4.2. Representación gráfica para el caso de funciones de dos variables independientes. Curvas de nivel.

Definición

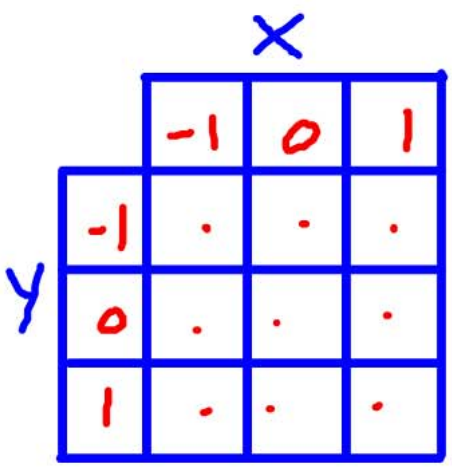
Sea la función escalar de variable vectorial

$f : S \subset \mathbb{R}^n \to \mathbb{R}$, se define la gráfica de f como el conjunto

$$\left\{(x_1, x_2, \ldots, x_n, w) \in \mathbb{R}^{n+1} \;\middle|\; (x_1, x_2, \ldots, x_n) \in S,\, w = f(x_1, x_2, \ldots, x_n)\right\}$$

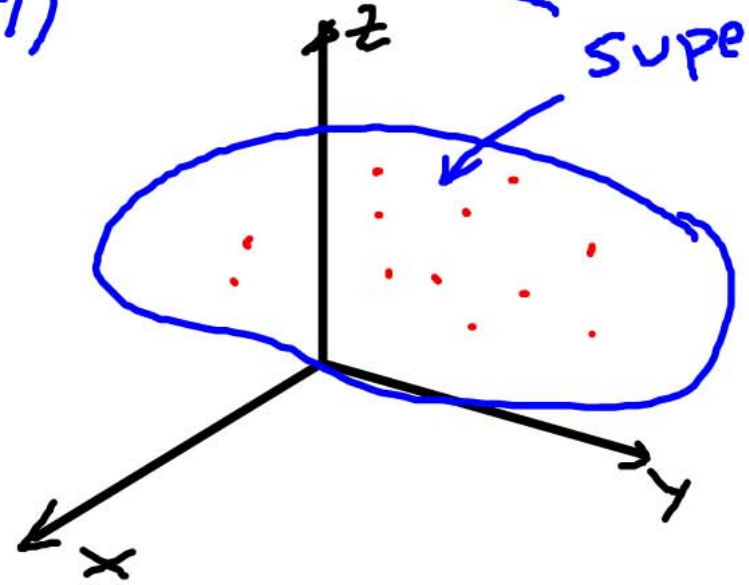
$$n = 2, \quad f : S \subset \mathbb{R}^2 \to \mathbb{R}$$

$$\text{gráfica de } f = \left\{(x, y, z) \in \mathbb{R}^3 \;\middle|\; (x, y) \in S,, z = f(x, y)\right\}$$



$z = f(x, y)$

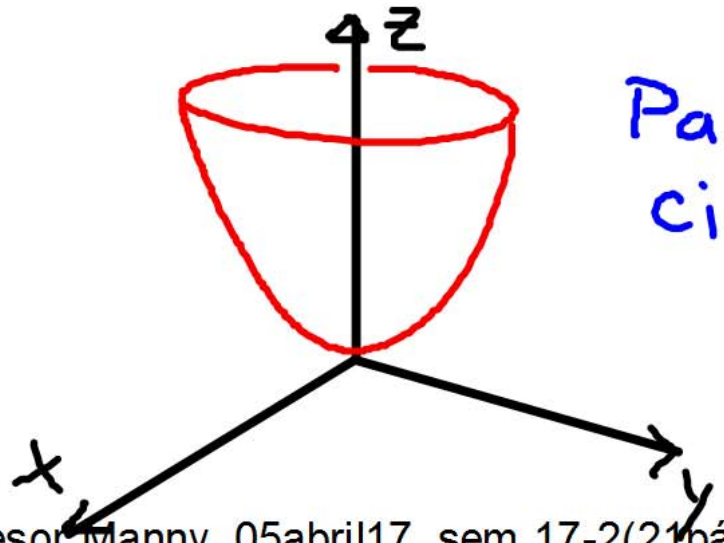
superficie



Ejemplo:

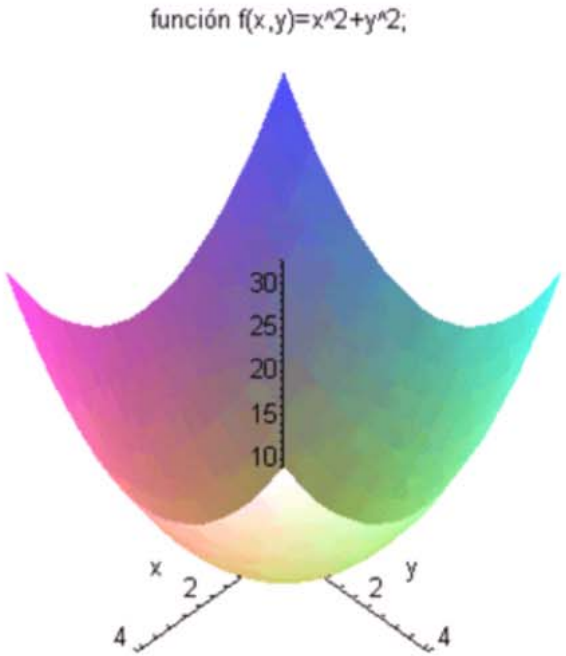
$$z = x^2 + y^2$$

$z = f(x, y)$



Paraboloide circular

```
> restart;
  plot3d(x^2+y^2,x=-4..4,y=-4..4,style=patchnogrid,title="funci
  ón f(x, y) = x^2 + y^2");
```



Definición. Nivel constante de una función.

Sea la función $f : S \subset \mathbb{R}^n \to \mathbb{R}$ y el número k en el recorrido de f, se define el nivel k de la función como el conjunto de puntos de S que f manda a k. Es decir

$$\left\{ (x_1, x_2, \ldots, x_n) \in S \;\middle|\; f(x_1, x_2, \ldots, x_n) = k \right\}$$

$$n = 2, \quad f : S \subset \mathbb{R}^2 \to \mathbb{R}$$

$$\{(x, y) \in S \mid f(x, y) = k\}$$ – Curvas de nivel

Ejemplo:

$$f(x, y) = x^2 + y^2 = k \quad , \quad k \in R \quad , \quad R = [0, \infty)$$

$x^2 + y^2 = k$

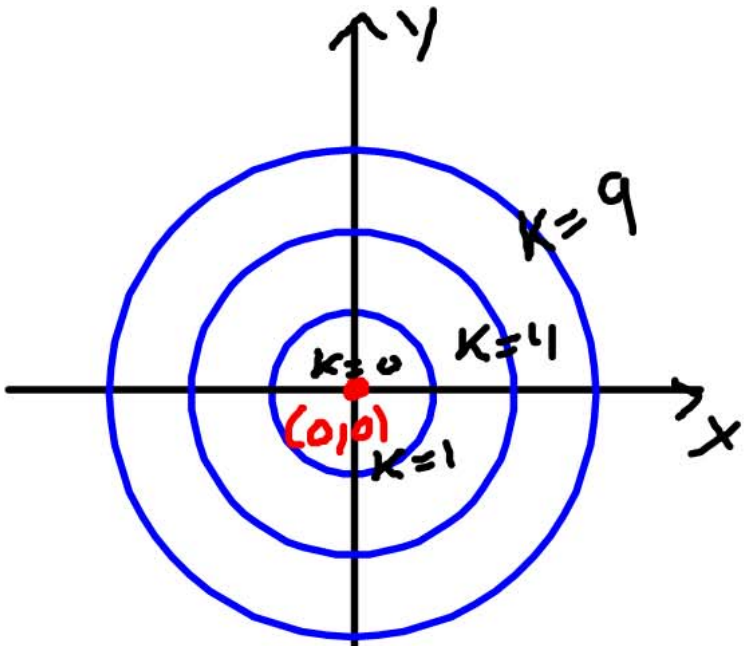
$k = 0$, $x^2 + y^2 = 0$ Punto: $(0, 0)$

$k = 1$, $x^2 + y^2 = 1$

$k = 4$, $x^2 + y^2 = 4$

$k = 9$, $x^2 + y^2 = 9$

} Curvas de nivel



Mapa de contorno

$z = x^2 + y^2 = k$

$z = k$, $k = \text{const}$

$k=0$, $z=0$

$k=1$, $z=1$

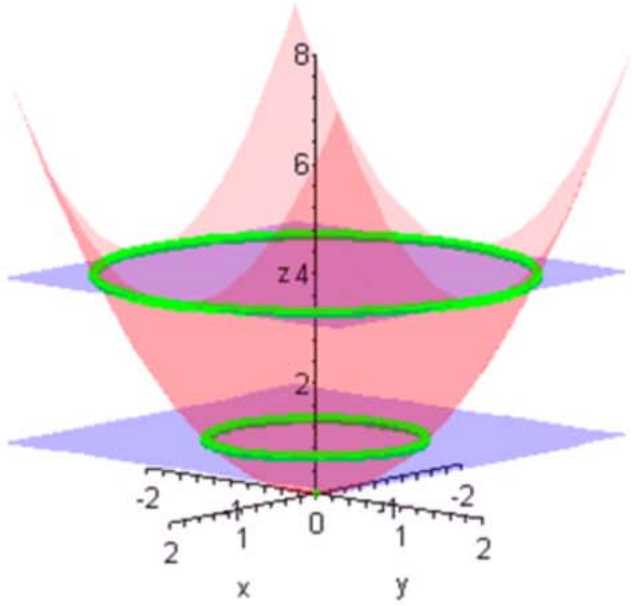
$k=4$, $z=4$

$k=9$, $z=9$

} Planos horizontales.

```
> restart;
  with(Student[MultivariateCalculus]):
  CrossSection(x^2+y^2,z=[0,1,4],x=-2..2,y=-2..2,
  title="función f(x,y)=x^2+y^2 y los planos z=0, z=1, z=4");
```

función f(x,y)=x^2+y^2; y los planos z=0, z=1, z=4



$$n = 3, \quad f : S \subset \mathbb{R}^3 \to \mathbb{R}$$

$$\{(x, y, z) \in S \mid f(x, y, z) = k\}$$ – Superficies de nivel

Ejemplo:

$$f(x, y, z) = x^2 + y^2 + z^2 = k \;,\; k \in R \;,\; R = [0, \infty)$$

$$x^2 + y^2 + z^2 = k$$

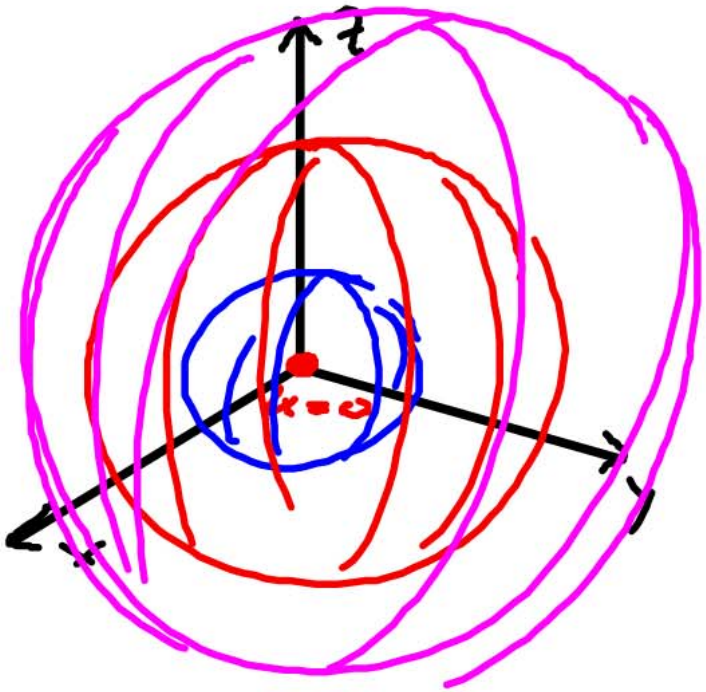
$k = 0$, $x^2 + y^2 + z^2 = 0$ Punto: $(0, 0, 0)$

$k = 1$, $x^2 + y^2 + z^2 = 1$

$k = 4$, $x^2 + y^2 + z^2 = 4$

$k = 9$, $x^2 + y^2 + z^2 = 9$

} Superficies de nivel

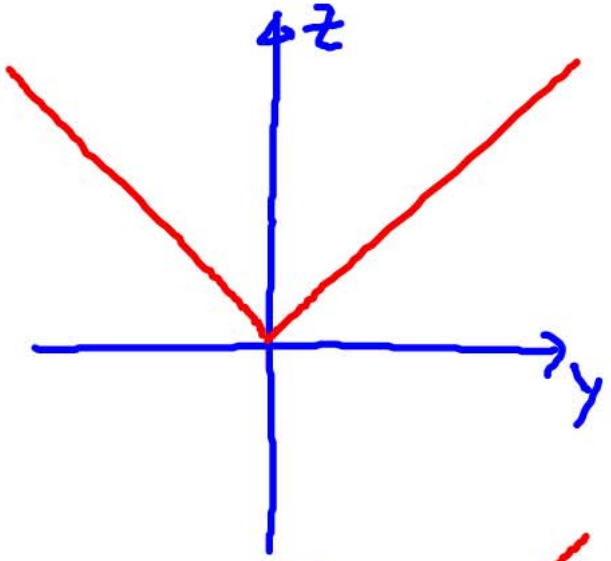

z
x = c
y

Ejemplos:

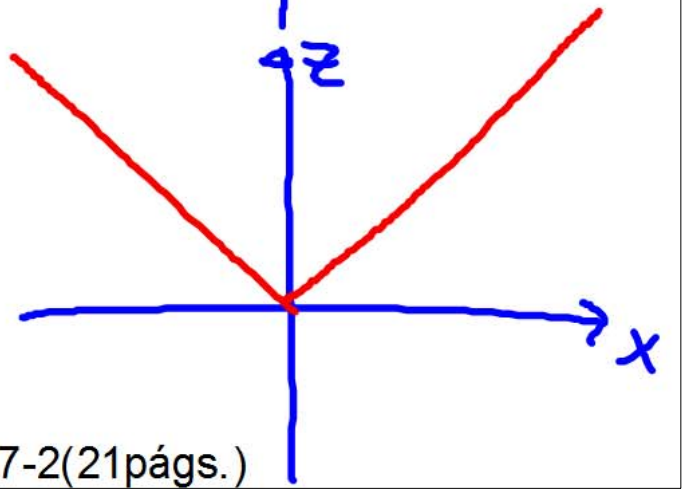
$$f(x,y)=\sqrt{x^2+y^2} \longrightarrow z=\sqrt{x^2+y^2}$$

Trazas

$x=0,\quad z=\sqrt{y^2} \rightarrow z=|y|$



$y=0,\quad z=\sqrt{x^2} \rightarrow z=|x|$



Curvas de nivel

$z = \sqrt{x^2+y^2} = k$, $k \in R$, $R = [0, \infty)$

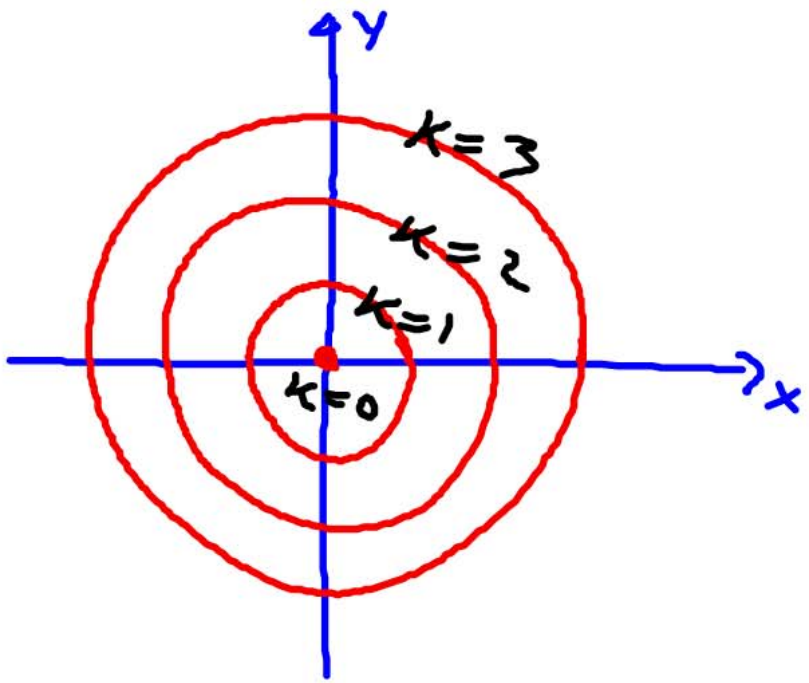
$x^2+y^2 = k^2$

$k=0$, $x^2+y^2=0$ Punto : $(0,0)$

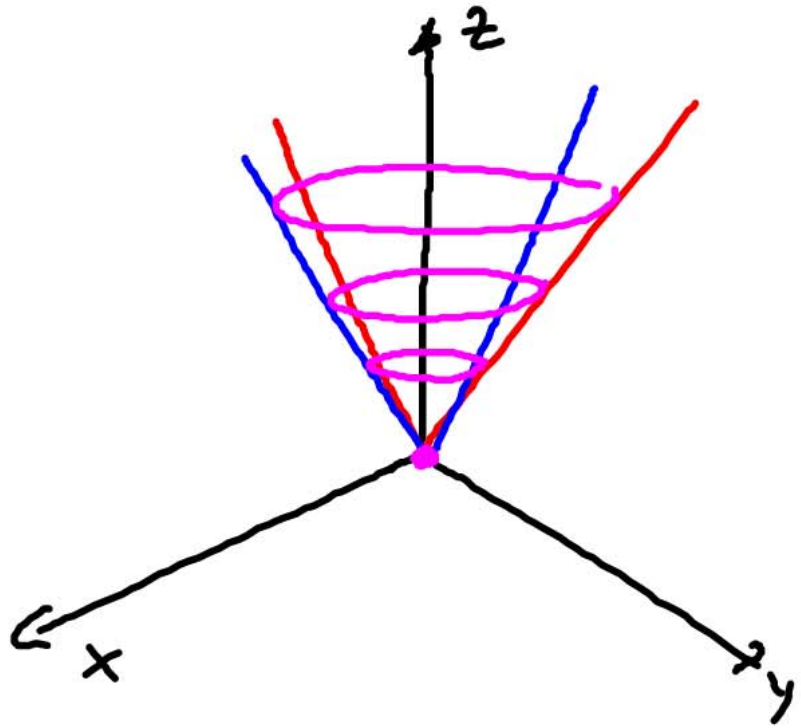
$k=1$, $x^2+y^2=1$

$k=2$, $x^2+y^2=4$

$k=3$, $x^2+y^2=9$

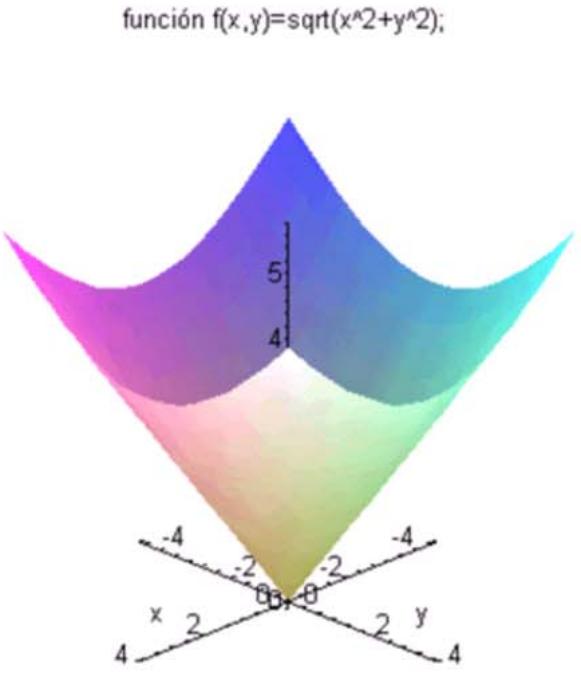


Gráfica



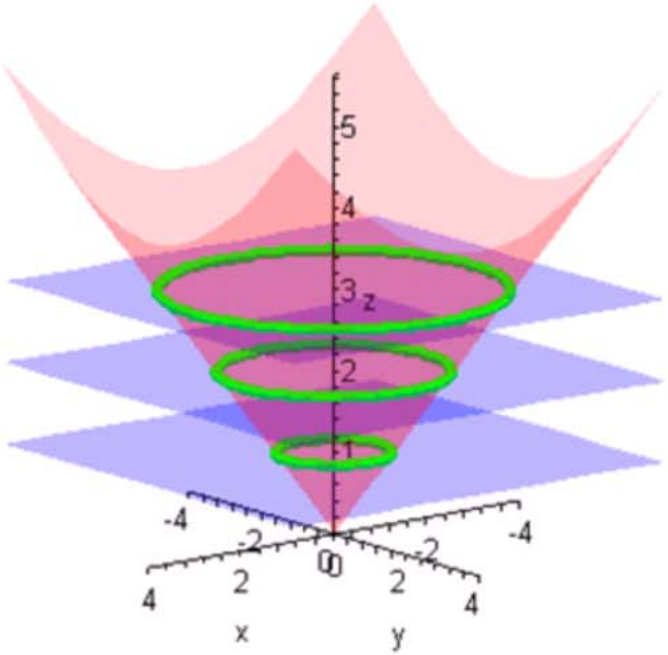
semicono

```
> restart;
  plot3d(sqrt(x^2+y^2),x=-4..4,y=-4..4,style=patchnogrid,title=
  "función f(x,y)=sqrt(x^2+y^2)");
```



```
> restart;
  with(Student[MultivariateCalculus]):
  CrossSection(sqrt(x^2+y^2),z=[0,1,2,3],x=-4..4,y=-4..4,
  title="función f(x,y)=√(x^2+y^2) y los planos z=0, z=1, z=2,
  z=3");
```

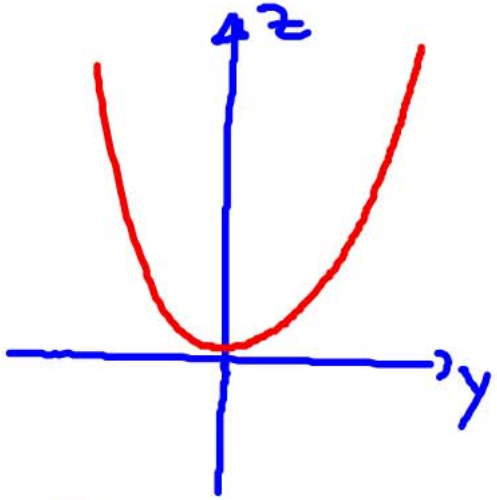
función f(x,y)=sqrt(x^2+y^2); y los planos z=0, z=1, z=2, z=3



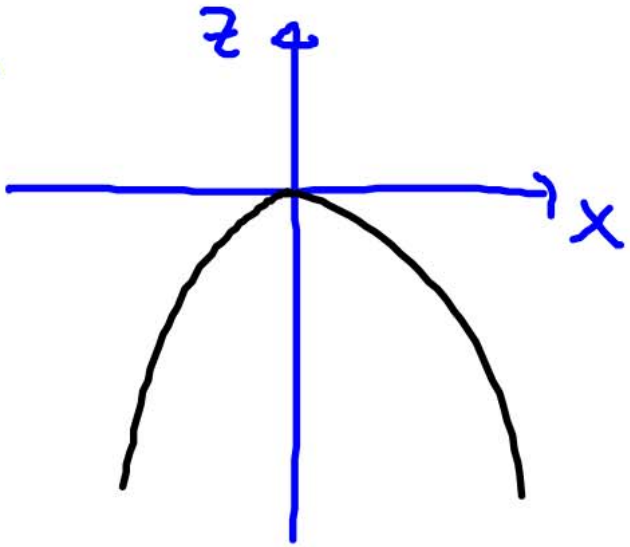
$$f(x,y) = y^2 - x^2 \quad \rightarrow z = y^2 - x^2$$

Trazas

$x = 0, \quad z = y^2$



$y = 0, \quad z = -x^2$



Curvas de nivel

$z = y^2 - x^2 = k$, $k \in R$, $R = \mathbb{R}$

$y^2 - x^2 = k$

$k = 0$, $y^2 - x^2 = 0 \rightarrow (y - x)(y + x) = 0$

$y = x$ ó $y = -x$

$k = 1$, $y^2 - x^2 = 1$

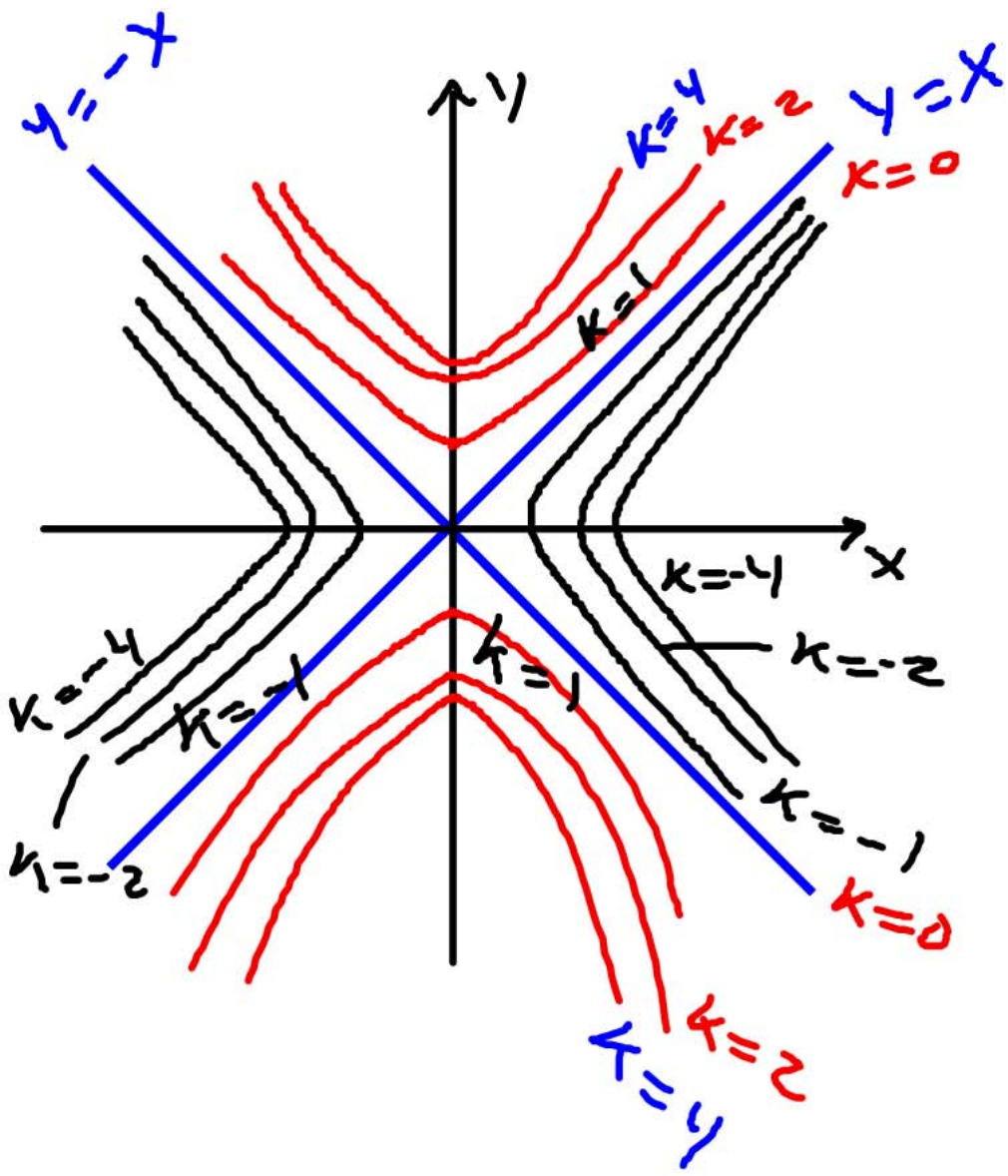
$k = -1$, $(y^2 - x^2 = -1) \rightarrow x^2 - y^2 = 1$

$k = 2$, $y^2 - x^2 = 2$

$k = -2$, $y^2 - x^2 = -2 \rightarrow x^2 - y^2 = 2$

$k = 4$, $y^2 - x^2 = 4$

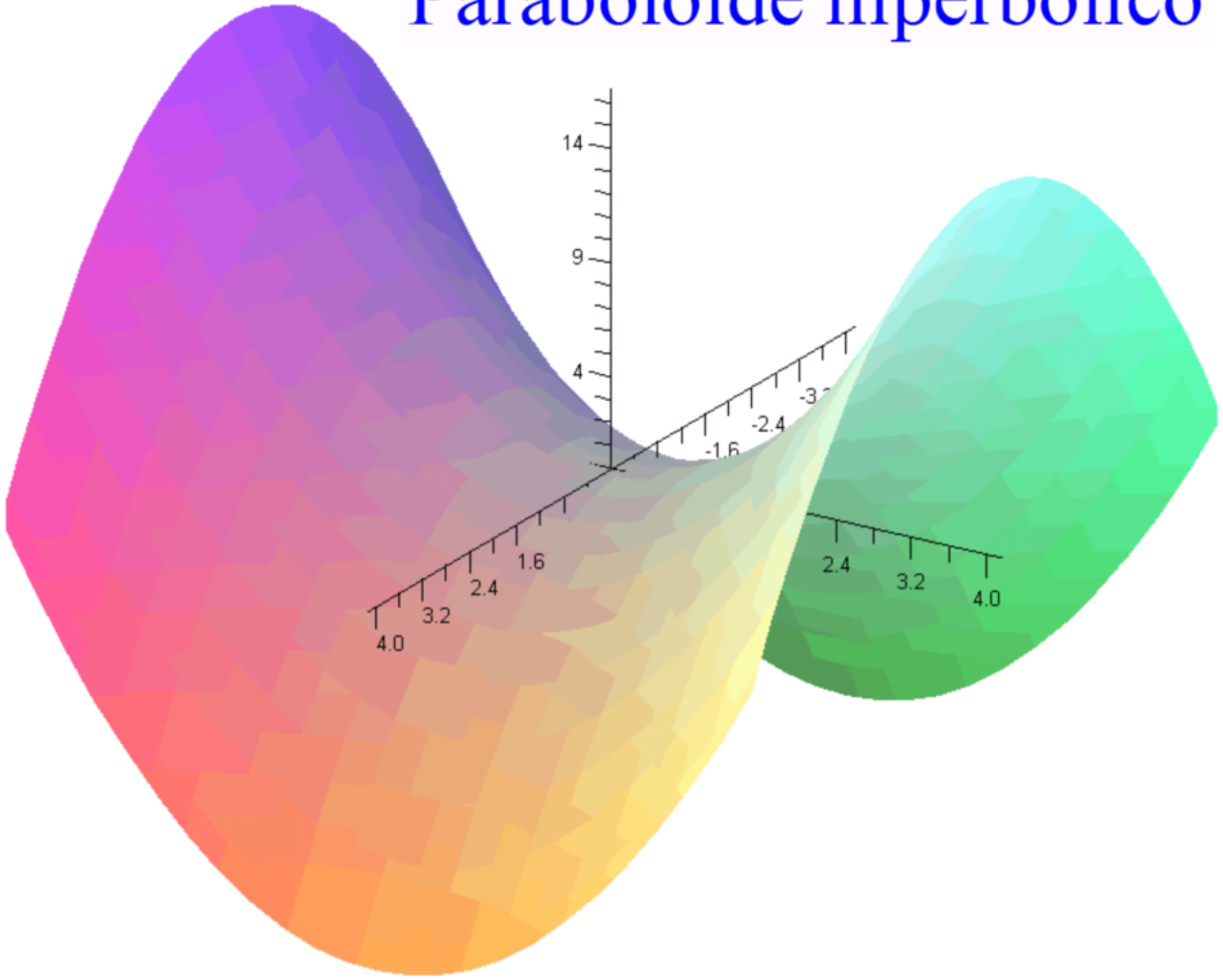
$k = -4$, $y^2 - x^2 = -4 \rightarrow x^2 - y^2 = 4$


y = -x
y
k=4
k=2
y = x
k=0
k=1
x
k=-4
k=-2
k=-4
k=-1
k=1
k=-1
k=-2
k=0
k=2
k=4

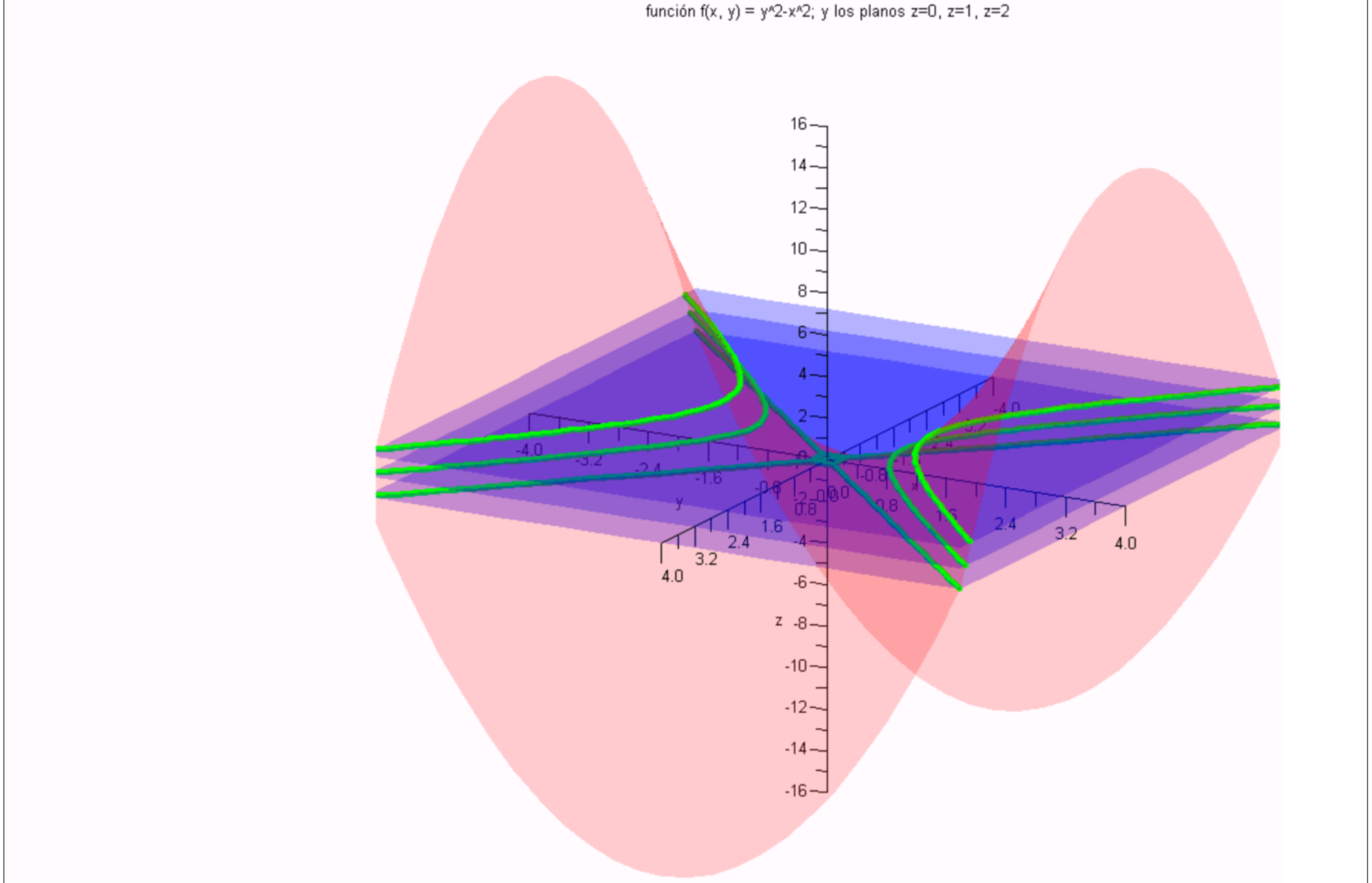
```
restart;
plot3d(y^2-x^2 ,x=-4..4,y=-4..4,style=patchnogrid,title="función f(x,y) = y^2 - x^2 ");
```

función f(x, y) = y^2-x^2;

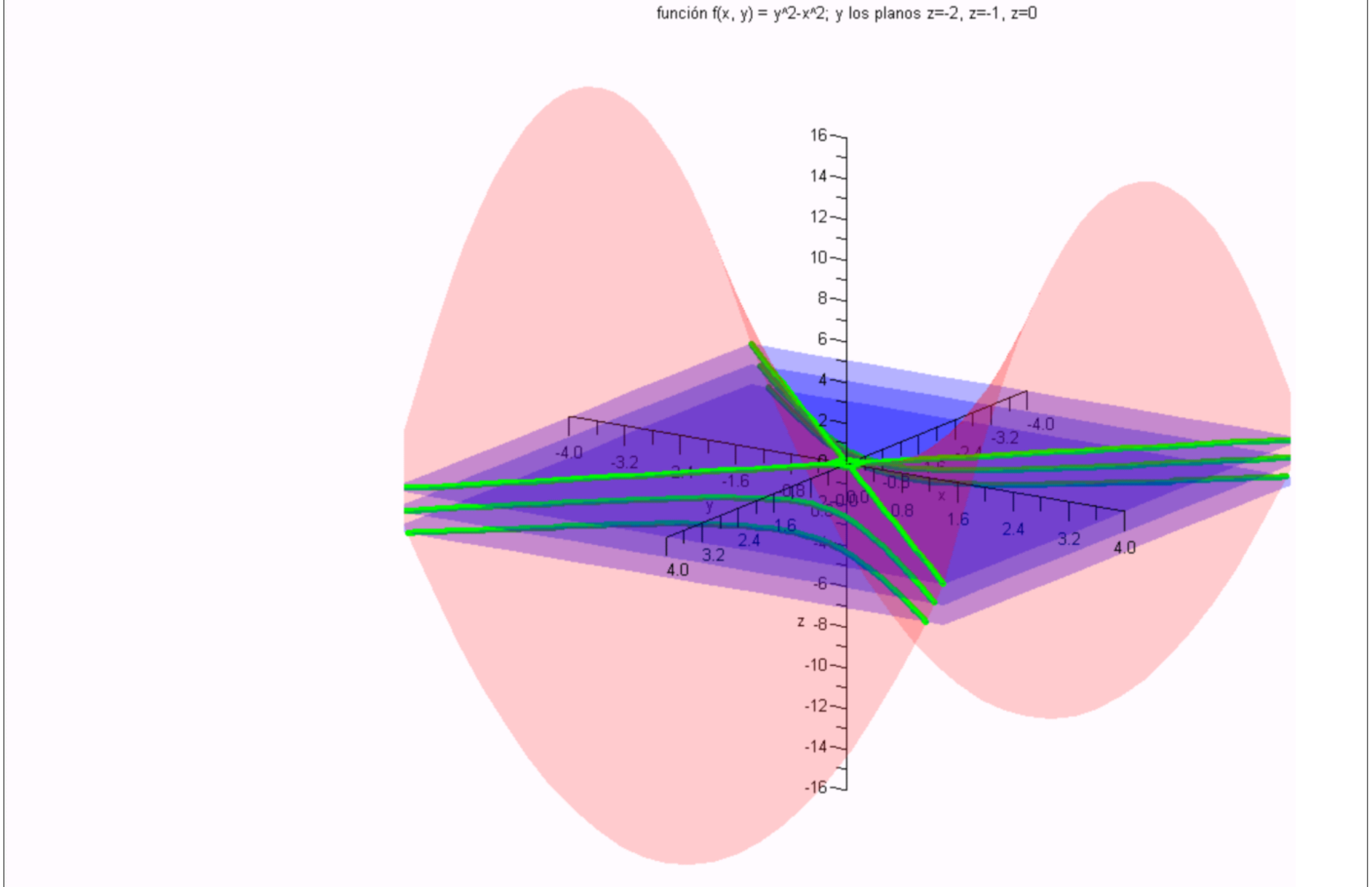
Paraboloide hiperbólico



```
restart;
with(Student[MultivariateCalculus]):
CrossSection(y^2-x^2,z=[0,1,2],x=-4..4,y=-4..4, title="función f(x,y) = y^2 - x^2 y los planos z=0, z=1, z=2");
```



```
restart;
with(Student[MultivariateCalculus]):
CrossSection(y^2-x^2,z=[-2,-1,0],x=-4..4,y=-4..4, title="función f(x,y) = y^2 − x^2 y los planos z=-2, z=-1, z=0");
```



```
restart;
with(Student[MultivariateCalculus]):
CrossSection(y^2-x^2,z=[-2,-1,0,1,2],x=-4..4,y=-4..4, title="función f(x,y) = y^2 - x^2 y los planos z=-2, z=-1, z=0, z=1, z=2");
```

función f(x, y) = y^2-x^2; y los planos z=-2, z=-1, z=0, z=1, z=2