MCG 4322[A]
FSAE analysis Dossier
FSAE 2

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Design Solution

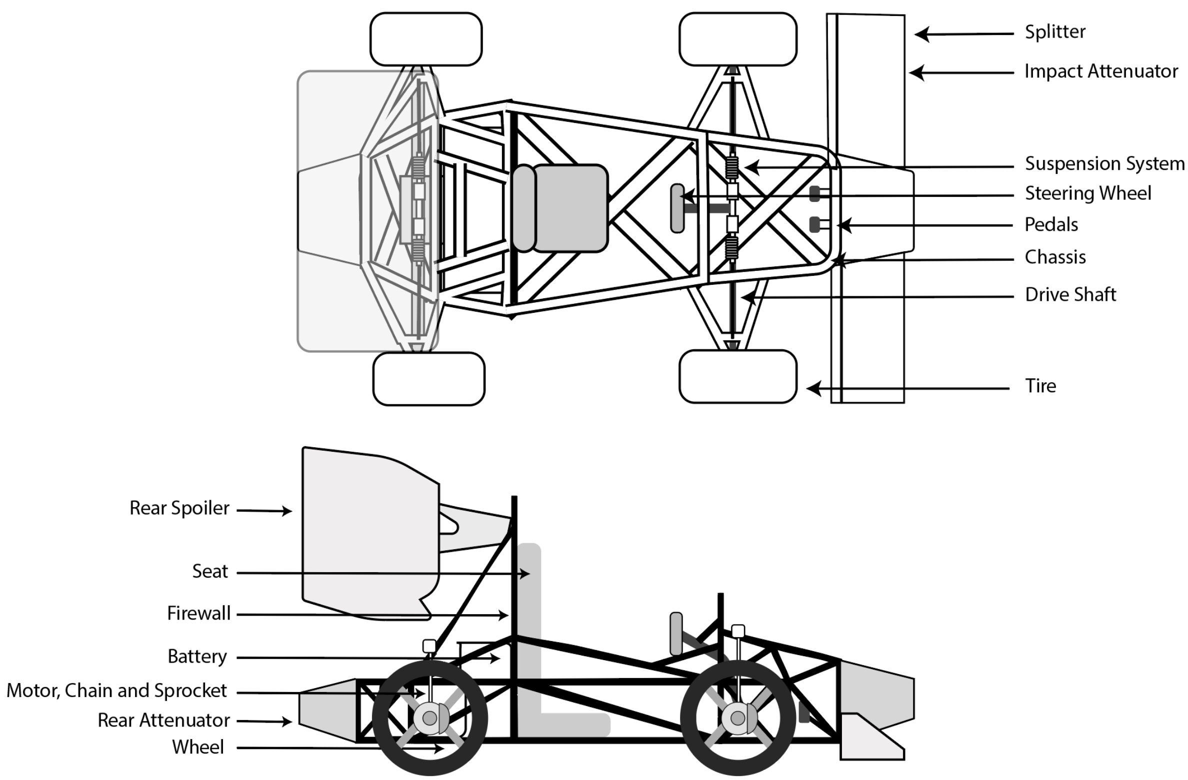


Figure 1.1: Updated Solution of the vehicle with labeled components

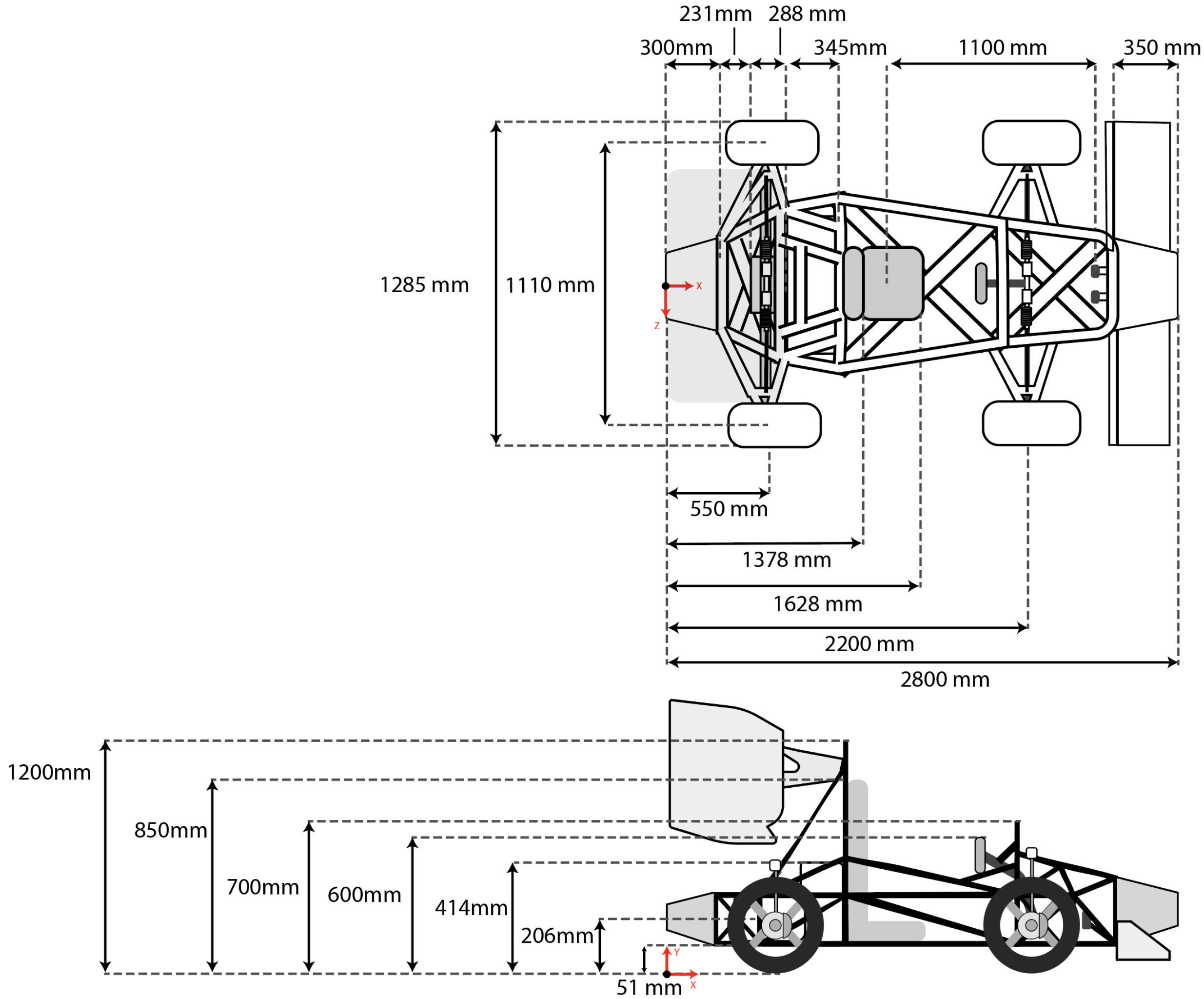


Figure 1.2: Updated Solution of the vehicle with dimensions

Chapter 2

System Modelling

2.1 Chassis

This is the updated design of the subsection

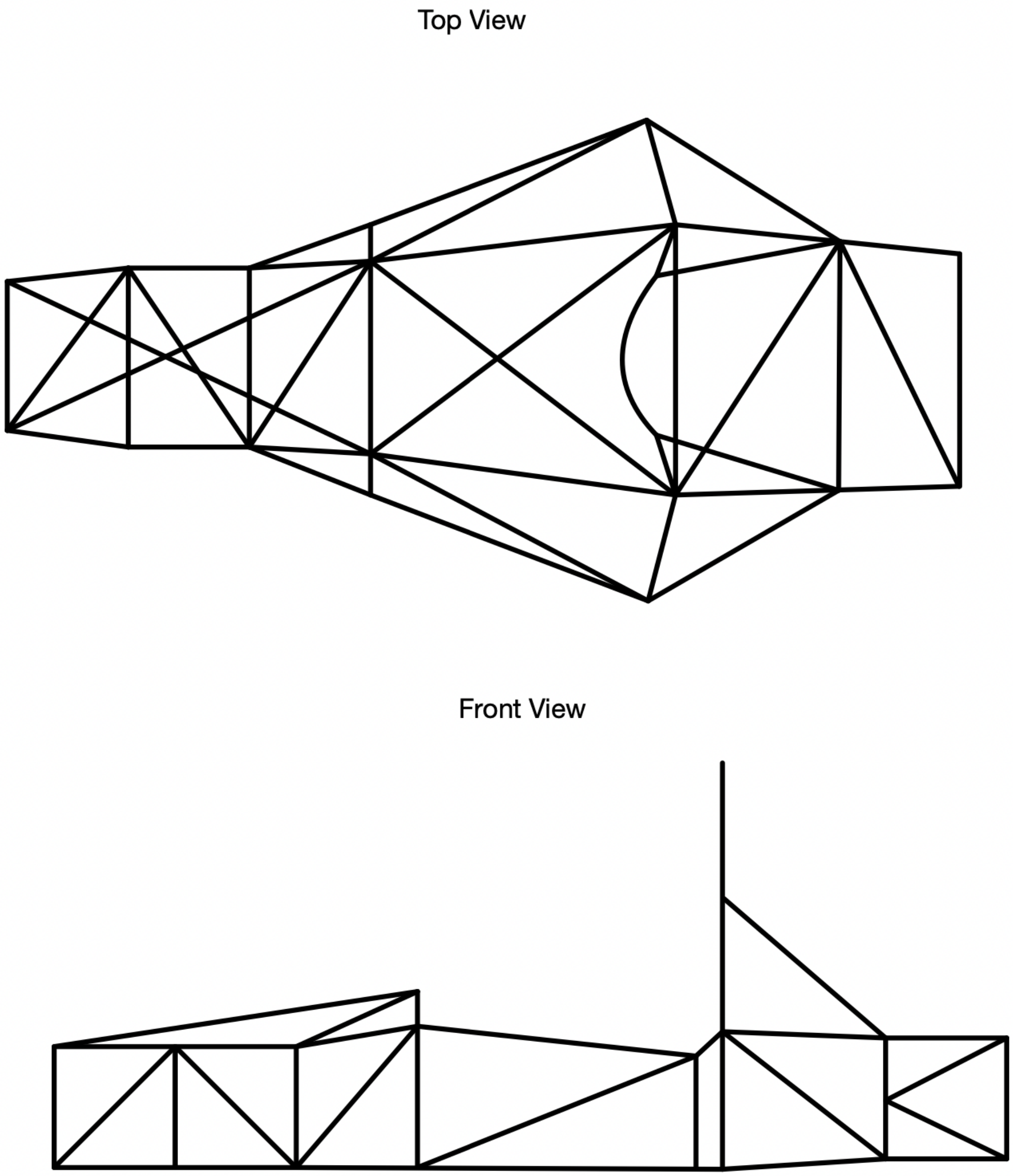


Figure 2.1: Front and Top View of the Updated Chassis Design

Furthermore, the FSAE rule-book specifies the minimum dimensions required for the chassis's tubing which will be applied to this design to reduce cost and weight of the vehicle.

Table 2.1: Chassis Tubing Dimensions

Member	**Cross Section (mm2)**	**Outer Diameter (mm2)**	**Wall Thickness (mm)**
Front Bulkhead	114	25.0	1.2
Front Bulkhead Support	91	25.0	1.2
Front Hoop	173	25.0	2.0
Front Hoop Bracing	114	25.0	1.2
Main Hoop	173	25.0	2.0
Main Hoop Bracing	114	25.0	1.2
Main Hoop Bracing Support	91	25.0	1.2
Side Impact Structure	114	25.0	1.2
Shoulder Harness Mounting Bar	173	25.0	2.0
Other Structural Tubing	91	25.0	1.2

2.2 Approximations

2.2.1 Vehicle Assumptions

Table 2.2: Assumptions of vehicle operation

Parameter	**Approximation**
Maximum Speed	105 km/h
Maximum Acceleration	6 m/s^2
Maximum Force Applied on the Brake Pedal	2000 N
Average Impact Deceleration	196 m/s^2

2.2.2 Mass Approximations

Table 2.3: Weight Calculation of the Vehicle Approximation [1]

Item	**Weight (kg)**
Steel Spaceframe	31.80
Maximum Driver Weight	111.58
Front Attenuator	1
Front Wing	3.10
Rear Wing	2.80

Wing Mounting	1.32
Battery	32
Emraxx 228 Motor	12
Chain and Sprocket Kit	3.60
Motor Mount	0.17
Limited Slip Differential	2.60
Chain and Sprocket Kit	3.58
Half Axle Shaft	0.7
Tripod Joints	0.2
Front Control Arm Assemblies	2.6
Rear Control Arm Assemblies	2.4
Front Uprights	1.8
Rear Uprights	8.1
Rockers Assemblies	1.8
Shock Absorber	2.6
Swaybar Assemblies	0.6
Rotors	1.22
Breakline Assembly	2.8

Break Pads	0.2
Break Calipers	0.85
Rack and Pinion Subsystem	3.1
Steering Wheel	1.97
Quick Release	0.99
Steering Shaft	2.2
Intermediate shaft	0.82
Steering column bushing	1.09
Tie rod	0.63
Tires	17.96
Wheels	13.3
Front Wheel Hubs	2.4
Rear Wheel Hubs	10.4
Wiring and Instruments	2.3
Seat	2.07
Firewall	2.02
Harness	1.04
Throttle and Brake Pedal System	2.4

Total Mass of the Car	**297**

2.2.3 Accumulator Approximations

It is important that the accumulator parameters are estimated so that the geometric and mass properties of the vehicle can be calculated. The table below illustrates the approximate dimensions and weight of the battery. The calculations for how this model was calculated are illustrated in appendix **??**

Table 2.4: Sony VTC6 Battery Specification [2]

Parameter	**Value**
Continuous Max. Discharge (A)	15.0
Peak Max. Discharge (A)	30.0
Nominal Voltage (V)	3.6
Max Voltage (V)	4.2
Capacity (Ah)	3.0
Diameter (mm)	18.3
Height (mm)	65.0
Mass (g)	48.0

Table 2.5: Sony VTC6 Accumulator Design 5 sections connected in series consisting of (4px28s)

Parameter	**Value**
Capacity per Module (Ah)	12
Nominal Voltage per Module (V)	100.8
Nominal Capacity per Module (Wh)	1209.6
Peak Voltage per Module (V)	117.6
Peak Capacity per Module (Wh)	1411.2
Total Nominal Voltage (V)	504
Accumulator Peak Voltage (V)	588
Max Discharge Current (A)	120
Accumulator Peak Power (W)	70560
Accumulator Capacity (kWh)	6.048
Dimensions X (mm)	693.4
Dimensions Y (mm)	345
Dimensions Z (mm)	123.2
Weight (Batteries) (kg)	26.88
Accumulator Estimated Weight (kg)	30.38

2.3 Geometric Relations & Properties

2.3.1 Wheelbase to Track Ratio

The wheelbase to track ratio is a mathematical relationship that characterizes the handling of the vehicle. The track width determines the weight that is transferred by the mass of the car in cornering. The wheelbase determines how much weight is transferred by the mass of the car in acceleration and braking.

$$\frac{\text{Wheelbase Length}}{\text{Track Length}} = \frac{1650mm}{1110mm} = 1.49 \tag{2.1}$$

The FSAE car's wheelbase length is designed to be 1650 mm while the track width is the 1110 mm, giving a ratio of 1.49. This ratio nearly falls into the range of optimum ratio which is 1.5 to 1.7. An optimum ratio of wheelbase to track ensures proper mass distribution throughout the car, resulting in high handling performance.

2.3.2 Center of Mass Calculation

A vehicle's center of mass is the point where the sum of all of the subassembly masses act. The center of mass is used to determine overall forces when the car is turning, accelerating or decelerating. Most FSAE vehicles are designed to have a relatively low center of mass. To measure the center of mass of the car, the subassemblies were split into volumes. Each volume had its own center of mass which was approximated using the dimensions and the origin in the table below [3].

Table 2.6: Volumes and their Respective Approximate Center of Mass

Volume	**Weight (kg)**	**x - Coordinate (mm)**	**y - Coordinate (mm)**	**z - coordinate (mm)**

Battery, Motor, and Powertrain	51.244	866.5	207	0
Front Suspension Assembly	10.656	2150	375	0
Rear Suspension Assembly	16.712	550	550	0
Top Right Wheel, Tire, and Brake Assembly	9.843	2150	206	625
Top Left Wheel, Tire, and Brake Assembly	9.843	2150	206	-625
Bottom Right Wheel, Tire, and Brake Assembly	13.843	550	206	625
Bottom Left Wheel, Tire, and Brake Assembly	13.843	550	206	-625
Steering Assembly	11.796	2000	415	0
Driver, Firewall, Harness, and Seat	116.71	1400	425	0
Spaceframe and Wings	38.972	1400	600	0

Refer to **??** for the locations of the center of mass of each volume.

$$x_{mm} = \frac{\sum_{i=1}^{N} m_i x_i}{M} \Rightarrow COM_x = \frac{381439.38mm}{293.46kg} = 1299.79mm \tag{2.2}$$

$$y_{mm} = \frac{\sum_{i=1}^{N} m_i y_i}{M} \Rightarrow COM_y = \frac{111434.019mm}{293.46kg} = 379.72mm \tag{2.3}$$

$$z_{mm} = \frac{\sum_{i=1}^{N} m_i z_i}{M} \Rightarrow COM_z = \frac{0mm}{293.46kg} = 0mm \tag{2.4}$$

2.3.3 Load Transfer

2.3.3.1 Longitudinal Load Transfer

The center of gravity is the location of rotation for breaking and acceleration inputs. The vehicle will squat when accelerating and dive when braking relative to the center of gravity. To mitigate this rotation, a counter rotation needs to be applied to the center of gravity. This opposing rotation is induced by anti-squat and anti-dive geometry.

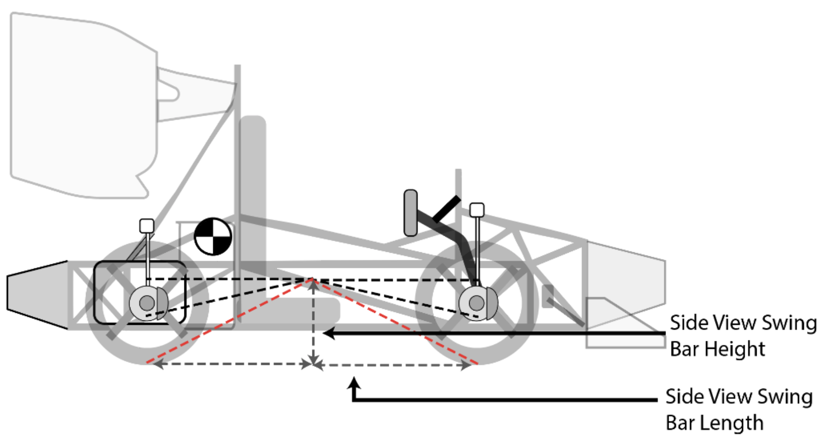


Figure 2.2: Anti-squat Geometry (left red line) and Anti-dive Geometry (right red line)

Anti-squat applications are added to the rear since the rear wheels squat while accelerating. The anti-squat geometry is determined using the swing arm length and height, wheelbase, and height of the center of gravity. The rear suspension's compression percentage can be modeled using the height of the center of gravity (h), wheelbase of the vehicle (L), and the dimensions of the swing arm length.

$$\%\text{ Anti Squat } = \frac{\tan \emptyset \mathrm{A}}{\mathrm{h/L}} \times 100 \tag{2.5}$$

Where,

$$\tan \emptyset_{\mathrm{R}} = \frac{\text{Side View Swing Arm Height}}{\text{Side View Swing Arm Length}} \tag{2.6}$$

Anti-dive geometry is applied for the front wheels since the front wheels experience compression whenever the car brakes. Anti-dive geometry prevents the car from diving onto the brakes and deflecting vertically. The percentage of front suspension's compression can be mathematically modeled using the height of the center of gravity (h), wheelbase of the vehicle (L) and the dimensions of the virtual side view swing arm length [4].

$$\%\text{ Front Anti Dive } = \%\text{ Front Braking } \times \tan \emptyset_{\mathrm{A}} \times \frac{\mathrm{L}}{\mathrm{h}} \tag{2.7}$$

2.3.3.2 Lateral Load Transfer

The car experiences rotation around a virtual point being the roll centre. The roll centre convention assumes that a load is applied on the center of the wheel. The roll center can be located using the geometry in figure 2.17.

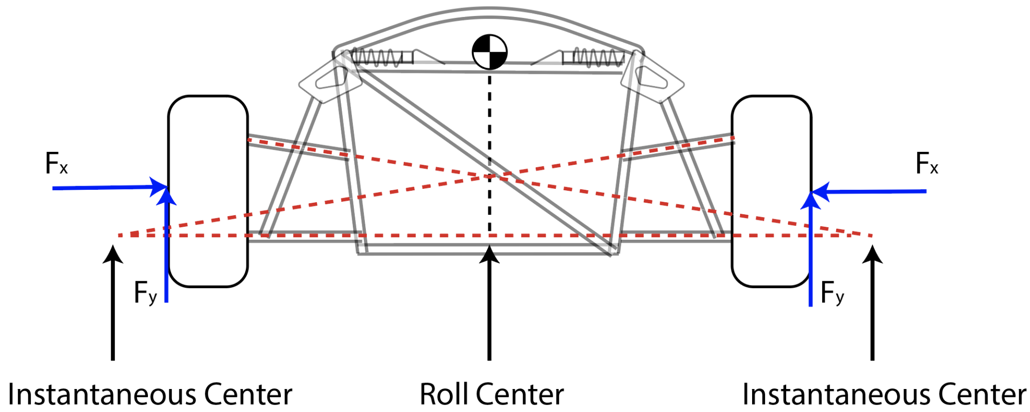


Figure 2.3: Suspension Geometry to Locate Instantaneous Points and Roll Centre

The instantaneous points act as a pivot where the force rotates on the imaginary red lines, subsequently causing a rotation around the line. As the vehicle accelerates, the x component of the force is applied on the control arms and the y component force moves the suspension around its instantaneous center [5].

2.3.4 Suspension Geometry

To determine the geometry of the suspension components, the following parameters are considered and fine tuned:

• Camber: As described in the literature review, camber is usually tuned to be negative for an FSAE vehicle. For the sake of calculations, camber will be assumed to be zero degrees.

• Roll Center: Roll center is to be kept between the center of gravity and the ground. The roll center is best optimized when it is in the range between 15 and 30 percent of the height of the center of gravity.

• Swing Arm Length: For FSAE cars, swing arm length is best optimized when it is within the range of 1000 to 1800 mm. As such, a swing arm length (or instantaneous center) was assumed to be 1500

• King Pin Inclination (KPI): KPI was assumed to be zero for ease of calculations. Like the camber angle, KPI should be negative for a race car application.

• Scrub Radius: Assumed to be 30 mm.

• Wishbone Spread: Should be as large as possible so that the wishbones don't collide with the wheels at a maximum steering angle. This can be determined with a SolidWorks model of the car, but is assumed to be 40 degrees for now.

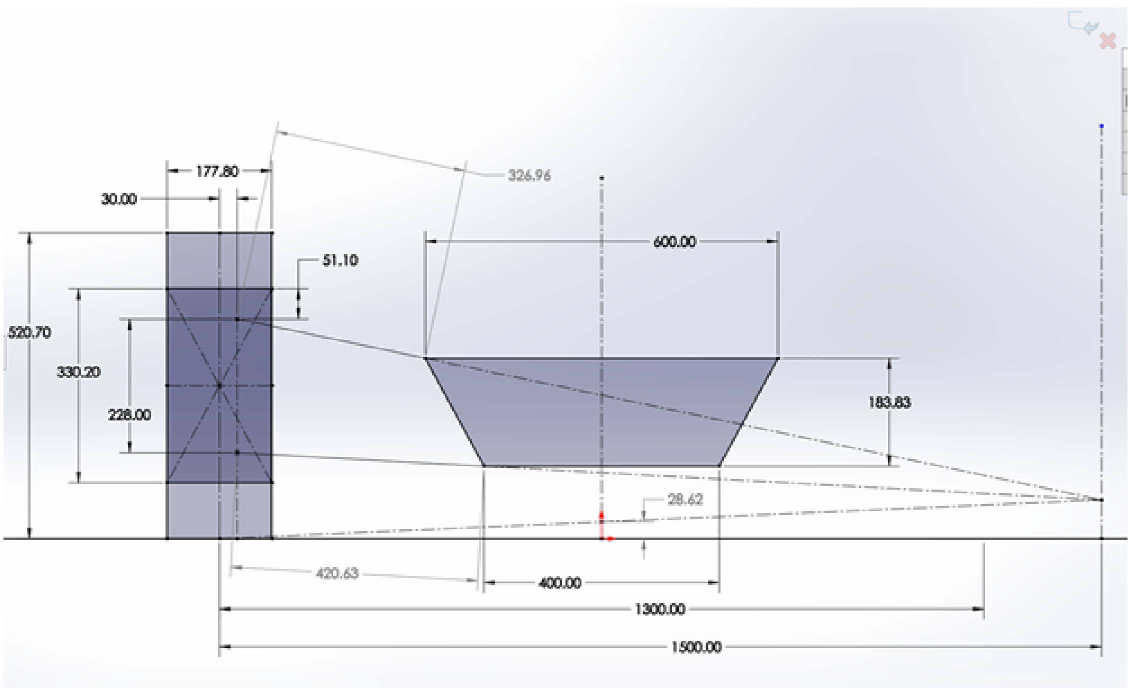


Figure 2.4: SolidWorks 2D Sketch Suspension Geometry

2.3.5 Assumptions and Working Environment

The FSAE vehicle is assumed to operate under clear weather conditions while being susceptible to high impact crashes. The driver of the vehicle is assumed to be a 95th percentile male with a weight of 111.58 kg. The total weight of the vehicle with the driver is calculated to a total of 297 kg. The vehicle is assumed to be operating using its maximum speed and acceleration, 105 km/hr and 6 m/s^2 respectively.

2.4 Kinematics and Applied Forces

2.4.1 2 Dimensional Kinematic Model of Vehicle

Illustrated below is a figure which represents a 2 dimensional model of a car in motion. The parameters can be defined as:

$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \tag{2.8}$$

The 4 Degrees of Freedom are represented as the vehicles position on the x-axis, y-axis and along the vehicle's steering angle θ

Assuming small velocities, the dynamics of the vehicle can be described using a single-track model [6]

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ \frac{1}{L} * tan(\phi) \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \omega \tag{2.9}$$

Where x and y define the position of the midpoint of the rear axle, θ refers to the orientation of the car relative to the x-axis, and ϕ refers to the steering angle. [7]

Hence, it is determined that the motion of the car can be determined using the equations below. It can easily be demonstrated that 2.10 and 2.11 are the velocity components of the car in the x and y coordinates, and 2.12 is the angular velocity of the vehicle.

$$\dot{x} = v\cos(\theta); \tag{2.10}$$

$$\dot{y} = v\cos(\theta); \tag{2.11}$$

$$\dot{\theta} = \frac{v}{L}\tan(\phi) \tag{2.12}$$

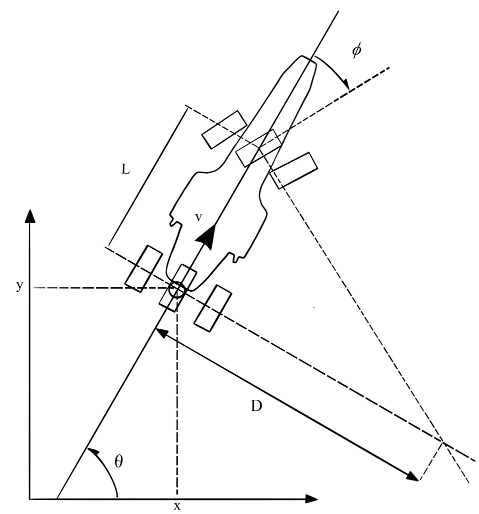


Figure 2.5: 2-D Kinematic Model of Vehicle

2.4.2 Steady State Low Speed Cornering Geometry and Vehicle Turning Radius

The figure below illustrates an Ackermann Geometry also known as a kinematic steering geometry. The Ackermann geometry states that for a basic steering system for a front wheel steering system, the difference of the cotangents of the angles of the outer front wheels to the inner front wheels should equate to the ratio of the width (T) and length (L) of the vehicle.

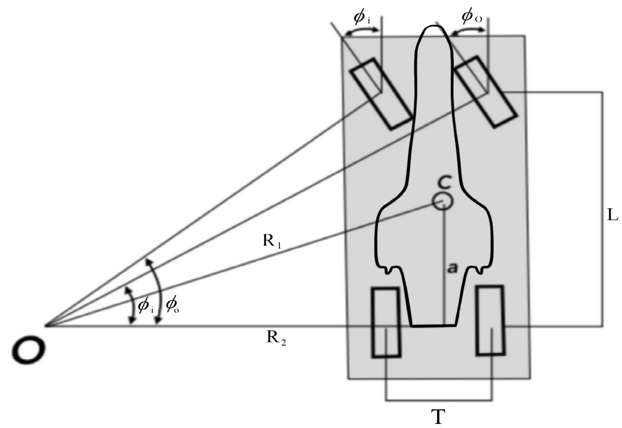


Figure 2.6: Vehicle Steering Ackermann Method

This condition can be simply modeled by the following set of equations:

$$\cot(\phi_i) - \cot(\phi_o) = \frac{T}{L} \tag{2.13}$$

Where, ϕ_i is the steering angle of the inner front wheel,ϕ_o is the steering angle of the outer front wheel, T is the vehicle's track width, and L is the wheelbase of the vehicle. As seen in figure 2.6 it can be seen that if a normal is drawn from all the wheels they intersect at a point O which is the center of rotation. The vehicle's center of mass will turn with a radius R_1. This is known as the turning radius, which can be geometrically defined as follows:

$$R_1 = \sqrt{a^2 + (L^2)(\frac{(\cot(\phi_o) + \cot(\phi_i))}{2})^2} \tag{2.14}$$

Where, a is the distance from the rear axle to the center of mass and is 0.739 m. Assuming $R_2 = 5m$, and provided $L = 1.65m$ and $T = 1.11m$. Then, the turning radius of the vehicle can be determined as follows:

$$\phi_o = \arctan\left(\frac{L}{R_2 + \frac{T}{2}}\right) = \arctan\left(\frac{1.65m}{5m + \frac{1.11m}{2}}\right) = 0.2804rad \tag{2.15}$$

$$\phi_i = \arctan\left(\frac{L}{R_2 - \frac{T}{2}}\right) = \arctan\left(\frac{1.65m}{5m - \frac{1.11m}{2}}\right) = 0.3455rad \tag{2.16}$$

$$R_1 = \sqrt{(.739m)^2 + (1.65m)^2(\frac{(\cot(.2804rad) + \cot(.3455rad)}{2})^2} = 5.209m \tag{2.17}$$

Hence, from the above series of equations it can be determined that the turning radius of the center mass of the vehicle would be 5.21 meters.

2.4.3 Static Applied Forces

2.4.3.1 Forces Applied on a Car Parked on a Level Road

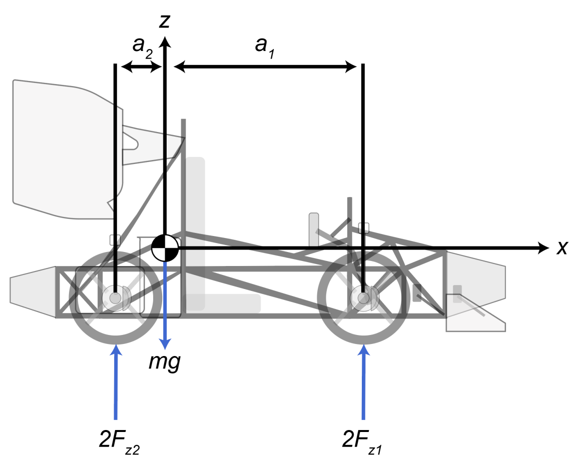


Figure 2.7: FBD of Car on a Level Road

The reaction forces on the wheels of a car parked on a level road can be found as follows:

$$F_{z_1} = \frac{1}{2}mg\frac{a_2}{(a_2 + a_1)} = \frac{1}{2}(297kg)(9.81\frac{m}{s^2})\frac{(0.75m)}{(0.75m + 0.9m)} = 662.1 \text{ N} \tag{2.18}$$

$$F_{z_2} = \frac{1}{2}mg\frac{a_1}{(a_2 + a_1)} = \frac{1}{2}(297kg)(9.81\frac{m}{s^2})\frac{(0.9m)}{(0.75m + 0.9m)} = 794.6 \text{ N} \tag{2.19}$$

2.4.3.2 Forces Applied on a Car Parked on an Inclined Road

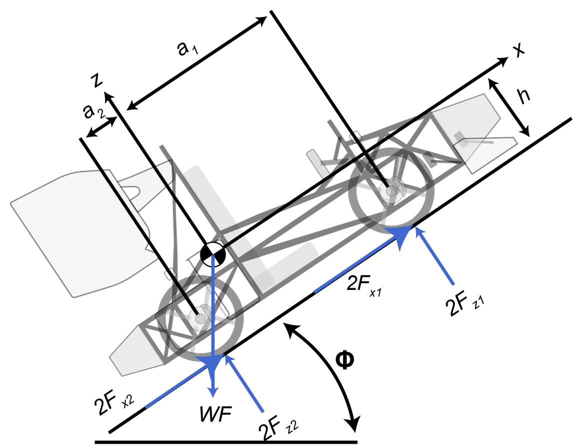


Figure 2.8: FBD of Car on an Inclined Road

The normal forces acting on the wheels of a car parked on an inclined road can be found as follows:

$$\mathrm{F}_{z_1} = \frac{1}{2}mg\left(\frac{a_2}{a_2+a_1}\cos\phi - \frac{h}{a_2+a_1}\sin\phi\right) \tag{2.20}$$

$$\mathrm{F}_{z_2} = \frac{1}{2}mg\left(\frac{a_1}{a_2+a_1}\cos\phi + \frac{h}{a_2+a_1}\sin\phi\right) \tag{2.21}$$

Assuming a coefficient of static friction $\mu_s = 0.7$, the friction forces at the wheels are found to be:

$$F_{x_1} = \mu_s F_{z_1} = (0.7)(594.01 \text{ N}) = 415.81 \text{ N} \tag{2.22}$$

$$F_{x_2} = \mu_s F_{z_2} = (0.7)(840.64 \text{ N}) = 588.45 \text{ N} \tag{2.23}$$

Assuming a typical angle of incline as $\phi = 10°$, the reaction forces can be calculated as follows:

$$\mathrm{F}_{z_1} = \frac{1}{2}(297kg)(9.81\frac{m}{s^2})(\frac{(0.75m)}{(0.75m+0.9m)}\cos(10°) - \frac{(0.379m)}{(0.75m+0.9m)}\sin(10°)) = 594.01 \text{ N} \tag{2.24}$$

$$F_{z_2} = \frac{1}{2}(297kg)(9.81\frac{m}{s^2})(\frac{(0.9m)}{(0.75m+0.9m)}\cos(10°) + \frac{(0.379m)}{(0.75m+0.9m)}\sin(10°)) = 840.64 \text{ N} \tag{2.25}$$

2.4.3.3 Forces Applied on a Car Parked on a Banked Road

When a car is parked on a banked road, the load is distributed differently between each side of the car, with a larger load experienced by the lower tires. To illustrate this, an FBD for the front wheels on a banked road is used, where the load experienced on the front wheels corresponds to the load on the front axle (W_F).

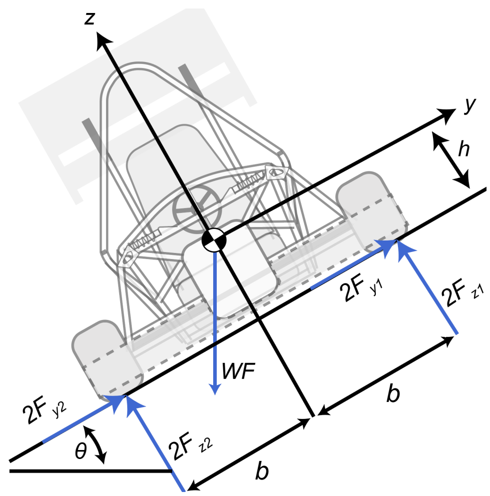


Figure 2.9: FBD of Car on a Banked Road

The reaction forces above are found as follows [8]:

$$F_{z_1} = \frac{W_F}{2b}(b\cos\theta - h\sin\theta) \tag{2.26}$$

$$F_{z_2} = \frac{W_F}{2b}(b\cos\theta + h\sin\theta) \tag{2.27}$$

To calculate for the normal forces experienced by the front wheels, W_F is first found by multiplying the value for F_{z_1} from section 2.4.3.1 by the cosine of the bank angle θ.

Assuming the bank angle to be $\theta = 10°$, thus giving $W_F = 652.04N$.

$$F_{z_1 FRONT} = \frac{652.04N}{2(0.625m)}(0.625\cos(10°) - 0.379\sin(10°)) = 286.76 \text{ N} \tag{2.28}$$

$$F_{z_2 FRONT} = \frac{652.04N}{2(0.625m)}(0.625\cos(10°) + 0.379\sin(10°)) = 355.43 \text{ N} \tag{2.29}$$

Using a coefficient of friction $\mu_s = 0.7$, the following friction forces can be found:

$$F_{y_1 FRONT} = (\mu_s)(F_{z_1 FRONT}) = (0.7)(286.76N) = 200.73 \text{ N} \tag{2.30}$$

$$F_{y_2 FRONT} = (\mu_s)(F_{z_2 FRONT}) = (0.7)(355.43N)248.8 \text{ N} \tag{2.31}$$

Similarly, calculating for the normal forces experienced by the rear wheels on a banked surface uses the same equations as the front. However, the normal forces experienced in the rear wheels correspond to the load on the rear axle (W_R) which is the value for F_{z_2} from equation (2.19) multiplied by the cosine of the bank angle. Again it is assumed $\theta = 10°$, thus giving $W_R = 782.53N$.

$$F_{z_1 REAR} = \frac{782.53N}{2(0.625m)}(0.625\cos(10°) - 0.379\sin(10°)) = 343.93 \text{ N} \tag{2.32}$$

$$F_{z_2 REAR} = \frac{782.53N}{2(0.625m)}(0.625\cos(10°) + 0.379\sin(10°)) = 426.28 \text{ N} \tag{2.33}$$

Using $\mu_s = 0.7$ the following friction forces can be found:

$$F_{y_1 REAR} = (\mu_s)(F_{z_1 REAR}) = (0.7)(343.93N) = 240.75 \text{ N} \tag{2.34}$$

$$F_{y_2 REAR} = (\mu_s)(F_{z_2 REAR}) = (0.7)(426.28N) = 298.4 \text{ N} \tag{2.35}$$

2.4.4 Dynamic Applied Forces

2.4.4.1 Acceleration on a Level Road

An accelerating car on a level road gives has reaction forces on the wheels as seen in the figure below. In this situation load is transferred longitudinally shifting the load more to the driven wheels, or rear wheels, causing the car to squat. Additionally, the friction force is only considered for the driven wheels since the front wheels are free-rolling and thus friction on them is assumed negligible [8].

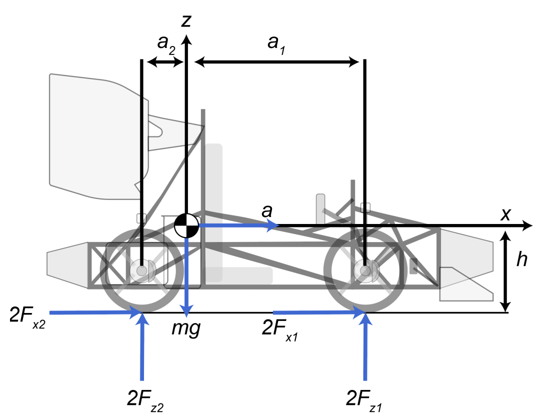


Figure 2.10: FBD of an Accelerating Car on a Level Road

The forces in the FBD above can be found as follows, where it is assumed $\mu_k = 0.6$ [8]:

$$F_{z_1} = \frac{1}{2}(mg)((\frac{a_2}{a_2 + a_1}) - (\frac{h}{a_1 + a_2})(\frac{a}{g})) \tag{2.36}$$

$$F_{z_2} = \frac{1}{2}(mg)((\frac{a_1}{a_2 + a_1}) + (\frac{h}{a_1 + a_2})(\frac{a}{g})) \tag{2.37}$$

$$F_{x_2} = \mu_k F_{z_2} = 0.6 \times 589.95N = 353.97 \text{ N} \tag{2.38}$$

Assuming an acceleration of $a = 6\frac{m}{s^2}$, the normal forces are calculated as:

$$F_{z_1} = \frac{1}{2}(297kg \times 9.81\frac{m}{s^2})((\frac{0.75m}{0.9m + 0.75m}) - ((\frac{0.379m}{0.9m + 0.75m})(\frac{6m}{9.81\frac{m}{s^2}})) = 457.52 \text{ N} \quad (2.39)$$

$$F_{z_2} = \frac{1}{2}(297kg \times 9.81\frac{m}{s^2})((\frac{0.9m}{0.9m + 0.75m}) + ((\frac{0.379m}{0.9m + 0.75m})(\frac{6m}{9.81\frac{m}{s^2}})) = 589.95 \text{ N} \quad (2.40)$$

2.4.4.2 Acceleration on an Inclined Road

Below is a depiction of the applied forces on a car while accelerating on an incline. The principle stays the same as acceleration on a level road, however the extent of load transfer and thus magnitude of each reaction force is changed due to the additional acceleration induced by gravity [8].

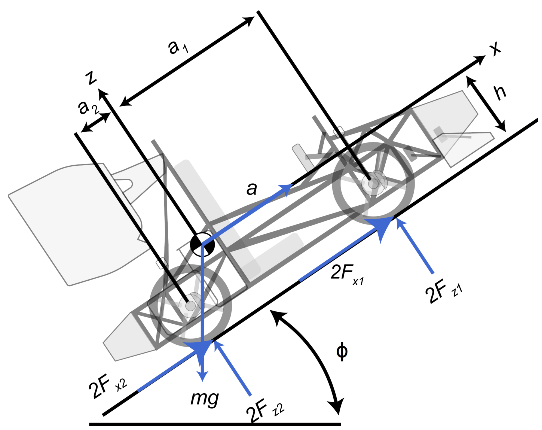


Figure 2.11: FBD of an Accelerating Car on an Inclined Road

$$F_{z_1} = \frac{1}{2}mg\left(\frac{a_2}{a_1 + a_2}\cos\phi - \frac{h}{a_1 + a_2}\sin\phi\right) - \frac{1}{2}(ma)\frac{h}{a_1 + a_2} \quad (2.41)$$

$$F_{z_2} = \frac{1}{2}mg\left(\frac{a_1}{a_1 + a_2}\cos\phi + \frac{h}{a_1 + a_2}\sin\phi\right) + \frac{1}{2}(ma)\frac{h}{a_1 + a_2} = \quad (2.42)$$

Assuming a typical angle of incline as $\phi = 10°$ and a typical acceleration as $a = 6\frac{m}{s^2}$, the values for F_z can be calculated as:

$$F_{z_1} = 389.35 \text{ N} \tag{2.43}$$

$$F_{z_2} = 1045.30 \text{ N} \tag{2.44}$$

Assuming a coefficient of kinetic friction $\mu_k = 0.6$, the friction force on the rear wheels is:

$$F_{x_2} == \mu_k F_{z_2} = 0.6 \times 1045.3N = 627.18 \text{ N} \tag{2.45}$$

2.4.4.3 High Speed Cornering

During a turn at a relatively high velocity, a centripetal force is produced which transfers the load laterally to the side of the car inside a turn. For a car moving at a reasonable velocity v, around a corner with a radius of curvature R, there is an applied centripetal force (Fc).With the assumption that $v = 15\frac{m}{s}$ and that the radius of curvature is $R = 30m$, the centripetal force can be calculated as follows below. It should be noted that this radius of curvature is close to the maximum value of R that is allowed in the FSAE guidelines [9].

$$\mathrm{F_C} = \frac{mv^2}{R} = (297 \text{ kg}) \left(\frac{(15\frac{\mathrm{m}}{\mathrm{s}})^2}{30 \text{ m}} \right) = 2227.5 \text{ N} \tag{2.46}$$

A consequence of F_C when turning, is that the load is transferred laterally from one side of the car to the other. That is, more load is placed on the wheels on the inside of a turn, causing the car to roll. To illustrate this, an FBD for the front wheels during a corner is used, where the load experienced on the front wheels corresponds to the load on the front axle (W_F). The value for W_F is equal to the value from section 2.4.3.1 found in section section 2.4.3.1.

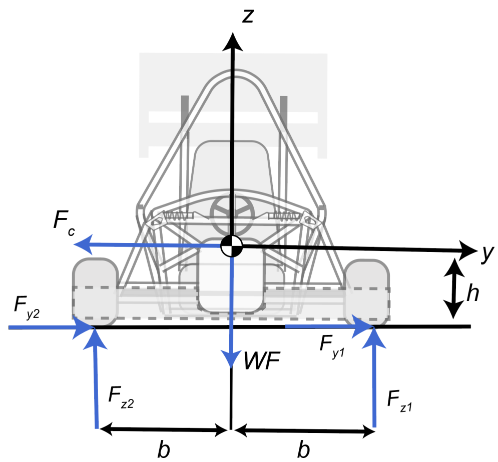


Figure 2.12: Reaction at Wheels Due to Applied Centripetal Force

The reaction forces above are found as follows:

$$F_{z_1FRONT} = \frac{bW_F - hF_C}{2b} = -344.3 \text{ N} \tag{2.47}$$

$$F_{z_2FRONT} = \frac{bW_F + hF_C}{2b} = 1006.46 \text{ N} \tag{2.48}$$

Assuming a kinetic friction coefficient $\mu_k = 0.6$, its found:

$$F_{y_1FRONT} = (\mu_s)(F_{z_1FRONT}) = (0.6)(-344.3N) = -206.58 \text{ N} \tag{2.49}$$

$$F_{y_2FRONT} = (\mu_s)(F_{z_2FRONT}) = (0.6)(1006.46N) = 603.88 \text{ N} \tag{2.50}$$

Similarly, calculating for the normal forces experienced by the rear wheels during a turn uses the same equations as the front. However, the normal forces experienced in the rear wheels correspond to the load on the rear axle (W_R) which is equal to the value for F_{z_2} from equation (2.19).

$$F_{z_1REAR} = \frac{bW_R - hF_C}{2b} = -278.07 \text{ N} \tag{2.51}$$

$$F_{z_2 REAR} = \frac{bW_R + hF_C}{2b} = 1027.68 \text{ N} \tag{2.52}$$

Assuming a kinetic friction coefficient $\mu_k = 0.6$, friction forces can be found:

$$F_{y_1 REAR} = (\mu_s)(F_{z_1 REAR}) = (0.6)(-278.07N) = -166.84 \text{ N} \tag{2.53}$$

$$F_{y_2 REAR} = (\mu_s)(F_{z_2 REAR}) = (0.6)(1027.68N) = 643.61 \text{ N} \tag{2.54}$$

2.4.4.4 Braking

As per the Formula SAE rules and regulations, the brake pedal must be designed to withstand a force of 311.38N without any failure of the brake system or pedal box. In this analysis, it is assumed that all parts are perfectly rigid, fluids are incompressible, and heat loss is minimal. By summing the moments about the pivot tube shown below, the force output can be found [10].

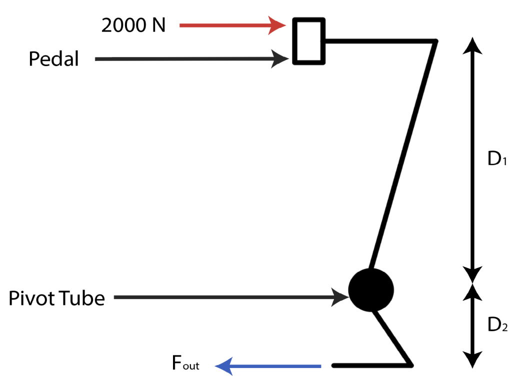


Figure 2.13: FBD of the maximum force exerted by the driver on the brake pedal

$$0 = M_{\text{out}} + M_{\text{in}} \tag{2.55}$$

$$F_{\text{out}}\, D_2 = -F_{\text{in}}\, D_1 \tag{2.56}$$

$$F_{\text{out}} = -311.38N\left(\frac{0.2m}{0.05m}\right) = -1245.5N \tag{2.57}$$

A balance bar divides the force from the brake pedal to the two master cylinders. The torque on one side of the bar must balance the torque on the other side of the bar. By summing the moments, the force going into the master cylinders can be determined.

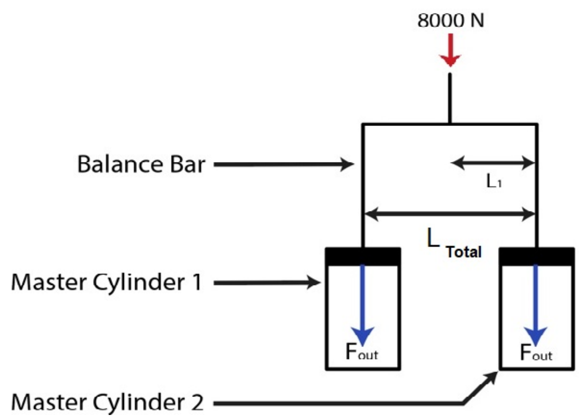


Figure 2.14: FBD of the balance bar and the two master brake cylinders

$$F_{\text{out}}\, L_{\text{total}} = F_{\text{in}}\, L_1 \tag{2.58}$$

$$F_{\text{out}} = 1245.5 \text{ N}\left(\frac{0.05 \text{ m}}{0.1 \text{ m}}\right) = 622.8N \text{ N} \tag{2.59}$$

Each master cylinder is connected to two calipers. Using Pascal's Law, a relation is developed depending on the master cylinder bore size and the caliper's piston diameter to solve for the force acting on a single rotor. The master cylinder bore size was assumed to be 20mm and the diameter of the piston was assumed to be 40mm.

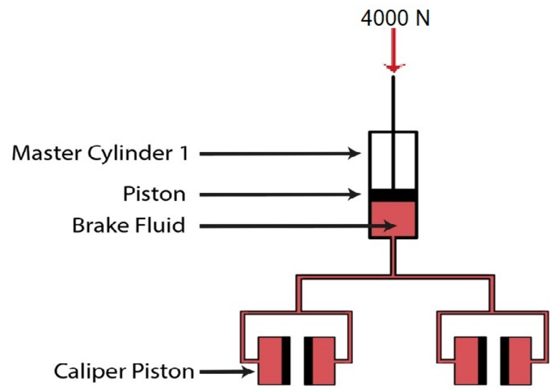


Figure 2.15: FBD of one master cylinder and the two calipers

For each individual caliper with two pistons, the total force at both pistons is:

$$F_{cal} = 2F_{cyl}\left(\frac{A_{cal}}{A_{cyl}}\right) = 2(622.76N)\left(\frac{\pi(0.02m)^2}{\pi(0.01m)^2}\right) = 4982.08N \text{ N} \tag{2.60}$$

The caliper's pistons are pushed against the brake pads, where the actual braking force can be determined depending on the brake pads coefficient of friction. In this calculation, the brake pad coefficient of friction is assumed to be 0.45.

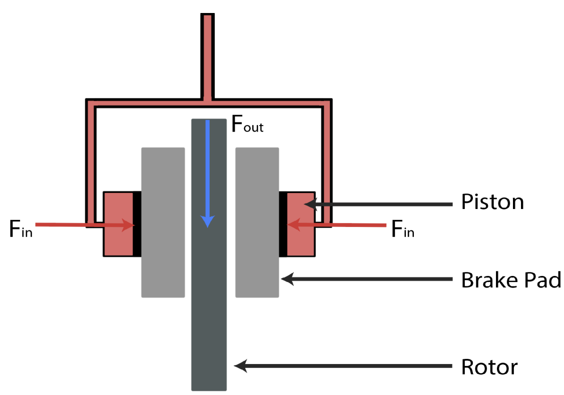


Figure 2.16: FBD of the caliper and rotor

$$F_{\text{out}} = \mu F_{\text{in}} = 0.45(4982.08 \text{ N}) = 2241.9N \text{ N} \tag{2.61}$$

The braking force is dependent on the radius of the rotor and wheel. A larger rotor radius will increase the brake torque produced by the brake system. It is assumed that the wheel has a radius (R) of 206mm and the rotor has a radius (r) of 124mm.

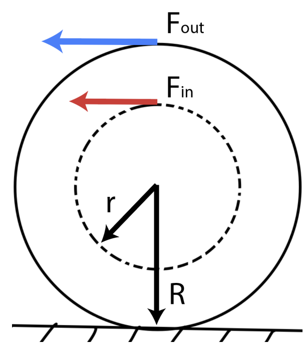


Figure 2.17: FBD of the wheel on the road

$$F_{\text{out}} = F_{\text{in}} \left(\frac{r}{R}\right) = 2241.9N \text{ N} \left(\frac{124 \text{ mm}}{206 \text{ mm}}\right) = 1349.5N \text{ N} \tag{2.62}$$

A braking force of 1349.5N is exerted on each wheel for a total braking force of 5398N on the vehicle. The analysis shows that a decrease in master cylinder size, increase in caliper piston area, or increase in the pedal ratio will ultimately increase the braking force on the wheels [11].

2.4.4.5 Impacts and Collisions

Impacts and Collisions:

The average deceleration of the vehicle can be calculated using:

$$a_{avg} = \frac{F_{braking}}{m} = \frac{5398N}{297Kg} = 18.18\ m/s^2$$

Assuming worst case scenario, the maximum impact velocity is the maximum velocity of the car which is 105m/s.

[12]

Solving for kinetic energy yields:

$$K_e = \frac{1}{2}(297)(105)^2 = 1.64 \text{ MJ}$$

Using conservation of energy:

$$K_e = P_e$$

Solving for time of impact

$$t = \frac{105 m/s}{18.18 m/s^2} = 5.78 s$$

Solving for impulse for vehicle to come to a complete stop:

$$I_m = m(V_{impact} - V_{final}) = 297(105)$$

$$\Rightarrow I_m = 31185 \text{ N.s}$$

Lastly, front impact force $\Rightarrow F = \frac{I_m}{t} = \frac{31185}{5.78}$

$$\Rightarrow \boxed{5395.33N}$$

2.4.4.6 Slip Angle

The slip angle of a vehicle describes the ratio of transverse and radial velocities in the form of an angle. It is the angle in between the wheel actual direction of motion and the direction that it is pointing towards [13].

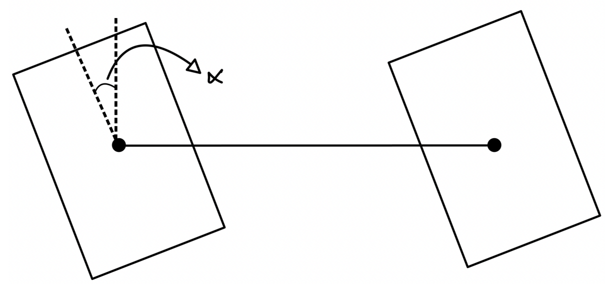


Figure 2.18: Slip Angle Representation

The slip angle is dependant on the load on the tire which differs from the rear and front tires. The slip angle can be mathematically modeled as the following equation [14].

$$\alpha = \frac{W \cdot v^2}{C_\alpha \cdot g \cdot R} \tag{2.63}$$

Where W is the load on the tire, v is the velocity of the tire, C is the tire stiffness coefficient which will be assumed to be 867 N/° [15], g is gravity, and R is the turning radius of the center of mass.

$$\alpha_{fr} = \frac{330.8N \cdot 29.2^2 m/s}{867N/^\circ \cdot 9.8m/s^2 \cdot 5.21m} = 6.37^\circ \tag{2.64}$$

$$\alpha_{re} = \frac{396.9N \cdot 29.2^2 m/s}{867N/^\circ \cdot 9.8m/s^2 \cdot 5.21m} = 7.64^\circ \tag{2.65}$$

2.4.5 Aerodynamics

2.4.5.1 Aerodynamic Effects at Max Speed

To analyse the aerodynamics effects of the vehicle appropriately, a model needs to be created for when the vehicle is at its top speed which represents the highest aerodynamic forces acting on the vehicle at any given point in time. A car's acceleration can be modeled according to newton's second law as:

$$m\ddot{x} = F - \frac{1}{2}\rho C_D A \dot{x}^2 \tag{2.66}$$

where F is the force propelling the vehicle forward, ρ is the density of air, C_D is the drag coefficient, and A is the frontal area of the vehicle. The driving force F can be expressed in terms of motor power and the car velocity where $v = \dot{x}$. This, can be simplified to when the vehicle is in equilibrium and is not accelerating to become [**?**]:

$$P = \frac{C_D A v^3}{1.633} \tag{2.67}$$

Now provided the motor used to estimate our battery calculations can produce a maximum of 100 kW peak power, and assuming transmission losses. Then the power at the wheels is assumed to be 84 kW. Moreover, assuming a drag coefficient $C_D = 0.63$ for a car with no aerodynamic packages, [16] and a frontal area of $A = 1.34m^2$. Then solving for the max speed of the vehicle one can find that the max velocity which the motor can produce to the tires is:

$$v = \sqrt[3]{\frac{1.633P}{C_D A}} = \sqrt[3]{\frac{1.633 \times 84000KW}{.63 \times 1.34m^2}} = 54.56m/s = 196.4km/hr \tag{2.68}$$

To determine the maximum drag coefficient desired whilst the vehicle is competing in its endurance races, a maximum top speed of $105km/hr$ can be assumed. Solving for the coefficient of drag:

$$C_D = \frac{1.633P}{Av^3} \Rightarrow C_D = \frac{1.633 \times 84000KW}{1.34m^2 \times (29.17m/s)^3} \Rightarrow C_D = 4.125 \tag{2.69}$$

Thus, the maximum allowable coefficient of drag is therefore 2.85. Which is a very high coefficient of drag, which indicates that the drag induced would not limit the top speed of the vehicle beyond its intended purpose.

2.4.5.2 Aerodynamic Effects on Cornering Performance

This section will analyse the aerodynamic effects induced on the tyres whilst cornering. This can be demonstrated by analysing the maximum allowed velocity a vehicle can corner at without losing its grip. Which is the velocity at which the fictional force is equal to the centripetal force [17].

$$\mu F_z = \mu(mg + \frac{1}{2}\rho C_L A v^2) = \frac{mv^2}{R} \tag{2.70}$$

$$v = \sqrt{\frac{\mu mg}{\frac{m}{R} - \frac{1}{2}\rho C_L A \mu}} \tag{2.71}$$

Where, g is the gravitational acceleration, μ is the coefficient of friction, ρ is the air density, C_L is the coefficient of lift, A is the frontal area, and R is the radius of the corner. According to the FSAE rules the corners will vary in radii from 4.5 to 30 m for dynamic events. Thus, assuming a lift coefficient when no aerodynamic packages are installed is $C_L = 0.29$, and the lift coefficient when aerodynamic packages are installed is $C_L = 2.34$ [16], and the density of air to be $\rho = 1.225$, assuming a constant coefficient of friction of $\mu = 0.65$, and provided the mass of the car is 298 kg, the frontal area $A = 1.34m^2$, then the following can be calculated from a 9-30 m turn radius [9]:

Table 2.7 illustrates the different speeds which can be achieved as the vehicle is cornering with an aerodynamic package installed, and without one. Clearly, higher speeds can be

Table 2.7: Cornering Speed differences with and without an Aerodynamic Package

Corner RadiusR (m)	With Aerodynamic Package	W/O Aerodynamic Package
	Velocity (km/hr)	Velocity (km/hr)
4.5	19.42	19.30
8	26.04	25.75
12	32.10	31.56
16	37.31	36.48
20	41.99	40.81
24	46.31	44.74
28	50.36	48.37
30	52.30	50.08

achieved whilst cornering when an aerodynamic package is installed, as there is more downforce acting on the vehicle.

2.4.5.3 Load Distribution Model

This section serves the purpose of illustrating how different forces and moments act on the vehicle. It will also introduce how the reaction forces for the rear wheels can be calculated. The sum of forces and moments for the diagram illustrated in figure 2.19 are illustrated below [17]:

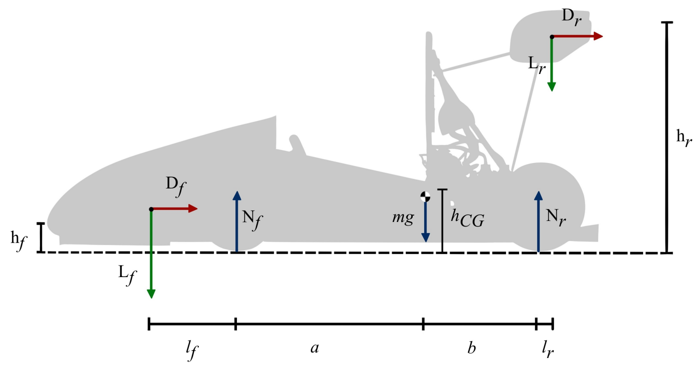


Figure 2.19: FBD for forces acting on the vehicle

$$\Sigma F = 0 \Rightarrow N_f + N_r - mg - L_F - L_r \tag{2.72}$$

$$\Sigma M_{CG} = 0 \Rightarrow bN_r + (a+l_f)L_F - (a)N_f - (b+l_r)L_r - (h_{CG} - h_r)D_r + (h_{CG} - h_f)D_f \tag{2.73}$$

The following can be determined after simplifying the equations of motion above:

$$N_r = \frac{(l_r + b)\, L_r - (l_f + a)\, L_f + (h_{CG} - h_r)\, \mathrm{D}_r - (h_{CG} - h_f)\, \mathrm{D}_f + amg + aL_f + aL_r}{a + b} \tag{2.74}$$

This equation gives the force on the rear tyres, which can be used to determine the load distribution of aerodynamic forces. The forces experienced on the rear tyres of the vehicle are therefore:

$$N_r = -0.363L_f + 1.1303L_r + 1493.825 + 229.455\mathrm{D}_r - 230.051\mathrm{D}_f \tag{2.75}$$

2.4.5.4 Lift Forces

The lift forces on the vehicle can easily be calculated. Assuming a lift force of -2.34 [16] with an aerodynamic package installed, and 0.29 [16] when no aerodynamic package is installed. Then, the lift force would be:

$$F_{L1} = 0.29 \times 1.34m^2 \times 1.225kg/m^2 \times \frac{(29.16m/s)^2}{2} = 202.38N \tag{2.76}$$

With an aerodynamic package, the lift force acts as a down-force.

$$F_{L2} = -2.34 \times 1.34m^2 \times 1.225kg/m^2 \times \frac{(29.16m/s)^2}{2} = -1633.05(N) \tag{2.77}$$

Force required to lift car off the ground.

$$F = 298kg \times 9.81m/s^2 = 2923.8N \tag{2.78}$$

Chapter 3

Analysis

3.1 Progress Summary

Table 3.1: Summary of Chassis Analysis Progress

Name of Part	**Priority**	**Completion**
Steel Tubes	high	Partial
Mounting Tabs	med	No

Table 3.2: Steering Assembly Parts with Priority and Completion Status

Name of Part	**Priority**	**Completion Status**
Steering Wheel	high	Partial
Primary Steering Shaft	med	No
Secondary Steering Shaft	med	No
Steering Knuckle	high	No
Steering Mounting Knuckle	high	No

Heim Joint	high	No
Tie Rod	med	No
Rack and Pinion Joint	high	No
Rack	med	No
Pinion	med	No
Pinion Key	med	No
Universal Joint	med	No

Table 3.3: Brake Assembly Parts with Priority and Completion Status

Name of Part	**Priority**	**Completion Status**
Rotor	high	No
Spindle	med	No
Wheel Hub	med	No
Caliper	high	No
Lug Bolts	high	No
Brake Pad	high	No
Brake Line	high	No
Axle Bearing	med	No

Table 3.4: Motor Assembly Parts with Priority and Completion Status

Name of Part	**Priority**	**Completion Status**
Motor	med	Partial
Motor mount	high	Partial

Table 3.5: Drivetrain Assembly Parts with Priority and Completion Status

Name of Part	**Priority**	**Completion Status**
Driving sprocket	Med	Partial
Chain	Med	Partial
Driven sprocket	Med	Partial
Differential	Med	Partial
CV joint	Med	Partial
Half Shaft	Med	Partial

Table 3.6: Suspension Assembly Parts with Priority and Completion Status

Name of Part	**Priority**	**Completion Status**
Upper Control Arm	Medium	Partial
Lower Control Arm	High	Partial
Push Rod	High	Partial

Rocker	High	None
Mounts	Medium	None
Bushings	Low	None
Bearings	Low	None
Ball Joints	Low	None
Fasteners	Low	None
Shock Absorbers	Medium	None
Front Anti-roll bar	Medium	None

3.2 Component Analysis

3.2.1 Chassis

3.2.1.1 Description of Inputs and Outputs

A stress analysis will be conducted on the chassis by applying the concept of Finite Element Analysis (FEA). The points where the tubes intersect are the nodes and the tubes will be modeled as 3-D frame elements. The inputs in this case would be forces that are applied to the nodes of the structure which would define the type of analysis being performed. The following are the inputs being applied to this analysis:

- Front collision forces
- Rear collision forces

- Side collision forces
- Side collision forces
- Torsion couples

  The outputs that would result from this analysis are wall thicknesses of the tubes that are being subjected to the respective force or torque.

3.2.1.2 Assumptions and Materials

To perform the analysis, assumptions need to be made in order to simplify the calculations. The assumptions are the following:

- Frame elements do not experience any shear stress
- The impacted elements are in static (This assumption helps the production of a worst case scenario)
- The elements will deform elastically only
- The safety factor is 1.5 (The safety factor will change after performing the analysis)

The material selected for the vehicles space-frame is 4130 steel which is more commonly known as chromoly steel. This grade of steel has a low carbon content which is more suitable for welding practices.

3.2.1.3 Stress Analysis

As mentioned, the chassis will be split into nodes and elements to help model the structure and perform a stress analysis. To demonstrate the theory of FEA, sample calculations will be performed on element 1 shown in figure 3.1. The rest of the calculations will be performed using MatLab.

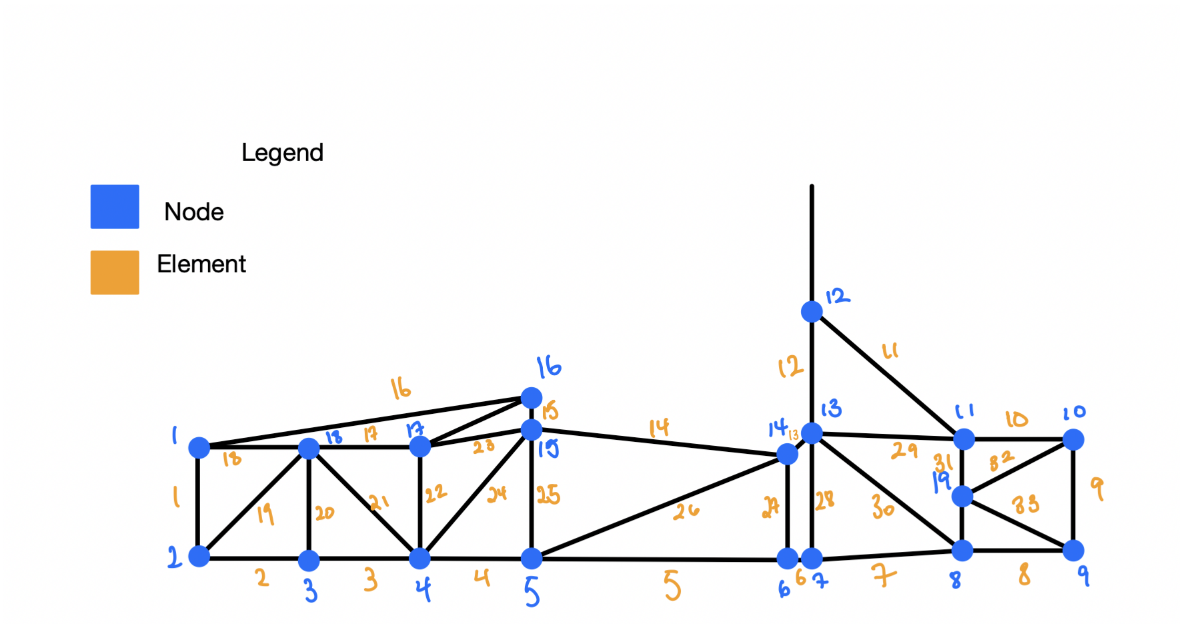
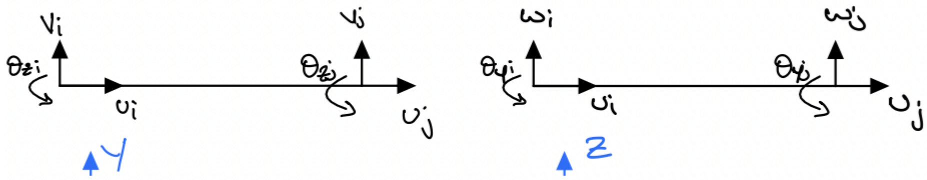


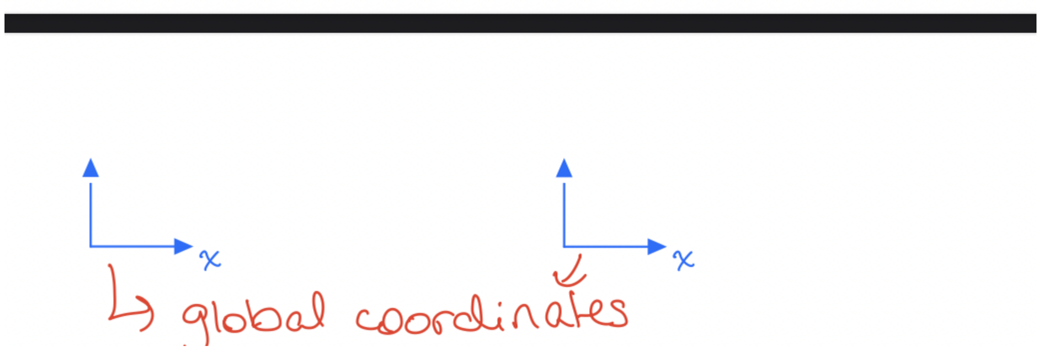
Figure 3.1: Nodes and Elements on the Chassis

Modelling the elements as 3-D frame elements will include the following:

- axial behaviours along x-axis
- bending behaviour in the xy-plane
- bending behaviour in xz-plane
- torsional behaviour about the x axis

3-D frames have 3 degrees of freedom represented in the free body-diagrams below;





The general equation regarding FEA is $[K]\{U\} = \{F\}$; where $[K]$ is the stiffness matrix, $\{U\}$ is the nodal displacement vector, and $\{F\}$ is the external load matrix (our input).

For a 3-D frame, the respective equation is the following:

$$\begin{bmatrix} [K'_{bar}] & [0] & [0] & [0] \\ [0] & [K'_{beam}]_{xy} & [0] & [0] \\ [0] & [0] & [K'_{beam}]_{xz} & [0] \\ [0] & [0] & [0] & [K'_{torsion}] \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ v_1 \\ \theta_{z_1} \\ v_2 \\ \theta_{z_2} \\ w_1 \\ \theta_{y_1} \\ w_2 \\ \theta_{y_2} \\ \theta_{x_1} \\ \theta_{x_2} \end{Bmatrix} = \begin{Bmatrix} F_{x_1} \\ F_{x_2} \\ F_{y_1} \\ M_{z_1} \\ F_{y_2} \\ M_{z_2} \\ F_{z_1} \\ M_{y_1} \\ F_{z_2} \\ M_{y_2} \\ M_{x_1} \\ M_{x_1} \end{Bmatrix}$$

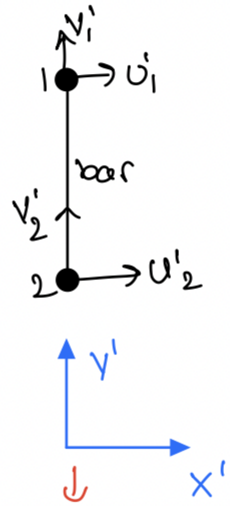
Stiffness matrix of element 1

To model the stiffness matrix we need

$[K'_{bar}]_1$, $[K'_{beam}]_{xy}$, $[K'_{beam}]_{xz}$, & $[K'_{torsion}]$

element 1

For $[K'_{bar}]$: To model axial loads



local coordinates for element 1

Hooke's law: $\sigma = \varepsilon \cdot E$

$$\Rightarrow \frac{F}{A} = E \cdot \frac{\delta}{L}$$

$$\Rightarrow F = \frac{(A)(E)}{L}\,\delta$$; where

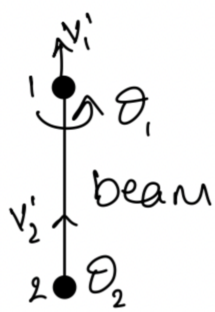
F is the axial load
A is cross section of element 1
E is the elastic modulus
L is the length of element
δ is the nodal displacement

In matrix form:

$$\frac{AE}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} u_i' \\ v_i' \\ u_j' \\ v_j' \end{Bmatrix} = \begin{Bmatrix} F'_{xi} \\ F'_{yi} \\ f'_{xj} \\ F'_{yj} \end{Bmatrix}$$

$$\therefore \; [k'_{bar}] = \frac{AE}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

<u>For $[k'_{beam}]_{xy}$:</u> To model bending



Equation of equilibrium for beam:

$$M_z(x) = EI\frac{d^2v}{dx^2} \; ; \text{ where}$$

M is the bending and I is the moment of inertia

Derive twice:

$$q(x) = EI\frac{d^4v}{dx^4} \; ;$$

where $q(x)$ is a distributed load

The applied loads are considered point load, there are no distributed loads $\Rightarrow g(x)=0$

$$\therefore \; V(x) = C_4 x^3 + C_3 x^2 + C_2 x + C_1$$

where $v(x)$ is the deflection

With boundary conditions:

$$V(0) = v_i = C_1$$
$$\frac{dV(0)}{dx} = \theta_i = C_2$$
$$V(L) = v_j = C_1 + C_2 L + C_3 L^2 + C_4 L^3$$
$$\frac{dV(L)}{dx} = \theta_j = C_2 + 2C_3 L + 3C_4 L^2$$

After applying interpolation and the principle of virtual work

In matrix format;

$$\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_1' \\ \theta_1' \\ v_2' \\ \theta_2' \end{Bmatrix} = \begin{Bmatrix} F_1' \\ M_1' \\ F_2' \\ M_2' \end{Bmatrix}$$

$$\Rightarrow [k'_{beam}]_{xy} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

For $[k'_{beam}]_{xz}$:

Same procedure as $[k'_{beam}]_{xy}$ but

$$M_y = -EI\frac{d^2y}{dx^2}$$

this results in;

$$\Rightarrow [K'_{beam}]_{\times 2} = \frac{EI}{L^3}\begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}$$

For $[K'_{torsion}]$:

It is represented by

$$[K'_{torsion}] = \frac{JG}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

where; J is the polar moment of inertia

G is the shear modulus

Now, we can assemble our stiffness matrix for element 1.

$$K' = \begin{bmatrix} [K'_{bar}] & [0] & [0] & [0] \\ [0] & [K'_{beam}]_{xy} & [0] & [0] \\ [0] & [0] & [K'_{beam}]_{xz} & [0] \\ [0] & [0] & [0] & [K'_{torsion}] \end{bmatrix}$$

By using the equations derived above, we can conclude a 12×12 matrix as such:

$$K^e_{1\ldots6,1\ldots6} = \begin{bmatrix} AE/L & 0 & 0 & 0 & 0 & 0 \\ 0 & 12EI_z/L^3 & 0 & 0 & 0 & 6EI_z/L^2 \\ 0 & 0 & 12EI_y/L^3 & 0 & -6EI_y/L^2 & 0 \\ 0 & 0 & 0 & GJ/L & 0 & 0 \\ 0 & 0 & -6EI_y/L^2 & 0 & 4EI_y/L & 0 \\ 0 & 6EI_z/L^2 & 0 & 0 & 0 & 4EI_z/L \end{bmatrix}$$

$$K^e_{1\ldots 6,7\ldots 12} = \begin{bmatrix} -AE/L & 0 & 0 & 0 & 0 & 0 \\ 0 & -12EI_z/L^3 & 0 & 0 & 0 & 6EI_z/L^2 \\ 0 & 0 & -12EI_y/L^3 & 0 & -6EI_y/L^2 & 0 \\ 0 & 0 & 0 & -GJ/L & 0 & 0 \\ 0 & 0 & 6EI_y/L^2 & 0 & 2EI_y/L & 0 \\ 0 & -6EI_z/L^2 & 0 & 0 & 0 & 2EI_z/L \end{bmatrix}$$

$$K^e_{7\ldots 12,1\ldots 6} = \begin{bmatrix} -AE/L & 0 & 0 & 0 & 0 & 0 \\ 0 & -12EI_z/L^3 & 0 & 0 & 0 & -6EI_z/L^2 \\ 0 & 0 & -12EI_y/L^3 & 0 & 6EI_y/L^2 & 0 \\ 0 & 0 & 0 & -GJ/L & 0 & 0 \\ 0 & 0 & -6EI_y/L^2 & 0 & 2EI_y/L & 0 \\ 0 & 6EI_z/L^2 & 0 & 0 & 0 & 2EI_z/L \end{bmatrix}$$

$$K^e_{7\ldots 12,7\ldots 12} = \begin{bmatrix} AE/L & 0 & 0 & 0 & 0 & 0 \\ 0 & 12EI_z/L^3 & 0 & 0 & 0 & -6EI_z/L^2 \\ 0 & 0 & 12EI_y/L^3 & 0 & 6EI_y/L^2 & 0 \\ 0 & 0 & 0 & GJ/L & 0 & 0 \\ 0 & 0 & 6EI_y/L^2 & 0 & 4EI_y/L & 0 \\ 0 & -6EI_z/L^2 & 0 & 0 & 0 & 4EI_z/L \end{bmatrix}$$

It is important to note that the matrix calculated from the above will be on the local axis which is only for element 1. As a result, every stiffness matrix will need to be rotated to match the global coordinates so that all elements share the same axes. The global stiffness matrix for element 1 can be obtained by:

$$\underbrace{[K']}_{\text{global}} = [R]^T \underbrace{[K'']}_{\text{local}} \underbrace{[R]}_{\text{Rotation matrix}}$$

$$R = \begin{bmatrix} T_{3D} & [0] & [0] & [0] \\ [0] & T_{3D} & [0] & [0] \\ [0] & [0] & T_{3D} & [0] \\ [0] & [0] & [0] & T_{3D} \end{bmatrix}$$

Where T_{3D} is the transformation matrix which is mathematically modelled using directional cosines;

$$T_{3D} = \begin{bmatrix} \cos(x', x) & \cos(x', y) & \cos(x', z) \\ \cos(y', x) & \cos(y', y) & \cos(y', z) \\ \cos(z', x) & \cos(z', y) & \cos(z', z) \end{bmatrix}$$

Computing the directional cosines with respect to the global coordinates gives you a rotational matrix that can be applied to every element

$$R_{1\ldots6,1\ldots6} = \begin{bmatrix} l & m & \omega & 0 & 0 & 0 \\ -m/L_y & l/L_y & 0 & 0 & 0 & 0 \\ -(l\omega)/L_y & -m\omega/L_y & L_y & 0 & 0 & 0 \\ 0 & 0 & 0 & l & m & \omega \\ 0 & 0 & 0 & -m/L_y & l & 0 \\ 0 & 0 & 0 & -l\omega/L_y & L_y & L_y \end{bmatrix}$$

$$R_{1\ldots6,\,7\ldots12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_{7\ldots12,\,1\ldots6} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_{7\ldots12,\,7\ldots12} = \begin{bmatrix} l & m & \omega & 0 & 0 & 0 \\ -m/L_y & l/L_y & 0 & 0 & 0 & 0 \\ -(l\omega)/L_y & -m\omega/L_y & L_y & 0 & 0 & 0 \\ 0 & 0 & 0 & l & m & \omega \\ 0 & 0 & 0 & -m/L_y & l & 0 \\ 0 & 0 & 0 & -l\omega/L_y & L_y & L_y \end{bmatrix}$$

where L_y is the length of the normalized x' & y' components of x' and L is the length of the element nodes

where $\lambda_y = l^2 + m^2$; $l = \dfrac{y_2 - y_1}{2}$; $\omega = \dfrac{z_2 - z_1}{2}$

Once the element has been rotated ; the global stiffness matrix is found and can be used to find the displacement matrix and external force matrix which are multiplied by [R] to represent them on the global coordinates.

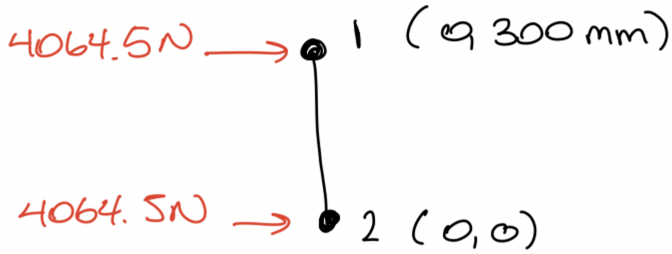
$$[K']\{U'\} = \{F'\}$$

$$\underbrace{\begin{matrix}[U'] \\ [F']\end{matrix}}_{\text{global}} \begin{matrix} = [R] \\ = [R]\end{matrix} \underbrace{\begin{matrix}[U''] \\ [F'']\end{matrix}}_{\text{local}}$$

Sample Calculation on Element 1:

The impact force on the front 4 nodes is 5395.33 N as calculated in the modelling section. Assuming a safety factor of 3, the impact force is 16186N.

On Element 1, nodes 1 and 2 will be subjected to 4046.5N of the impact force each.



$$d_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow d_1 = 300 \text{ mm} \quad ; \quad J = 25555 \text{ mm}^4$$

$$E = 205 \text{ GPa} \quad ; \quad I = 12{,}777 \text{ mm}^4$$

$$A = 114 \text{ mm}^2 \quad ; \quad G = 75842 \text{ MPa}$$

$\therefore$ The stiffness matrix can be represented using the matrices above:

$$K^1_{1\ldots6,1\ldots6} = \begin{bmatrix} 7.79\times10^7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.1641\times10^6 & 0 & 0 & 0 & 1.75\times10^5 \\ 0 & 0 & 1.1641\times10^6 & 0 & -1.75\times10^5 & 0 \\ 0 & 0 & 0 & 6.46\times10^3 & 0 & 0 \\ 0 & 0 & -1.75\times10^5 & 0 & 3.49\times10^4 & 0 \\ 0 & 1.75\times10^5 & 0 & 0 & 0 & 3.49\times10^4 \end{bmatrix}$$

$$K^1_{1\ldots6,7\ldots12} = \begin{bmatrix} -7.79\times10^7 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.1641\times10^6 & 0 & 0 & 0 & 1.75\times10^8 \\ 0 & 0 & -1.1641\times10^6 & 0 & -1.75\times10^5 & 0 \\ 0 & 0 & 0 & -6.46\times10^3 & 0 & 0 \\ 0 & 0 & 1.75\times10^5 & 0 & 3.49\times10^4 & 0 \\ 0 & -1.75\times10^5 & 0 & 0 & 0 & 3.49\times10^4 \end{bmatrix}$$

$$K'_{7\ldots12,1\ldots6} = \begin{bmatrix} -7.79\times10^{7} & 0 & 0 & 0 & 0 & 0 \\ 0 & -1.1641\times10^{6} & 0 & 0 & 0 & -1.75\times10^{5} \\ 0 & 0 & -1.1641\times10^{6} & 0 & 1.75\times10^{5} & 0 \\ 0 & 0 & 0 & -6.46\times10^{8} & 0 & 0 \\ 0 & 0 & -1.75\times10^{5} & 0 & 3.49\times10^{4} & 0 \\ 0 & 1.75\times10^{5} & 0 & 0 & 0 & 3.49\times10^{4} \end{bmatrix}$$

$$K'_{7\ldots12,7\ldots12} = \begin{bmatrix} 7.79\times10^{7} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.1641\times10^{6} & 0 & 0 & 0 & -1.75\times10^{5} \\ 0 & 0 & 1.1641\times10^{6} & 0 & 1.75\times10^{5} & 0 \\ 0 & 0 & 0 & 6.46\times10^{3} & 0 & 0 \\ 0 & 0 & 1.75\times10^{5} & 0 & 3.49\times10^{4} & 0 \\ 0 & -1.75\times10^{5} & 0 & 0 & 0 & 3.49\times10^{4} \end{bmatrix}$$

Now, we need to determine the rotation matrix;

$$l = \frac{x_j - x_i}{L} = 0 \quad ; \quad m = \frac{y_j - y_i}{L} = \frac{300}{300} = 1$$

$$\omega = 0 \quad ; \quad L_y = \sqrt{300^2} = \sqrt{9{,}000} = 300\text{mm}$$

$$R_{1\cdots 6,1\cdots 6} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1/0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 1 & 0 \\ 0 & 0 & 0 & -1/0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 \end{bmatrix}$$

$$R_{1\cdots 6,7\cdots 12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_{7\cdots 12,1\cdots 6} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_{7\ldots12,7\ldots12} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1/0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 1 & 0 \\ 0 & 0 & 0 & -1/0.3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3 \end{bmatrix}$$

The global stiffness matrix can be calculated to give:

$$K'_{1\ldots6,1\ldots6} = 10^7 \begin{bmatrix} 0.1164 & 0 & 0 & 0 & 0 & -0.0053 \\ 0 & 7.79 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0105 & 0.0175 & 0 & 0 \\ 0 & 0 & 0.0175 & 0.0888 & 0.0002 & 0 \\ 0 & 0 & 0 & 0.0002 & 0.0006 & 0 \\ -0.0053 & 0 & 0 & 0 & 0 & 0.0003 \end{bmatrix}$$

$$K'_{1\ldots6,7\ldots12} = 10^7 \begin{bmatrix} 0.1164 & 0 & 0 & 0 & 0 & -0.0053 \\ 0 & -7.79 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0105 & 0.0175 & 0 & 0 \\ 0 & 0 & 0.0175 & 0.0888 & 0.0002 & 0 \\ 0 & 0 & 0 & 0.0002 & 0.0006 & 0 \\ 0.0053 & 0 & 0 & 0 & 0 & 0.0003 \end{bmatrix}$$

$$K'_{1\ldots6,\,7\ldots12} = 10^7 \begin{bmatrix} 0.1164 & 0 & 0 & 0 & 0 & -0.0053 \\ 0 & -7.79 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0105 & -0.0175 & 0 & 0 \\ 0 & 0 & 0.0175 & 0.0888 & -0.0002 & 0 \\ 0 & 0 & 0 & 0 & -0.0006 & 0 \\ -0.0053 & 0 & 0 & 0 & 0 & 0.0003 \end{bmatrix}$$

$$K'_{1\ldots6,\,7\ldots12} = 10^7 \begin{bmatrix} 0.1164 & 0 & 0 & 0 & 0 & 0.0053 \\ 0 & 7.79 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0105 & 0.0175 & 0 & 0 \\ 0 & 0 & -0.0175 & -0.0888 & -0.0002 & 0 \\ 0 & 0 & 0 & 0 & 0.0006 & 0 \\ 0.0053 & 0 & 0 & 0 & 0 & 0.0003 \end{bmatrix}$$

External force vector $\{F'\}$

$$\Rightarrow \{F'\} = \begin{Bmatrix} 4064.5N \\ 4064.5N \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Now, we can compute $\{U'\}$ in the global coordinates

$$\{U'\} = [K']^{-1}\{F'\}$$

$$\Rightarrow \{U'\} = 10^{3}\begin{Bmatrix} 1.59 \\ 9.58 \\ 0 \\ 0 \\ 0 \\ -0.0215 \\ -1.5911 \\ -9.583 \\ 0 \\ 0 \\ 0 \\ -0.0215 \end{Bmatrix}$$

Rotate on local coordinates by $\{U^{e'}\}^{\rightarrow \text{local}} = [R]\{U^{e}\}$

$$\Rightarrow \{U^{e}\} = 10^{3} \begin{Bmatrix} 9.583 \\ -5.3038 \\ 0 \\ 0 \\ 0 \\ -0.0065 \\ -9.5830 \\ 5.3038 \\ 0 \\ 0 \\ 0 \\ -0.0065 \end{Bmatrix}$$

Now, we can obtain the local external forces & reactions by $\{F'\} = [K']\{U'\}$

$$\{F''\} = 10^6 \begin{Bmatrix} 1.493 \\ -0.0124 \\ 0 \\ 0 \\ 0 \\ -0.0019 \\ -1.4930 \\ 0.0124 \\ 0 \\ 0 \\ 0 \\ -0.0019 \end{Bmatrix}$$

This was done using Matlab

$F_x = 1.493 \times 10^6 N$
$F_y = -0.0124 \times 10^6 N$
$M_z = -0.0019 \times 10^6 N.m$

$$\sigma_{axial} = \frac{F_x}{A} = \frac{1.493 \times 10^6 N}{114 mm^2} = 130965 MPa$$

Too high; will have to recalculate

$$\tau_{shear} = \frac{2(0.0124 \times 10^6)\ N}{114\ mm^2} = 217.54\ MPa$$

$$\tau_{bending} = \frac{(0.0019 \times 10^6)(1150)}{12777} = 171\ MPa$$

Where x is the distance from neutral axis to COG

These stresses will be compared to the material's maximum allowable stress to produce a safety factor

For example;

Bendin safety factor on element 1;

$S_y = 460\ MPa$ for AISI 4130 Steel

$$n = \frac{S_y}{\tau_{bending}} = \frac{757\ MPa}{171\ MPa} = 4.43$$

The same method will be applied to other characteristics.

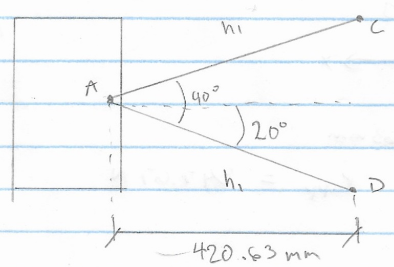
[18]

3.2.2 Suspension

3.2.2.1 Forces acting on Suspension Control Arms and Pushrod

The forces being analyzed should be the forces that arise from critical conditions. In this case, the forces being used are those induced by maximum acceleration of the car, which is found in the modelling section.

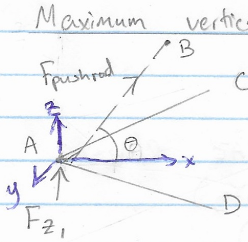
Lower Control Arm Top View:



$$\cos(20) = \frac{420.63}{h_1}$$

$$h_1 = 447.63 \text{ mm}$$

Maximum vertical load case



F_{z_1} ⇒ vertical force on front wheel during max acceleration

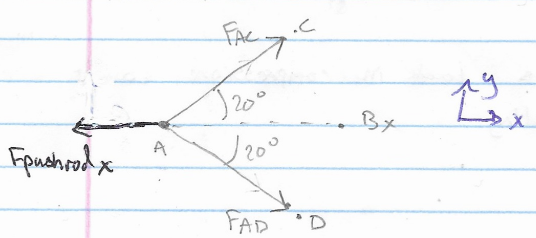
$$F_{z_1} = 457.52 \text{ N}$$

$$\uparrow + \sum F_y = F_{z_1} - F_{pushrod}(\sin\theta) = 0$$

↳ Assume pushrod angle $\theta = 35°$

$$F_{pushrod} = \frac{457.52}{\sin(35)} = 797.7 \text{ N}$$

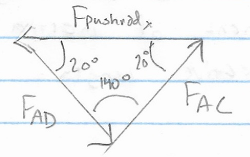
Horizontal:



$$\sum F_x = 0$$

$$F_{pushrod_x} = F_{AC_x} + F_{AD_x}$$

Rearrange forces:



$$F_{pushrod\,x} = 797.7\cos(35) = 653.4\ \text{N}$$

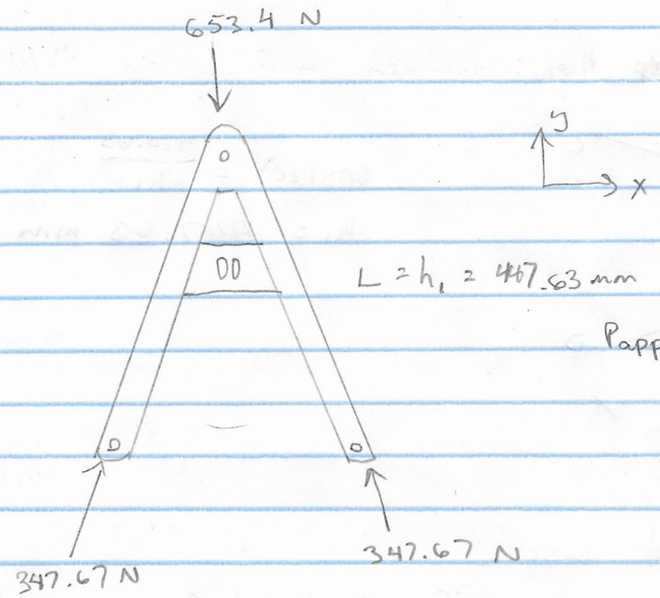
$$\frac{F_{AC}}{\sin 20} = \frac{F_{pushrod\,x}}{\sin 140}$$

$$F_{AC} = F_{AD} = \frac{(653.4)\sin 20}{\sin 140} = 347.67\ \text{N}$$

Hilroy

3.2.2.2 Component Analysis: Lower Control Arm

Lower Control Arm FBD under maximum load:



$L = h_1 = 467.63$ mm

$P_{app} = 347.67$ N

- Steel properties specified by the FSAE Rules :

$E = 200$ GPa (modulus of elasticity)

$S_y = 305$ MPa (yield strength)

- Wishbones are critical component, so safety factor used will exceed 1.5 :

$\eta = 1.6$

- Bottom wishbone experiences load in compression so i may fail due to buckling.

Euler buckling load :

$$P_E = \frac{\pi^2 E I}{L^2}$$; E is modulus of elasticity
I is moment of inertia
L is effective length

Allowable Euler buckling :

$$P_{cr} = \frac{\pi^2 E I}{\eta L^2}$$; η is safety factor

Moment of inertia of hollow tube :

$$I = \frac{\pi}{64}\left(D^4 - d^4\right) \text{ ; where } D \text{ is outer diameter, } d \text{ is inner diameter}$$

Yield stress :

$$\sigma_y = \frac{n \cdot F_t}{A} \text{ , } A \text{ is minimum area}$$

Area of hollow tube :

$$A = \pi\left(\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right)$$

Buckling Safety Factor :

$$SF = \frac{P_{cr}}{P_{app}} \text{ , } P_{cr} \text{ is maximum pressure, } P_{app} \text{ is current pressure}$$

Parameterization of Wishbone :

$$SF = \frac{P_{cr}}{P_{app}}$$

$$SF = \frac{\left(\frac{\pi^2 E I}{L^2}\right)}{P_{app}} = \frac{\left(\frac{\pi^2 E \left(\frac{\pi}{64}\left(D^4 - d^4\right)\right)}{L^2}\right)}{P_{app}}$$

$$SF = \frac{\left[\frac{\pi^2 E \left(\frac{\pi}{64}\left(D^4 - (0.8D)^4\right)\right)}{L^2}\right]}{P_{app}} \text{ ; Assume that } d \text{ is } 0.8D$$

→ substitute values :

$$1.6 = \frac{\left[\frac{\pi^2 (200\,000)\left(\frac{\pi}{64}\left(.5904\, D^4\right)\right)}{(447.63)^2}\right]}{347.67 \text{ N}} \Rightarrow D = 6.644 \text{ mm}, \quad d = 0.8D = 5.32 \text{ mm}$$

3.2.3 Steering

3.2.3.1 Steering Wheel Analysis

Description of Inputs and Outputs

The weight of the driver will determine all of the forces exerted on the steering wheel.

Constants and Safety Factors

The chosen safety factor is 1.7

Assumptions and Materials

The steering wheel will be made out of AISI 1018 due to its weldability and balance of toughness, strength and ductility.

Stress Analysis

Assume the maximum weight of the driver is 111 kg. Assume the maximum pulling force of each hand to be 270 N, or 1/4 of the weight of the driver.

The first type of stress that will be analysed is the force of the driver pushing towards the steering wheel. This will happen when the vehicle suddenly comes to a halt. The failure point of the pushing force will depend on the quality of the welding, which will not be analysed in this report.

The second type of stresses that will be analysed is the force of the driver pulling on the steering wheel. This stress analysis will depend on the bolt used to secure the steering wheel to the steering shaft.

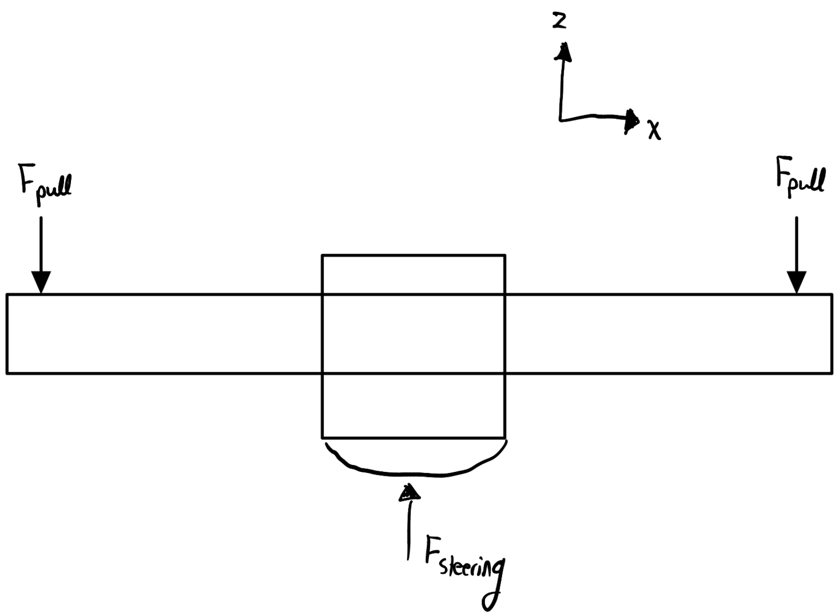


Figure 3.2: FBD of pulling forces on steering wheel

Assume a M10 bolt with 1.5mm pitch will be used. The stress area for this bolt is 58mm^2. Since the force of this bolt is occurring axially, the following equation can be used:

$$\sigma_{bolt} = \frac{F_{pull}}{A_t} = \frac{270N \times 2}{58mm^2} = 9.31Mpa \tag{3.1}$$

Assume that the bolt is made out of SAE class 4.6 steel. This would provide a yield strength of S_y = 232 Mpa. The safety factor can then be calculated:

$$\eta = \frac{S_y}{\sigma_{bolt}} = \frac{240Mpa}{9.31Mpa} = 25.77 \tag{3.2}$$

This safety is much greater than 1.7. Thus, the size of the bolt can be reduced. However, this bolt is not very large and cheap, and is a suitable option.

The third type of stress that will be analysed is the torque transferred from the steering wheel to the steering key.

Flowchart for parameterization

This bolt can be used for all designs since the maximum weight of the driver was used.

1349 N

3.2.4 Brakes

3.2.5 Powertrain

3.2.5.1 Motor

The selected motor is estimated to have a peak torque of 230 Nm and a continuous torque up to 120 Nm. The. motor has a peak power of 109 kW at 6500 rpm and a continuous power up to 62 kW. The motor is estimated to weigh around 12 kg with a casing diameter of 228 mm.

3.2.5.2 Chain and Sprocket Drive

The drive was provided by chain and sprockets with the sprocket mounted to the shaft by a flange. The sprocket is built to take the linear force of the drive chain and produce rotational energy from it to rotate the differential. Thus, the diameter of the sprocket helps to produce a moment arm to produce a torque on the differential.

The maximum torque produced in the rear axle is limited by the amount of torque the tires can transmit before slipping. Determining the force at which the tyres will slip is controlled by the normal force and coefficient of friction.

- Assume top speed of 105 km/h (29.17 m/s)
- Assume tire diameter of 406mm
- Assume vehicle weight to be 300 kg including a 111 kg driver
- Assume vehicle acceleration time from 0 to 100 km/h (27.78 m/s) is 5s
- Assume a coefficent of friction of 0.7
- Assume a front to back weight distribution of 35:65
- Chosen gear ratio of 4 (44/11)
- Material selection:
  - Sprockets: Aluminum 7075 Tempered Alloy
    - ↳ tensile strength: 510 MPa
    - ↳ Yield strength: 410 MPa
    - ↳ elasticity of modulus: 70 GPa
    - ↳ Shear modulus: 26 GPa
    - ↳ Poisson's ratio: 0.32
  - Shafts: mild Steel AISI 1020
    - ↳ Yield Strength: 331 MPa
    - ↳ ultimate strength: 448 MPa

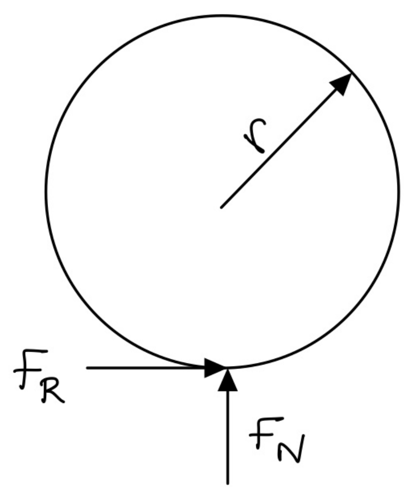
$$\boxed{\text{Acceleration}}$$

$$a = \frac{V_2 - V_1}{t}$$

<u>where</u> V_2 = final velocity (m/s)
V_1 = initial velocity (m/s)
t = time (s)

$$a = \frac{27.78\,m/s}{5s} = 5.56\,m/s^2$$

<u>Tire</u>



$$\boxed{\text{Normal force on the rear tires}}$$

$$F_N = mg$$

where F_N = normal force (N)
m = mass (kg)
g = acceleration due to gravity (m/s²)

$$F_N = 300\,kg \cdot 9.81\,m/s^2 \cdot \frac{70}{100} = 2060\,N$$

$$\boxed{\text{Frictional Force provided by rear tires}}$$

$$F_R = F_N \mu$$

where F_R = frictional force (N)
F_N = normal force (N)
μ = coefficient of friction

$$F_R = 2060N \cdot 0.7 = 1442\,N$$

$\boxed{\text{Torque produced in shaft}}$

$$T = F_R r$$

where T = torque (Nm)
F_R = frictional force (N)
r = radius (m)

$$T = 1442\,N \cdot 0.203\,m = 292.73\ Nm$$

$\boxed{\text{Sprocket Selection}}$

$$v = \omega r$$

where v = velocity (m/s)
ω = angular velocity (rad/s)
r = radius (m)

Angular velocity of the rear wheel, R:

$$\omega_R = \frac{v_R}{r_R} = \frac{29.17\,m/s}{0.203\,m} = 143.7\ rad/s$$

Angular velocity of the Motor Sprocket:

$$\omega_M = 143.7\,\text{rad/s} \times \frac{44}{11} = 574.8\,\text{rad/s}$$

$$\frac{\omega_M}{\omega_R} = \frac{t_D}{t_M}$$

where ω_M = angular velocity of the output motor shaft (rad/s)

ω_R = angular velocity of the wheel (rad/s)

t_D = number of teeth of the differential sprocket

t_M = number of teeth of the motor sprocket

$$\frac{\omega_M}{\omega_R} = \frac{t_D}{t_M} = \frac{574.8\,\text{rad/s}}{143.7\,\text{rad/s}} = 4$$

A 44-tooth rear differential sprocket with a 11-tooth motor sprocket give a gear ratio of 4.

A chain # of 520 with a pitch 0.625" is used and therefore, the motor sprocket is estimated to have a diameter of 56.4mm and the differential sprocket to have a diameter of 222.5mm.

The maximum tangential force the chain can transmit to the differential sprocket is

$$F = \frac{T}{r} = \frac{292.73\ Nm}{0.11\ m} = 2661.2\ N$$

Stress Analysis

$\boxed{\text{Shear stress due to torsion}}$

$$\tau = \frac{Tr}{J}$$

where τ = torsional shear stress (MPa)
T = torque (Nm)
r = radius (m)
J = polar moment of inertia (m^4)

$\boxed{\text{Polar Moment Of Inertia}}$

$$J = \frac{\pi d^4}{32}$$

where d = diameter of circle

Under normal loading of $\frac{292.73\,Nm}{2}$, the shaft shear stress is

Assuming the splines root diameter is 25mm

<u>Normal Loading.</u>

$$T = \frac{146.37\,Nm \times 0.0125\,mm}{\frac{\pi \times 0.025^4}{32}}$$

$$= 47.7\ MPa$$

$\boxed{\text{Maximum Distortion Energy Criterion}}$

$$\sigma_y = \frac{\tau_{max}}{0.58}$$

<u>where</u> σ_y = yield strength (MPa)

τ_{max} = maximum shear stress (MPa)

<u>equivalent tensile strength under normal loading</u>

$$\sigma = \frac{47.7\,MPa}{0.58} = 82.24\ MPa$$

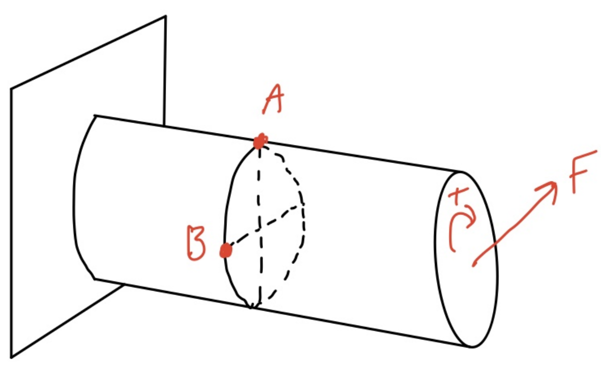
Extreme Loading

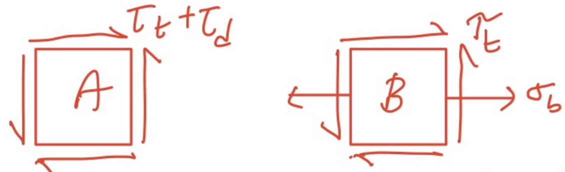
$$\tau = \frac{292.73\,\text{Nm} \times 0.0125\,\text{mm}}{\frac{\pi \times 0.025^4}{32}}$$

$$= 95.42\ \text{MPa}$$

equivalent tensile strength under extreme loading

$$\sigma = \frac{95.42}{0.58} = 164.52\,\text{MPa}$$





Max shear element A, Max bending element B

3.3 Accelerator Pedal Analysis

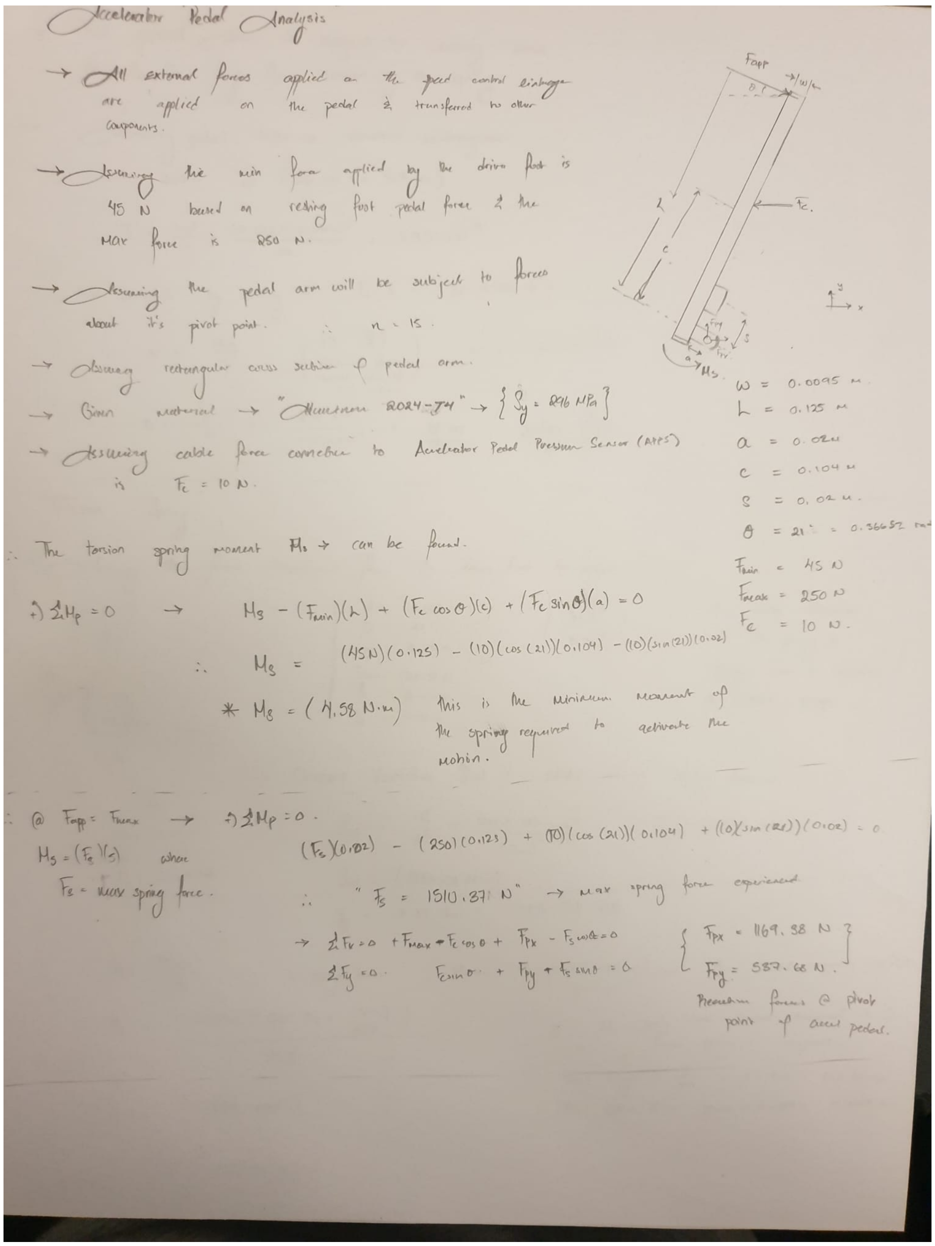
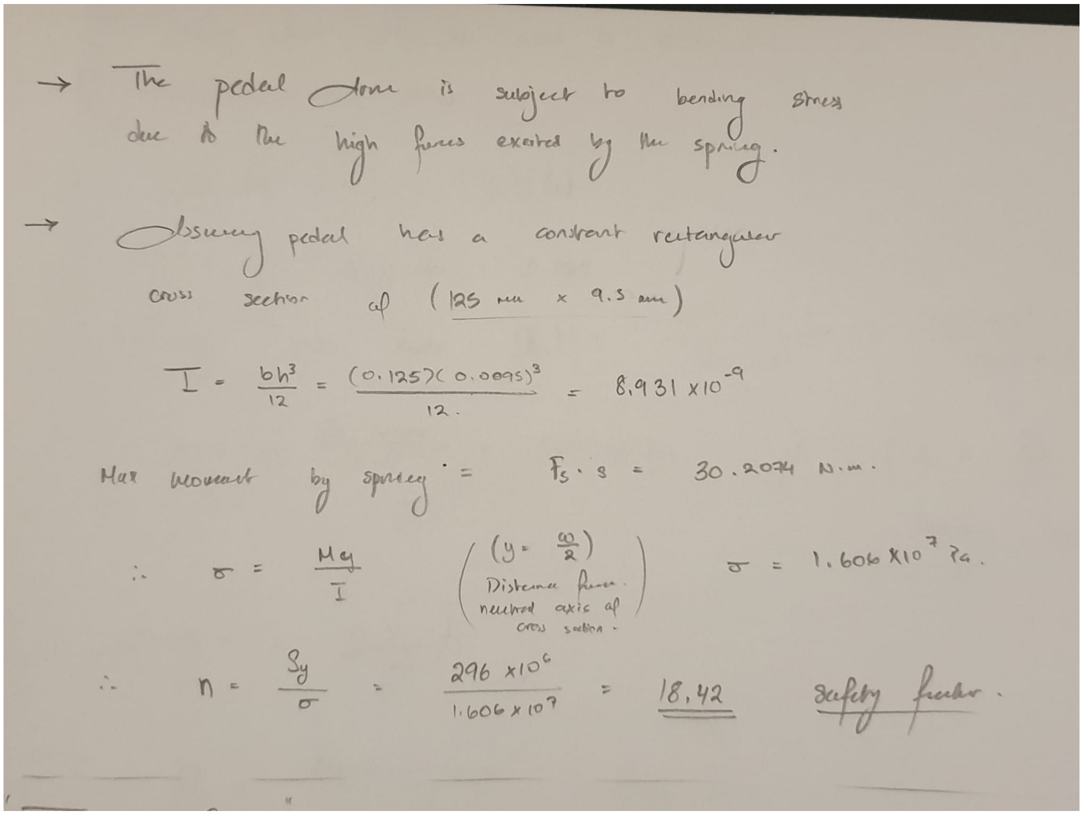


Figure 3.3: Accelerator Pedal Analysis pg.1



→ The pedal arm is subject to bending stress due to the high forces exerted by the spring.

→ Assuming pedal has a constant rectangular cross section of (125 mm × 9.5 mm)

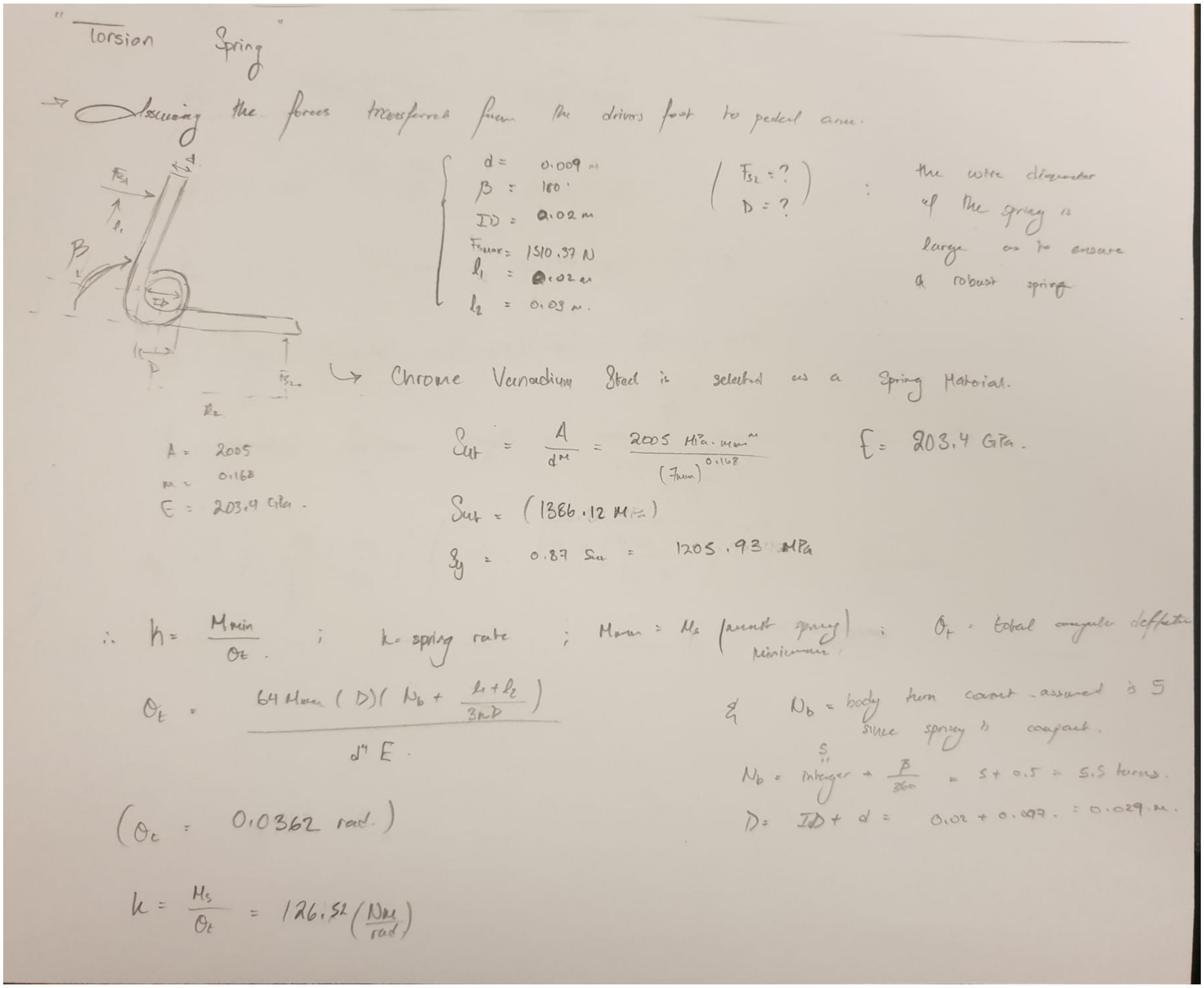
$$I = \frac{bh^3}{12} = \frac{(0.125)(0.0095)^3}{12} = 8.931 \times 10^{-9}$$

Max moment by spring = $F_s \cdot s$ = 30.2074 N.m.

$$\therefore \sigma = \frac{My}{I} \quad \left(y = \frac{w}{2} \text{ Distance from neutral axis of cross section.} \right) \quad \sigma = 1.606 \times 10^7 \text{ Pa.}$$

$$\therefore n = \frac{S_y}{\sigma} = \frac{296 \times 10^6}{1.606 \times 10^7} = \underline{\underline{18.42}} \text{ safety factor.}$$

Figure 3.4: Accelerator Pedal Analysis pg.2


"Torsion Spring"

→ Assuming the forces transferred from the drivers foot to pedal arm.

$d = 0.009$ m
$\beta = 180°$
$ID = 0.02$ m
$F_{smax} = 1510.37$ N
$l_1 = 0.02$ m
$l_2 = 0.03$ m.

$\left(F_{s2} = ? \quad D = ? \right)$: the wire diameter of the spring is large as to ensure a robust spring

↳ Chrome Vanadium Steel is selected as a Spring Material.

$A = 2005$
$m = 0.168$
$E = 203.4$ GPa.

$$S_{ut} = \frac{A}{d^m} = \frac{2005 \text{ MPa} \cdot \text{mm}^m}{(7\text{mm})^{0.168}}$$

$E = 203.4$ GPa.

$S_{ut} = (1386.12 \text{ MPa})$

$S_y = 0.87 S_{ut} = 1205.93$ MPa

$\therefore k = \dfrac{M_{min}}{\theta_t}$; k = spring rate ; $M_{min} = M_s$ (minimum spring) ; θ_t = total angular deflection

$$\theta_t = \frac{64 M_{min} (D)\left(N_b + \frac{l_1 + l_2}{3\pi D}\right)}{d^4 E}$$

& N_b = body turn count assumed is 5 since spring is compact.

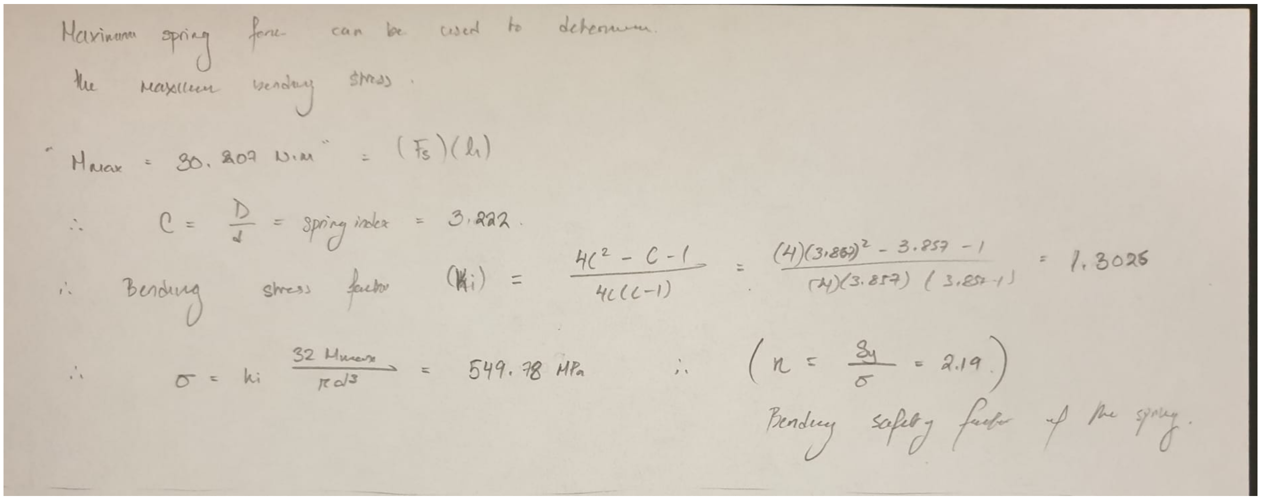
$N_b = \underset{5}{\text{integer}} + \dfrac{\beta}{360} = 5 + 0.5 = 5.5$ turns.

$D = ID + d = 0.02 + 0.009 = 0.029$ m.

$(\theta_t = 0.0362 \text{ rad})$

$$k = \frac{M_s}{\theta_t} = 126.52 \left(\frac{\text{Nm}}{\text{rad}}\right)$$

Figure 3.5: Accelerator Pedal Analysis pg.3



Maximum spring force can be used to determine the maximum bending stress.

$M_{max} = 30.207 \text{ N.m} = (F_s)(l_1)$

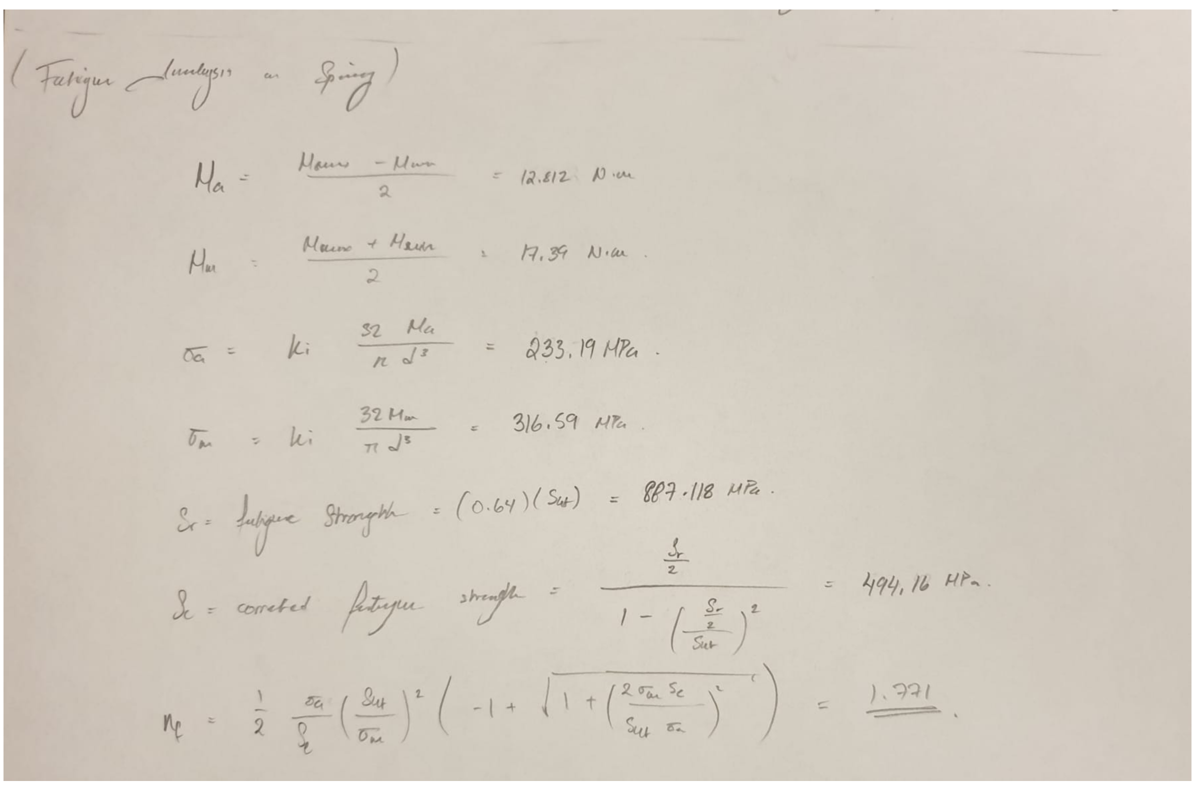
$\therefore \quad C = \frac{D}{d} = \text{spring index} = 3.222.$

$\therefore$ Bending stress factor $(K_i) = \frac{4C^2 - C - 1}{4C(C-1)} = \frac{(4)(3.857)^2 - 3.857 - 1}{(4)(3.857)(3.857-1)} = 1.3025$

$\therefore \quad \sigma = k_i \frac{32 M_{max}}{\pi d^3} = 549.78 \text{ MPa} \quad \therefore \left(n = \frac{S_y}{\sigma} = 2.19 \right)$

Bending safety factor of the spring.

Figure 3.6: Accelerator Pedal Analysis pg.4



(Fatigue Analysis on Spring)

$M_a = \frac{M_{max} - M_{min}}{2} = 12.812 \text{ N.m}$

$M_m = \frac{M_{max} + M_{min}}{2} = 17.39 \text{ N.m}.$

$\sigma_a = k_i \frac{32 M_a}{\pi d^3} = 233.19 \text{ MPa}.$

$\sigma_m = k_i \frac{32 M_m}{\pi d^3} = 316.59 \text{ MPa}.$

$S_r =$ fatigue Strength $= (0.64)(S_{ut}) = 887.118 \text{ MPa}.$

$S_e =$ corrected fatigue strength $= \frac{\frac{S_r}{2}}{1 - \left(\frac{\frac{S_r}{2}}{S_{ut}}\right)^2} = 494.16 \text{ MPa}.$

$n_f = \frac{1}{2} \frac{\sigma_a}{S_e} \left(\frac{S_{ut}}{\sigma_m}\right)^2 \left(-1 + \sqrt{1 + \left(\frac{2 \sigma_m S_e}{S_{ut} \sigma_a}\right)^2} \right) = 1.771.$

Figure 3.7: Accelerator Pedal Analysis pg.5

3.4 Brake Pedal

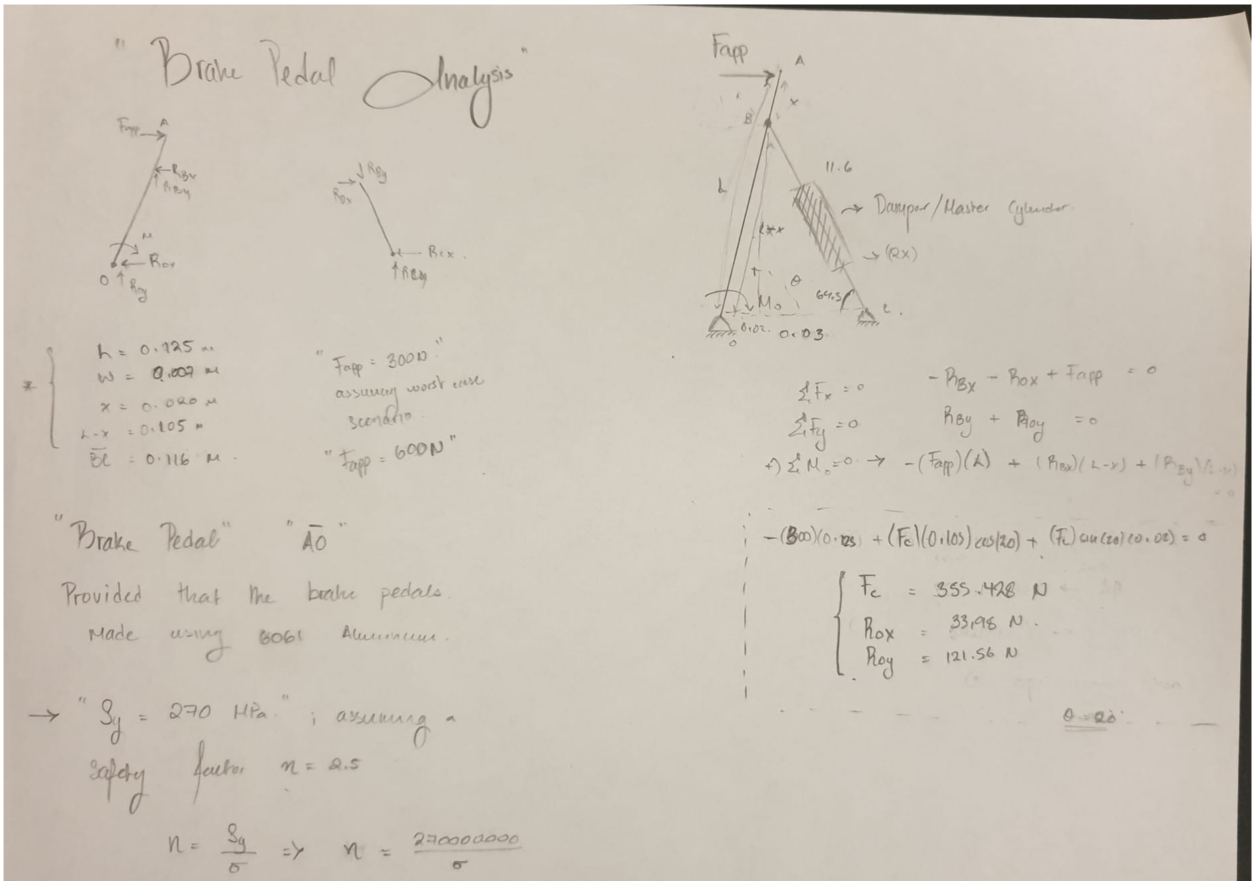
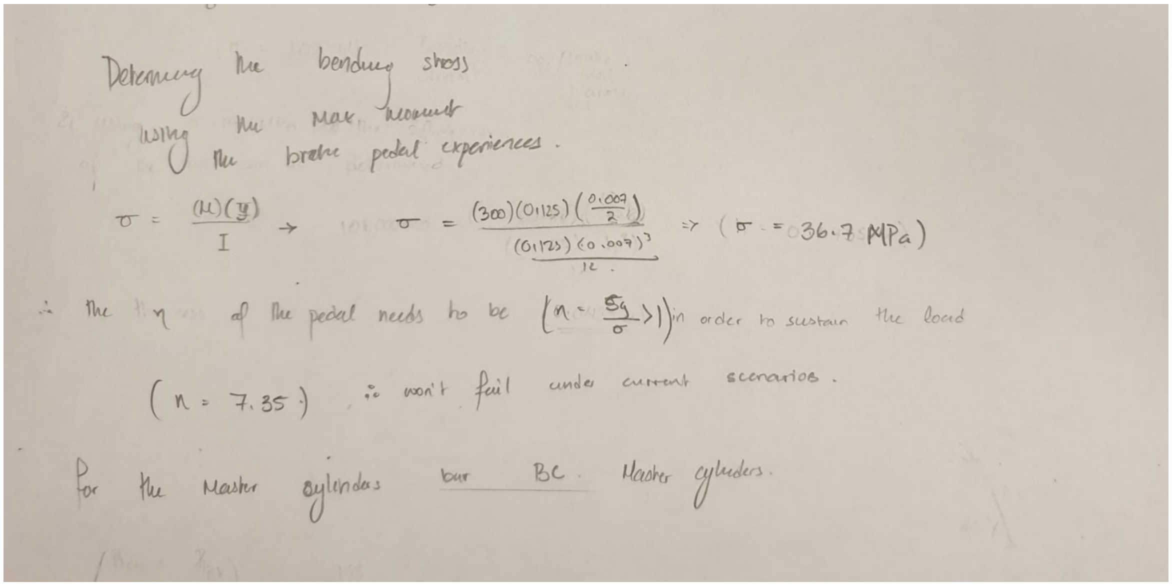


Figure 3.8: Brake Pedal Analysis Pg.1



Determining the bending stress using the max moment the brake pedal experiences.

$$\sigma = \frac{(M)(y)}{I} \rightarrow \sigma = \frac{(300)(0.125)\left(\frac{0.007}{2}\right)}{\frac{(0.125)(0.007)^3}{12}} \Rightarrow (\sigma = 36.7\ \text{MPa})$$

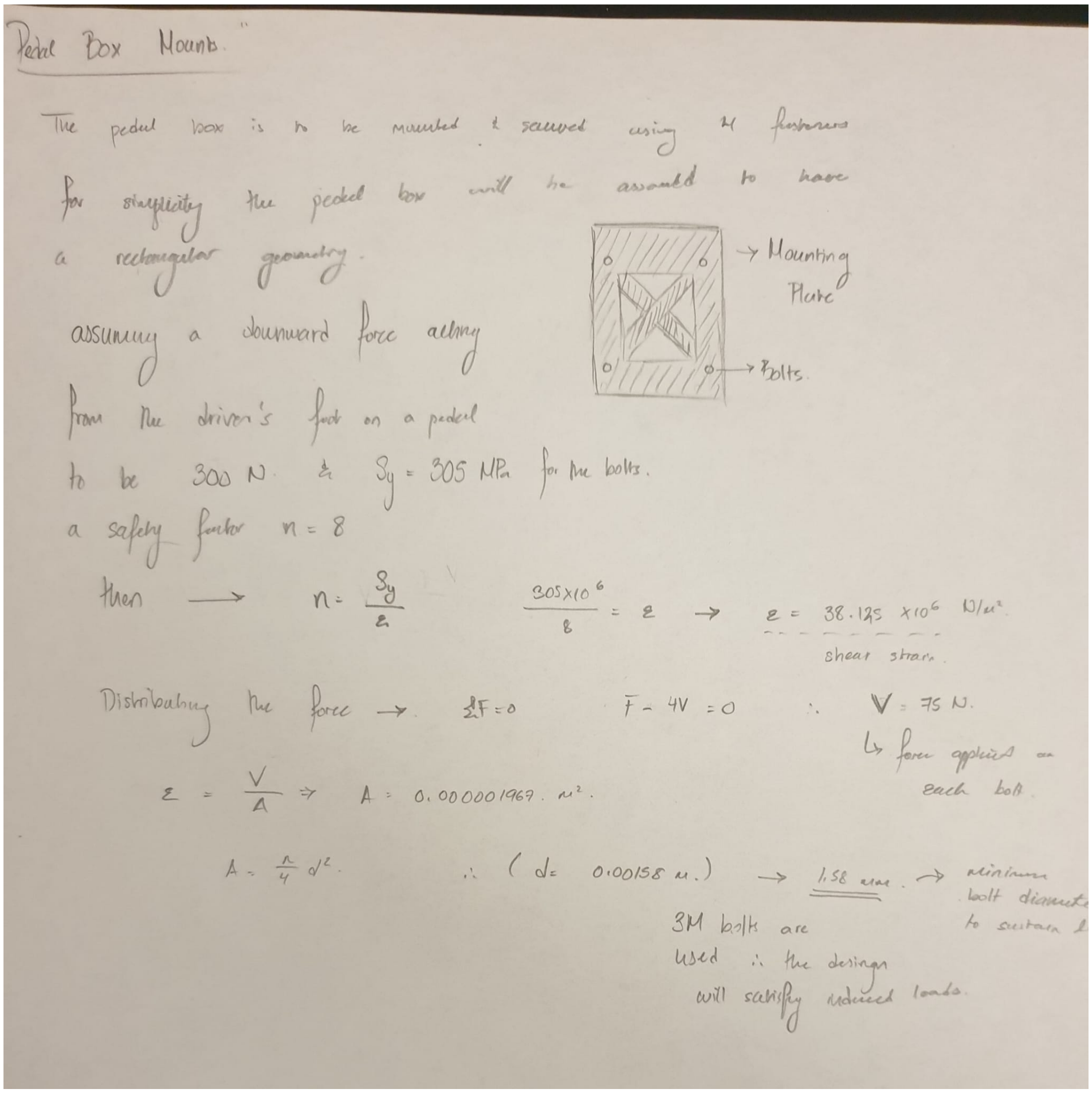
$\therefore$ The η of the pedal needs to be $\left(n = \frac{S_y}{\sigma} > 1\right)$ in order to sustain the load

$(n = 7.35)$ $\therefore$ won't fail under current scenarios.

For the master cylinders bar BC. Master cylinders.

Figure 3.9: Brake Pedal Analysis Pg.2

3.5 Pedal Box Mount



Pedal Box Mounts

The pedal box is to be mounted & secured using 4 fasteners

for simplicity the pedal box will be assumed to have a rectangular geometry.

assuming a downward force acting from the driver's foot on a pedal to be 300 N & $S_y = 305$ MPa for the bolts.

a safety factor $n = 8$

then $\longrightarrow$ $n = \frac{S_y}{\tau}$ $\qquad \frac{305\times10^6}{8} = \tau \rightarrow \tau = 38.125\times10^6$ N/m².

shear strain.

Distributing the force $\rightarrow$ $\sum F = 0$ $\qquad F - 4V = 0$ $\quad \therefore V = 75$ N.

↳ force applied on each bolt.

$\tau = \frac{V}{A} \Rightarrow A = 0.000001967$ m².

$A = \frac{\pi}{4} d^2$. $\quad \therefore (d = 0.00158$ m.) $\rightarrow$ 1.58 mm. $\rightarrow$ minimum bolt diameter to sustain l

3M bolts are used ∴ the design will satisfy induced loads.

Figure 3.10: Pedal Box Mount Analysis Pg.1

3.6 Battery Mount Analysis

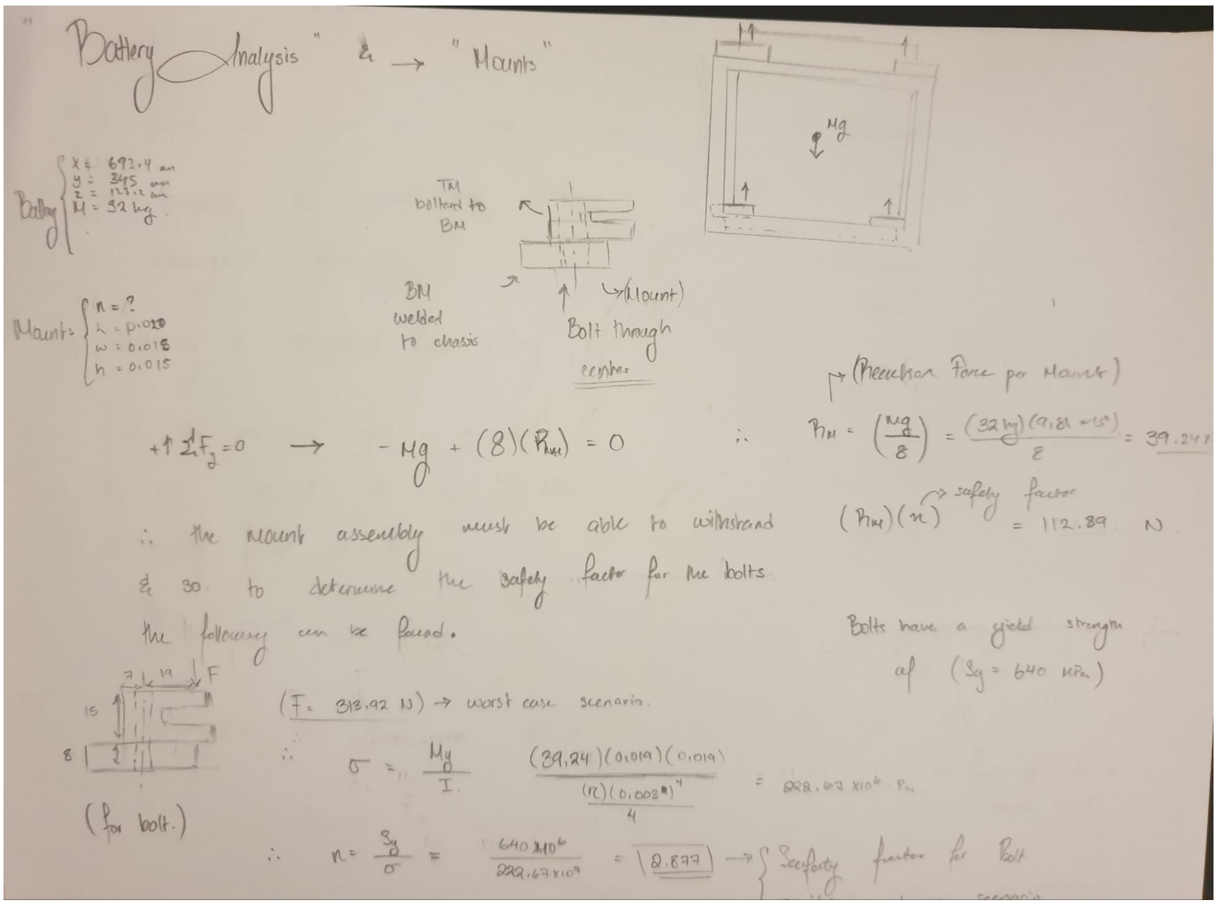


Figure 3.11: Battery Mount Analysis Pg.1

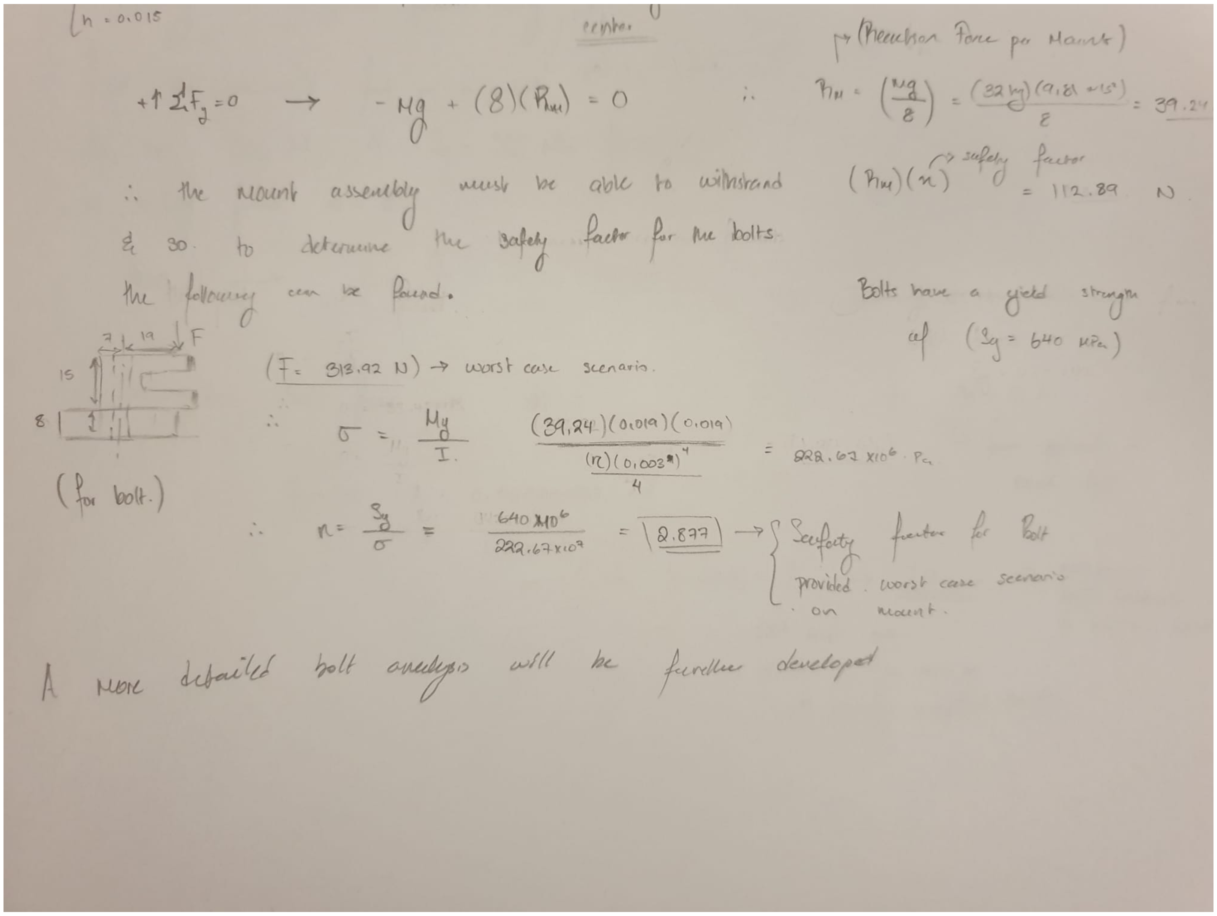


Figure 3.12: Battery Mount Analysis Pg.2

3.7 Rear Aerodynamic Wing Mounts

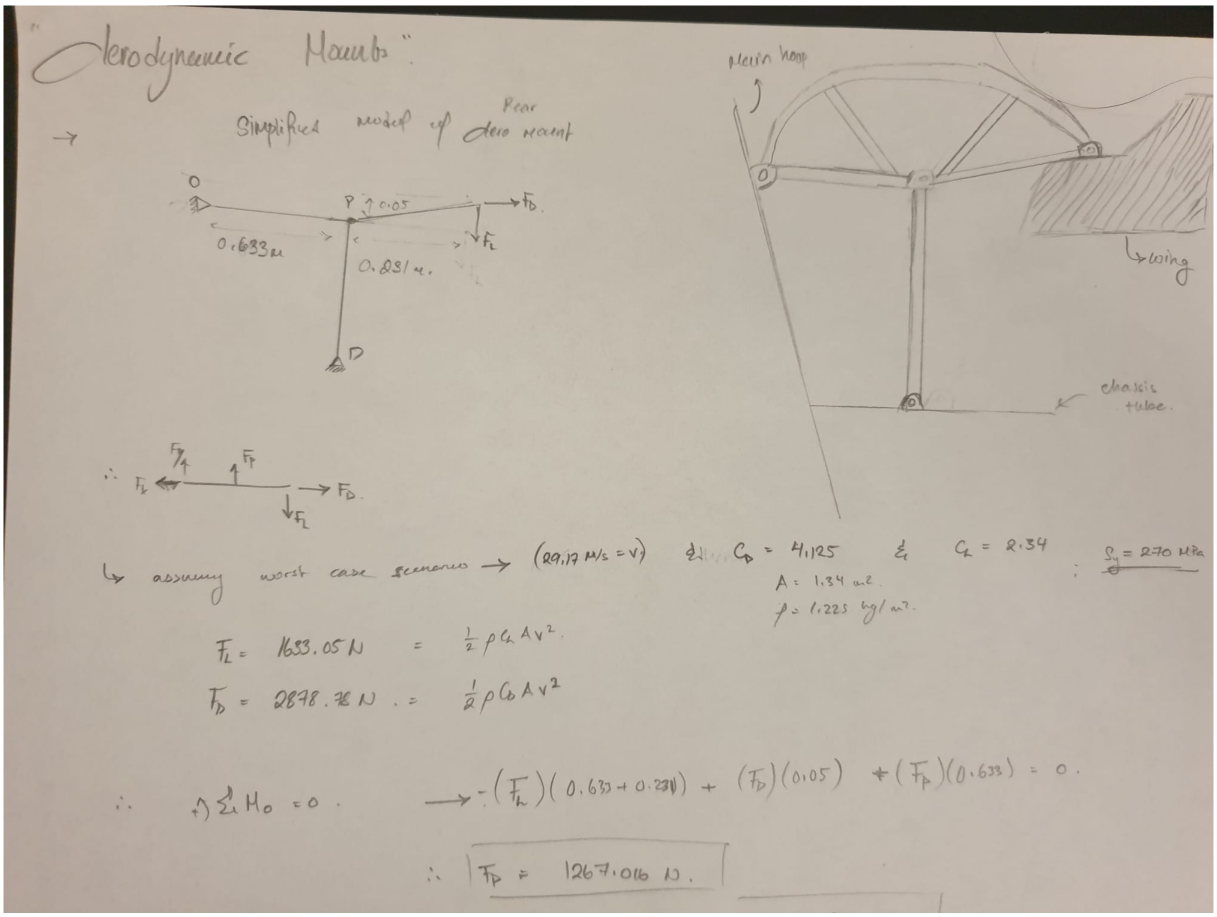


Figure 3.13: Rear Aero Mount Analysis pg.1

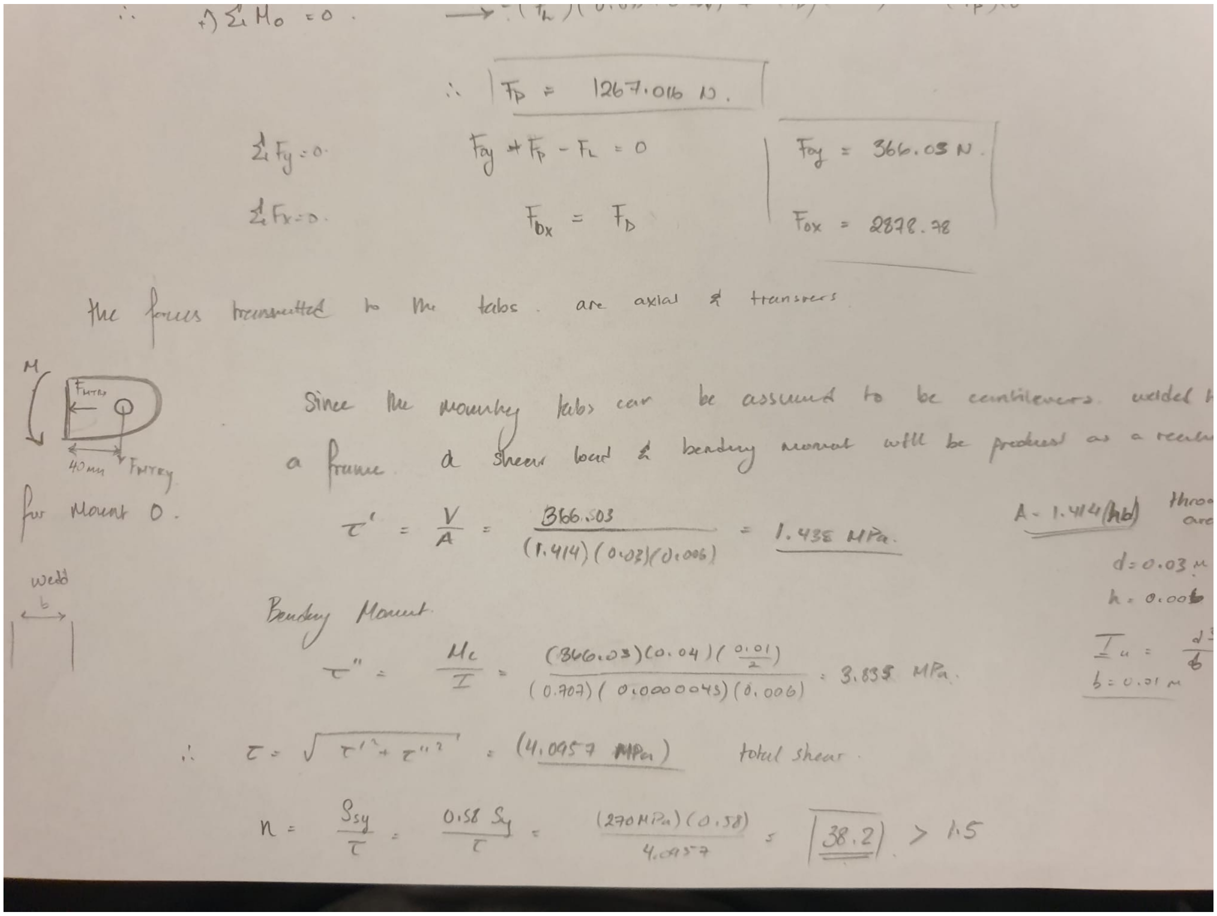


Figure 3.14: Rear Aero Mount Analysis pg.2

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APPENDICES