EXAMPLE 8.1
Calculation of Range, Line of sight, Range-rate, and Elevation

INPUTS:

Satellite position (Cartesian ECEF)	$\mathbf{r}^e_{es}\left(t^s_{st,a}\right) =$	26580000	m		
		0	m		
		0	m		
Satellite velocity (ECEF)	$\mathbf{v}^e_{es}\left(t^s_{st,a}\right) =$	0	m s^{-1}		
		1755	m s^{-1}		
		3170	m s^{-1}		
User latitude	L_a =	45	deg	0.785398	rad
User longitude	λ_a =	10	deg	0.174533	rad
User height	h_a =	1000	m		
Equatorial radius	R_0 =	6378137	m		
Ellipsoid eccentricity	e =	0.08181919			
Earth rate	ω_{ie} =	7.29E-05	rad s^{-1}		
Speed of light	c =	299792458	m s^{-1}		

Convert user position to Cartesian ECEF
Transverse Radius of Curvature
From (2.106),
$$R_E(L_a) = \frac{R_0}{\sqrt{1 - e^2 \sin^2 L_a}}$$

R_E = 6388838.29 m

Cartesian Position (ECEF-frame resolved and referenced)
From (2.112),
$$\begin{aligned} x^e_{ea} &= \left(R_E(L_a) + h_a\right)\cos L_a \cos \lambda_a \\ y^e_{ea} &= \left(R_E(L_a) + h_a\right)\cos L_a \sin \lambda_a \\ z^e_{ea} &= \left[\left(1 - e^2\right)R_E(L_a) + h_a\right]\sin L_a \end{aligned}$$

$\mathbf{r}^e_{ea}\left(t^s_{sa,a}\right) =$	4448958.522	m
	784471.4236	m
	4487348.409	m

Range
From (8.31),
$$r_{as} = \left|\mathbf{r}^e_{es}(t^s_{st,a}) - \mathbf{r}^e_{ea}(t^s_{sa,a})\right| + \delta\rho^s_{ie,a}$$

$\mathbf{r}^e_{es}\left(t^s_{st,a}\right) - \mathbf{r}^e_{ea}\left(t^s_{sa,a}\right) =$	22131041	m
	-784471.4	m
	-4487348	m

Sagnac correction:
From (8.32),
$$\delta\rho^s_{ie,a} \approx \frac{\omega_{ie}}{c}\left[y^e_{es}(t^s_{st,a})x^e_{ea}(t^s_{sa,a}) - x^e_{es}(t^s_{st,a})y^e_{ea}(t^s_{sa,a})\right]$$

$\delta\rho^s_{ie,a} \approx$ -5.0718326 m

r_{as} = 22595009.6 m

Line-of-sight unit vector

From (8.41), $\mathbf{u}^e_{as} \approx \dfrac{\mathbf{r}^e_{es}(t^s_{st,a}) - \mathbf{r}^e_{ea}(t^s_{sa,a})}{\left|\mathbf{r}^e_{es}(t^s_{st,a}) - \mathbf{r}^e_{ea}(t^s_{sa,a})\right|}$

$$\mathbf{u}^e_{as} \approx \begin{pmatrix} 0.979465683 \\ -0.03471878 \\ -0.19859905 \end{pmatrix}$$

Range rate

From (8.45), $\dot{r}_{as} = \mathbf{u}^{e\,\mathrm{T}}_{as}\left(\mathbf{v}^e_{es}(t^s_{st,a}) - \mathbf{v}^e_{ea}(t^s_{sa,a})\right) + \delta\dot{\rho}^s_{ie,a}$

In this example, $\mathbf{v}^e_{ea}(t^s_{sa,a}) = \mathbf{0}$

Sagnac correction:

From (8.46), $\delta\dot{\rho}^s_{ie,a} \approx \dfrac{\omega_{ie}}{c}\begin{pmatrix} v^e_{es,y}(t^s_{st,a})x^e_{ea}(t^s_{sa,a}) + y^e_{es}(t^s_{st,a})v^e_{ea,x}(t^s_{sa,a}) \\ - v^e_{es,x}(t^s_{st,a})y^e_{ea}(t^s_{sa,a}) - x^e_{es}(t^s_{st,a})v^e_{ea,y}(t^s_{sa,a}) \end{pmatrix}$

$\delta\dot{\rho}^s_{ie,a} \approx$ 0.001899189 m s^{-1}

$\dot{r}_{as} =$ -690.488549 m s^{-1}

Transformation of line of sight to north, east, down

ECEF to NED coordinate transformation matrix

From (2.150), $\mathbf{C}^n_e = \begin{pmatrix} -\sin L_b \cos\lambda_b & -\sin L_b \sin\lambda_b & \cos L_b \\ -\sin\lambda_b & \cos\lambda_b & 0 \\ -\cos L_b \cos\lambda_b & -\cos L_b \sin\lambda_b & -\sin L_b \end{pmatrix}$

$$\mathbf{C}^n_e = \begin{pmatrix} -0.69636424 & -0.122788 & 0.70710678 \\ -0.17364818 & 0.9848078 & 0 \\ -0.69636424 & -0.122788 & -0.70710678 \end{pmatrix}$$

From (8.39), $\mathbf{u}^n_{as} = \mathbf{C}^n_e \mathbf{u}^e_{as}$

$$\mathbf{u}^n_{as} = \begin{pmatrix} -0.81823257 \\ -0.20427376 \\ -0.5373711 \end{pmatrix}$$

Elevation and azimuth

From (8.57), $\theta^{as}_{nu} = -\arcsin\left(u^n_{as,D}\right)$, $\psi^{as}_{nu} = \arctan_2\left(u^n_{as,E}, u^n_{as,N}\right)$

$\theta^{as}_{nu} =$ 0.567316775 rad 32.5048569 deg

$\psi^{as}_{nu} =$ -2.89694113 rad -165.9825 deg

Note: The arguments of the Excel ATAN2 function are the opposite way round