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Saturation Protection with Pseudo Control Hedging: A Control Allocation Perspective

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In this paper, we derive a simple direct relationship between the pseudo control hedging signal and the pseudo control command, for a linear feedforward-feedback structure with a hedged linear reference model. The derivation indicates that, the hedging signal will slow down the pseudo control command with the low pass filtered hedging signal, given appropriately chosen bandwidths of the reference model and the error controller. Based on this observation, we propose a hedging structure with the saturation-induced algebraic control allocation error, which automatically triggers the protection, yet only when input saturations enter. In a steady state flight, the protection should exactly reduce the pseudo control command with the allocation error. Simulation results show that the hedged structure performs identically to the unhedged structure during nominal unsaturated operations, whereas the closed-loop performance with hedging shows a significant improvement, when input saturations are provoked due to harsh maneuvers and effector failures.

I. Introduction and Objective

Actuator saturations are critical hazard factors for flight safety [1]. Especially in effector failure cases, the system is subject to severe saturation where even an optimization control allocation technique can lead to allocation errors and pseudo control distortion between commanded and actual value [2–6]. Not to mention that many of them are highly nonlinear, computationally demanding and bring about challenges to the closed-loop analysis and certifiable requirements. Pseudo control hedging (PCH) was developed to protect against actuator saturation and dynamics and violation of flight envelope, by "hiding" the actuator dynamics from the reference path [7, 8]. The benefit is to prevent "over commands" into the actuators even given a relatively high feedback bandwidth. On the downside, hedging changes the open-loop transfer function by introducing the feedback path into reference dynamics and vice versa, making it less straightforward to understand the structure of the loop design.

One point to address is that, in existing literature there seems to miss a direct link between the pseudo control command and the hedging itself, e.g. how exactly does the pseudo control hedging affect the commanded pseudo control. This thought of such a link is provoked since pseudo controls are effectiveness-wise on the same algebraic level with the control effectors, thus subject to their physical dynamics and saturations. A desired protection of the actuators is hence intuitively connected and can be illustrated with its influence on the pseudo control command.

We will show in this paper that, considering a simple structure, hedging might not be able to always help protect from actuator dynamics and saturation. It only takes a positively protective effect if the reference model and error controller bandwidth are chosen appropriately, e.g. slower dynamics of reference model compared to the error controller. Luckily, this argument can be assumed true since it makes no sense for a system to track the reference dynamics faster than its physical capability. Additionally, the reference dynamics are generally designed to comply with pilot handling qualities whereas the error controller needs to fulfill disturbance rejection requirements [9, 10].

With the appropriate choice of bandwidths, the derivation shows that a hedging signal will slow down the pseudo control command with a low-passed version of the hedging signal itself. With this observation, we propose a novel hedging structure to focus on protection against saturation. The desired behavior is to keep the closed-loop performance identical to its unhedged counterpart when no effector saturation is induced, whereas it will automatically trigger the protection when input saturation arises. Compared to the conventional hedging method found in literature which always affect the loop transfer behavior, this method will preserve the closed-loop dynamics as in nominal non-saturation conditions and conserve the ease of straightforward loop design and analysis. In abnormal saturation

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situations, the loop transfer behavior will be influenced by hedging but it is anyway distorted by the nonlinear saturation effect, and the method will positively preserve the margin due to its protective effect.

The proposed method is verified with a closed-loop F-16 model controlled by an incremental nonlinear dynamic inversion (INDI) baseline structure. Simulations show the desired behavior mentioned above with an identical performance during nominal operations and significant improvements during high saturation periods compared to the unhedged test case.

II. Derivation: Relating the Hedging Signal $\boldsymbol{\nu}_{\mathrm{h}}$ and Pseudo Control Command $\boldsymbol{\nu}_{\mathrm{cmd}}$

Consider the simple structure shown as in Fig.1: hedged linear reference model with feedforward and linear feedback design. The system considered here for derivation is a *SISO* case, where $\boldsymbol{K}_r$ and $\boldsymbol{K}_e$ are gain of the reference model and the error controller, respectively. Subscripts "*cmd*", "*ref*" and "*ff*" stand for "command", "reference" and "feed-forward" signals. $\widehat{\boldsymbol{y}}$ is the feedback of measured outputs, and $\boldsymbol{\nu}$ is pseudo control.

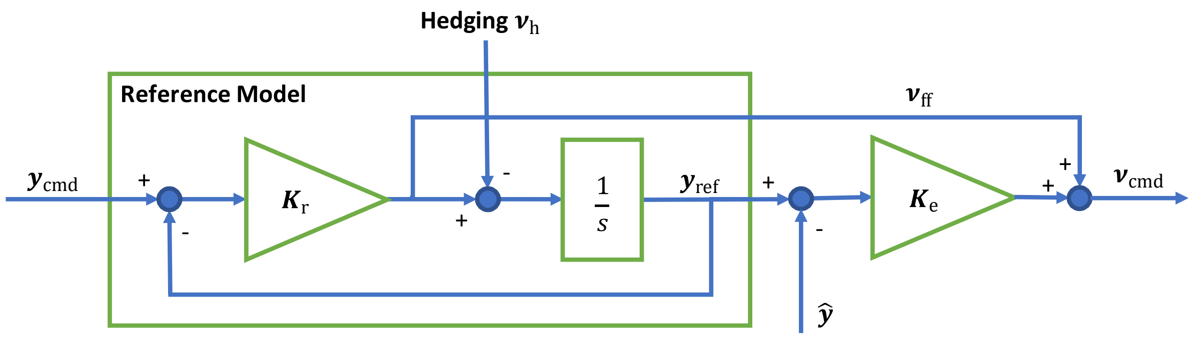

Fig. 1 Linear Reference Model with Hedging: feedforward and feedback

We would like to understand the relationship between the pseudo controls command $\boldsymbol{\nu}_{\mathrm{cmd}}$ and hedging signal $\boldsymbol{\nu}_{\mathrm{h}}$. Starting with the $\boldsymbol{\nu}_{cmd}$ by combining both the feedforward and feedback parts, we have:

$$\begin{aligned}\boldsymbol{\nu}_{\mathrm{cmd}} &= \boldsymbol{K}_e(\boldsymbol{y}_{\mathrm{ref}} - \widehat{\boldsymbol{y}}) + \boldsymbol{\nu}_{\mathrm{ff}} \\ &= \boldsymbol{K}_e\left(\frac{1}{s}(\boldsymbol{\nu}_{\mathrm{ff}} - \boldsymbol{\nu}_{\mathrm{h}}) - \widehat{\boldsymbol{y}}\right) + \boldsymbol{\nu}_{\mathrm{ff}} \\ &= \frac{\boldsymbol{K}_e}{s}(\boldsymbol{\nu}_{\mathrm{ff}} - \boldsymbol{\nu}_h) + \boldsymbol{\nu}_{\mathrm{ff}} - \boldsymbol{K}_e\widehat{\boldsymbol{y}} = \left(\frac{\boldsymbol{K}_e}{s} + 1\right)\boldsymbol{\nu}_{\mathrm{ff}} - \frac{\boldsymbol{K}_e}{s}\boldsymbol{\nu}_h - \boldsymbol{K}_e\widehat{\boldsymbol{y}}\end{aligned} \tag{1}$$

From the reference model, we can write $\boldsymbol{\nu}_{\mathrm{ff}}$ as:

$$\boldsymbol{\nu}_{\mathrm{ff}} = \boldsymbol{K}_r(\boldsymbol{y}_{\mathrm{cmd}} - \boldsymbol{y}_{\mathrm{ref}}) = \boldsymbol{K}_r\boldsymbol{y}_{\mathrm{cmd}} - \frac{\boldsymbol{K}_r}{s}(\boldsymbol{\nu}_{\mathrm{ff}} - \boldsymbol{\nu}_{\mathrm{h}})$$

which gives,

$$\left(1 + \frac{\boldsymbol{K}_r}{s}\right)\boldsymbol{\nu}_{\mathrm{ff}} = \boldsymbol{K}_r\boldsymbol{y}_{\mathrm{cmd}} + \frac{\boldsymbol{K}_r}{s}\boldsymbol{\nu}_{\mathrm{h}}$$

Therefore,

$$\boldsymbol{\nu}_{\mathrm{ff}} = \frac{\boldsymbol{K}_r s}{s + \boldsymbol{K}_r}\boldsymbol{y}_{\mathrm{cmd}} + \frac{\boldsymbol{K}_r}{s + \boldsymbol{K}_r}\boldsymbol{\nu}_{\mathrm{h}} \tag{2}$$

By putting Eq. (2) into Eq. (1), we have

$$\boldsymbol{\nu}_{\text{cmd}} = \left(\frac{\boldsymbol{K}_e}{s} + 1\right)\left(\underbrace{\frac{\boldsymbol{K}_r s}{s + \boldsymbol{K}_r}\boldsymbol{y}_{\text{cmd}} + \frac{\boldsymbol{K}_r}{s + \boldsymbol{K}_r}\boldsymbol{\nu}_{\text{h}}}_{\boldsymbol{\nu}_{\text{ff}}}\right) - \frac{\boldsymbol{K}_e}{s}\boldsymbol{\nu}_{\text{h}} - \boldsymbol{K}_e\hat{\boldsymbol{y}}$$

Grouping together the $\boldsymbol{\nu}_{\text{h}}$ terms, it becomes:

$$\boldsymbol{\nu}_{\text{cmd}} = \boldsymbol{K}_r\frac{s + \boldsymbol{K}_e}{s + \boldsymbol{K}_r}\boldsymbol{y}_{\text{cmd}} - \boldsymbol{K}_e\hat{\boldsymbol{y}} + \left(\frac{s + \boldsymbol{K}_e}{s + \boldsymbol{K}_r}\frac{\boldsymbol{K}_r}{s} - \frac{\boldsymbol{K}_e}{s}\right)\boldsymbol{\nu}_{\text{h}} = \boldsymbol{K}_r\frac{s + \boldsymbol{K}_e}{s + \boldsymbol{K}_r}\boldsymbol{y}_{\text{cmd}} - \boldsymbol{K}_e\hat{\boldsymbol{y}} + \frac{\boldsymbol{K}_r - \boldsymbol{K}_e}{s + \boldsymbol{K}_r}\boldsymbol{\nu}_{\text{h}} \tag{3}$$

Note the first term on the RHS $\boldsymbol{K}_r\frac{s+\boldsymbol{K}_e}{s+\boldsymbol{K}_r}\boldsymbol{y}_{\text{cmd}}$. It represents the signals in an *unhedged* linear reference model, as:

$$\boldsymbol{K}_r\frac{s + \boldsymbol{K}_e}{s + \boldsymbol{K}_r}\boldsymbol{y}_{\text{cmd}} = \dot{\boldsymbol{y}}_{\text{ref,unhedged}} + \boldsymbol{K}_e\boldsymbol{y}_{\text{ref,unhedged}}$$

Therefore Eq. (3) becomes,

$$\boldsymbol{\nu}_{\text{cmd}} = \dot{\boldsymbol{y}}_{\text{ref}} + \boldsymbol{K}_e\boldsymbol{y}_{\text{ref}} - \boldsymbol{K}_e\hat{\boldsymbol{y}} + \frac{\boldsymbol{K}_r - \boldsymbol{K}_e}{s + \boldsymbol{K}_r}\boldsymbol{\nu}_{\text{h}} = \underbrace{\dot{\boldsymbol{y}}_{\text{ref,unhedge}}}_{\text{ff,unhedged}} + \underbrace{\boldsymbol{K}_e\left(\boldsymbol{y}_{\text{ref,unhedge}} - \hat{\boldsymbol{y}}\right)}_{\text{fb,unhedged}} + \underbrace{\frac{\boldsymbol{K}_r - \boldsymbol{K}_e}{s + \boldsymbol{K}_r}\boldsymbol{\nu}_h}_{\text{hedging}} \tag{4}$$

Note again in Eq. (4), the $\dot{\boldsymbol{y}}_{\text{ref,unhedge}}$ and $\boldsymbol{y}_{\text{ref,unhedge}}$ represents signals in a *linear unhedged* reference model. Eq. (4) expresses that, for a structure shown in Fig-1, compared to its unhedged counterpart, hedging adds an additional part $\frac{\boldsymbol{K}_r-\boldsymbol{K}_e}{s+\boldsymbol{K}_r}\boldsymbol{\nu}_h$ to the pseudo controls command $\boldsymbol{\nu}_{\text{cmd}}$, *irrelevant of what signal is being used as* $\boldsymbol{\nu}_h$. The conclusion we arrive at here provides a means to analyze the structure with hedging in a linear manner. At this point, three cases can be discerned as follows:

- If $\boldsymbol{K}_r = \boldsymbol{K}_e$: then the hedged structure reduces to an unhedged structure with a pure feedback path $\boldsymbol{\nu}_{\text{cmd}} = \boldsymbol{K}_e(\boldsymbol{y}_{\text{cmd}} - \hat{\boldsymbol{y}})$ (Since $\dot{\boldsymbol{y}}_{\text{ref,unhedge}} = \boldsymbol{K}_r(\boldsymbol{y}_{\text{cmd}} - \boldsymbol{y}_{\text{ref}})$)
- If $\boldsymbol{K}_r > \boldsymbol{K}_e$: then hedging will "add-on" $\boldsymbol{\nu}_{\text{cmd}}$ by $\frac{\boldsymbol{K}_r-\boldsymbol{K}_e}{s+\boldsymbol{K}_r}\boldsymbol{\nu}_h$
- If $\boldsymbol{K}_r < \boldsymbol{K}_e$: then hedging will "slow-down" $\boldsymbol{\nu}_{\text{cmd}}$ by $\frac{\boldsymbol{K}_e-\boldsymbol{K}_r}{s+\boldsymbol{K}_r}\boldsymbol{\nu}_h$

From a protection perspective, only the third situation makes sense. In case of saturation, we would always like to slow down the $\boldsymbol{\nu}_{\text{cmd}}$ to avoid further distortion. It is also consistent with a general control design philosophy: the the reference model is usually slower than the error controller to be in favor of handling qualities and avoid overloading the error controller. Therefore, Eq. (4) is rearranged below for better intuition:

$$\boldsymbol{\nu}_{\text{cmd}} = \dot{\boldsymbol{y}}_{\text{ref}} + \boldsymbol{K}_e\boldsymbol{y}_{\text{ref}} - \boldsymbol{K}_e\hat{\boldsymbol{y}} + \frac{\boldsymbol{K}_r - \boldsymbol{K}_e}{s + \boldsymbol{K}_r}\boldsymbol{\nu}_{\text{h}} = \underbrace{\dot{\boldsymbol{y}}_{\text{ref,unhedge}}}_{\text{ff,unhedged}} + \underbrace{\boldsymbol{K}_e\left(\boldsymbol{y}_{\text{ref,unhedge}} - \hat{\boldsymbol{y}}\right)}_{\text{fb,unhedged}} - \underbrace{\frac{\boldsymbol{K}_e - \boldsymbol{K}_r}{s + \boldsymbol{K}_r}\boldsymbol{\nu}_h}_{\text{hedging}} \tag{5}$$

III. Proposed Hedging Structure

In controller designs for systems where actuator dynamics are fast enough to be ignored or actively accounted for, actuator saturations are the most harmful phenomenon. Even when a re-allocation method is utilized onboard, pseudo control commands can still exceed the Attainable Moment Set (AMS) and degrade the closed-loop stability [2, 6, 3]. On the other hand, during normal operations where saturations are not triggered, an unhedged behavior is more desirable for ease of closed-loop performance evaluation. Summarizing all the observations and assumptions above, we propose a hedging structure to address saturation, which remains disabled during nominal operations and is enabled only when input saturation is triggered. The proposed structure is shown in Fig. 2.

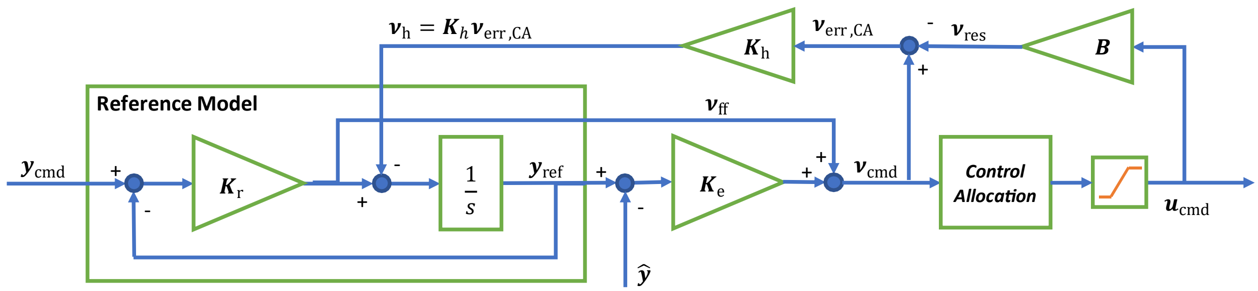

Fig. 2 Proposed Hedging Structure

In this figure:

- $\boldsymbol{B}$: Effectiveness matrix used in control allocation
- $\boldsymbol{u}_{\mathrm{cmd}}$: input commands, saturated with actuator limits
- $\boldsymbol{\nu}_{\mathrm{res}}$: restored pseudo controls or expected pseudo controls reaction, $\boldsymbol{\nu}_{\mathrm{res}} = \boldsymbol{B}\boldsymbol{u}_{\mathrm{cmd}}$
- $\boldsymbol{\nu}_{\mathrm{err,CA}}$: allocation error due to saturations, or in other words, distorted pseudo controls reaction compared to pseudo control commands, $\boldsymbol{\nu}_{\mathrm{err,CA}} = \boldsymbol{\nu}_{\mathrm{cmd}} - \boldsymbol{\nu}_{\mathrm{res}}$
- $\boldsymbol{K}_h$: gain used on the hedging signal.

The difference compared to existing hedging implementation is to use the arithmetic allocation error $\boldsymbol{\nu}_{\mathrm{err,CA}}$ with an additional gain as the hedging signal $\boldsymbol{\nu}_{\mathrm{h}}$. The computation of $\boldsymbol{\nu}_{\mathrm{h}}$ is therefore completely on an algebraic level, requiring no dynamical signals such as estimated input positions $\hat{\boldsymbol{u}}$ to be introduced. The benefit is that, when no allocation error is made due to saturation, the hedging signal will be set to 0 automatically and the closed-loop performance reverts to the nominal design case.

Selection of $\boldsymbol{K}_h$ is given below. Note the hedging part on the RHS of Eq. (5): $\frac{\boldsymbol{K}_e - \boldsymbol{K}_r}{s+\boldsymbol{K}_r}\boldsymbol{\nu}_h$, the hedging signal contribution to $\boldsymbol{\nu}_{\mathrm{cmd}}$ is low-passed, with a steady-state gain of $\frac{\boldsymbol{K}_e - \boldsymbol{K}_r}{\boldsymbol{K}_r}$. From a protection point of view, the desired slow-down effect is to free from $\boldsymbol{\nu}_{\mathrm{cmd}}$ all the error that is made due to saturation, therefore, the desired steady-state gain on the $\boldsymbol{\nu}_{\mathrm{err,CA}}$ should be one. Thus, we can select $\boldsymbol{K}_h$ as:

$$\boldsymbol{K}_h = \frac{\boldsymbol{K}_r}{\boldsymbol{K}_e - \boldsymbol{K}_r}$$

As a result, the expression of $\boldsymbol{\nu}_{\mathrm{cmd}}$ with the proposed hedging structure becomes Eq. (6).

$$\boldsymbol{\nu}_{\mathrm{cmd}} = \underbrace{\dot{\boldsymbol{y}}_{\mathrm{ref,unhedge}}}_{\mathrm{ff,unhedged}} + \underbrace{\boldsymbol{K}_e\left(\boldsymbol{y}_{\mathrm{ref,unhedge}} - \hat{\boldsymbol{y}}\right)}_{\mathrm{fb,unhedged}} - \underbrace{\frac{\boldsymbol{K}_r}{s+\boldsymbol{K}_r}\boldsymbol{\nu}_{\mathrm{err,CA}}}_{\mathrm{hedging}} \tag{6}$$

Interpretation for Eqn. (6) is: due to saturation, the control allocation is distorted by an error of $\boldsymbol{\nu}_{\mathrm{err,CA}} = \boldsymbol{\nu}_{\mathrm{cmd}} - \boldsymbol{\nu}_{\mathrm{res}}$. When this happens, hedging will slow down the reference dynamics which directly prompts reduction of the pseudo control command by $\frac{\boldsymbol{K}_r}{s+\boldsymbol{K}_r}\boldsymbol{\nu}_{\mathrm{err,CA}}$, that is the allocation error itself at a steady state. The control allocation is therefore discharged from the saturation burden. Consequently, the distortion effect is mitigated. In other words, the reference dynamics is slowed down and synchronized to the system's control capability, preventing further level of saturations to aggravate.

IV. Simulation Model, Baseline Controller and Control Allocation

In this paper, an INDI baseline controller is implemented, for an extended F-16 model. Thorough details for the implementation of this simulation model and baseline controller can be found in Ref. [11]. The plant is highly redundant in its effectors, whose configuration is shown in Fig. 4.

Regarding the baseline controller, the control variables are the rotation rates in the body frame. A linear reference model (RM) and error controller (EC) are used. As mentioned earlier, the bandwidth of the error controller is faster than that of the reference model to activate the protection effect of hedging. The on-board-plant-model (OBPM) calculates and updates the effectiveness matrix $\boldsymbol{B}$ in real time, based on state and input feedback. The pseudo controls in this case are the rotation rates, which are assumed directly measurable.

The allocation method here functions also incrementally, allocating incremental pseudo controls command $\Delta\boldsymbol{\nu}_{\mathrm{cmd}}$ into incremental input commands $\Delta\boldsymbol{u}_{\mathrm{cmd}}$. The hedging signal is correspondingly chosen to be the incremental allocation error. The complete closed-loop diagram is shown in Fig. 3.

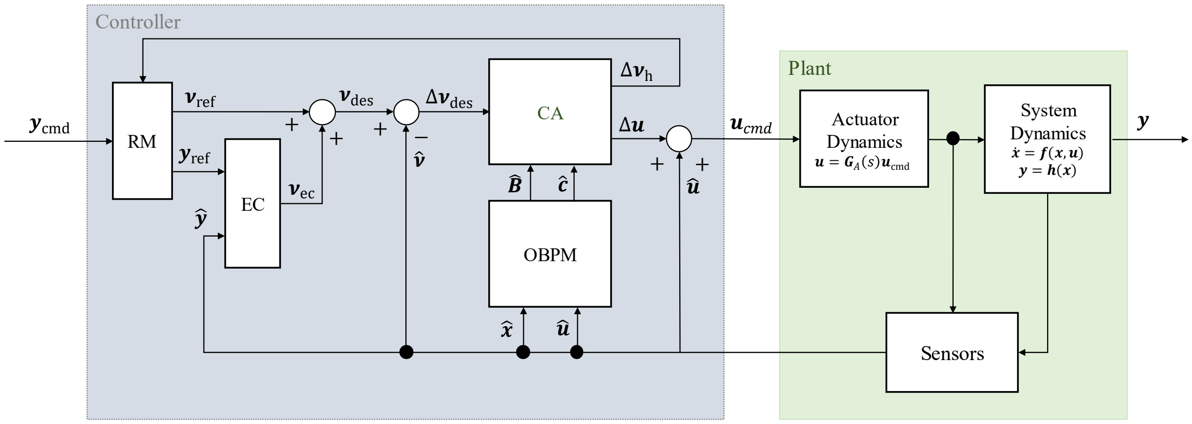

Fig. 3 Baseline Controller Structure

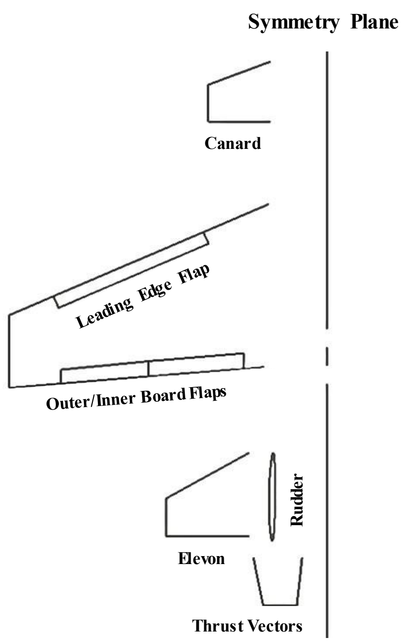

Fig.4 Effectors on the Extended F-16 Model

The control allocation used here is the redistributed pseudo-inverse method. System redundancy related issues such as trajectory dependency for incremental controllers are addressed with a null space transition algorithm, which shifts the allocation solution in the null space of the effectiveness matrix $\boldsymbol{B}$, slowly converging to a set of desired solutions in a steady-state flight, denoted by $\boldsymbol{c}$ as an output of the OBPM [11].

V. Test Setup and Simulation Results

Simulations are setup to compare the closed-loop performance with and without the proposed hedging. During the simulation, one control surface failure is manually triggered during harsh maneuvers to induce multiple saturations simultaneously. The failed control surface is blocked and fixed at the current position of the failure time. The failed surface is known to the allocation process, meaning that allocation removes the failed surface from the $\boldsymbol{B}$ matrix. Fig. 5 compares the responses of control variable commands, references, and measurements in the hedged and unhedged simulation.

At the start of the simulation, in the first 25 seconds of simulation, where commands are relatively gentle, the two simulation results are almost identical, proving that hedging does not result in any tracking deficit during nominal operations. From 27 seconds on, where both the harsh multi-axis commands are given, and the effector failure is triggered, the tracking error in the unhedged case is significantly greater than the hedged counterpart, in both the commanded direction and cross direction. For example, yaw rate divergence is observed at 28 seconds when pitch and roll rate are commanded, while at 33 seconds, the yaw rate command leads to divergence of pitch and roll rate responses from the reference value. At 30 seconds, the yaw reference signal in the hedged case stays very closely to the measurement, indicating a strong slow-down effect of hedging, which essentially helps to reduce cross errors in the other two axes.

To give more proof of the hedging effect, the control input responses are examined closely in Fig. 6 to Fig. 9. In these figures, the red line always stands for the unhedged simulation results and the blue line for the hedged ones. Dotted lines represent upper and lower lines of the effectors. First, as mentioned earlier, the system behavior before 27 seconds are almost identical in both simulations. At 27 second, in the upper left subplot of Fig. 6, the elevator position freezes from then on, indicating the failed control surface.

Starting from 27 seconds, the unhedged result encounters a large extent of saturations, whereas the hedged results stay only shortly at the saturation positions and then reduce to unsaturated positions. This is especially clear in the responses of ailerons, rudders, and flaps. When the pitch thrust vector completely saturated for almost 5 seconds in the unhedged results, the hedged results sees minor saturations in this effector. Fig.9 gives a zoomed-in look at effectors group 1, which clearly depicts the recovery of the hedged ones compared to fully saturated unhedged results.

Finally, in Fig. 10, a comparison of the incremental allocation error is provided. Due to the hedging protection, the allocation error made is much smaller than the unhedged case, which explains the cross error observed in the unhedged result.

VI. Conclusion

In this paper, we start from a simple linear reference model and feedback structure with hedging, and derive a direct relationship between the hedging signal and pseudo controls command, which shows that given appropriately chosen reference model and error controller bandwidths, the hedging slows down the reference model with the low-passed hedging signal, for whatever signal used as hedging signal. From this observation, we proposed a hedging structure using the algebraic control allocation error with a properly designed gain, which leads to steady state pseudo control commands protection due to this allocation error. The closed-loop performance with this hedging structure will remain identical to the unhedged case when no input saturates, but automatically trigger the protection when any allocation error occurs due to input saturation. Closed-loop simulation shows significantly improved tracking performance given simultaneous harsh pilot command and effector failure.

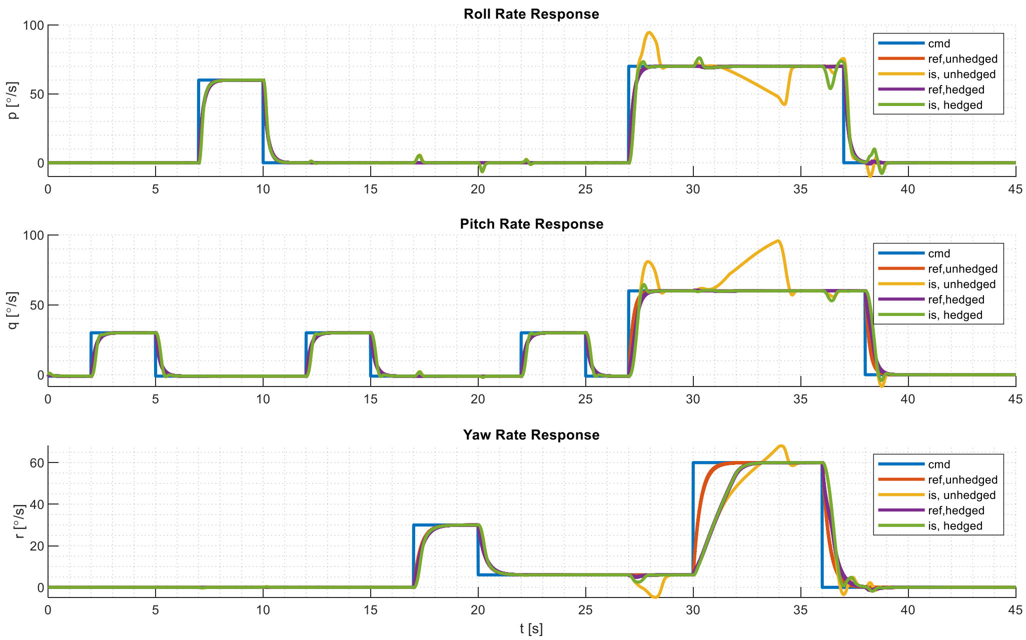

Fig. 5 Comparison of Hedged and Unhedged Closed-loop Performance

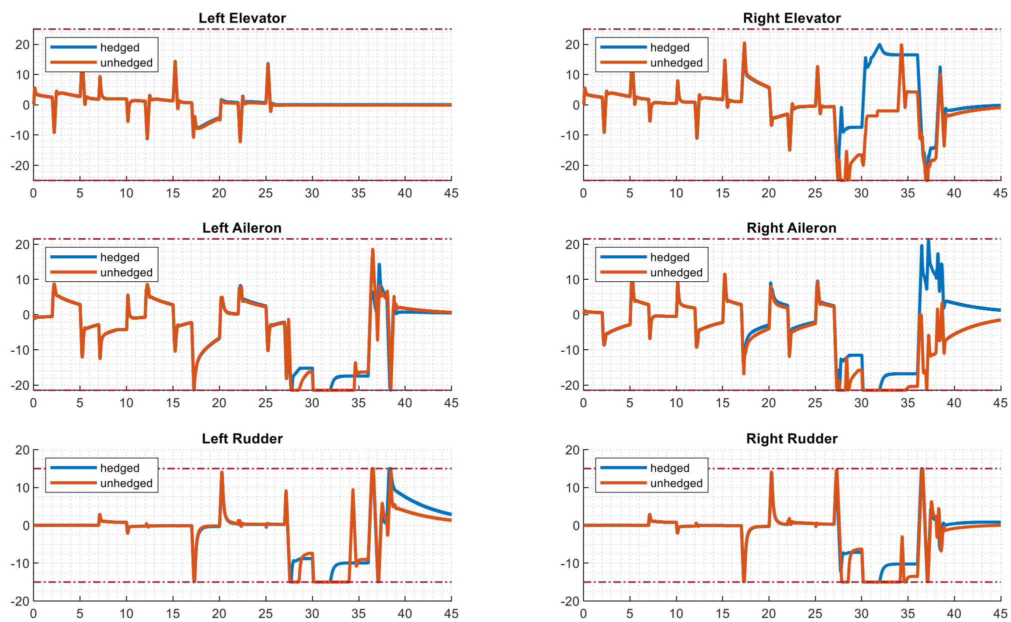

Fig. 6 Control Surfaces Group 1 (Elevator, Aileron and Rudder)

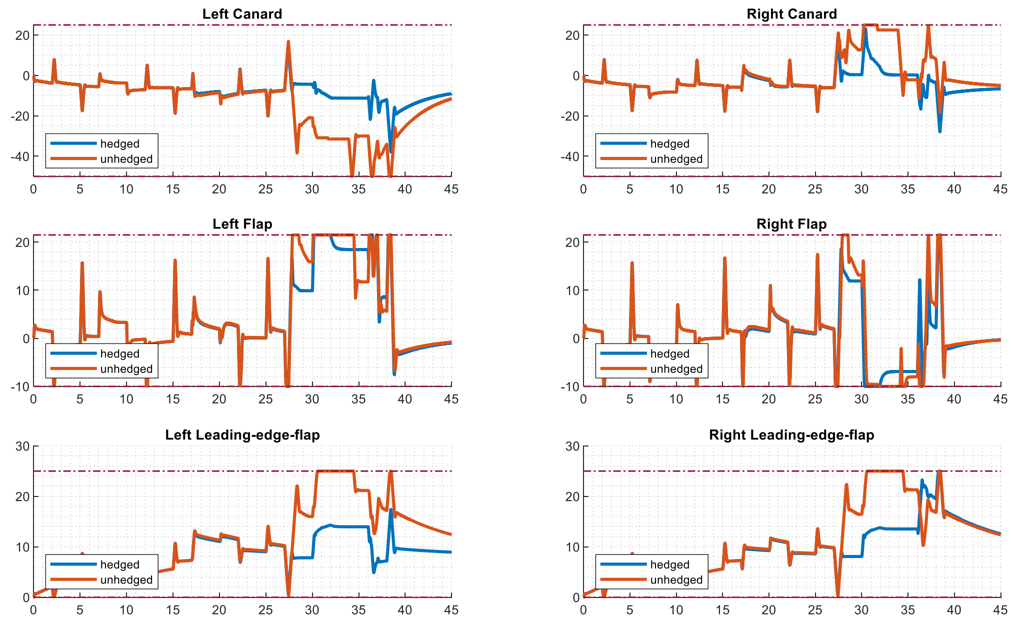

Fig. 7 Control Surfaces Group 2 (Canard, Flap and Leading-edge-flap)

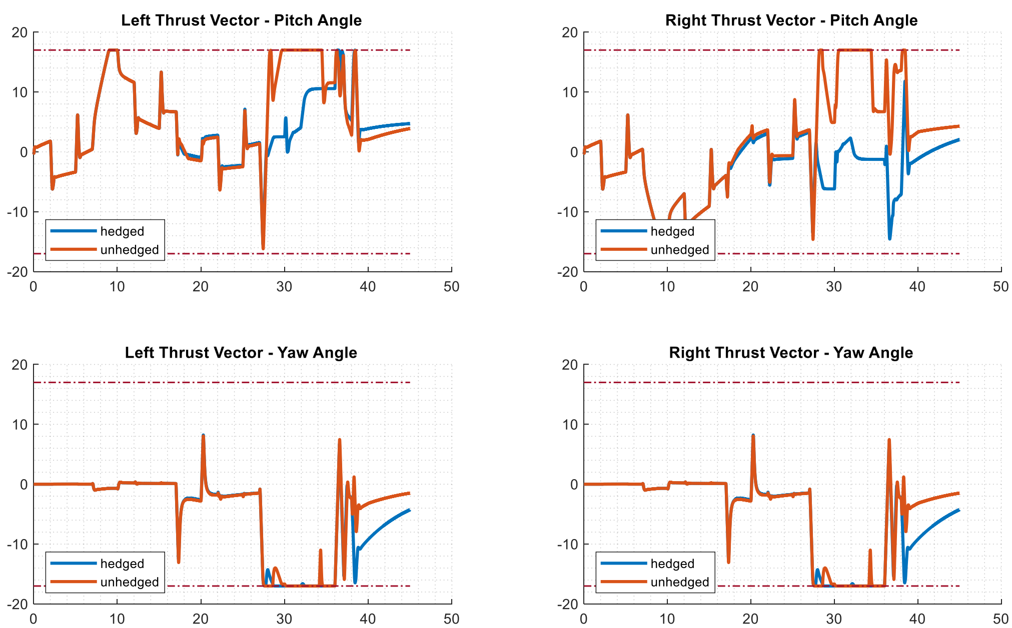

Fig. 8 Thrust Vectoring Angles (Pitch and Yaw Direction)

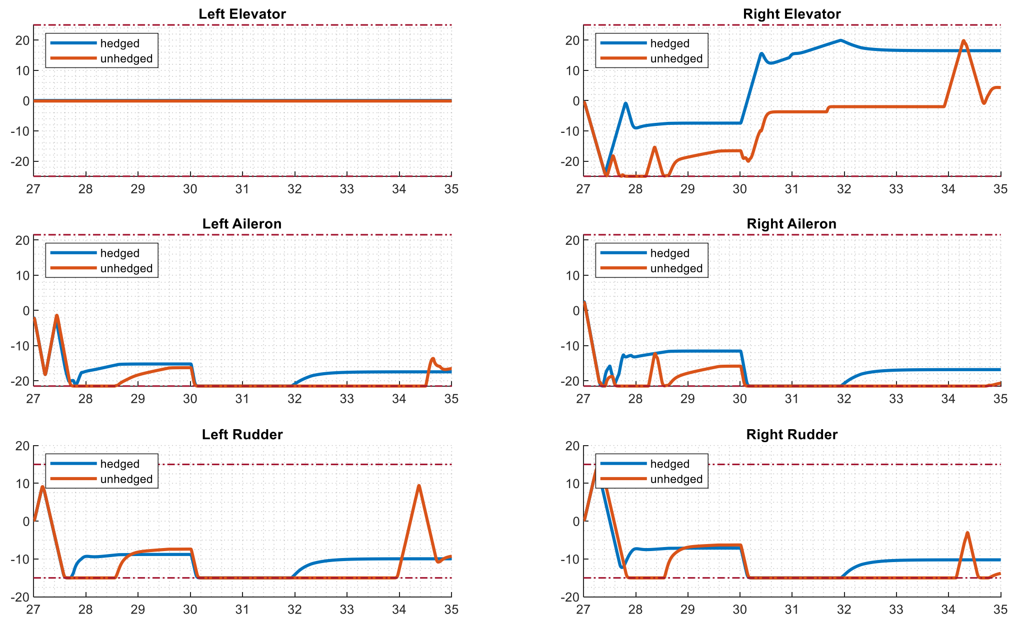

Fig. 9 Control Surfaces Group 1, Zoom in 27-35 Seconds

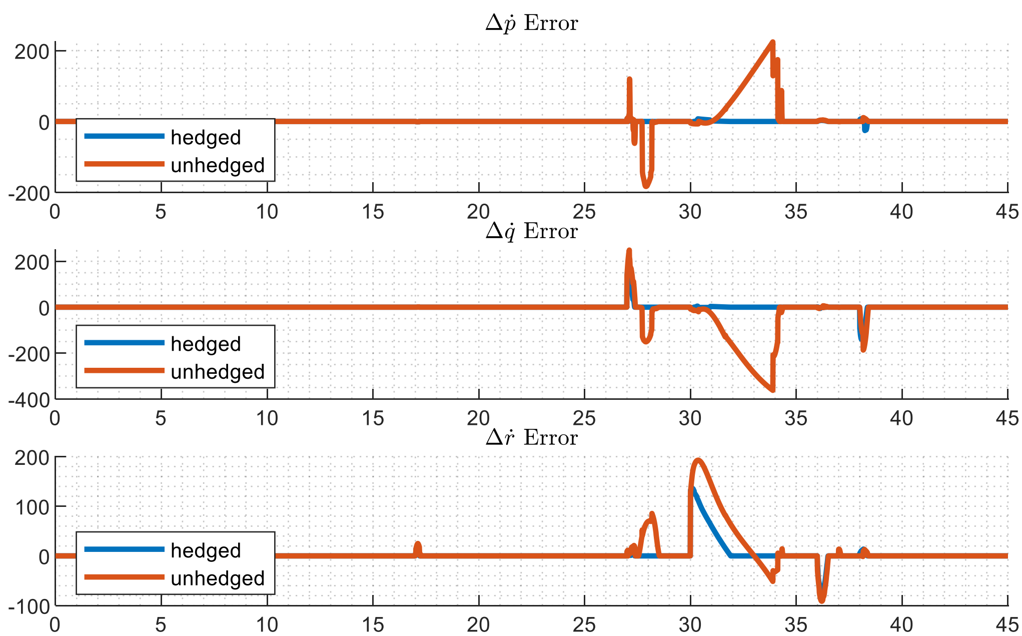

Fig. 10 $\nu_{\mathbf{error,CA}}$ Comparison

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