EXAMPLE 9.2(a)
Dilution of Precision Calculation for Optimal Four-satellite GNSS Geometry

INPUTS:

Satellite	Azimuth	Elevation	Azimuth	Elevation
1	0	0	0	0
2	120	0	2.0943951	0
3	240	0	4.1887902	0
4	0	-90	0	-1.5708
	degrees	degrees	radians	radians

Calculate Measurement (Geometry) Matrix

From (9.174),

$$\mathbf{H}_G^n = \begin{pmatrix} -u_{a1,N}^n & -u_{a1,E}^n & -u_{a1,D}^n & 1 \\ -u_{a2,N}^n & -u_{a2,E}^n & -u_{a2,D}^n & 1 \\ -u_{a3,N}^n & -u_{a3,E}^n & -u_{a3,D}^n & 1 \\ -u_{a4,N}^n & -u_{a4,E}^n & -u_{a4,D}^n & 1 \end{pmatrix}$$

$\mathbf{H}_G^n =$

-1	0	0	1
0.5	-0.866025	0	1
0.5	0.8660254	0	1
-6.1257E-17	0	1	1

Calculate Local-navigation-frame Cofactor Matrix

From (9.180), $\mathbf{\Pi}^n = \left(\mathbf{H}_G^{n\,\mathrm{T}} \mathbf{H}_G^n\right)^{-1}$

$\mathbf{\Pi}^n =$

0.666666667	-1.73E-16	9.0182E-17	-4.9E-17
-1.727E-16	0.6666667	-7.4015E-17	7.4E-17
9.01815E-17	-7.4E-17	1.33333333	-0.33333
-4.9343E-17	7.401E-17	-0.33333333	0.333333

Dilutions of Precision

From (9.180),

$$\begin{pmatrix} D_N^2 & \cdot & \cdot & \cdot \\ \cdot & D_E^2 & \cdot & \cdot \\ \cdot & \cdot & D_V^2 & \cdot \\ \cdot & \cdot & \cdot & D_T^2 \end{pmatrix} = \mathbf{\Pi}^n$$

NDOP = 0.816496581
EDOP = 0.816496581
VDOP = 1.154700538
TDOP = 0.577350269

From (9.181),

$$D_H = \sqrt{D_N^2 + D_E^2}$$
$$D_P = \sqrt{D_N^2 + D_E^2 + D_V^2}$$
$$D_G = \sqrt{D_N^2 + D_E^2 + D_V^2 + D_T^2}$$

HDOP = 1.154700538
PDOP = 1.632993162
GDOP = 1.732050808

EXAMPLE 9.2(b)
Dilution of Precision Calculation for Poor GNSS Geometry due to Lack of Azimuth Variation

INPUTS:

Satellite	Azimuth	Elevation	Azimuth	Elevation
1	60	-30	1.04719755	-0.5236
2	90	-30	1.57079633	-0.5236
3	120	-30	2.0943951	-0.5236
4	90	-60	1.57079633	-1.0472
	degrees	degrees	radians	radians

Calculate Measurement (Geometry) Matrix

From (9.174),

$$\mathbf{H}_G^n = \begin{pmatrix} -u_{a1,N}^n & -u_{a1,E}^n & -u_{a1,D}^n & 1 \\ -u_{a2,N}^n & -u_{a2,E}^n & -u_{a2,D}^n & 1 \\ -u_{a3,N}^n & -u_{a3,E}^n & -u_{a3,D}^n & 1 \\ -u_{a4,N}^n & -u_{a4,E}^n & -u_{a4,D}^n & 1 \end{pmatrix}$$

$\mathbf{H}_G^n =$

-0.4330127	-0.75	0.5	1
-5.305E-17	-0.866025	0.5	1
0.433012702	-0.75	0.5	1
-3.0629E-17	-0.5	0.8660254	1

Calculate Local-navigation-frame Cofactor Matrix

From (9.180), $\boldsymbol{\Pi}^n = \left(\mathbf{H}_G^{n\,\mathrm{T}} \mathbf{H}_G^n\right)^{-1}$

$\boldsymbol{\Pi}^n =$

2.666666667	1.207E-14	-1.0109E-14	1.49E-14
1.20747E-14	111.42563	-87.8786205	131.8179
-1.0109E-14	-87.87862	79.2598183	-109.848
1.48734E-14	131.81793	-109.848276	159.6743

Dilutions of Precision

From (9.180),

$$\begin{pmatrix} D_N^2 & \cdot & \cdot & \cdot \\ \cdot & D_E^2 & \cdot & \cdot \\ \cdot & \cdot & D_V^2 & \cdot \\ \cdot & \cdot & \cdot & D_T^2 \end{pmatrix} = \boldsymbol{\Pi}^n$$

NDOP =	1.632993162
EDOP =	10.55583374
VDOP =	8.902798341
TDOP =	12.63623113

From (9.181),

$$D_H = \sqrt{D_N^2 + D_E^2}$$
$$D_P = \sqrt{D_N^2 + D_E^2 + D_V^2}$$
$$D_G = \sqrt{D_N^2 + D_E^2 + D_V^2 + D_T^2}$$

HDOP =	10.68139937
PDOP =	13.90511096
GDOP =	18.78899806

EXAMPLE 9.2(c)
Dilution of Precision Calculation for Poor GNSS Geometry due to High Elevations

INPUTS:

Satellite	Azimuth	Elevation	Azimuth	Elevation
1	0	60	0	1.047198
2	120	60	2.0943951	1.047198
3	240	60	4.1887902	1.047198
4	0	90	0	1.570796
	degrees	degrees	radians	radians

Calculate Measurement (Geometry) Matrix

From (9.174),

$$\mathbf{H}_G^n = \begin{pmatrix} -u_{a1,N}^n & -u_{a1,E}^n & -u_{a1,D}^n & 1 \\ -u_{a2,N}^n & -u_{a2,E}^n & -u_{a2,D}^n & 1 \\ -u_{a3,N}^n & -u_{a3,E}^n & -u_{a3,D}^n & 1 \\ -u_{a4,N}^n & -u_{a4,E}^n & -u_{a4,D}^n & 1 \end{pmatrix}$$

$\mathbf{H}_G^n =$

-0.5	0	-0.8660254	1
0.25	-0.433013	-0.8660254	1
0.25	0.4330127	-0.8660254	1
-6.1257E-17	0	-1	1

Calculate Local-navigation-frame Cofactor Matrix

From (9.180), $\mathbf{\Pi}^n = \left(\mathbf{H}_G^{n\,\mathrm{T}} \mathbf{H}_G^n\right)^{-1}$

$\mathbf{\Pi}^n =$

2.666666667	-7.89E-16	-4.5076E-15	-4.1E-15
-7.8949E-16	2.6666667	-3.3147E-15	-2.9E-15
-4.5076E-15	-3.31E-15	74.2837506	66.81965
-4.0878E-15	-2.87E-15	66.8196489	60.35555

Dilutions of Precision

From (9.180),

$$\begin{pmatrix} D_N^2 & \cdot & \cdot & \cdot \\ \cdot & D_E^2 & \cdot & \cdot \\ \cdot & \cdot & D_V^2 & \cdot \\ \cdot & \cdot & \cdot & D_T^2 \end{pmatrix} = \mathbf{\Pi}^n$$

NDOP =	1.632993162
EDOP =	1.632993162
VDOP =	8.618802154
TDOP =	7.768883274

From (9.181),

$$D_H = \sqrt{D_N^2 + D_E^2}$$
$$D_P = \sqrt{D_N^2 + D_E^2 + D_V^2}$$
$$D_G = \sqrt{D_N^2 + D_E^2 + D_V^2 + D_T^2}$$

HDOP =	2.309401077
PDOP =	8.922840573
GDOP =	11.83100297

EXAMPLE 9.2(d)
Dilution of Precision Calculation for Poor GNSS Geometry due to Signal Reception from Opposing Directions Only

INPUTS:

Satellite	Azimuth	Elevation	Azimuth	Elevation
1	-10	-30	-0.17453293	-0.5236
2	10	-75	0.17453293	-1.309
3	-170	-75	-2.96705973	-1.309
4	170	-30	2.96705973	-0.5236
	degrees	degrees	radians	radians

Calculate Measurement (Geometry) Matrix

From (9.174),

$$\mathbf{H}_G^n = \begin{pmatrix} -u_{a1,N}^n & -u_{a1,E}^n & -u_{a1,D}^n & 1 \\ -u_{a2,N}^n & -u_{a2,E}^n & -u_{a2,D}^n & 1 \\ -u_{a3,N}^n & -u_{a3,E}^n & -u_{a3,D}^n & 1 \\ -u_{a4,N}^n & -u_{a4,E}^n & -u_{a4,D}^n & 1 \end{pmatrix}$$

$\mathbf{H}_G^n =$

-0.85286853	0.1503837	0.5	1
-0.254887	-0.044943	0.96592583	1
0.254887002	0.0449435	0.96592583	1
0.852868532	-0.150384	0.5	1

Calculate Local-navigation-frame Cofactor Matrix

From (9.180), $\mathbf{\Pi}^n = \left(\mathbf{H}_G^{n\,\mathrm{T}}\mathbf{H}_G^n\right)^{-1}$

$\mathbf{\Pi}^n =$

2.095890915	9.9371851	-9.4362E-17	-1.1E-17
9.937185106	67.411056	2.0591E-15	-2.2E-15
-9.4362E-17	2.059E-15	4.60645075	-3.37636
-1.057E-17	-2.17E-15	-3.37635756	2.724745

Dilutions of Precision

From (9.180),

$$\begin{pmatrix} D_N^2 & \cdot & \cdot & \cdot \\ \cdot & D_E^2 & \cdot & \cdot \\ \cdot & \cdot & D_V^2 & \cdot \\ \cdot & \cdot & \cdot & D_T^2 \end{pmatrix} = \mathbf{\Pi}^n$$

NDOP =	1.447719211
EDOP =	8.210423643
VDOP =	2.14626437
TDOP =	1.650680124

From (9.181),

$$D_H = \sqrt{D_N^2 + D_E^2}$$
$$D_P = \sqrt{D_N^2 + D_E^2 + D_V^2}$$
$$D_G = \sqrt{D_N^2 + D_E^2 + D_V^2 + D_T^2}$$

HDOP =	8.337082661
PDOP =	8.608913872
GDOP =	8.765736873