MCG 4322[A]
FSAE Modelling Report
FSAE 2

by

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Chapter 1

Project Charter

1.1 Mandate

Formula SAE (Society of Automotive Engineers) is a series of international competitions in which university teams compete to design and manufacture the best performing race cars. Our design team has been approached by a car manufacturer and contracted to develop a small electric Formula-style race car. The prototype should have high performance and be sufficiently durable to successfully complete all the static and dynamic events at the Formula SAE competitions. The prototype will be evaluated as it must follow all Formula SAE rules and regulations.

1.2 Requirements

1. The tractive system must be completely isolated from the chassis and any other conductive parts of the vehicle.

2. The tractive system motor(s) must be connected to the accumulator through a motor controller.

3. Only electrical motors of any type are allowed and the number of motors is not limited.

4. All accumulator containers must be designed to withstand forces from deceleration in all directions.

5. The accumulator container(s) must be completely closed at all times without the need to install extra protective covers.
6. Exposed high speed final drivetrain equipment must be fitted with scatter shields that may be composed of multiple pieces.
7. Coolant for electric motors and accumulators must be either plain water with no additives or oil.
8. The braking system must be operated by a single control and act on all four wheels. It must have two independent hydraulic circuits that have their own fluid reserve.
9. Vehicles may have either dry or wet tires.
10. The steering wheel must be mechanically connected to the front wheels.
11. The chassis must include both a main hoop and a front hoop.

1.3 Constraints

1. The maximum power drawn from the accumulator must not exceed 80 kW.
2. The maximum permitted voltage that may occur between any two points must not exceed 600 V DC.
3. A maximum of 12 kg is allowed in any accumulator container section.
4. Regenerating energy is allowed and unrestricted when the vehicle speed is more than 5 km/hr.
5. The brake pedal and system must withstand a minimum force of 2000 N
6. Wheels must be 203.2 mm (8.0 inches) or more in diameter.

7. The vehicle must be equipped with an operational suspension system with usable wheel travel of at least 50 mm, with a driver seated.

1.4 Criteria

1. **Performance**: Increase in handling, response, and tractive capability of the steering, suspension, and tires. Increase in acceleration and traction force of the vehicle.
2. **Serviceability**: Ease of repair, subsystems accessibility, parts interchangeability, low manufacturing complexity, and standardization of fasteners across the vehicle.
3. **Safety**: Visibility, cockpit protection, firewall, rollover protection, and scatter shields. Wiring is safely routed, color coded, and marked for function.
4. **Ergonomics**: Driver comfort, arm room, leg room, head restraint, ease of control, seat adjustability, and readability of essential instruments.
5. **Reliability**: Consistent and reliable braking system, drivetrain, and motor.
6. **Aerodynamics**: Drag reduction, lift reduction, noise elimination, and downforce gain.
7. **Cost**: Raw material selection, manufacturing process selection, and design optimizations for simpler solutions.
8. **Efficiency**: Lightweight design, increase in range, and improve in performance and structural integrity.

1.5 Parameterization Outline

The design of the vehicle is driven by the length and weight to be able to accommodate drivers of sizes ranging from 5th percentile female up to 95th percentile male. Accommodation will include driver position, driver controls, and driver equipment. Primarily, the wheel base will be altered to change the length and weight of the vehicle to accommodate the driver. To alter the wheel base, the geometrical properties of the space frame and its members will be adjusted and hence, the overall weight of the vehicle will vary.

Chapter 2

Design Solution

2.1 Updated Solution

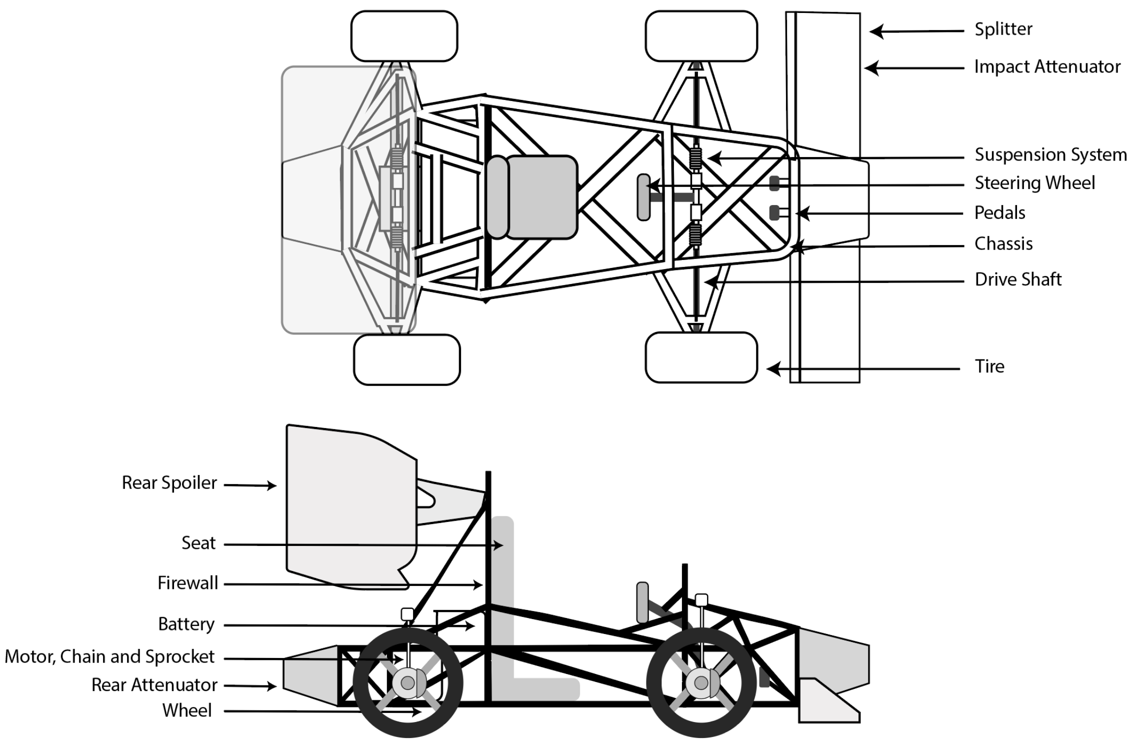

Figure 2.1: Updated Solution of the vehicle with labeled components

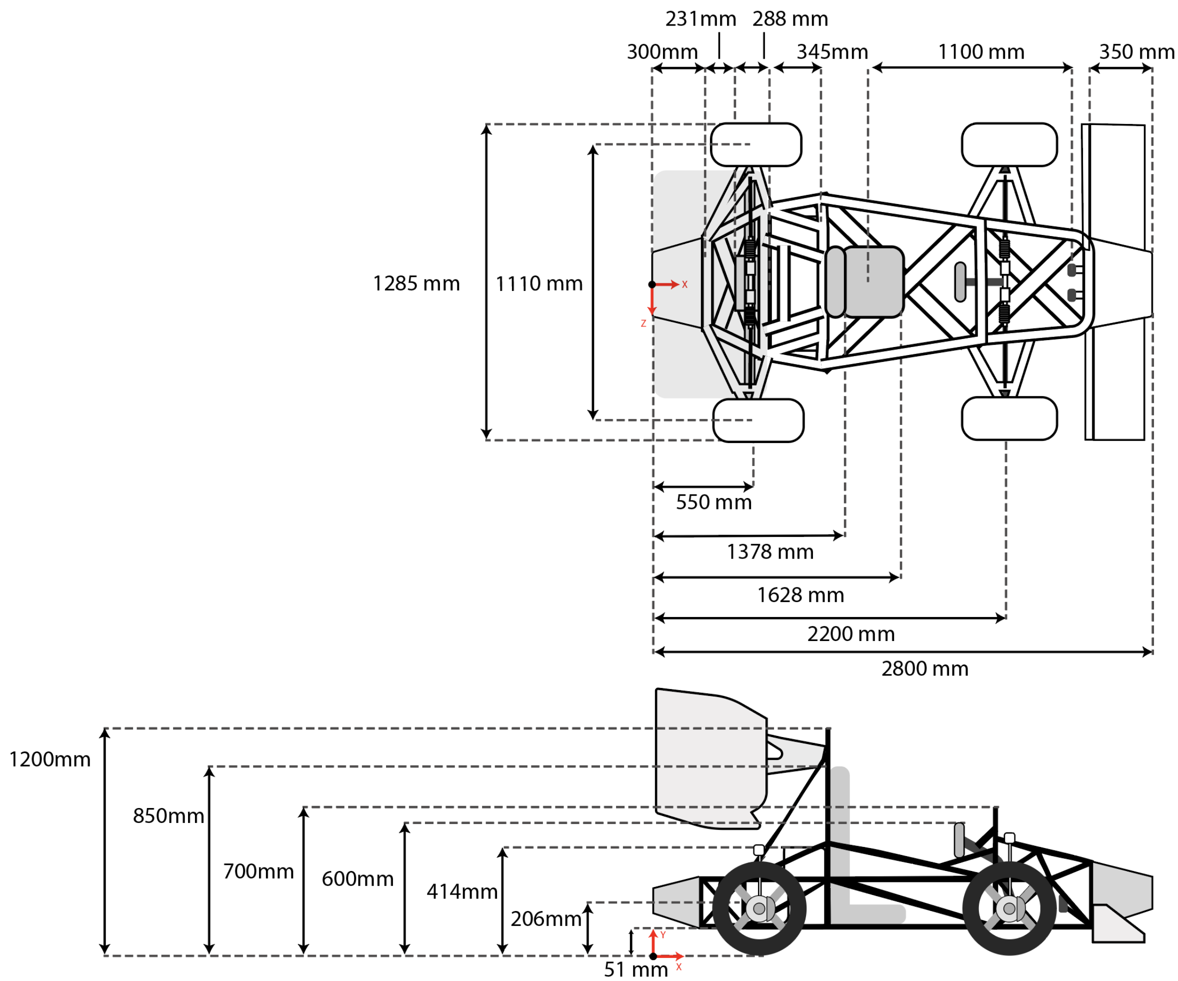

Figure 2.2: Updated Solution of the vehicle with dimensions

The updated design of the FSAE car is shown in figure 2.1 and figure 2.2 is based on the decision criteria tables in the Concept Report. Additional design concepts include a rear spoiler and a front splitter which will improve the aerodynamic performance of the car. Furthermore, the tripod joints that couple the motor shaft and axle half shaft are going to be replaced with CV joints as they are less expensive to purchase and maintain.

2.2 Approximations

2.2.1 Vehicle Assumptions

Table 2.1: Assumptions of vehicle operation

Parameter	Approximation
Maximum Speed	105 km/h
Maximum Acceleration	6 m/s^2
Maximum Force Applied on the Brake Pedal	2000 N
Average Impact Deceleration	196 m/s^2

2.2.2 Mass Approximations

Table 2.2: Weight Calculation of the Vehicle Approximation [1]

Item	Weight (kg)
Steel Spaceframe	31.80
Maximum Driver Weight	111.58
Front Attenuator	1
Front Wing	3.10
Rear Wing	2.80

Wing Mounting	1.32
Battery	32
Emraxx 228 Motor	12
Chain and Sprocket Kit	3.60
Motor Mount	0.17
Limited Slip Differential	2.60
Chain and Sprocket Kit	3.58
Half Axle Shaft	0.7
Tripod Joints	0.2
Front Control Arm Assemblies	2.6
Rear Control Arm Assemblies	2.4
Front Uprights	1.8
Rear Uprights	8.1
Rockers Assemblies	1.8
Shock Absorber	2.6
Swaybar Assemblies	0.6
Rotors	1.22
Breakline Assembly	2.8

Break Pads	0.2
Break Calipers	0.85
Rack and Pinion Subsystem	3.1
Steering Wheel	1.97
Quick Release	0.99
Steering Shaft	2.2
Intermediate shaft	0.82
Steering column bushing	1.09
Tie rod	0.63
Tires	17.96
Wheels	13.3
Front Wheel Hubs	2.4
Rear Wheel Hubs	10.4
Wiring and Instruments	2.3
Seat	2.07
Firewall	2.02
Harness	1.04
Throttle and Brake Pedal System	2.4

Total Mass of the Car	**297**

2.2.3 Accumulator Approximations

It is important that the accumulator parameters are estimated so that the geometric and mass properties of the vehicle can be calculated. The table below illustrates the approximate dimensions and weight of the battery. The calculations for how this model was calculated are illustrated in appendix B.1

Table 2.3: Sony VTC6 Battery Specification [2]

Parameter	**Value**
Continuous Max. Discharge (A)	15.0
Peak Max. Discharge (A)	30.0
Nominal Voltage (V)	3.6
Max Voltage (V)	4.2
Capacity (Ah)	3.0
Diameter (mm)	18.3
Height (mm)	65.0
Mass (g)	48.0

Table 2.4: Sony VTC6 Accumulator Design 5 sections connected in series consisting of (4px28s)

Parameter	**Value**
Capacity per Module (Ah)	12
Nominal Voltage per Module (V)	100.8
Nominal Capacity per Module (Wh)	1209.6
Peak Voltage per Module (V)	117.6
Peak Capacity per Module (Wh)	1411.2
Total Nominal Voltage (V)	504
Accumulator Peak Voltage (V)	588
Max Discharge Current (A)	120
Accumulator Peak Power (W)	70560
Accumulator Capacity (kWh)	6.048
Dimensions X (mm)	693.4
Dimensions Y (mm)	345
Dimensions Z (mm)	123.2
Weight (Batteries) (kg)	26.88
Accumulator Estimated Weight (kg)	30.38

Chapter 3

Geometric Relations & Properties

3.1 Wheelbase to Track Ratio

The wheelbase to track ratio is a mathematical relationship that characterizes the handling of the vehicle. The track width determines the weight that is transferred by the mass of the car in cornering. The wheelbase determines how much weight is transferred by the mass of the car in acceleration and braking.

$$\frac{\text{Wheelbase Length}}{\text{Track Length}} = \frac{1650mm}{1110mm} = 1.49 \tag{3.1}$$

The FSAE car's wheelbase length is designed to be 1650 mm while the track width is the 1110 mm, giving a ratio of 1.49. This ratio nearly falls into the range of optimum ratio which is 1.5 to 1.7. An optimum ratio of wheelbase to track ensures proper mass distribution throughout the car, resulting in high handling performance.

3.2 Center of Mass Calculation

A vehicle's center of mass is the point where the sum of all of the subassembly masses act. The center of mass is used to determine overall forces when the car is turning, accelerating or decelerating. Most FSAE vehicles are designed to have a relatively low center of mass. To measure the center of mass of the car, the subassemblies were split into volumes. Each volume had its own center of mass which was approximated using the dimensions and the origin in the table below [3].

Table 3.1: Volumes and their Respective Approximate Center of Mass

Volume	**Weight (kg)**	**x - Coordinate (mm)**	**y - Coordinate (mm)**	**z - coordinate (mm)**
Battery, Motor, and Powertrain	51.244	866.5	207	0
Front Suspension Assembly	10.656	2150	375	0
Rear Suspension Assembly	16.712	550	550	0
Top Right Wheel, Tire, and Brake Assembly	9.843	2150	206	625
Top Left Wheel, Tire, and Brake Assembly	9.843	2150	206	-625
Bottom Right Wheel, Tire, and Brake Assembly	13.843	550	206	625
Bottom Left Wheel, Tire, and Brake Assembly	13.843	550	206	-625
Steering Assembly	11.796	2000	415	0
Driver, Firewall, Harness, and Seat	116.71	1400	425	0
Spaceframe and Wings	38.972	1400	600	0

Refer to B.2 for the locations of the center of mass of each volume.

$$x_{mm} = \frac{\sum_{i=1}^{N} m_i x_i}{M} \Rightarrow COM_x = \frac{381439.38mm}{293.46kg} = 1299.79mm \tag{3.2}$$

$$y_{mm} = \frac{\sum_{i=1}^{N} m_i y_i}{M} \Rightarrow COM_y = \frac{111434.019mm}{293.46kg} = 379.72mm \tag{3.3}$$

$$z_{mm} = \frac{\sum_{i=1}^{N} m_i z_i}{M} \Rightarrow COM_z = \frac{0mm}{293.46kg} = 0mm \tag{3.4}$$

3.3 Load Transfer

3.3.1 Longitudinal Load Transfer

The center of gravity is the location of rotation for breaking and acceleration inputs. The vehicle will squat when accelerating and dive when braking relative to the center of gravity. To mitigate this rotation, a counter rotation needs to be applied to the center of gravity. This opposing rotation is induced by anti-squat and anti-dive geometry.

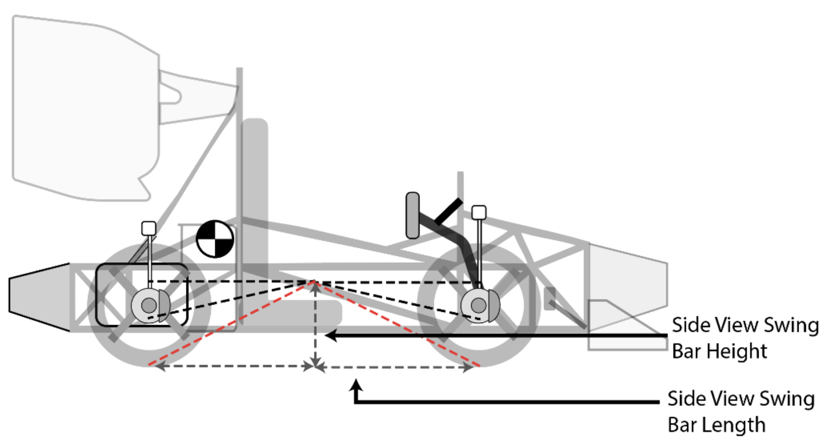

Figure 3.1: Anti-squat Geometry (left red line) and Anti-dive Geometry (right red line)

Anti-squat applications are added to the rear since the rear wheels squat while accelerating. The anti-squat geometry is determined using the swing arm length and height, wheelbase, and height of the center of gravity. The rear suspension's compression percentage can be modeled using the height of the center of gravity (h), wheelbase of the vehicle (L), and the dimensions of the swing arm length.

$$\%\text{ Anti Squat } = \frac{\tan \emptyset \mathrm{A}}{\mathrm{h/L}} \times 100 \tag{3.5}$$

Where,

$$\tan \emptyset_{\mathrm{R}} = \frac{\text{Side View Swing Arm Height}}{\text{Side View Swing Arm Length}} \tag{3.6}$$

Anti-dive geometry is applied for the front wheels since the front wheels experience compression whenever the car brakes. Anti-dive geometry prevents the car from diving onto the brakes and deflecting vertically. The percentage of front suspension's compression can be mathematically modeled using the height of the center of gravity (h), wheelbase of the vehicle (L) and the dimensions of the virtual side view swing arm length [4].

$$\%\text{ Front Anti Dive } = \%\text{ Front Braking } \times \tan \emptyset_{\mathrm{A}} \times \frac{\mathrm{L}}{\mathrm{h}} \tag{3.7}$$

3.3.2 Lateral Load Transfer

The car experiences rotation around a virtual point being the roll centre. The roll centre convention assumes that a load is applied on the center of the wheel. The roll center can be located using the geometry in figure 4.13.

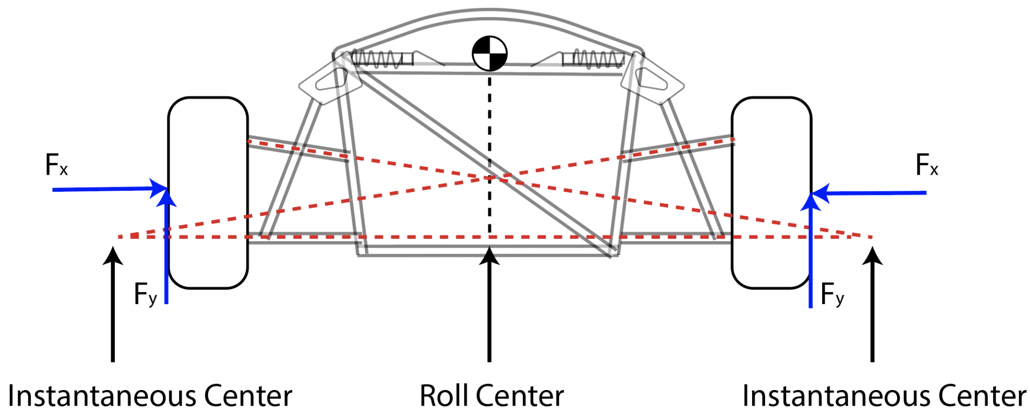

Figure 3.2: Suspension Geometry to Locate Instantaneous Points and Roll Centre

The instantaneous points act as a pivot where the force rotates on the imaginary red lines, subsequently causing a rotation around the line. As the vehicle accelerates, the x component of the force is applied on the control arms and the y component force moves the suspension around its instantaneous center [5].

3.4 Assumptions and Working Environment

The FSAE vehicle is assumed to operate under clear weather conditions while being susceptible to high impact crashes. The driver of the vehicle is assumed to be a 95th percentile male with a weight of 111.58 kg. The total weight of the vehicle with the driver is calculated to a total of 297 kg. The vehicle is assumed to be operating using its maximum speed and acceleration, 105 km/hr and 6 m/s^2 respectively.

Chapter 4

Kinematics and Applied Forces

4.1 2 Dimensional Kinematic Model of Vehicle

Illustrated below is a figure which represents a 2 dimensional model of a car in motion. The parameters can be defined as:

$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \tag{4.1}$$

The 4 Degrees of Freedom are represented as the vehicles position on the x-axis, y-axis and along the vehicle's steering angle θ

Assuming small velocities, the dynamics of the vehicle can be described using a single-track model [6]

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ \frac{1}{L} * tan(\phi) \\ 0 \end{bmatrix} v = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \omega \tag{4.2}$$

Where x and y define the position of the midpoint of the rear axle, θ refers to the orientation of the car relative to the x-axis, and ϕ refers to the steering angle. [7]

Hence, it is determined that the motion of the car can be determined using the equations below. It can easily be demonstrated that 4.3 and 4.4 are the velocity components of the car in the x and y coordinates, and 4.5 is the angular velocity of the vehicle.

$$\dot{x} = v\cos(\theta); \tag{4.3}$$

$$\dot{y} = v\cos(\theta); \tag{4.4}$$

$$\dot{\theta} = \frac{v}{L}\tan(\phi) \tag{4.5}$$

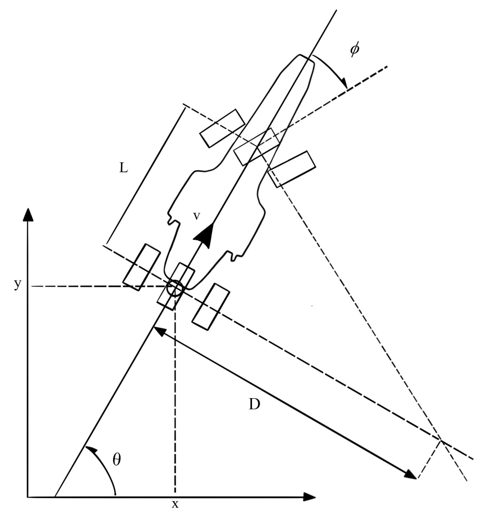

Figure 4.1: 2-D Kinematic Model of Vehicle

4.2 Steady State Low Speed Cornering Geometry and Vehicle Turning Radius

The figure below illustrates an Ackermann Geometry also known as a kinematic steering geometry. The Ackermann geometry states that for a basic steering system for a front wheel steering system, the difference of the cotangents of the angles of the outer front wheels to the inner front wheels should equate to the ratio of the width (T) and length (L) of the vehicle.

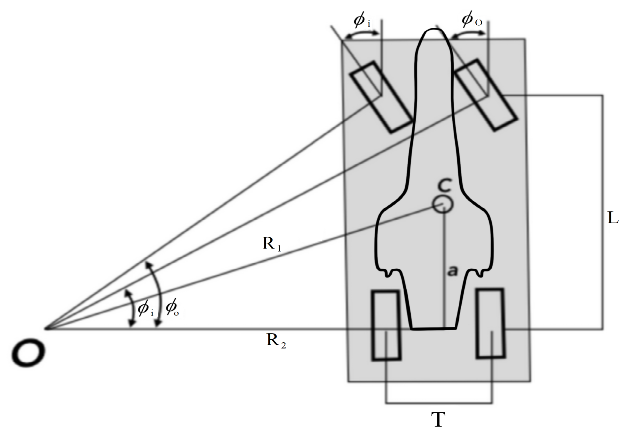

Figure 4.2: Vehicle Steering Ackermann Method

This condition can be simply modeled by the following set of equations:

$$\cot(\phi_i) - \cot(\phi_o) = \frac{T}{L} \tag{4.6}$$

Where, ϕ_i is the steering angle of the inner front wheel,ϕ_o is the steering angle of the outer front wheel, T is the vehicle's track width, and L is the wheelbase of the vehicle. As seen in figure 4.2 it can be seen that if a normal is drawn from all the wheels they intersect at a point O which is the center of rotation. The vehicle's center of mass will turn with a radius R_1. This is known as the turning radius, which can be geometrically defined as follows:

$$R_1 = \sqrt{a^2 + (L^2)(\frac{(\cot(\phi_o) + \cot(\phi_i))}{2})^2} \tag{4.7}$$

Where, a is the distance from the rear axle to the center of mass and is 0.739 m. Assuming $R_2 = 5m$, and provided $L = 1.65m$ and $T = 1.11m$. Then, the turning radius of the vehicle can be determined as follows:

$$\phi_o = \arctan\left(\frac{L}{R_2 + \frac{T}{2}}\right) = \arctan\left(\frac{1.65m}{5m + \frac{1.11m}{2}}\right) = 0.2804rad \tag{4.8}$$

$$\phi_i = \arctan\left(\frac{L}{R_2 - \frac{T}{2}}\right) = \arctan\left(\frac{1.65m}{5m - \frac{1.11m}{2}}\right) = 0.3455rad \tag{4.9}$$

$$R_1 = \sqrt{(.739m)^2 + (1.65m)^2(\frac{(\cot(.2804rad) + \cot(.3455rad)}{2})^2} = 5.209m \tag{4.10}$$

Hence, from the above series of equations it can be determined that the turning radius of the center mass of the vehicle would be 5.21 meters.

4.3 Static Applied Forces

4.3.1 Forces Applied on a Car Parked on a Level Road

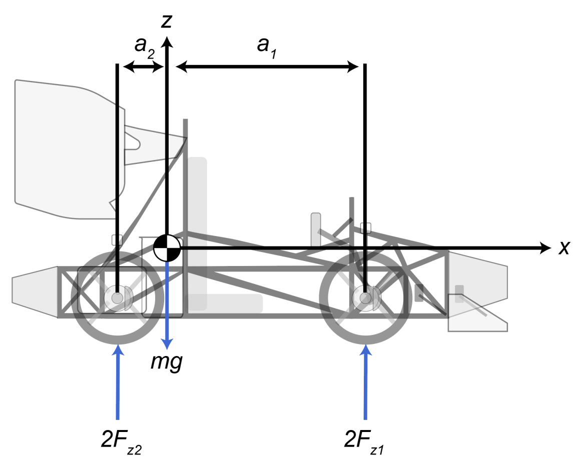

Figure 4.3: FBD of Car on a Level Road

The reaction forces on the wheels of a car parked on a level road can be found as follows:

$$F_{z_1} = \frac{1}{2}mg\frac{a_2}{(a_2+a_1)} = \frac{1}{2}(297kg)(9.81\frac{m}{s^2})\frac{(0.75m)}{(0.75m+0.9m)} = 662.1 \text{ N} \tag{4.11}$$

$$F_{z_2} = \frac{1}{2}mg\frac{a_1}{(a_2+a_1)} = \frac{1}{2}(297kg)(9.81\frac{m}{s^2})\frac{(0.9m)}{(0.75m+0.9m)} = 794.6 \text{ N} \tag{4.12}$$

4.3.2 Forces Applied on a Car Parked on an Inclined Road

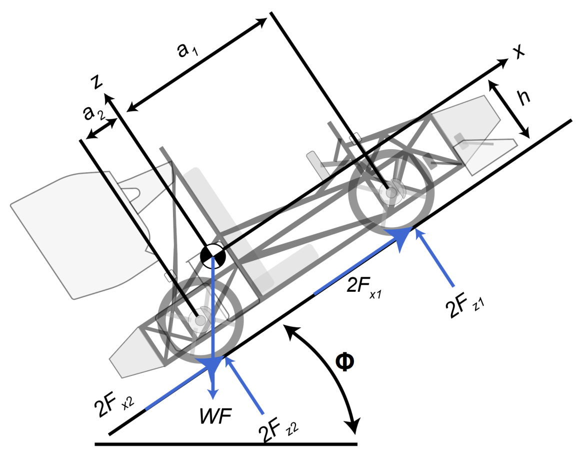

Figure 4.4: FBD of Car on an Inclined Road

The normal forces acting on the wheels of a car parked on an inclined road can be found as follows:

$$\mathrm{F}_{z_1} = \frac{1}{2}mg\left(\frac{a_2}{a_2+a_1}\cos\phi - \frac{h}{a_2+a_1}\sin\phi\right) \tag{4.13}$$

$$\mathrm{F}_{z_2} = \frac{1}{2}mg\left(\frac{a_1}{a_2+a_1}\cos\phi + \frac{h}{a_2+a_1}\sin\phi\right) \tag{4.14}$$

Assuming a coefficient of static friction $\mu_s = 0.7$, the friction forces at the wheels are found to be:

$$F_{x_1} = \mu_s F_{z_1} = (0.7)(594.01 \text{ N}) = 415.81 \text{ N} \tag{4.15}$$

$$F_{x_2} = \mu_s F_{z_2} = (0.7)(840.64 \text{ N}) = 588.45 \text{ N} \tag{4.16}$$

Assuming a typical angle of incline as $\phi = 10°$, the reaction forces can be calculated as follows:

$$\text{F}_{z_1} = \frac{1}{2}(297kg)(9.81\frac{m}{s^2})(\frac{(0.75m)}{(0.75m + 0.9m)}\cos(10°) - \frac{(0.379m)}{(0.75m + 0.9m)}\sin(10°)) = 594.01 \text{ N} \tag{4.17}$$

$$\text{F}_{z_2} = \frac{1}{2}(297kg)(9.81\frac{m}{s^2})(\frac{(0.9m)}{(0.75m + 0.9m)}\cos(10°) + \frac{(0.379m)}{(0.75m + 0.9m)}\sin(10°)) = 840.64 \text{ N} \tag{4.18}$$

4.3.3 Forces Applied on a Car Parked on a Banked Road

When a car is parked on a banked road, the load is distributed differently between each side of the car, with a larger load experienced by the lower tires. To illustrate this, an FBD for the front wheels on a banked road is used, where the load experienced on the front wheels corresponds to the load on the front axle (W_F).

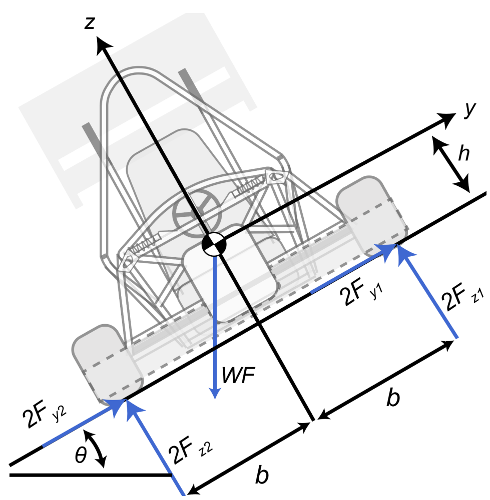

Figure 4.5: FBD of Car on a Banked Road

The reaction forces above are found as follows [8]:

$$F_{z_1} = \frac{W_F}{2b}(b\cos\theta - h\sin\theta) \tag{4.19}$$

$$F_{z_2} = \frac{W_F}{2b}(b\cos\theta + h\sin\theta) \tag{4.20}$$

To calculate for the normal forces experienced by the front wheels, W_F is first found by multiplying the value for F_{z_1} from section 4.3.1 by the cosine of the bank angle θ. Assuming the bank angle to be $\theta = 10°$, thus giving $W_F = 652.04N$.

$$F_{z_1FRONT} = \frac{652.04N}{2(0.625m)}(0.625\cos(10°) - 0.379\sin(10°)) = 286.76 \text{ N} \tag{4.21}$$

$$F_{z_2FRONT} = \frac{652.04N}{2(0.625m)}(0.625\cos(10°) + 0.379\sin(10°)) = 355.43 \text{ N} \tag{4.22}$$

Using a coefficient of friction $\mu_s = 0.7$, the following friction forces can be found:

$$F_{y_1FRONT} = (\mu_s)(F_{z_1FRONT}) = (0.7)(286.76N) = 200.73 \text{ N} \tag{4.23}$$

$$F_{y_2FRONT} = (\mu_s)(F_{z_2FRONT}) = (0.7)(355.43N)248.8 \text{ N} \tag{4.24}$$

Similarly, calculating for the normal forces experienced by the rear wheels on a banked surface uses the same equations as the front. However, the normal forces experienced in the rear wheels correspond to the load on the rear axle (W_R) which is the value for F_{z_2} from equation (4.12) multiplied by the cosine of the bank angle. Again it is assumed $\theta = 10°$, thus giving $W_R = 782.53N$.

$$F_{z_1REAR} = \frac{782.53N}{2(0.625m)}(0.625\cos(10°) - 0.379\sin(10°)) = 343.93 \text{ N} \tag{4.25}$$

$$F_{z_2REAR} = \frac{782.53N}{2(0.625m)}(0.625\cos(10°) + 0.379\sin(10°)) = 426.28 \text{ N} \tag{4.26}$$

Using $\mu_s = 0.7$ the following friction forces can be found:

$$F_{y_1 REAR} = (\mu_s)(F_{z_1 REAR}) = (0.7)(343.93N) = 240.75 \text{ N} \tag{4.27}$$

$$F_{y_2 REAR} = (\mu_s)(F_{z_2 REAR}) = (0.7)(426.28N) = 298.4 \text{ N} \tag{4.28}$$

4.4 Dynamic Applied Forces

4.4.1 Acceleration on a Level Road

An accelerating car on a level road gives has reaction forces on the wheels as seen in the figure below. In this situation load is transferred longitudinally shifting the load more to the driven wheels, or rear wheels, causing the car to squat. Additionally, the friction force is only considered for the driven wheels since the front wheels are free-rolling and thus friction on them is assumed negligible [8].

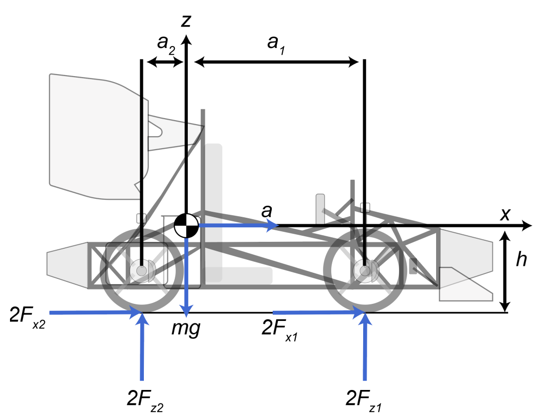

Figure 4.6: FBD of an Accelerating Car on a Level Road

The forces in the FBD above can be found as follows, where it is assumed $\mu_k = 0.6$ [8]:

$$F_{z_1} = \frac{1}{2}(mg)((\frac{a_2}{a_2 + a_1}) - (\frac{h}{a_1 + a_2})(\frac{a}{g})) \tag{4.29}$$

$$F_{z_2} = \frac{1}{2}(mg)((\frac{a_1}{a_2 + a_1}) + (\frac{h}{a_1 + a_2})(\frac{a}{g})) \tag{4.30}$$

$$F_{x_2} = \mu_k F_{z_2} = 0.6 \times 589.95N = 353.97 \text{ N} \tag{4.31}$$

Assuming an acceleration of $a = 6\frac{m}{s^2}$, the normal forces are calculated as:

$$F_{z_1} = \frac{1}{2}(297kg \times 9.81\frac{m}{s^2})((\frac{0.75m}{0.9m + 0.75m}) - ((\frac{0.379m}{0.9m + 0.75m})(\frac{6m}{9.81\frac{m}{s^2}})) = 457.52 \text{ N} \tag{4.32}$$

$$F_{z_2} = \frac{1}{2}(297kg \times 9.81\frac{m}{s^2})((\frac{0.9m}{0.9m + 0.75m}) + ((\frac{0.379m}{0.9m + 0.75m})(\frac{6m}{9.81\frac{m}{s^2}})) = 589.95 \text{ N} \tag{4.33}$$

4.4.2 Acceleration on an Inclined Road

Below is a depiction of the applied forces on a car while accelerating on an incline. The principle stays the same as acceleration on a level road, however the extent of load transfer and thus magnitude of each reaction force is changed due to the additional acceleration induced by gravity [8].

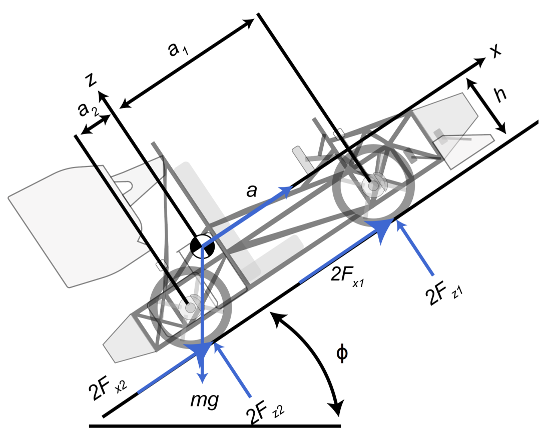

Figure 4.7: FBD of an Accelerating Car on an Inclined Road

$$F_{z_1} = \frac{1}{2}mg\left(\frac{a_2}{a_1 + a_2}\cos\phi - \frac{h}{a_1 + a_2}\sin\phi\right) - \frac{1}{2}(ma)\frac{h}{a_1 + a_2} \tag{4.34}$$

$$F_{z_2} = \frac{1}{2}mg\left(\frac{a_1}{a_1+a_2}\cos\phi + \frac{h}{a_1+a_2}\sin\phi\right) + \frac{1}{2}(ma)\frac{h}{a_1+a_2} = \tag{4.35}$$

Assuming a typical angle of incline as $\phi = 10°$ and a typical acceleration as $a = 6\frac{m}{s^2}$, the values for F_z can be calculated as:

$$F_{z_1} = 389.35 \text{ N} \tag{4.36}$$

$$F_{z_2} = 1045.30 \text{ N} \tag{4.37}$$

Assuming a coefficient of kinetic friction $\mu_k = 0.6$, the friction force on the rear wheels is:

$$F_{x_2} == \mu_k F_{z_2} = 0.6 \times 1045.3N = 627.18 \text{ N} \tag{4.38}$$

4.4.3 High Speed Cornering

During a turn at a relatively high velocity, a centripetal force is produced which transfers the load laterally to the side of the car inside a turn. For a car moving at a reasonable velocity v, around a corner with a radius of curvature R, there is an applied centripetal force (Fc).With the assumption that $v = 15\frac{m}{s}$ and that the radius of curvature is $R = 30m$, the centripetal force can be calculated as follows below. It should be noted that this radius of curvature is close to the maximum value of R that is allowed in the FSAE guidelines [9].

$$\mathrm{F_C} = \frac{mv^2}{R} = (297\text{ kg})\left(\frac{(15\frac{\text{m}}{\text{s}})^2}{30\text{ m}}\right) = 2227.5 \text{ N} \tag{4.39}$$

A consequence of F_C when turning, is that the load is transferred laterally from one side of the car to the other. That is, more load is placed on the wheels on the inside of a turn, causing the car to roll. To illustrate this, an FBD for the front wheels during a corner is used, where the load experienced on the front wheels corresponds to the load on the front axle (W_F). The value for W_F is equal to the value from section 4.3.1 found in section section 4.3.1.

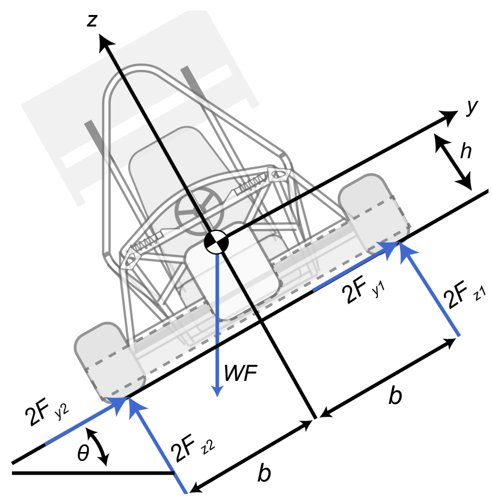

Figure 4.8: Reaction at Wheels Due to Applied Centripetal Force

The reaction forces above are found as follows:

$$F_{z_1FRONT} = \frac{bW_F - hF_C}{2b} = -344.3 \text{ N} \tag{4.40}$$

$$F_{z_2FRONT} = \frac{bW_F + hF_C}{2b} = 1006.46 \text{ N} \tag{4.41}$$

Assuming a kinetic friction coefficient $\mu_k = 0.6$, its found:

$$F_{y_1FRONT} = (\mu_s)(F_{z_1FRONT}) = (0.6)(-344.3N) = -206.58 \text{ N} \tag{4.42}$$

$$F_{y_2FRONT} = (\mu_s)(F_{z_2FRONT}) = (0.6)(1006.46N) = 603.88 \text{ N} \tag{4.43}$$

Similarly, calculating for the normal forces experienced by the rear wheels during a turn uses the same equations as the front. However, the normal forces experienced in the rear wheels correspond to the load on the rear axle (W_R) which is equal to the value for F_{z_2} from equation (4.12).

$$F_{z_1REAR} = \frac{bW_R - hF_C}{2b} = -278.07 \text{ N} \tag{4.44}$$

$$F_{z_2REAR} = \frac{bW_R + hF_C}{2b} = 1027.68 \text{ N} \tag{4.45}$$

Assuming a kinetic friction coefficient $\mu_k = 0.6$, friction forces can be found:

$$F_{y_1REAR} = (\mu_s)(F_{z_1REAR}) = (0.6)(-278.07N) = -166.84 \text{ N} \tag{4.46}$$

$$F_{y_2REAR} = (\mu_s)(F_{z_2REAR}) = (0.6)(1027.68N) = 643.61 \text{ N} \tag{4.47}$$

4.4.4 Braking

As per the Formula SAE rules and regulations, the brake pedal must be designed to withstand a force of 2000N without any failure of the brake system or pedal box. In this analysis, it is assumed that all parts are perfectly rigid, fluids are incompressible, and heat loss is minimal. By summing the moments about the pivot tube shown below, the force output can be found [10].

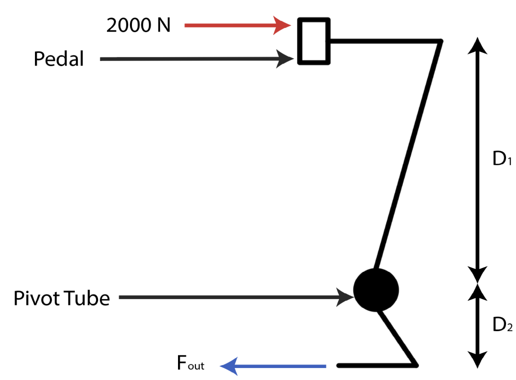

Figure 4.9: FBD of the maximum force exerted by the driver on the brake pedal

$$0 = M_{\text{out}} + M_{\text{in}} \tag{4.48}$$

$$F_{\text{out}}\, D_2 = -F_{\text{in}}\, D_1 \tag{4.49}$$

$$F_{\text{out}} = -2000N \left(\frac{0.2m}{0.05m}\right) = -8000N \tag{4.50}$$

A balance bar divides the force from the brake pedal to the two master cylinders. The torque on one side of the bar must balance the torque on the other side of the bar. By summing the moments, the force going into the master cylinders can be determined.

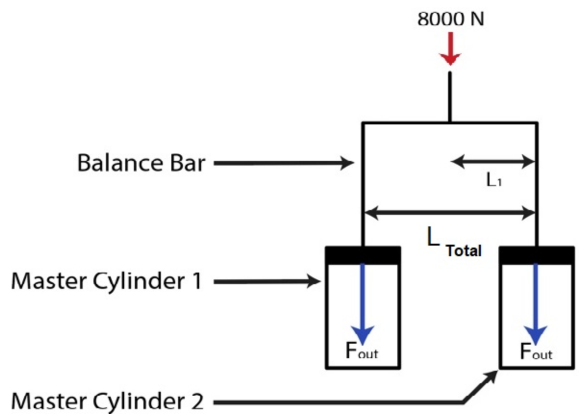

Figure 4.10: FBD of the balance bar and the two master brake cylinders

$$F_{\text{out}}\, L_{\text{total}} = F_{\text{in}}\, L_1 \tag{4.51}$$

$$F_{\text{out}} = 8000 \text{ N} \left(\frac{0.05 \text{ m}}{0.1 \text{ m}}\right) = 4000 \text{ N} \tag{4.52}$$

Each master cylinder is connected to two calipers. Using Pascal's Law, a relation is developed depending on the master cylinder bore size and the caliper's piston diameter to solve for the force acting on a single rotor. The master cylinder bore size was assumed to be 20mm and the diameter of the piston was assumed to be 40mm.

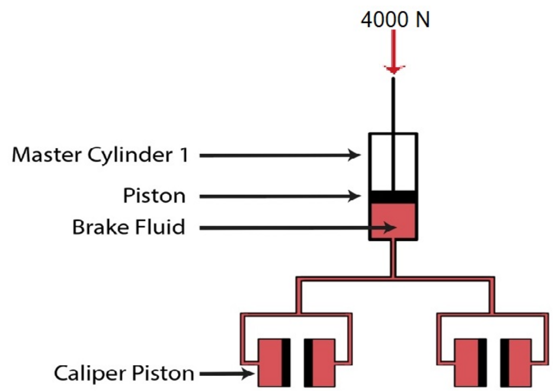

Figure 4.11: FBD of one master cylinder and the two calipers

For each individual caliper with two pistons, the total force at both pistons is:

$$F_{cal} = 2F_{cyl}\left(\frac{A_{cal}}{A_{cyl}}\right) = 2(4000N)\left(\frac{\pi(0.02m)^2}{\pi(0.01m)^2}\right) = 32000 \text{ N} \tag{4.53}$$

The caliper's pistons are pushed against the brake pads, where the actual braking force can be determined depending on the brake pads coefficient of friction. In this calculation, the brake pad coefficient of friction is assumed to be 0.45.

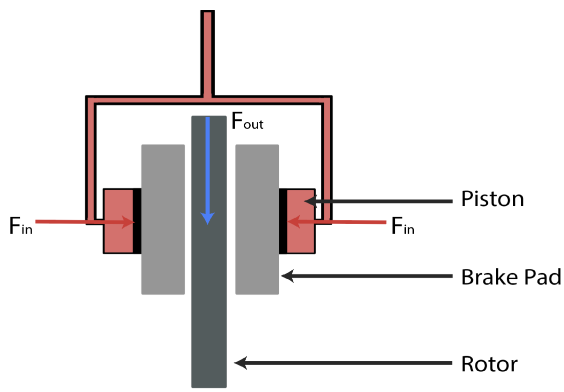

Figure 4.12: FBD of the caliper and rotor

$$F_{\text{out}} = \mu F_{\text{in}} = 0.45(32000 \text{ N}) = 14400 \text{ N} \tag{4.54}$$

The braking force is dependent on the radius of the rotor and wheel. A larger rotor radius will increase the brake torque produced by the brake system. It is assumed that the wheel has a radius (R) of 206mm and the rotor has a radius (r) of 124mm.

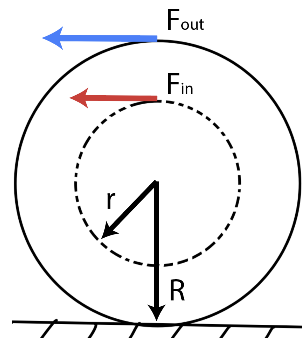

Figure 4.13: FBD of the wheel on the road

$$F_{\text{out}} = F_{\text{in}}\left(\frac{r}{R}\right) = 14400\text{ N}\left(\frac{124\text{ mm}}{206\text{ mm}}\right) = 8668\text{ N} \tag{4.55}$$

A braking force of 8668N is exerted on each wheel for a total braking force of 34672N on the vehicle. The analysis shows that a decrease in master cylinder size, increase in caliper piston area, or increase in the pedal ratio will ultimately increase the braking force on the wheels [11].

4.4.5 Impacts and Collisions

As per FSAE rules and regulations, the impact attenuator must decelerate the vehicle at a rate not exceeding 20g average. The energy absorbed must meet or exceed 7350J when the total mass of the vehicle is 300kg at an impact velocity of 7.0m/s [12].

$$a_{\text{avg}} = 20\left(9.81\text{ m/s}^2\right) = 196.2\text{ m/s}^2 \tag{4.56}$$

Solving for kinetic energy yields:

$$K_e = \frac{1}{2}mv_{\text{impact}}^2 = \frac{1}{2}(300\text{ kg})(7\text{ m/s})^2 = 7350\text{ J} \tag{4.57}$$

By conservation of energy, kinetic energy is equal to potential energy.

$$K_e = P_e \tag{4.58}$$

Solving for time of impact at a deceleration of 20g yields:

$$t = \frac{v_{\text{impact}}}{a_{\text{avg}}} = \frac{7\text{ m/s}}{196.2\text{ m/s}^2} = 0.036\text{ s} \tag{4.59}$$

Solving for impulse as the vehicle comes to a full stop from 7m/s yields:

$$I_m = m\left(v_{\text{impact}} - v_{\text{final}}\right) = 300\text{ kg}(7\text{ m/s} - 0\text{m/s}) = 2100\text{ N}\cdot\text{s} \tag{4.60}$$

Lastly, the impact force that the impact attenuator will be subjected to at 20g is:

$$F = \frac{I_m}{t} = \frac{2100\text{ N}\cdot\text{s}}{0.036\text{ s}} = 58,333\text{ N} \tag{4.61}$$

4.4.6 Slip Angle

The slip angle of a vehicle describes the ratio of transverse and radial velocities in the form of an angle. It is the angle in between the wheel actual direction of motion and the direction that it is pointing towards [13].

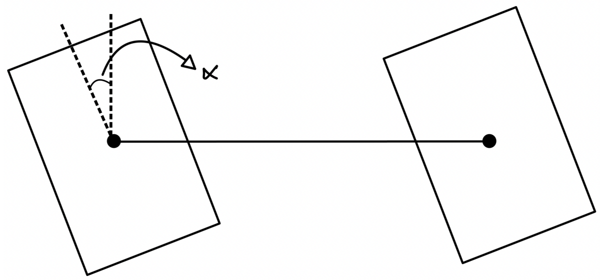

Figure 4.14: Slip Angle Representation

The slip angle is dependant on the load on the tire which differs from the rear and front tires. The slip angle can be mathematically modeled as the following equation [14].

$$\alpha = \frac{W \cdot v^2}{C_\alpha \cdot g \cdot R} \tag{4.62}$$

Where W is the load on the tire, v is the velocity of the tire, C is the tire stiffness coefficient which will be assumed to be 867 N/° [15], g is gravity, and R is the turning radius of the center of mass.

$$\alpha_{fr} = \frac{330.8N \cdot 29.2^2 m/s}{867N/^\circ \cdot 9.8m/s^2 \cdot 5.21m} = 6.37^\circ \tag{4.63}$$

$$\alpha_{re} = \frac{396.9N \cdot 29.2^2 m/s}{867N/^\circ \cdot 9.8m/s^2 \cdot 5.21m} = 7.64^\circ \tag{4.64}$$

4.4.7 Aerodynamics

4.4.7.1 Aerodynamic Effects at Max Speed

To analyse the aerodynamics effects of the vehicle appropriately, a model needs to be created for when the vehicle is at its top speed which represents the highest aerodynamic forces acting on the vehicle at any given point in time. A car's acceleration can be modeled according to newton's second law as:

$$m\ddot{x} = F - \frac{1}{2}\rho C_D A\dot{x}^2 \tag{4.65}$$

where F is the force propelling the vehicle forward, ρ is the density of air, C_D is the drag coefficient, and A is the frontal area of the vehicle. The driving force F can be expressed in terms of motor power and the car velocity where $v = \dot{x}$. This, can be simplified to when the vehicle is in equilibrium and is not accelerating to become [?]:

$$P = \frac{C_D A v^3}{1.633} \tag{4.66}$$

Now provided the motor used to estimate our battery calculations can produce a maximum of 100 kW peak power, and assuming transmission losses. Then the power at the wheels is assumed to be 84 kW. Moreover, assuming a drag coefficient $C_D = 0.63$ for a car with no aerodynamic packages, [16] and a frontal area of $A = 1.34m^2$. Then solving for the max speed of the vehicle one can find that the max velocity which the motor can produce to the tires is:

$$v = \sqrt[3]{\frac{1.633P}{C_D A}} = \sqrt[3]{\frac{1.633 \times 84000KW}{.63 \times 1.34m^2}} = 54.56m/s = 196.4km/hr \tag{4.67}$$

To determine the maximum drag coefficient desired whilst the vehicle is competing in its endurance races, a maximum top speed of $105km/hr$ can be assumed. Solving for the coefficient of drag:

$$C_D = \frac{1.633P}{Av^3} \Rightarrow C_D = \frac{1.633 \times 84000KW}{1.34m^2 \times (29.17m/s)^3} \Rightarrow C_D = 4.125 \tag{4.68}$$

Thus, the maximum allowable coefficient of drag is therefore 2.85. Which is a very high coefficient of drag, which indicates that the drag induced would not limit the top speed of the vehicle beyond its intended purpose.

4.4.7.2 Aerodynamic Effects on Cornering Performance

This section will analyse the aerodynamic effects induced on the tyres whilst cornering. This can be demonstrated by analysing the maximum allowed velocity a vehicle can corner at without losing its grip. Which is the velocity at which the fictional force is equal to the centripetal force [17].

$$\mu F_z = \mu(mg + \frac{1}{2}\rho C_L A v^2) = \frac{mv^2}{R} \tag{4.69}$$

$$v = \sqrt{\frac{\mu mg}{\frac{m}{R} - \frac{1}{2}\rho C_L A \mu}} \tag{4.70}$$

Where, g is the gravitational acceleration, μ is the coefficient of friction, ρ is the air density, C_L is the coefficient of lift, A is the frontal area, and R is the radius of the corner. According to the FSAE rules the corners will vary in radii from 4.5 to 30 m for dynamic events. Thus, assuming a lift coefficient when no aerodynamic packages are installed is $C_L = 0.29$, and the lift coefficient when aerodynamic packages are installed is $C_L = 2.34$ [16], and the density of air to be $\rho = 1.225$, assuming a constant coefficient of friction of $\mu = 0.65$, and provided the mass of the car is 298 kg, the frontal area $A = 1.34m^2$, then the following can be calculated from a 9-30 m turn radius [9]:

Table 4.1 illustrates the different speeds which can be achieved as the vehicle is cornering with an aerodynamic package installed, and without one. Clearly, higher speeds can be achieved whilst cornering when an aerodynamic package is installed, as there is more downforce acting on the vehicle.

Table 4.1: Cornering Speed differences with and without an Aerodynamic Package

Corner RadiusR (m)	With Aerodynamic Package	W/O Aerodynamic Package
	Velocity (km/hr)	Velocity (km/hr)
4.5	19.42	19.30
8	26.04	25.75
12	32.10	31.56
16	37.31	36.48
20	41.99	40.81
24	46.31	44.74
28	50.36	48.37
30	52.30	50.08

4.4.7.3 Load Distribution Model

This section serves the purpose of illustrating how different forces and moments act on the vehicle. It will also introduce how the reaction forces for the rear wheels can be calculated. The sum of forces and moments for the diagram illustrated in figure 4.15 are illustrated below [17]:

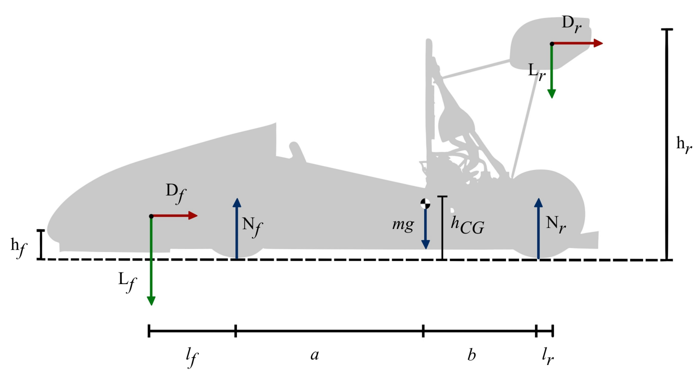

Figure 4.15: FBD for forces acting on the vehicle

$$\Sigma F = 0 \Rightarrow N_f + N_r - mg - L_F - L_r \tag{4.71}$$

$$\Sigma M_{CG} = 0 \Rightarrow bN_r + (a+l_f)L_F - (a)N_f - (b+l_r)L_r - (h_{CG}-h_r)D_r + (h_{CG}-h_f)D_f \tag{4.72}$$

The following can be determined after simplifying the equations of motion above:

$$N_r = \frac{(l_r + b)\, L_r - (l_f + a)\, L_f + (h_{CG} - h_r)\, \mathrm{D}_r - (h_{CG} - h_f)\, \mathrm{D}_f + amg + aL_f + aL_r}{a + b} \tag{4.73}$$

This equation gives the force on the rear tyres, which can be used to determine the load distribution of aerodynamic forces. The forces experienced on the rear tyres of the vehicle are therefore:

$$N_r = -0.363L_f + 1.1303L_r + 1493.825 + 229.455\mathrm{D}_r - 230.051\mathrm{D}_f \tag{4.74}$$

4.4.7.4 Lift Forces

The lift forces on the vehicle can easily be calculated. Assuming a lift force of -2.34 [16] with an aerodynamic package installed, and 0.29 [16] when no aerodynamic package is installed. Then, the lift force would be:

$$F_{L1} = 0.29 \times 1.34m^2 \times 1.225kg/m^2 \times \frac{(29.16m/s)^2}{2} = 202.38N \tag{4.75}$$

With an aerodynamic package, the lift force acts as a down-force.

$$F_{L2} = -2.34 \times 1.34m^2 \times 1.225kg/m^2 \times \frac{(29.16m/s)^2}{2} = -1633.05(N) \tag{4.76}$$

Force required to lift car off the ground.

$$F = 298kg \times 9.81m/s^2 = 2923.8N \tag{4.77}$$

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APPENDICES

Appendix A

Data Sheets

A.1 Sony VTC 6 Data Sheet summary

2. Cell Rating セル定格

Item 項目		Rating 定格	Note 備考
2.1 Rated Capacity 定格容量		3000mAh	Discharge at 0.2ItA,2.0Vcutoff after Standard Charge 0.2ItA, 2.0V 終止放電での容量規格値、充電は標準充電
2.2 Maximum Charge Voltage 最大充電電圧		4.25V	
2.3 Cut Off Voltage 放電終止下限電圧		2.0V	
2.4 Continuous Maximum Charge Current 連続最大充電電流		5.0A	Continuous 連続
		6.0A	Pulse パルス
2.5 Continuous Maximum Discharge Current 連続最大放電電流		30A	(With 80 deg temperature cut) (温度カット 80℃あり)
		15A	(Without 80 deg temperature cut) (温度カット 80℃なし)
2.6 Allowable Environment Temperature 使用雰囲気温度	Charge 充電	0～+60℃	Refer to the cell temperature spec of 2.8 for cell surface temperature. セル表面温度に関しては 2.8 のセル温度規格を参照のこと。
	Discharge 放電	-20～+60℃	
2.7 Weight 質量		46.6+/-1.5 g	

※ Cell condition at the shipment About 75% discharged. セル 出荷状態 約 75%放電

Figure A.1: Data Sheet for Sony VTC6 Battery Cell (1)

2.8 Cell Temperature Spec セル温度規格

2.8.1 Charge Conditions 充電条件

Temperature Range / Cell Surface Temperature Range 温度範囲 / セル表面温度				Upper Limited Charging Voltage 上限充電電圧	Maximum Charging Current 最大充電電流	Charging Current Recommendation 推奨充電電流
1	Low Charging Temperature Range 低温度域	0℃≦T＜10℃	A	4.25V	2.00A	1.00A
			B	4.15V	4.00A	2.00A
2	Standard Charging Temperature Range 標準温度域	10℃≦T≦45℃		4.25V	5.00A	3.00A
3	High Charging Temperature Range 高温度域	45℃＜T≦60℃		4.20V	5.00A	2.00A

At Low Charging Temperature range, condition A and B are both available.
Recommended condition is B.
低温度域は条件 A または条件 B のどちらかを選択できる。推奨は条件 B。

2.8.2 Discharge Conditions 放電条件

Discharge at cell surface temperature below 80℃.
セル表面温度が 80℃以下で放電をおこなうこと。

3. Cell Nominal Value セル公称値

Item 項目	Nominal 公称	Note 備考
3.1 Nominal Capacity 公称容量	3120mAh	Discharge at 0.2ItA,2.0Vcutoff after Standard Charge 0.2ItA, 2.0V終止放電での容量規格値、充電は標準充電
3.2 Nominal Voltage 公称電圧	3.6V	
3.3 Charge Voltage 充電電圧	4.20V	
3.4 Energy Density エネルギー密度	631Wh/l	

4. Shape/Dimension and Appearance 形状 / 寸法と外観

4.1 Shape/Dimension (Ref. P11 7. Outline) 形状 / 寸法 (参照:P11 7. 外形)

Diameter of crimp クリンプ部外形	18.35 +0.15 / -0.20mm
Diameter of trunk 胴部外形	18.35 +0.15 / -0.20mm (excluding wrinkle on the tube)
Total Height 総高	65.00 +/- 0.2mm

4.2Appearance 外観

It shall be free from any defects such as remarkable scratches, breaks, cracks, discoloration, leakage, or deformation. It shall be clean, and have equality and product value.
著しい傷、破損、ひび、変色、液漏れ、変形のないものとし、清潔、均一で製品価値を持つものとする。

Figure A.2: Data Sheet for Sony VTC6 Battery Cell (2)

A.2 Emrax 228 Data Sheet Summary

EMRAX INNOVATIVE E-MOTORS

User's Manual for Advanced Axial Flux Synchronous Motors and Generators

EMRAX 228 Technical Data Table (dynamometer test data)

Technical data / Type	EMRAX 228 High Voltage			EMRAX 228 Medium Voltage			EMRAX 228 Low Voltage		
Air cooled = AC Liquid cooled = LC Combined cooled = Air + Liquid cooled = CC	AC	LC	CC	AC	LC	CC	AC	LC	CC
Ingress protection	IP21	IP65	IP21	IP21	IP65	IP21	IP21	IP65	IP21
Cooling medium specification (Air Flow = AF; Inlet Water/glycol Flow = WF; Ambient Air = AA) If inlet WF temperature and/or AA temperature are lower, then continuous power is higher.	AF=20m/s ; AA=25°C	WF=8l/min at 50°C; AA=25°C	WF=8l/min at 50°C; AA=25°C	AF=20m/s ; AA=25°C	WF=8l/min at 50°C; AA=25°C	WF=8l/min at 50°C; AA=25°C	AF=20m/s ; AA=25°C	WF=8l/min at 50°C; AA=25°C	WF=8l/min at 50°C; AA=25°C
Weight [kg]	12,0	12,3	12,3	12,0	12,3	12,3	12,0	12,3	12,3
Diameter ø / width [mm]	228/86								
Maximal battery voltage [Vdc] and full load/no load RPM	670 Vdc (5300/6500 RPM)			470 Vdc (5170/6500 RPM)			130 Vdc (4400/5200 RPM)		
Peak motor power at max RPM (few min at cold start / few seconds at hot start) [kW]	100								
Continuous motor power (at 3000-5000 RPM) depends on the motor RPM [kW]	28 - 42	28 - 42	35 - 55	28 - 42	28 - 42	35 - 55	28 - 42	28 - 42	35 - 55
Maximal rotation speed [RPM]	5500 (6500 RPM peak for a few seconds)								
Maximal motor current (for 2 min if cooled as described in Manual) [Arms]	240			340			900		
Continuous motor current [Arms]	115			160			450		
Maximal motor torque (for a few seconds) [Nm]	240								
Continuous motor torque [Nm]	125								
Torque / motor current [Nm/1Aph rms]	1,1			0,75			0,27		
Maximal temperature of the copper windings in the stator and max. temperature of the magnets [°C]	120								
Motor efficiency [%]	92 – 98								
Internal phase resistance at 25 °C [mΩ]	18			8,0			1,12		
Input phase wire cross-section [mm²]	10,2			15,2			38		
Wire connection	star								
Induction in Ld/Lq [µH]	177/183			76/79			10,3/10,6		
Controller / motor signal	sine wave								
AC voltage between two phases [Vrms/1RPM]	0,0730			0,0478			0,0176		
Specific idle speed (no load RPM) [RPM/1Vdc]	9,8			14			40		
Specific load speed (depends on the controller settings) [RPM/1Vdc]	8 – 9,8			11 – 14			34 – 40		
Magnetic field weakening (for higher RPM at the same power and lower torque) [%]	up to 100								
Magnetic flux – axial [Vs]	0,0542			0,0355			0,0131		
Temperature sensor in the motor	kty 81/210								
Number of pole pairs	10								
Rotor inertia (mass dia=175mm, m=5,5kg) [kg*cm²]	421								
Bearings (front:back) - SKF/FAG	6206:6206 (for radial forces) or 6206:7206 (for axial-radial forces; for pull mode; e.g. for air propeller) or 6206:3206 (for axial-radial forces; for pull-push mode; »O« orientation, α=25°); other bearings are possible (exceptionally)								

Figure A.3: Data Sheet for Emrax 228 Motor

Appendix B

Additional material

B.1 Accumulator Calculations

This section illustrates all the calculations used to model the battery and to estimate its parameters.

The battery can be designed accordingly provided the following FSAE restrictions for the accumulator.

Table B.1: FSAE Accumulator Restrictions and Guidelines

Accumulator Design Restrictions and Guidelines	
EV.4.1.1	"The maximum power drawn from the accumulator must not exceed 80 kW"
EV.4.1.2	"The maximum permitted voltage that may occur between any two points must not exceed 600 V DC"
EV.6.1.2	"Maximum static voltage of less than 120 V DC" "Maximum energy of 6 MJ"
F.10.3.4	Cells and Segments "The cells and/or segments must be appropriately secured against moving inside the Container" "This mounting system design must withstand the following accelerations: 40 g in the longitudinal direction (forward/aft) 40 g in the lateral direction (left/right) 20 g in the vertical direction (up/down)"

Firstly, assuming that the Voltage that the motor will require is 500 Vdc. Indicates that the required voltage should be within this range. Secondly, from previous FSAE electric vehicle designs [18] [18] [19] the required battery capacity can be assumed to be 6kWh.

Given these starting parameters the number of required cells can be calculated , as well as the amount of Ah needed. Therefore, for a 500 Vdc battery 12 Ah 's are required to have a 6kWh battery.

$$\frac{6000\,(Wh)}{500\,(V)} = 12\,(Ah) \tag{B.1}$$

Next, the cell configuration must be determined.

Provided the nominal voltage of a single cell is 3.6 V and 3 Ah, as seen in table 2.3. The number of cells to be connected in series that will provide 500 Vdc would be the voltage required divided by the voltage produced by a single cell; and the number of cells to be connected in parallel to provide the required capacity would be the capacity required divided by the capacity of a single cell.

$$\frac{500\,(V)}{3.6\,(V)} = 139\,(cells) \tag{B.2}$$

$$\frac{12\,(Ah)}{3\,(Ah)} = 4\,(cells) \tag{B.3}$$

For the sake of design consistency, 140 cells will be connected in series rather than the 139 cells. Each cell will have to be connected to 4 cells in parallel to achieve the required battery capacity of 6 kWh.

Hence, the total number of cells required for the battery is:

$$140 * 4 = 560\,(cells) \tag{B.4}$$

Each cell section configurations will now be modelled according to the FSAE restrictions which state: "A Maximum energy of 6 MJ per section" and "No more than 12 kg per section"

$$6\,MJ = 1666\,Wh. \tag{B.5}$$

Assuming, the battery is divided into 5 sections and each section is connected in series, to produce the 504 V required. It can be determined that each section will have 28 cells connected in series.

$$\frac{504\,(V)}{3.6\,(V)} = 28\,(cells) \tag{B.6}$$

$$28 * 4 = 112\,(cells) \tag{B.7}$$

Therefore, each section has 112 cells. To verify that each section satisfies the 6 MJ energy constriction, the peak voltage and peak current discharge of the cells are taken and multiplied to find the energy produced per section.

$$4.2\,(V) * 28\,(cells) * 3\,(Ah) * 4\,(cells) = 1411.2\,(Wh) \leq 1666.7\,(Wh) \tag{B.8}$$

To prove that the weight restrictions per section is achieved, the total number of cells per section/section can be multiplied by the mass of a single cell from 2.3.

$$112\,(cells) * .048\,(kg) = 5.376\,(kg) \tag{B.9}$$

Finally, given the restriction: "The maximum power drawn from the accumulator must not exceed 80 kW" can be verified using the values obtained from the cell specification sheet from table 2.3. Each cell has a peak voltage of 4.2 V, and peak current 30 A.

Thus,

$$140 * 4.2\,(V) * 30 * 4\,(A) = 70.56\,(kW) \leq 80\,(kW) \tag{B.10}$$

Therefore, with all the electrical restrictions cleared, this battery model can be utilized as a sample for our design with its final properties illustrated in table 2.4

B.2 Center of Mass Calculations

$$\text{COM}_{\text{ x-coordinate }} = \frac{\begin{gathered}(51.2kg \times 866.5mm) + (10.7kg \times 2150mm) \\ + (16.7kg \times 550mm) + (9.8kg \times 2150mm) \\ + (9.8kg \times 2150mm) + (13.8kg \times 550mm) \\ + (13.8kg \times 550mm) + (11.8kg \times 2000mm) \\ + (116.71kg \times 1450mm) + (39kg \times 1400mm)\end{gathered}}{293.5kg} = 1299.7mm \tag{B.11}$$

$$\text{COM}_{\text{ y-coordinate }} = \frac{\begin{gathered}(51.2kg \times 207mm) + (10.7kg \times 375mm) \\ + (16.7kg \times 550mm) + (9.8kg \times 206mm) \\ + (9.8kg \times 206mm) + (13.8kg \times 206mm) \\ + (13.8kg \times 206mm) + (11.8kg \times 415mm) \\ + (116.71kg \times 425mm) + (39kg \times 600mm)\end{gathered}}{293.5kg} = 379.7mm \tag{B.12}$$

$$\text{COM}_{\text{ z-coordinate }} = \frac{\begin{gathered}(51.2kg \times 0mm) + (10.7kg \times 0mm) \\ + (16.7kg \times 0mm) + (9.8kg \times 625mm) \\ + (9.8kg \times -625mm) + (13.8kg \times 625mm) \\ + (13.8kg \times -625mm) + (11.8kg \times 0mm) \\ + (116.71kg \times 0mm) + (39kg \times 0mm)\end{gathered}}{293.5kg} = 0mm \tag{B.13}$$

B.3 Nomenclature

Table B.2: Nomenclature for Sections 4.1 and 4.2

Section 4.1 & 4.2	
Variable	**Definition**
x	Position of the vehicle along the x-axis
y	Position of the vehicle along the y-axis
θ	Orientation of the car relative to x-axis
ϕ	Steering Angle
$\dot{x}$	Velocity component along the x-axis
$\dot{y}$	Velocity component along the y-axis
$\dot{\theta}$	Angular velocity of the vehicle
v	Velocity of the vehicle
C	Center of Mass
R_1	Turning radius for the center of mass
ϕ_i	Inner Steering Angle of front wheel
ϕ_o	Outer Steering Angle of front wheel
T	Vehicle's track width
L	Vehicle's wheel base
a	Distance from rear axle to C of M

Table B.3: Nomenclature for Aerodynamic Section

Section 4.5	
Variable	**Definition**
ρ	Density of Air
C_D	Coefficient of Drag
C_L	Coefficient of Lift
A	Frontal Area
v	Velocity of Vehicle
$\ddot{x}$	Acceleration along the x direction
F	Propulsion force of the Vehicle
μ	Coefficient of Friction
m	Mass of the Vehicle
R	Corner Radius
g	Gravitational Acceleration
N_f	Normal Force on front tyres
N_r	Normal Force on rear tyres
F_L	Lift Force
D_f	Drag force acting on front
D_r	Drag force acting on rear
l_f	Distance from front axle to front wing
l_r	Distance from rear axle to rear wing
a	Distance from center of gravity to front axle
b	Distance from center of gravity to rear axle
h_f	Height from ground to front wing
h_r	Height from ground to rear wing
h_{CG}	Height from ground to center of gravity

Table B.4: Nomenclature for Applied, Static and Dynamic Force Sections

Symbols for Sections 4.3, 4.4.1, 4.4.2, and 4.4.3	
m	Mass
g	Gravity
a	Acceleration
a_1	Distance in the x-direction from the car's center of mass to the front axle
a_2	Distance in the x-direction from the car's center of mass to the rear axle
b	Distance in the y-direction from the car's center of mass to the middle of a wheel on the side of an axle
h	Distance in the z-direction from the car's center of mass to the surface of the road
θ	Bank angle of the road from the horizontal
ϕ	Angle of inclination of the road from the horizontal
W_F	Weight acting on the front axle
W_R	Weight acting on the rear axle
μ_s	Coefficient of static friction
μ_k	Coefficient of kinematic friction
F_C	Centripetal Force
v	Velocity
R	Radius of curvature of road

Table B.5: Nomenclature for Slip Angles

Nomenclature for Slip Angles	
α	Slip Angle
C_α	Tire Stiffness
g	Gravitational acceleration
R	Turning Radius of the C of M of the vehicle
v	Velocity
W_r	Force acting on the axles front and rear

Table B.6: Nomenclature for Braking

4.4.4 braking	
M_{in}	Input moment
M_{out}	Output moment
F_{in}	Input force
F_{out}	Output force
D_1	Distance from pivot tube to brake pedal
D_2	Distance from pivot tube to balance bar
L_1	Horizontal distance from master cylinder piston to the center of the balance bar
L_{Total}	horizontal distance between the two master cylinder pistons
F_{cal}	Force exerted on both caliper pistons
F_{cyl}	Force exerted on master cylinder
A_{cal}	Area of caliper piston
A_{cyl}	Area of master cylinder piston
μ	Coefficient of friction
r	Radius of rotor
R	Radius of wheel

Table B.7: Nomenclature for Impact Forces

4.4.5 Impacts and Collisions	
a_{avg}	average deceleration
$K(e)$	kinetic energy
$P(e)$	potential energy
t	time of impact
$v(impact)$	impact velocity
m	mass
g	gravitational acceleration
$v(final)$	final velocity
$l(m)$	impulse
F	impact force