An Engineering Re-entry Guidance Method for Fixed-trim Vehicles with Terminal Constraints

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ABSTRACT
Fixed-trim vehicles are characterized by uncontrollable lift-force magnitude but controllable direction with single-channel bank-to-turn mode. It is difficult to obtain high landing precision with traditional guidance laws, especially in cases with terminal constraints and kinds of disturbances. Consequently, optimization algorithms are usually applied to find a feasible trajectory for re-entry guidance. But these optimization methods are not applicable for engineering application due to the large amount of computation. In this paper, an engineering re-entry guidance method is proposed for fixed-trim vehicles. For simplicity, it is designed based on the traditional proportional guidance law. The key of this method is a parameter-expansion item augmented to the proportional guidance law. This item is designed to adjust the trajectory steepness and realize both of the terminal velocity and angle constraints. The parameter is expanded as functions of states deviations to guarantee a high landing precision under kinds of disturbances. Finally, the effectiveness and robustness of this method are illustrated by numerical simulations. It is rather simple and potential for engineering application.

CCS Concepts
Computing methodologies→Modeling methodologies

Keywords
fixed-trim; engineering guidance; robustness

1. INTRODUCTION
The fixed-trim re-entry vehicle has attracted a lot of attention as its control mode is very simple. That is the so-called single-rolling control mode [1- 3]. With one moving mass placed inside the body, the fixed-trim vehicle is able to maintain a good aerodynamic configuration without rudders. And the moving mass is of greater control efficiency than conventional rudders or reaction control system [2, 4, 5].

The MK500 re-entry body of the United States is a typical representative of fixed-trim vehicles. Its asymmetric aerodynamic configuration produces a non-zero trimmed angle of attack. And the one-dimensional moving mass system is used for the rolling control to realize its program flight [2, 6]. The typical characteristic of fixed-trim vehicles is the single-rolling control mode. In other words, without active control, the pitch motion and yaw motion can automatically converge to the non-zero trimmed angle of attack and zero sideslip angle, respectively. As a result, the magnitude of overload cannot be controlled. However, by changing the moving mass position in the vehicle, the direction of the overload can be controlled.

vehicles by a feasible rolling motion to consume excess lift force [1]. Ref. [2] introduced a virtual target for the guidance law, which guided the fixed-trim vehicle to the desired landing site with the terminal angle constraint fulfilled. Ref. [6] combined the rolling guidance law and three-dimensional circumferential guidance law together to deal with terminal angle constraint. However, there are few effective guidance methods to guide the re-entry fixed-trim vehicle to the target with both of the terminal velocity and flight-path angle constraints fulfilled. Trajectory optimization methods have been studied for such guidance problems with multiple constraints. But its drawback lies on the large amount of calculation. As a result, trajectory optimization methods are usually not suitable for engineering application.

In this paper, a simple re-entry guidance method is proposed for fixed-trim vehicle. It is based on the conventional proportional guidance law and suitable for engineering application. An parameter-expansion item is augmented to realize the terminal constraints and guarantee a high terminal precision. Its effectiveness is finally illustrated by numerical simulations.

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2. PRELIMINARY

2.1 Dynamic Equations
A fixed-trim vehicle with a single moving mass is studied in this paper. The rolling channel is controlled with a moving mass on *z* axis of the body. The three-dimension dynamic equations of the mass point of the vehicle are established based on the following assumptions:

- The Earth curvature and rotation are ignored.
- The angle of attack and the angle of sideslip in trimmed state are small. There are $\cos\alpha \approx 1$, $\cos\beta \approx 1$, $\sin\alpha \approx 0$, $\sin\beta \approx 0$.
- The moving mass relative velocity and acceleration to the vehicle are rather small. Thus the disturbances caused by the relative velocity and acceleration are ignored.

Based on the above assumptions, the fixed-trim vehicle's motion equations are

$$\dot{V} = -\frac{D}{m} - g\sin\theta \tag{1}$$

$$\dot{\theta} = \frac{L}{mV}\cos\gamma_V - \frac{g}{V}\cos\theta \tag{2}$$

$$\dot{\psi}_V = \frac{-L}{mV}\sin\gamma_V \sec\theta \tag{3}$$

$$\dot{x} = V\cos\theta\cos\psi_V \quad (4)$$

$$\dot{h} = V\sin\theta \quad (5)$$

$$\dot{z} = -V\cos\theta\sin\psi_V \quad (6)$$

where V, θ, and ψ_V are the velocity, flight-path angle and the azimuth angle, respectively. D and L are the aerodynamic drag force and aerodynamic lift fore. The vehicle mass is m. γ_V is the bank angle. The gravity acceleration is set as g =9.8 m/s^2.

2.2 Geometric Relation of the Vehicle and Target

The line of sight coordinate system $o-\xi_o\eta_o\varsigma_o$ can be got through rotating the re-entry coordinate system $o-x_e y_e z_e$ twice. The re-entry coordinate system $o-x_e y_e z_e$ firstly rotates around the axis $+oy_e$ with an azimuth angle of the line of sight λ_{T}, and then around the axis $+oz_e$ with an angle λ_{D} as shown in Fig. 1. Thus it is obtained that:

$$\begin{bmatrix} o\xi_o \\ o\eta_o \\ o\varsigma_o \end{bmatrix} = S_e \begin{bmatrix} ox_e \\ oy_e \\ oz_e \end{bmatrix} \quad (7)$$

where

$$S_e = \begin{bmatrix} \cos\lambda_{\mathrm{D}}\cos\lambda_{\mathrm{T}} & \sin\lambda_{\mathrm{D}} & -\cos\lambda_{\mathrm{D}}\sin\lambda_{\mathrm{T}} \\ -\sin\lambda_{\mathrm{D}}\cos\lambda_{\mathrm{T}} & \cos\lambda_{\mathrm{D}} & \sin\lambda_{\mathrm{D}}\sin\lambda_{\mathrm{T}} \\ \sin\lambda_{\mathrm{T}} & 0 & \cos\lambda_{\mathrm{T}} \end{bmatrix} \quad (8)$$

The angular velocity of the vehicle's velocity is then expressed in the line of sight coordinate system $o-\xi_o\eta_o\varsigma_o$ as:

$$\begin{bmatrix} \dot{\gamma}_\xi \\ \dot{\gamma}_{\mathrm{T}} \\ \dot{\gamma}_{\mathrm{D}} \end{bmatrix} = S_e \begin{bmatrix} -\dot{\psi}_V\sin\theta \\ \dot{\psi}_V\cos\theta \\ \dot{\theta} \end{bmatrix} \quad (9)$$

From Eq. (8) and Eq. (9), the angular velocity components are calculated with the following equations.

$$\begin{aligned} \dot{\gamma}_{\mathrm{T}} &= \dot{\psi}_V\sin\theta\sin\lambda_{\mathrm{D}}\cos\lambda_{\mathrm{T}} + \dot{\psi}_V\cos\theta\cos\lambda_{\mathrm{D}} \\ &\quad + \dot{\theta}\sin\lambda_{\mathrm{D}}\sin\lambda_{\mathrm{T}} \\ \dot{\gamma}_{\mathrm{D}} &= -\dot{\psi}_V\sin\theta\sin\lambda_{\mathrm{T}} + \dot{\theta}\cos\lambda_{\mathrm{T}} \end{aligned} \quad (10)$$

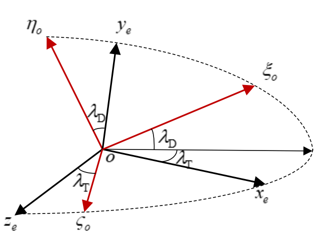

Figure 1. Geometric relation of the vehicle and target

3. RE-ENTRY GUIDANCE DESIGN

In this section, for engineering application, a fixed-trim re-entry guidance method is proposed based on the widely used proportional guidance method. The basic proportional guidance scheme is improved in two steps. Firstly, augmenting it for the enforcement of the terminal constraints. And secondly, expanding the guidance parameters to be functions of the flying states deviations so that the robustness of the method is enhanced. The terminal constraints of the fixed-trim vehicle can be realized with the velocity and the flight-path angle satisfied:

$$\begin{aligned} V_f &\geq V_f^* \\ \theta_f &\leq \theta_f^* \end{aligned} \quad (11)$$

where V_f^* is the minimum velocity and θ_f^* is the maximum flight-path angle desired at the terminal point.

3.1 Augmented Proportional Guidance

The proportional guidance method is widely used in engineering because it is very simple and reliable. The basic proportional guidance scheme is:

$$\begin{aligned} \dot{\gamma}_{\mathrm{D}} &= K_{\mathrm{GD}}\dot{\lambda}_{\mathrm{D}} \\ \dot{\gamma}_{\mathrm{T}} &= K_{\mathrm{GT}}\dot{\lambda}_{\mathrm{T}} \end{aligned} \quad (12)$$

where K_{GD} and K_{GT} are the guidance parameters to be selected. With the Equations of (12), the three-dimensional guidance problem is decomposed into two planes, which are the vertical plane and the horizontal plane respectively.

Concerning the enforcement of the desired terminal velocity and flight-path angle, the proportional guidance method is improved with an augmented item:

$$\begin{aligned} \dot{\gamma}_{\mathrm{D}} &= K_{\mathrm{GD}}\dot{\lambda}_{\mathrm{D}} + K_{\mathrm{LD}}(\lambda_{\mathrm{D}} + \theta_{\mathrm{DF}}^*)/T_g \\ \dot{\gamma}_{\mathrm{T}} &= K_{\mathrm{GT}}\dot{\lambda}_{\mathrm{T}} \end{aligned} \quad (13)$$

where λ_{D}, $\dot{\lambda}_{\mathrm{D}}$ and $\dot{\lambda}_{\mathrm{T}}$ are measured online by the navigation system. $K_{\mathrm{LD}}(\lambda_{\mathrm{D}} + \theta_{\mathrm{DF}}^*)/T_g$ is the augmented item to adjust the flight path for the desired terminal velocity and flight-path angle. K_{LD} is the guidance parameter to be designed. θ_{DF}^* is an angle value associated with the terminal flight-path angle constraint θ_f^*. The rest time for the fixed-trim vehicle to fly, denoted by T_g, is estimated with current velocity V.

From Eq.(10), it can be obtained that:

$$\begin{bmatrix} \dot{\psi}_V \\ \dot{\theta} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \dot{\gamma}_{\mathrm{T}} \\ \dot{\gamma}_{\mathrm{D}} \end{bmatrix} \quad (14)$$

where

$$\mathbf{M} = \begin{bmatrix} \sin\theta\sin\lambda_{\mathrm{D}}\cos\lambda_{\mathrm{T}} + \cos\theta\cos\lambda_{\mathrm{D}} & \sin\lambda_{\mathrm{D}}\sin\lambda_{\mathrm{T}} \\ -\sin\theta\sin\lambda_{\mathrm{T}} & \cos\lambda_{\mathrm{T}} \end{bmatrix}^{-1} \quad (15)$$

Substituting augmented guidance scheme as Eq.(13) into Eq.(14) yields that:

$$\begin{bmatrix} \dot{\psi}_V \\ \dot{\theta} \end{bmatrix} = \mathbf{M} \begin{bmatrix} K_{\mathrm{GT}} & 0 \\ 0 & K_{\mathrm{GD}} \end{bmatrix} \begin{bmatrix} \dot{\lambda}_{\mathrm{T}} \\ \dot{\lambda}_{\mathrm{D}} \end{bmatrix} + \mathbf{M} \begin{bmatrix} 0 \\ K_{\mathrm{LD}} \end{bmatrix} (\lambda_{\mathrm{D}} + \theta_{\mathrm{DF}}^*)/T_g \quad (16)$$

According to the dynamics of the vehicle, the demanded overload commands are determined as:

$$n_{yc} = \frac{\dot{\theta}\cdot V\cdot\cos\psi_V}{g} + \cos\theta \quad (17)$$

$$n_{zc} = \frac{-\dot{\psi}_V\cdot V}{g} + \sin\psi_V\sin\theta \quad (18)$$

The overload commands are obtained by substitute Eq.(16) into Eqs. (17) and (18). However, controls can only be acted in the rolling channel for the fixed-trim vehicle. Therefore, the direction of the overload can be controlled, while its magnitude cannot be controlled on the contrary. In other words, the overloads commands in Eq. (17) and Eq. (18) cannot be controlled at the same time. In consideration of the fixed-trim vehicle control mode, a bank angle command γ_{Vc} is derived from the above overload commands as:

$$\gamma_{Vc}=\begin{cases}\tan_2^{-1}\left(n_{zc},n_{yc}\right) & ,\sqrt{{n_{zc}}^2+{n_{yc}}^2}\ge\left|n_t\right| \\ \tan_2^{-1}\left(n_{zc},n_{yc}\right)-\cos^{-1}\left(\dfrac{\sqrt{{n_{zc}}^2+{n_{yc}}^2}}{\left|n_t\right|}\right) & ,\sqrt{{n_{zc}}^2+{n_{yc}}^2}<\left|n_t\right|\end{cases} \tag{19}$$

where n_t is the maximum normal overload perpendicular to the longitudinal axis of the vehicle. With the assumption of side-slip angle $\beta=0$ in the Bank-To-Turn (BTT) flight mode, The maximum normal overload is computed with $n_t = L/mg$. The inverse function $\tan_2^{-1}(\cdot)$ is defined as:

$$\tan_2^{-1}(n_{zc},n_{yc})=\begin{cases}\tan^{-1}\left(\dfrac{n_{zc}}{n_{yc}}\right) & ,n_{yc}>0 \\ \tan^{-1}\left(\dfrac{n_{zc}}{n_{yc}}\right)+\pi & ,n_{yc}<0\ \&\ n_{zc}\ge 0 \\ \tan^{-1}\left(\dfrac{n_{zc}}{n_{yc}}\right)-\pi & ,n_{yc}<0\ \&\ n_{zc}<0\end{cases} \tag{20}$$

The bank angle command derived from Eq. (19) is the closest solution to the proportional guidance commands in Eq. (17) and Eq. (18). As shown in Fig. 2 (a), if the maximum normal overload of the fixed-trim vehicle is smaller than the sum overload of the demanded commands n_{yc} and n_{zc} , the bank angle command is designed to guarantee the desired direction of the overload. On the other hand, if the maximum normal overload of the fixed-trim vehicle is greater than the joint overload of the demanded commands, the bank angle command is designed to guarantee the desired magnitude of the overload. The extra overload is perpendicular to the expected overload direction as shown in Fig.2 (b).

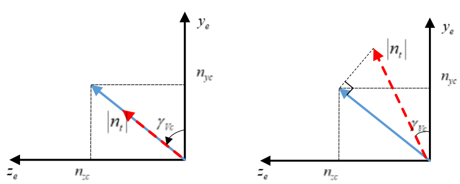

Figure 2. The Bank Angle Command Derived from Overload Commands

In this section, the augmented proportional guidance scheme is designed for the fixed-trim vehicle. The augmented item $K_{\rm LD}(\lambda_{\rm D}+\theta_{\rm DF}^{*})/T_g$ is introduced to adjust the flight path for the desired terminal velocity and flight-path angle. The greater is the parameter $K_{\rm LD}$, the steeper is the flight path. And the terminal flight-path angle will be smaller. The terminal velocity might be greater because the vehicle glides shorter and consume less kinetic energy. The desired terminal flight-path angle and velocity can be fulfilled by selecting a proper parameter $K_{\rm LD}$ for a proper value of the augmented item $K_{\rm LD}(\lambda_{\rm D}+\theta_{\rm DF}^{*})/T_g$.

3.2 Expansion Design for Guidance Parameter

In fact, it is rather difficult to select a proper value for the augmented item $K_{\rm LD}(\lambda_{\rm D}+\theta_{\rm DF}^{*})/T_g$ online when there are some disturbances during re-entry flight, such as the wind effect and the uncertainty of the vehicle itself. Consequently, the guidance scheme given above is utilized to guide the fixed-trim vehicle to the target with desired velocity and flight-path angle in nominal conditions. The guidance parameter $K_{\rm LD}$ is expanded to be a function of flight states deviations. Obviously, the expansion design of the guidance parameter is to enhance the robustness of the guidance method.

The guidance parameter $K_{\rm LD}$ is expanded to be a piecewise function as:

$$K_{\rm LD}=\begin{cases}K_{\rm LD0}{}^{*}+K_{\theta 0}(\theta_0^{*}-\theta_0) \\ \qquad \text{if}\quad h>h_1 \\ K_{{\rm LD}i}{}^{*}+K_{\theta i}(\theta_i^{*}-\theta_i)\ +K_{Vi}(V_i^{*}-V_i) \\ \qquad \text{if}\quad h_{i+1}\ge h>h_{i+1}\quad (i=1\cdots n-1) \\ K_{{\rm LD}n}{}^{*}+K_{\theta n}(\theta_n^{*}-\theta_n)+K_{Vn}(V_n^{*}-V_n) \\ \qquad \text{if}\quad h_n\ge h>0\end{cases} \tag{21}$$

where

- $h_i\ \ (i=1\cdots n-1)$ is the altitude points at which the guidance parameter $K_{{\rm LD}i}{}^{*}$, $K_{\theta i}$, and K_{Vi} are changed.
- $i=1\cdots n$ is the number of the altitude points at which the guidance parameter $K_{{\rm LD}i}{}^{*}$, $K_{\theta i}$, and K_{Vi} are changed.
- $K_{{\rm LD}i}{}^{*}$ is the guidance parameter $K_{\rm LD}$ at each altitude points under nominal conditions.
- θ_0^{*} is the nominal value of the flight-path angle at the re-entry initial interface.
- θ_0 is the real value of the flight-path angle at the re-entry initial interface.
- θ_i^{*}, V_i^{*} are the nominal values of the flight-path angle and velocity at each altitude points $h_i\ \ (i=1\cdots n-1)$.
- θ_i, V_i are the real values of the flight-path angle and velocity at each altitude points $h_i\ \ (i=1\cdots n-1)$.
- $K_{\theta i}$, K_{Vi} are the expanded parameters to correct the deviations of the flight states, including the flight-path angle and the velocity.

It is obvious that the guidance parameter $K_{\rm LD}$ is expanded to cope with the deviations of the flight states, which can be easily caused by the disturbances of the environment and the uncertainties of the vehicle itself. However, the negative derivations and the positive ones are asymmetrical. The selected values of $K_{\theta i}$ and K_{Vi} are related to the direction of the deviations. They should be designed as follows:

$$K_{\theta i}=\begin{cases}K_{\theta i}{}^{+} & \text{if}\ \theta_i^{*}-\theta_i>0 \\ K_{\theta i}{}^{-} & \text{if}\ \theta_i^{*}-\theta_i\le 0\end{cases}\quad (i=0\cdots n) \tag{22}$$

$$K_{Vi}=\begin{cases}K_{Vi}{}^{+} & \text{if}\ V_i^{*}-V_i>0 \\ K_{Vi}{}^{-} & \text{if}\ V_i^{*}-V_i\le 0\end{cases}\quad (i=0\cdots n) \tag{23}$$

where $K_{\theta i}{}^{+}$, $K_{\theta i}{}^{-}$, $K_{Vi}{}^{+}$, and $K_{Vi}{}^{-}$ are non-negative constants. And they satisfy $K_{\theta i}{}^{+}\ne K_{\theta i}{}^{-}$ and $K_{Vi}{}^{+}\ne K_{Vi}{}^{-}$.

The guidance method is based on the basic proportional guidance scheme in Eq.(12). It is rather simple for engineering application. What's more, with the expansion design of the parameters, the robustness of the improved guidance method is much stronger than the basic method.

4. NUMERICAL ANALYSIS

To illustrate the effectiveness of the proposed method, numerical simulations are conducted under both nominal conditions and disturbed conditions.

4.1 Nominal Conditions

Under the nominal conditions, the guidance parameter expansion is not needed. In this test bed, the terminal constraints are set to be:

$$\begin{aligned} V_f &\geq V_f^{*} = 500\text{m/s} \\ \theta_f &\leq \theta_f^{*} = -30\,\text{deg} \end{aligned} \tag{24}$$

The guidance parameters K_{GD}, K_{GT}, θ_{DF}^{*}, and $K_{\text{LD}i}^{*}$ are selected off-line.

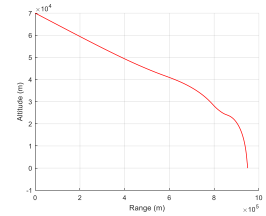

Figure 3. Re-entry Trajectory in Nominal Simulation

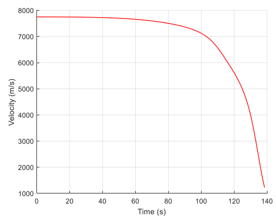

Figure 4. Velocity Curve in Nominal Simulation

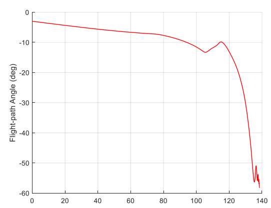

Figure 5. Flight-path Angle Curve in Nominal Simulation

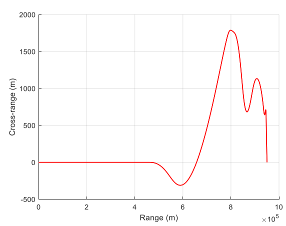

Figure 6. Cross-range Curve in Nominal Simulation

The numerical simulation is carried out in the Visual Studio 2013 environment. The simulation results are shown in Fig. 3-Fig. 6.

With the augmented proportional guidance scheme, the fixed-trim vehicle is guided to the ground target with a rather high precision. The terminal flight states are listed in Table 1It can be concluded that the desired terminal velocity and flight-path angle are well realized with the augmented item adjusting the flight path.

Table 1 Terminal States of the re-entry

Flight states	Landing precision (m)	Terminal velocity (m/s)	Terminal flight-path angle (deg)
Desired value	≤ 10	≥ 500	≤ -30
Real value	1.04	1223.7	-58.3

To verify the effectiveness of the augmented item in adjusting the flight path for the desired terminal velocity and flight-path angle. Different values are selected for the guidance parameter K_{LD1}^{*}. The simulation results are shown in Fig. 8-Fig. 9.

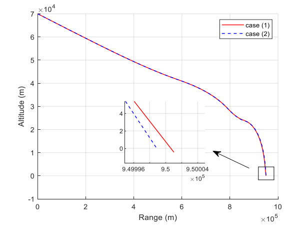

Figure 7. Re-entry Trajectories with Different Guidance Parameters

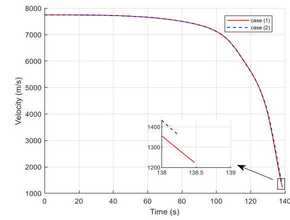

Figure 8. Velocity Curves with Different Guidance Parameters

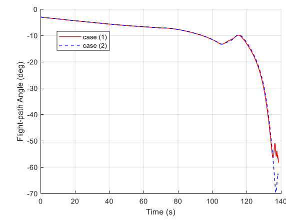

Figure 9. Flight-path Angle Curves with Different Guidance Parameters

Through comparing the terminal states it is demonstrated that the augmented item $K_{\text{LD}}(\lambda_{\text{D}} + \theta_{\text{DF}}^{*})/T_g$ is able to take the fixed-trim vehicle up or down to fulfill the desired terminal velocity and flight-path angle. It is verified that the greater is the parameter K_{LD}, the smaller will be the terminal flight-path angle. And the terminal velocity is greater.

4.2 Disturbed Conditions

In this section, numerical simulations with disturbances are conducted to test the robustness of the expansion design of guidance parameters. The disturbances of the fixed-trim vehicle's mass, aerodynamic forces, atmospheric density, and the initial velocity at the re-entry interface are concerned. The expanded guidance parameters are selected. It can be found that the expanded guidance parameters do not need to be changed at every altitude. And then t0he Monte Carlo simulation is carried.

Table 2 Disturbances Concerning in Simulation

Disturbance variables	Normal distribution	
	Expectation	**Standard deviation**
Vehicle's mass Δm	0	10%/3
Lift coefficient ΔC_L	0	15%/3
Drag coefficient ΔC_D	0	15%/3
Atmospheric density $\Delta\rho$	0	10%/3
Initial velocity ΔV_0 (m/s)	0	100/3

As shown in Fig. 11, high precision terminal landing is obtained. And the values of terminal velocity and flight-path angle in the Monte Carlo cases satisfy all of the terminal constraints of Eq. (24).

The expansion design of the guidance parameters is proved to be effective. In conclusion, the re-entry guidance method is of great robustness and it is rather simple for engineering application.

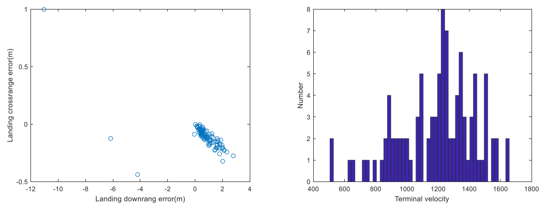

Figure 11. Distribution of the Landing Point & Terminal Velocity

5. CONCLUSION

This paper proposed a simple re-entry guidance method for the fixed-trim vehicle, which is rather suitable for engineering application. The effectiveness and robustness are verified by numerical simulations. The future work will focus on how to determine the guidance parameters automatically.

6. ACKNOWLEDGMENT

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