---|
| 0.707106781 | -0.70711 |
| 0.707106781 | 0.707107 |

After update

| | |
|---|---|
| 0.510184 | -0.86007 |
| 0.860066 | 0.510184 |

Average

| | |
|---|---|
| 0.608645154 | -0.78359 |
| 0.783586171 | 0.608645 |

$$\begin{pmatrix} a^p_{pb,x} \\ a^p_{pb,y} \end{pmatrix} =$$

| | |
|---|---|
| 1.138931691 | m s$^{-2}$ |
| 1.628036858 | m s$^{-2}$ |

**Update Velocity**

From (5.3),
$$\begin{pmatrix} v^p_{pb,x}(t+\tau) \\ v^p_{pb,y}(t+\tau) \end{pmatrix} = \begin{pmatrix} v^p_{pb,x}(t) \\ v^p_{pb,y}(t) \end{pmatrix} + \begin{pmatrix} a^p_{pb,x} \\ a^p_{pb,y} \end{pmatrix}\tau$$

| | | |
|---|---|---|
| $v^p_{pb,x} =$ | 0.569465845 | m s$^{-1}$ |
| $v^p_{pb,y} =$ | 0.814018429 | m s$^{-1}$ |

**Update Position**

From (5.4),
$$\begin{pmatrix} x^p_{pb}(t+\tau) \\ y^p_{pb}(t+\tau) \end{pmatrix} = \begin{pmatrix} x^p_{pb}(t) \\ y^p_{pb}(t) \end{pmatrix} + \left[\begin{pmatrix} v^p_{pb,x}(t) \\ v^p_{pb,y}(t) \end{pmatrix} + \begin{pmatrix} v^p_{pb,x}(t+\tau) \\ v^p_{pb,y}(t+\tau) \end{pmatrix}\right]\frac{\tau}{2}$$

| | | |
|---|---|---|
| $x^p_{pb} =$ | 0.142366461 | m |
| $y^p_{pb} =$ | 0.203504607 | m |

**Epoch 2 Measurements**

| | | | |
|---|---|---|---|
| Time interval | $\tau =$ | 0.5 | s |
| Angular rate | $\omega^b_{pb,z} =$ | 0.2 | rad $s^{-1}$ |
| Body x-axis acceleration | $a^b_{pb,x} =$ | 5 | m $s^{-2}$ |
| Body y-axis acceleration | $a^b_{pb,y} =$ | 0.1 | m $s^{-2}$ |

**Update Heading**

From (5.5), $\psi_{pb}(t+\tau) = \psi_{pb}(t) + \omega^b_{pb,z}\tau$ Assuming a constant angular rate

$\psi_{pb}$ = 1.135398163 rad 65.05352 deg

**Transform acceleration to *p* frame**

From (5.6), $\begin{pmatrix} a^p_{pb,x} \\ a^p_{pb,y} \end{pmatrix} = \begin{pmatrix} \cos\psi_{pb} & -\sin\psi_{pb} \\ \sin\psi_{pb} & \cos\psi_{pb} \end{pmatrix} \begin{pmatrix} a^b_{pb,x} \\ a^b_{pb,y} \end{pmatrix}$

$\begin{pmatrix} \cos\psi_{pb} & -\sin\psi_{pb} \\ \sin\psi_{pb} & \cos\psi_{pb} \end{pmatrix} =$

Before update

| | |
|---|---|
| 0.510183526 | -0.86007 |
| 0.860065561 | 0.510184 |

After update

| | |
|---|---|
| 0.421771 | -0.9067 |
| 0.906702 | 0.421771 |

Average

| | |
|---|---|
| 0.465977488 | -0.88338 |
| 0.883383871 | 0.465977 |

$\begin{pmatrix} a^p_{pb,x} \\ a^p_{pb,y} \end{pmatrix} =$

| | |
|---|---|
| 2.241549055 | m $s^{-2}$ |
| 4.463517102 | m $s^{-2}$ |

**Update Velocity**

From (5.3), $\begin{pmatrix} v^p_{pb,x}(t+\tau) \\ v^p_{pb,y}(t+\tau) \end{pmatrix} = \begin{pmatrix} v^p_{pb,x}(t) \\ v^p_{pb,y}(t) \end{pmatrix} + \begin{pmatrix} a^p_{pb,x} \\ a^p_{pb,y} \end{pmatrix}\tau$

$v^p_{pb,x} =$ 1.690240373 m $s^{-1}$

$v^p_{pb,y} =$ 3.04577698 m $s^{-1}$

**Update Position**

From (5.4), $\begin{pmatrix} x^p_{pb}(t+\tau) \\ y^p_{pb}(t+\tau) \end{pmatrix} = \begin{pmatrix} x^p_{pb}(t) \\ y^p_{pb}(t) \end{pmatrix} + \left[ \begin{pmatrix} v^p_{pb,x}(t) \\ v^p_{pb,y}(t) \end{pmatrix} + \begin{pmatrix} v^p_{pb,x}(t+\tau) \\ v^p_{pb,y}(t+\tau) \end{pmatrix} \right] \frac{\tau}{2}$

$x^p_{pb} =$ 0.707293016 m

$y^p_{pb} =$ 1.168453459 m

**Epoch 3 Measurements**

| | | | |
|---|---|---|---|
| Time interval | $\tau =$ | 0.5 | s |
| Angular rate | $\omega^b_{pb,z} =$ | 0.1 | rad $s^{-1}$ |
| Body x-axis acceleration | $a^b_{pb,x} =$ | 2 | m $s^{-2}$ |
| Body y-axis acceleration | $a^b_{pb,y} =$ | -0.05 | m $s^{-2}$ |

**Update Heading**

From (5.5), $\psi_{pb}(t+\tau) = \psi_{pb}(t) + \omega^b_{pb,z}\tau$ Assuming a constant angular rate

$\psi_{pb}$ = 1.185398163 rad 67.91831 deg

**Transform acceleration to *p* frame**

From (5.6), $$\begin{pmatrix} a^p_{pb,x} \\ a^p_{pb,y} \end{pmatrix} = \begin{pmatrix} \cos\psi_{pb} & -\sin\psi_{pb} \\ \sin\psi_{pb} & \cos\psi_{pb} \end{pmatrix} \begin{pmatrix} a^b_{pb,x} \\ a^b_{pb,y} \end{pmatrix}$$

$$\begin{pmatrix} \cos\psi_{pb} & -\sin\psi_{pb} \\ \sin\psi_{pb} & \cos\psi_{pb} \end{pmatrix} =$$

Before update

| | |
|---|---|
| 0.42177145 | -0.9067 |
| 0.90670218 | 0.421771 |

After update

| | |
|---|---|
| 0.375928 | -0.92665 |
| 0.926649 | 0.375928 |

Average

| | |
|---|---|
| 0.398849787 | -0.91668 |
| 0.916675503 | 0.39885 |

$$\begin{pmatrix} a^p_{pb,x} \\ a^p_{pb,y} \end{pmatrix} =$$

| | |
|---|---|
| 0.84353335 | m s$^{-2}$ |
| 1.813408516 | m s$^{-2}$ |

**Update Velocity**

From (5.3), $$\begin{pmatrix} v^p_{pb,x}(t+\tau) \\ v^p_{pb,y}(t+\tau) \end{pmatrix} = \begin{pmatrix} v^p_{pb,x}(t) \\ v^p_{pb,y}(t) \end{pmatrix} + \begin{pmatrix} a^p_{pb,x} \\ a^p_{pb,y} \end{pmatrix} \tau$$

| | | |
|---|---|---|
| $v^p_{pb,x}$ = | 2.112007048 | m s$^{-1}$ |
| $v^p_{pb,y}$ = | 3.952481238 | m s$^{-1}$ |

**Update Position**

From (5.4), $$\begin{pmatrix} x^p_{pb}(t+\tau) \\ y^p_{pb}(t+\tau) \end{pmatrix} = \begin{pmatrix} x^p_{pb}(t) \\ y^p_{pb}(t) \end{pmatrix} + \left[ \begin{pmatrix} v^p_{pb,x}(t) \\ v^p_{pb,y}(t) \end{pmatrix} + \begin{pmatrix} v^p_{pb,x}(t+\tau) \\ v^p_{pb,y}(t+\tau) \end{pmatrix} \right] \frac{\tau}{2}$$

| | | |
|---|---|---|
| $x^p_{pb}$ = | 1.657854871 | m |
| $y^p_{pb}$ = | 2.918018014 | m |

**Epoch 4 Measurements**

| | | | |
|---|---|---|---|
| Time interval | $\tau$ = | 0.5 | s |
| Angular rate | $\omega^b_{pb,z}$ = | -0.1 | rad s$^{-1}$ |
| Body x-axis acceleration | $a^b_{pb,x}$ = | 0 | m s$^{-2}$ |
| Body y-axis acceleration | $a^b_{pb,y}$ = | 0 | m s$^{-2}$ |

**Update Heading**

From (5.5), $\psi_{pb}(t+\tau) = \psi_{pb}(t) + \omega^b_{pb,z}\tau$ Assuming a constant angular rate

| | | | | |
|---|---|---|---|---|
| $\psi_{pb}$ = | 1.135398163 | rad | 65.05352 | deg |

**Transform acceleration to *p* frame**

From (5.6), $$\begin{pmatrix} a^p_{pb,x} \\ a^p_{pb,y} \end{pmatrix} = \begin{pmatrix} \cos\psi_{pb} & -\sin\psi_{pb} \\ \sin\psi_{pb} & \cos\psi_{pb} \end{pmatrix} \begin{pmatrix} a^b_{pb,x} \\ a^b_{pb,y} \end{pmatrix}$$

$$\begin{pmatrix} \cos\psi_{pb} & -\sin\psi_{pb} \\ \sin\psi_{pb} & \cos\psi_{pb} \end{pmatrix} =$$

Before update

| | |
|---|---|
| 0.375928124 | -0.92665 |
| 0.926648825 | 0.375928 |

After update

| | |
|---|---|
| 0.421771 | -0.9067 |
| 0.906702 | 0.421771 |

Average

| | |
|---|---|
| 0.398849787 | -0.91668 |
| 0.916675503 | 0.39885 |

$$\begin{pmatrix} a^p_{pb,x} \\ a^p_{pb,y} \end{pmatrix} =$$

| | |
|---|---|
| 0 | m s$^{-2}$ |
| 0 | m s$^{-2}$ |

**Update Velocity**

From (5.3),
$$\begin{pmatrix} v^p_{pb,x}(t+\tau) \\ v^p_{pb,y}(t+\tau) \end{pmatrix} = \begin{pmatrix} v^p_{pb,x}(t) \\ v^p_{pb,y}(t) \end{pmatrix} + \begin{pmatrix} a^p_{pb,x} \\ a^p_{pb,y} \end{pmatrix}\tau$$

| | | |
|---|---|---|
| $v^p_{pb,x} =$ | 2.112007048 | m s$^{-1}$ |
| $v^p_{pb,y} =$ | 3.952481238 | m s$^{-1}$ |

**Update Position**

From (5.4),
$$\begin{pmatrix} x^p_{pb}(t+\tau) \\ y^p_{pb}(t+\tau) \end{pmatrix} = \begin{pmatrix} x^p_{pb}(t) \\ y^p_{pb}(t) \end{pmatrix} + \left[\begin{pmatrix} v^p_{pb,x}(t) \\ v^p_{pb,y}(t) \end{pmatrix} + \begin{pmatrix} v^p_{pb,x}(t+\tau) \\ v^p_{pb,y}(t+\tau) \end{pmatrix}\right]\frac{\tau}{2}$$

| | | |
|---|---|---|
| $x^p_{pb} =$ | 2.713858395 | m |
| $y^p_{pb} =$ | 4.894258633 | m |