# 1] Trigonometría

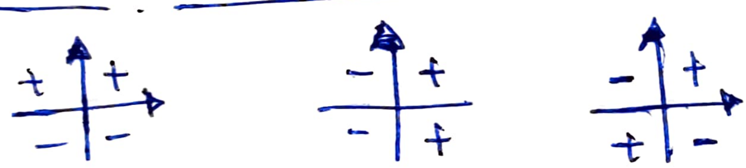
$360° = 2\pi$ radianes

- $\operatorname{sen}\alpha = \dfrac{c.op}{hipo}$ 
- $\cos\alpha = \dfrac{c.ad}{hipo}$ 
- $\tan = \dfrac{c.op}{c.ad}$

Recíprocas

- $\csc = \dfrac{hipo}{c.op}$ 
- $\sec = \dfrac{hipo}{c.ad}$ 
- $\cot = \dfrac{c.ad}{c.op}$

| | 0° | 30° | 45° | 60° | 90° |
|---|---|---|---|---|---|
| sen | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| cos | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| tan | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | $\infty$ |




- sen y csc
- cos y sec
- tan y cot

Metodología

1. cuadrante
2. Ángulo en el cuadrante
3. Signo del cuadrante
4. Valor númerico

$\beta = \_ + \_$

Identidades trigonométricas

- $\cos^2\alpha + \operatorname{sen}^2\alpha = 1$
- $\sec^2\alpha - \tan\alpha = 1$
- $\csc^2\alpha - \cot\alpha = 1$

$\operatorname{sen}(\alpha \pm \beta) = \operatorname{sen}\alpha\cos\beta \pm \cos\alpha\operatorname{sen}\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$$

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Ley senos | $\frac{\alpha}{\operatorname{sen}\alpha} = \frac{b}{\operatorname{sen}\beta} = \frac{c}{\operatorname{sen}\gamma}$ | Piden ángulo

Ley cosenos | $a^2 = b^2 + c^2 - 2bc\cos\alpha$ | 1 ángulo y lado

$a$ y $\alpha$ contrarios

Piden lado

# ...meros Reales

**SUMA**

$n=1 \quad 1+3+5+7+(2n-1)=n^2$

$n=K \quad 1+3+5+7+\dots 2K-1=K^2$ (A) Hipotesis

$n=K+1 \quad 1+3+5+7+\dots \underline{2K+1}=(K+1)^2$ (B) Tesis

sostituir (sumar B (Tesis) en (A)) para = Hipotesis modificada

$K^2+2K+1=(K+1)^2$

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**Multiplicación**

$n=1 \quad \left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\dots\left(1+\frac{1}{n}\right)=n+1$

$n=K \quad \left(1+\frac{1}{K}\right)=K+1$ (A)

$n=K+1 \quad \underline{\left(1+\frac{1}{K+1}\right)}=K+2$ (B)

Multiplicar $\left(1+\frac{1}{K+1}\right)(K+1)$ (C) e igualar en (B)

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**División**

$6^n-1$ divisible entre 5

$n=1 \quad \frac{6^1-1}{5}=1$ que sea divisible

$n=K \quad \frac{6^K-1}{5}=\alpha \; ; \; 6^K-1=5\alpha$ (A)

$n=K+1 \quad \frac{6^{K+1}-1}{5}=\beta \; ; \; 6^{K+1}-1=5\beta$ (B)

(A) ~~igualar~~ pasar a B

$6(6^K-1=5\alpha) \; ; \; 6^{K+1}-6=6(5\alpha)$

$6^{K+1}-6+5=6(5\alpha)+5$ Regla correspon

$\therefore 6^{K+1}-1=5(6\alpha+1) \quad \beta=(6\alpha+1)$

$K=1$ en (A) $\frac{6^1-1}{5}=\alpha=1$ (B) $\frac{6^2-1}{5}=\beta=7$

# Racionales

$\frac{a}{b} = 1.08333\ldots$

$100\frac{a}{b} = 108.333$

$1000\frac{a}{b} = 1083.333$

$900\frac{a}{b} = 1083.333 - 108.333 = 975$

$\frac{a}{b} = \frac{975}{900} \xrightarrow{\div 5} \frac{195}{180} \xrightarrow{\div 5} \frac{39}{36} \xrightarrow{\div 3} \frac{13}{12}$

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- $3x+5 > x-15$

$3x - x > -15-5$

$2x > -20$

$x > -10$

$20 > -2x$

$\frac{20}{-2} < x$

$-10 < x$

se invierte el signo

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- caso$_1$ $(+)(+)$, caso$_2$ $(-)(-)$ } si es mayor que $>$

caso$_1$ $(+)(-)$, caso$_2$ $(-)(+)$ } si es menor que $<$

$\frac{x+16}{x-4} > 0$

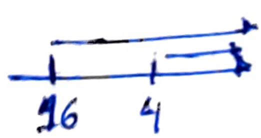
$\frac{x+5}{x-2} < 2$

- $\left|\frac{x-4}{2x+1}\right| < 1 = |x-4| < |2x+1|$

$\therefore \frac{x-4}{2x+1} < 1$ y $\frac{x-4}{2x+1} > -2$ } cambiamos signo y valor numérico

$C_1$ $(+)$ $x+16 > 0$ $\quad x > -16$

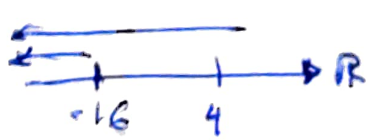
$(+)$ $x-4 > 0$ $\quad x > 4$




$(4, \infty)$

$C_2$ $(-)$ $x+16 < 0$ $\quad x < -16$

$(-)$ $x-4 < 0$ $\quad x < 4$




$(\infty, -16)$

$\therefore x \in R \quad (\infty, -16) \cup (4, \infty)$

# Números Complejos

**▶ Binómica** $z = a + bi$

$$\left| \begin{array}{l} a = r\cos\theta \\ b = r\,\text{sen}\,\theta\, i \end{array} \right.$$

$$\left| \begin{array}{l} r = \sqrt{a^2 + b^2} \\ \theta = \text{angtan}\left(\frac{b}{a}\right) \end{array} \right.$$

**▶ Polar** $z = r \operatorname{cis}\theta$

$0° < \theta \leq 360°$

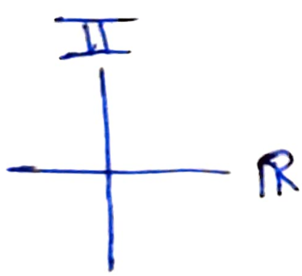
$$\left| \begin{array}{l} z = r_1 r_2 \operatorname{cis}(\alpha + \beta) \\ z = \frac{r_1}{r_2} \operatorname{cis}(\alpha - \beta) \\ z^n = r^n \operatorname{cis}(n\theta) \end{array} \right.$$

$$\sqrt[n]{z} = \sqrt[n]{r}\, \operatorname{cis}\left[\frac{\theta}{n} + \frac{k \cdot 360}{n}\right]$$

**▶ Exponencial** $z = r e^{i}$

$$\left| \begin{array}{l} z = r_1 r_2\, e^{(\alpha + \beta) i} \\ z = \frac{r_1}{r_2}\, e^{(\alpha - \beta) i} \\ z = r_1^{\,n}\, e^{(n\alpha) i} \end{array} \right.$$

$$\sqrt[n]{z} = \sqrt[n]{r_1}\; e^{\left[\frac{\alpha + k2\pi}{n}\right] i}$$

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$4z = 2\bar{z} + 8 \operatorname{cis} 210°$

$4z = 2\bar{z} - 4\sqrt{3} - 4i$

$4(a + bi) = 2(a - bi) - 4\sqrt{3} - 4i$

$4a + 4bi = 2a - 2bi - 4\sqrt{3} - 4i$

$2a + 6bi = -4\sqrt{3} - 4i \quad \left| \begin{array}{l} 2a = -4\sqrt{3} \;;\; a = -2\sqrt{3} \\ 6b\cancel{i} = -4\cancel{i} \;;\; b = -\frac{2}{3} \end{array} \right.$

Poten

$i^2 = -1$ $\quad * (-1)^{Par} = -1$

$i^3 = -i$ $\quad * (-i)^{Impar} = -1$

$i^{78} = (i^2)^{39} = (-1)^{39} = -1$

$i^{59} = (i^2)^{27} i = (-1) i = -i$

# Polinomios

cuando residuo = 0 es un factor

Factor $(x-\alpha)$ / Raiz $\alpha$ ← solo el número

- Posibles raices (reales) racionales

$a_0 \; . \; . \; . \; . \; a_n \rightarrow 2 \; . \; . \; . \; -3$

$a_0 = -2$ ; Factores $\pm(1 \;\; 2)$

$a_n = 3$ ; $\pm(1, 3)$

- combinaciones $\pm\left(1, \frac{1}{2}, \frac{3}{2}, 3\right)$

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$$F(x) = x^4 + x^3 - \frac{A}{2}x^2 - x + 6$$

P.R.R $\pm(1, 2, 3, 6)$

$\therefore$

| | | | | | |
|---|---|---|---|---|---|
| | 1 | 1 | $-\frac{A}{2}$ | $-1$ | $+6$ |
| 2 | | 2 | 6 | $12-A$ | $22-2A$ |
| | 1 | 3 | $6-\frac{A}{2}$ | $11-A$ | $\boxed{28-2A}$ |

$\therefore 28-2A = 0$ ; $A = 14$

continuamos evaluando hasta encontrar con el total de raices

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REGLA DE SIGNOS | Tabla de descartes

$P(x) = x^4 + x^3 - 7x^2 - x + 6$ R+ · 2

$P(-x) = x^4 - x^3 - 7x^2 + x + 6$ R- 2

| | | | | |
|---|---|---|---|---|
| R+ | 2 | 2 | 0 | 0 |
| R- | 2 | 0 | 2 | 0 |
| C | 0 | 2 | 2 | 4 |
| | 4 | 4 | 4 | 4 |

← Pares (conjugados)

* Pueden ser irracionales las raices

$p(x) = x^3 + Ax^2 + Bx + C$

$a_1 = 3 \quad a_2 = -1 \}$ raices

$p(0) = 6$

$\therefore p(x) = x^3 + Ax^2 + Bx + 6 \qquad \underline{c = 6}$

P.r.r. $\pm(1, 2, 3, 6)$

$$\begin{array}{r|rrrl} -1 & 1 & A & B & 6 \\ & & -1 & -A+1 & -B+A-1 \\ \hline & 1 & A-1 & B-A+1 & \boxed{-B+A+5} = 0 \end{array}$$

<u>resolver:</u> <u>sistema de</u> <u>ecuaciones</u> / <u>Alterna</u>

$p(x) = (x-3)(x+1)(x-\alpha)$

$p(0) = (-3)(+1)(-\alpha) = 3\alpha = 6 \; ; \alpha = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# Sistemas de ecuaciones

- Compatible determinado $\{$ Gauss Jordan
- Compatible indeterminado $\{$ un renglón igual todo 0
- Indeterminado $\{$ un renglón 0 pero igualado a algo

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- Compatible indeterminado | Solución general

① Escalonar

$$-4c+7d+7e=1$$

$$\blacktriangleright -4c=1-7d-7e;\ c=-\frac{1}{4}+\frac{7}{4}d+\frac{7}{4}e$$

$$\therefore\ a+b+\left[-\frac{1}{4}+\frac{7}{4}d+\frac{7}{4}e\right]+2d-e=1$$

$$\blacktriangleright a=-b+\frac{1}{4}d-\frac{3}{4}e+\frac{5}{4}$$

$$\therefore\ b=b,\ d=d,\ e=e$$

② El total de renglones = total de polinomios

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Solución Particular | $b=d=e=0$

$$\therefore\ a=\frac{5}{4},\quad c=-\frac{1}{4}$$

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$$\begin{aligned} x+3y+2z&=7\\ x+4y+(K+2)z&=10\\ x+(K+3)y+6z&=13 \end{aligned}$$

$$M_1=\left[\begin{array}{ccc|c} 1 & 3 & 2 & 7\\ 1 & 4 & K+2 & 10\\ 1 & K+3 & 6 & 13 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 7\\ 0 & 1 & K & 3\\ 0 & K & 4 & 6 \end{array}\right]\sim\left[\begin{array}{cccc} 1 & 3 & 2 & 7\\ 0 & 1 & K & 3\\ 0 & 0 & -K^2+4 & -3K+6 \end{array}\right]$$

$-k^2+4 = k^2-4 = (k-2)(k+2)$

$-3k+$

$3(k+2)$

- Compatible determinado $k \neq 2, -2$

- Si $k = -2$

$(-2-2)(-2+2)$ $\quad 3(-2+2)$ | compatible indeterminado $k = -2$

$0 \quad 0$

- Si $k = 2$

$(2-2)(2+2)$ $\quad 3(+2+2)$ | incompatible $k = 2$

$0 \quad 12$

# Matrices y Determinantes

Multiplicación | $A = \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}_{3\times 4}$ $B = \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}_{4\times 3}$ $AB = [\ ]_{3\times 3}$

* Asociatividad es posible

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Matriz identidad | $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$ | singular - no tiene inversa

Matriz inversa | $A A^{-1} = I_n$

* $\frac{1}{\lambda} A^{-1}$

determinante (escalar)

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Ecuaciones Matriciales

$Ax + B = 3x$ ; $Ax - 3x = -B$
$(A-3)(x) = -B$ ; $x = (A - 3I)^{-1} \cdot B$ * Respeta el orden

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Sistema ecuaciones | Ecuación Matricial

$A =$ Matriz coeficientes $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B =$ Terminos independientes

$x = [A^{-1}]_{3\times\cancel{3}} [B]_{\cancel{3}\times 1}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = [\ ]_{3\times 1}$

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Matriz inversa Cofactores

$A^{-1} = \left(\frac{1}{\det A}\right)(\text{Adj } A)$

$\text{Adj } A = \begin{bmatrix} \text{Matriz transpuesta de cofactores} \end{bmatrix}$

$\det A =$ Renglon $= 1C_{11} + 2C_{13}$

**Traza de una matriz**

$A = \begin{bmatrix} 1 & 2 & 3 \\ 9 & 4 & 1 \\ 2 & 0 & 1 \end{bmatrix}$ $Tr(A) = 6$

**Matriz triangular** | superior — 0 abajo
inferior — 0 arriba

si A es triangular

2] $\lambda A$ = se conserva el tipo

3] $AB$ = El mismo tipo

**Matriz Diagonal** | $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $dig(A) = [1, 2, 3]$.

**Matriz escalar** | $\alpha A = \alpha \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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**Transposición** { ▶ **Simétrica** { $\begin{bmatrix} 1 & -3 & -4 \\ -3 & 2 & -3 \\ -4 & -3 & 3 \end{bmatrix} = A^T$

▶ **Antisimétrica** { $\begin{bmatrix} 0 & -3 & -2 \\ 3 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix} = -A^T$

Antisimétrica { 1] Triangulo inverso 2] Diagonal 0

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**Conjugación** { $\bar{A}$ (real) ⇒ La misma
$-\bar{A}$ (imaginario) $\begin{bmatrix} i & -i \\ 2 & -i+2 \end{bmatrix}$ $-\bar{A} = \begin{bmatrix} -i & +i \\ 2 & +i+2 \end{bmatrix}$

* solo se conjugan los imaginarios

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**conjugación transposición** $A^* = (\bar{A})^T = (\overline{A^T})$

↳ Hermitiana { ① Diagonal real ② Triangulos conjugados } $A^*$

↳ Antihermitiana { ① Diagonal compleja ② Conjugados negativos } $-A^*$