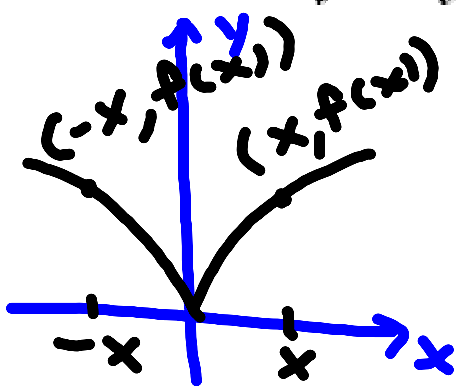
2.5.2 Funciones pares e impares

Criterio para funciones pares e impares

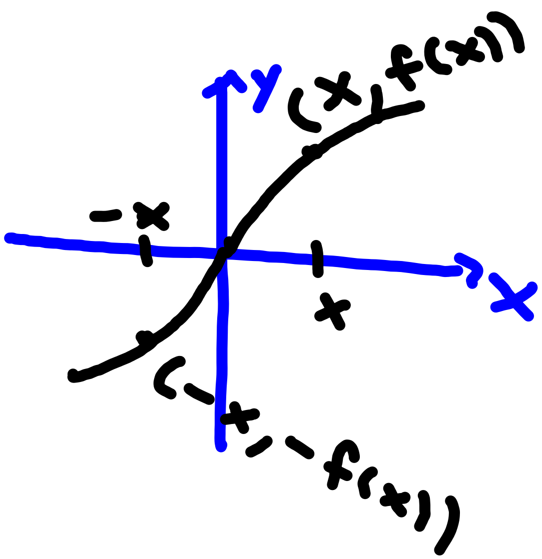
La función $y = f(x)$ es par si $f(-x) = f(x)$.

La función $y = f(x)$ es impar si $f(-x) = -f(x)$.



f - PAR

Es simétrica con respecto al eje "y"

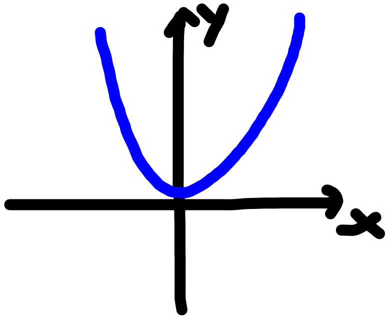


f - IMPAR

Es simétrica con respecto al origen.

Ejemplos:

1) $f(x) = x^2$

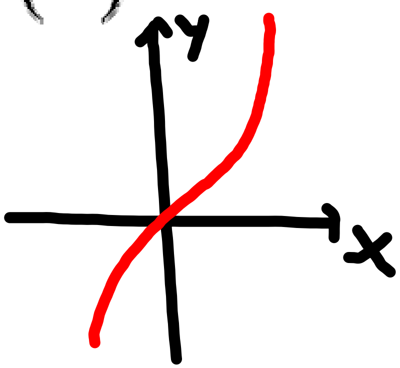


Es par.

$f(-x) = (-x)^2 = x^2 = f(x)$

$f(-x) = f(x)$

2) $f(x) = x^3$



Es impar

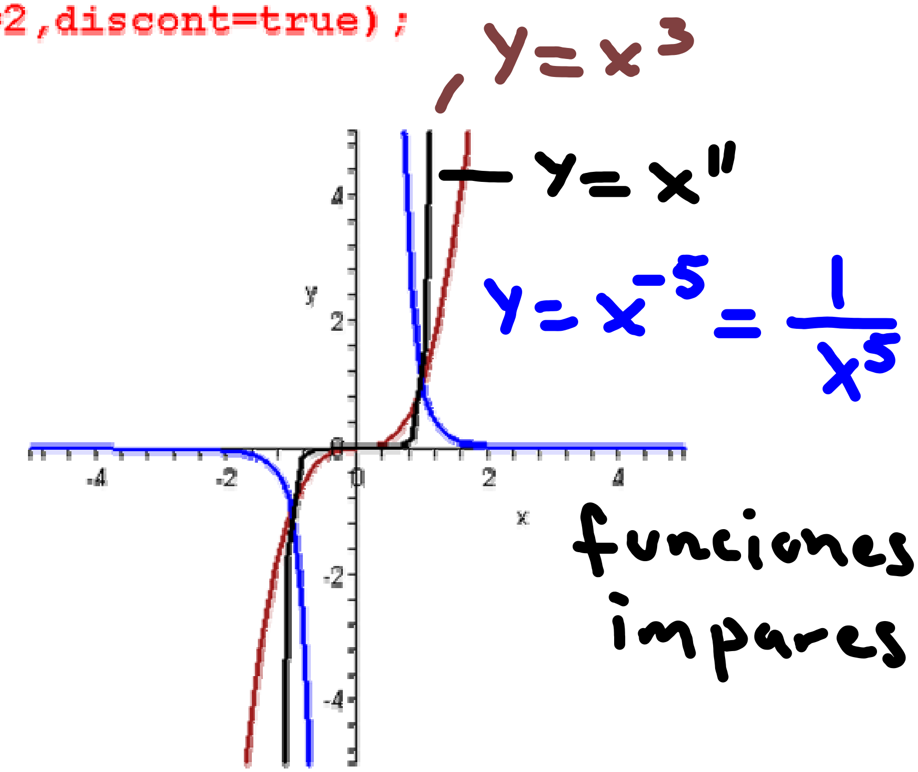
$f(-x) = (-x)^3 = -x^3 = -f(x)$

$f(-x) = -f(x)$

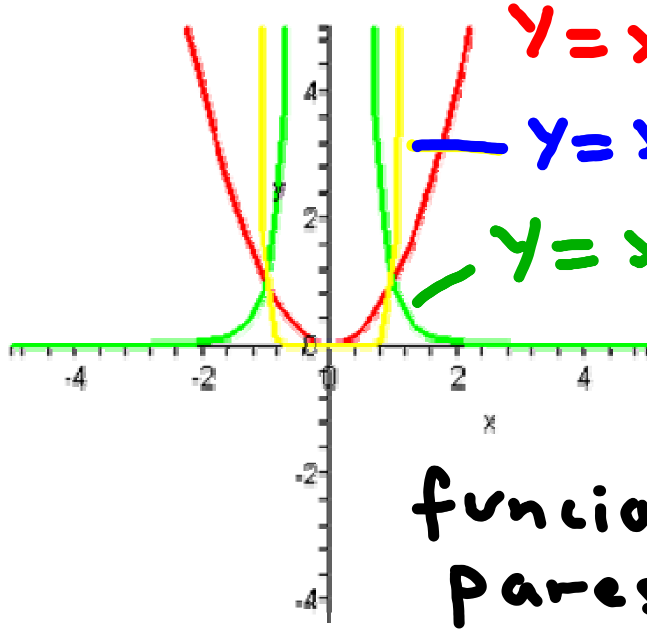
3) $f(x) = x^n$ donde n es un entero.

f es par si n es par, y es impar si n es impar.

```
> plot([x**3,x**(-5),x**11],x=-5..5,y=-5..5,color=[brown,blue,b
  lack],thickness=2,discont=true);
```



```
plot([x**2,x**(-4),x**12],x=-5..5,y=-5..5,color=[red,green,ye
llow],thickness=2,discont=true);
```



$y = x^2$

$y = x^{12}$

$y = x^{-4} = \frac{1}{x^4}$

funciones pares

Propiedades de las funciones pares e impares

Productos y sumas de funciones con el mismo dominio obedecen las siguientes reglas:

$$(\text{par})(par) = (impar)(impar) = par$$

$$(\text{par})(impar) = (impar)(par) = impar$$

$$(\text{par}) \pm (par) = par$$

$$\text{impar} \pm \text{impar} = \text{impar}$$

$f(x)$ - par $\quad g(x)$ - impar

$$F(x) = f(x)\,g(x)$$

$$F(-x) = f(-x)\,g(-x) = [f(x)][-g(x)]$$

$$F(-x) = -f(x)\,g(x) = -F(x)$$

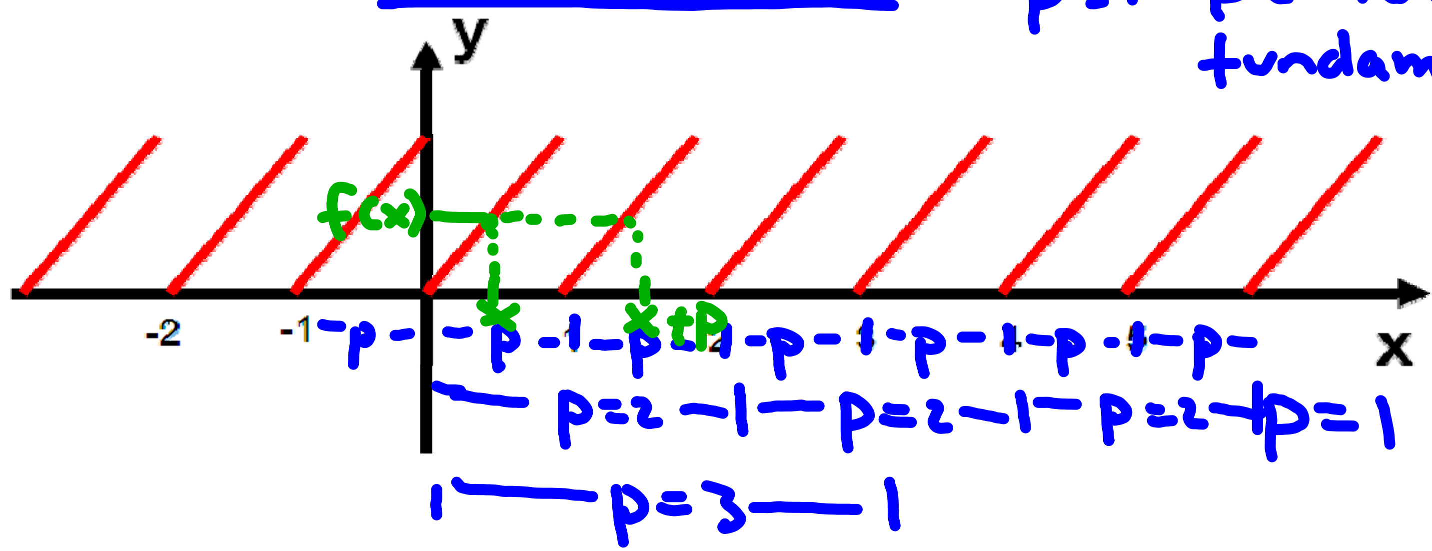
$$F(-x) = -F(x)$$

2.5.3 Funciones trigonométricas directas e inversas y su representación gráfica

Funciones periódicas

Una función f se dice que es periódica con periodo $p \neq 0$, si siempre que x esté en el dominio de f, entonces $x + p$ también está en el dominio de f, y se cumple que: $\underline{f(x+p) = f(x)}$

p=1 periodo fundamental



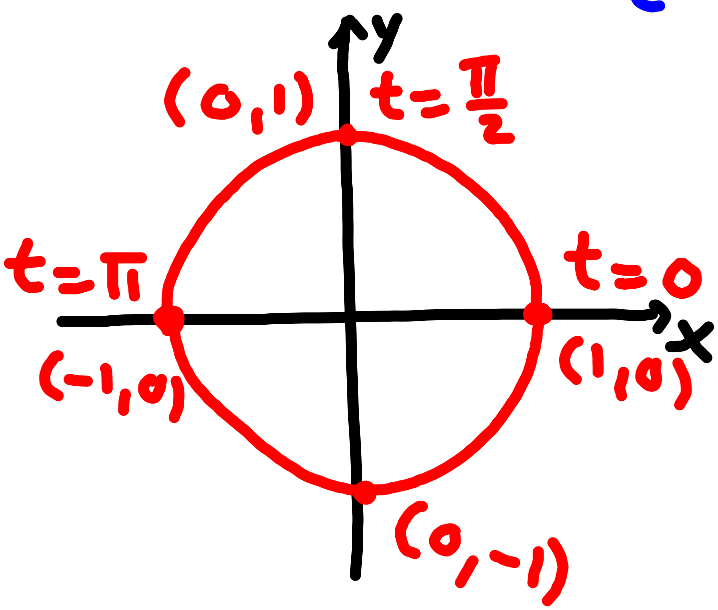
Funciones circulares

$$f=\{(x,y)\mid x=\cos t,\ y=sent,\ t\in\mathbb{R}\}$$ en forma paramétrica

t - parámetro

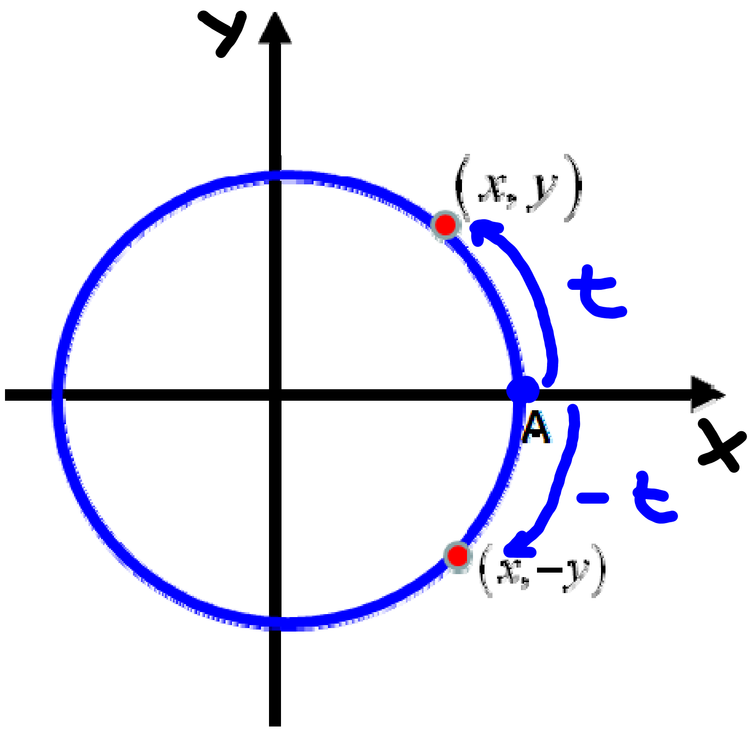
$$\begin{cases} x=\cos t \\ y=sen\,t \end{cases}$$

$$\begin{array}{rl} & (x)^2=(\cos t)^2 \\ + & (y)^2=(sen\,t)^2 \\ \hline & x^2+y^2=1 \end{array}$$



Dominio

$$\begin{cases} x = \cos t \\ y = sent \end{cases}$$



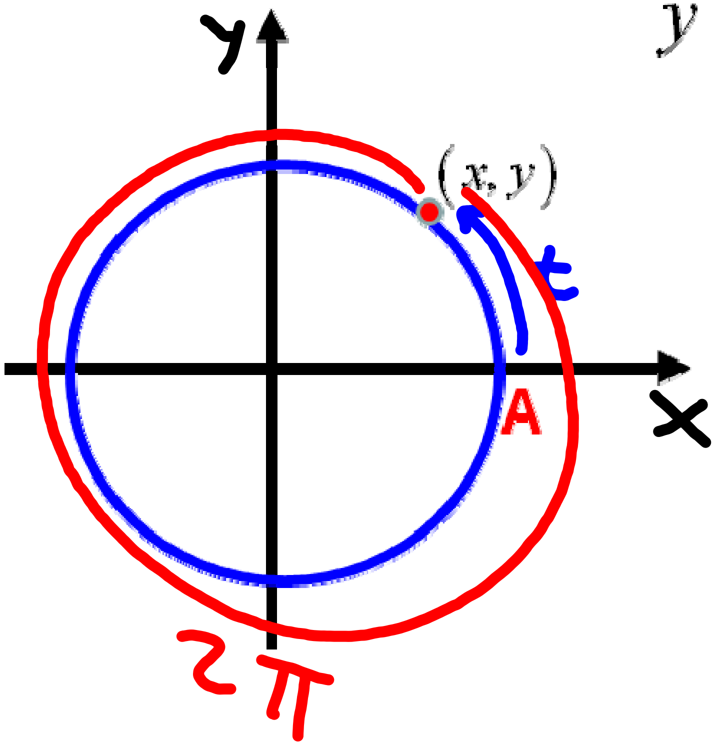
$D_{\cos t} = \mathbb{R}$

$D_{sen t} = \mathbb{R}$

$R = [-1, 1]$

Periodicidad

$$\begin{cases} x = \cos t \\ y = sent \end{cases}$$



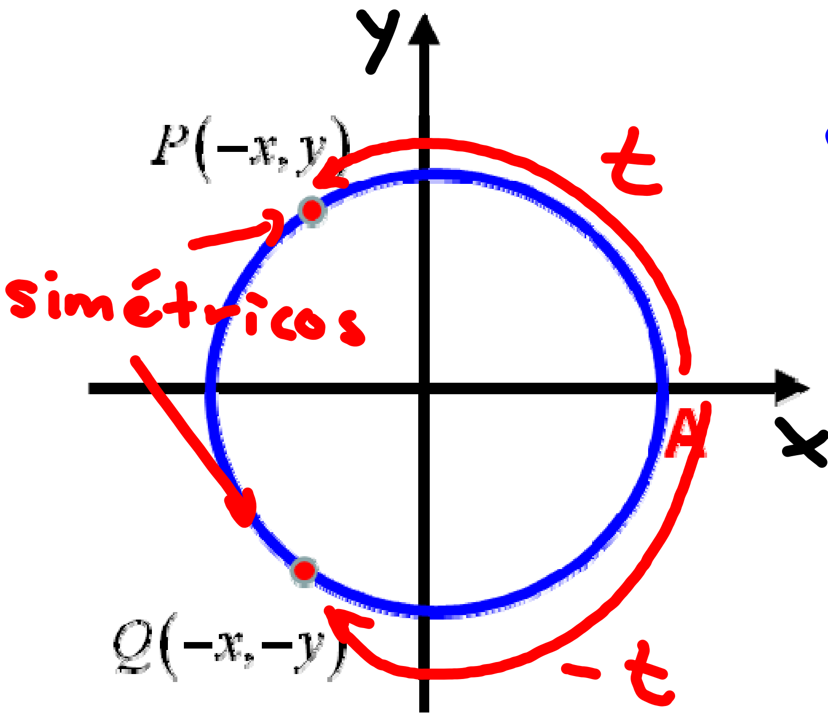
$$x = \cos\left(t + \underline{2\pi}\right) = \cos t$$

$$y = sen\left(t + \underline{2\pi}\right) = sent$$

Periodo = 2π

Paridad

$$\begin{cases} x = \cos t \\ y = sen\,t \end{cases}$$



$\cos(-t) = -x = \cos t$

$\cos(-t) = \cos t$

PAR

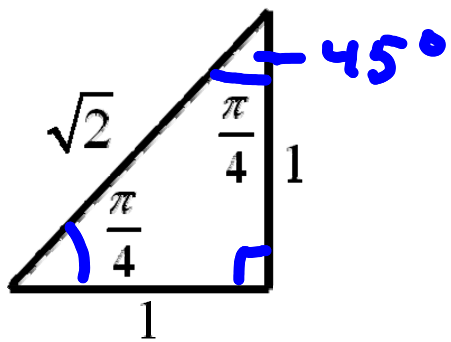
$sen(-t) = -y = -sen\,t$

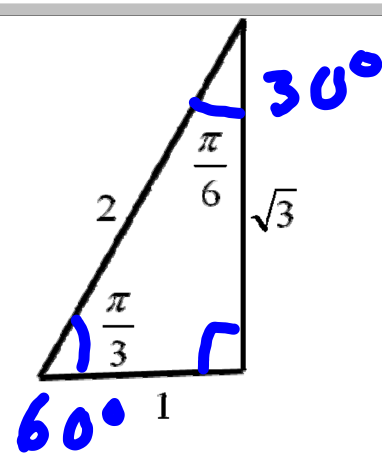
$sen(-t) = -sen\,t$

IMPAR

GRÁFICAS

t grados	0°	30°	45°	60°	90°	120°	135°	150°	180°
t radianes	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$2\frac{\pi}{3}$	$3\frac{\pi}{4}$	$5\frac{\pi}{6}$	π
sent	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
cost	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1





Identidades de cofunciones

(fórmulas de reducción)

$$\cos t = sen\left(\frac{\pi}{2} - t\right)$$

$$sent = \cos\left(\frac{\pi}{2} - t\right)$$

$$\operatorname{sen}(150^\circ) = \cos(90^\circ - 150^\circ) =$$

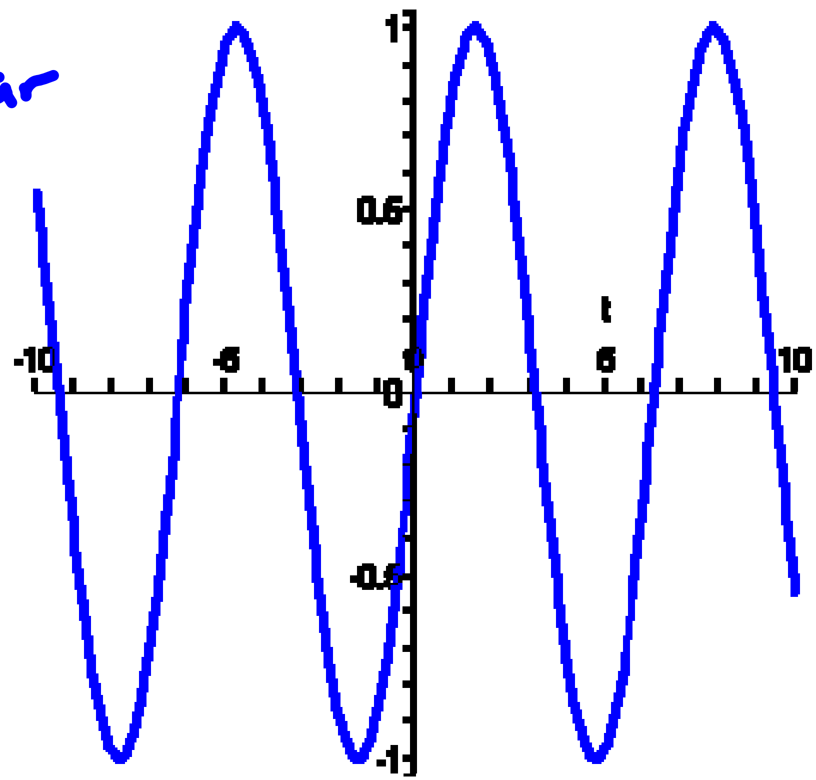
$$= \cos(-60^\circ) = \cos 60^\circ = \frac{1}{2}$$

$$\operatorname{sen}(150^\circ) = \frac{1}{2}$$

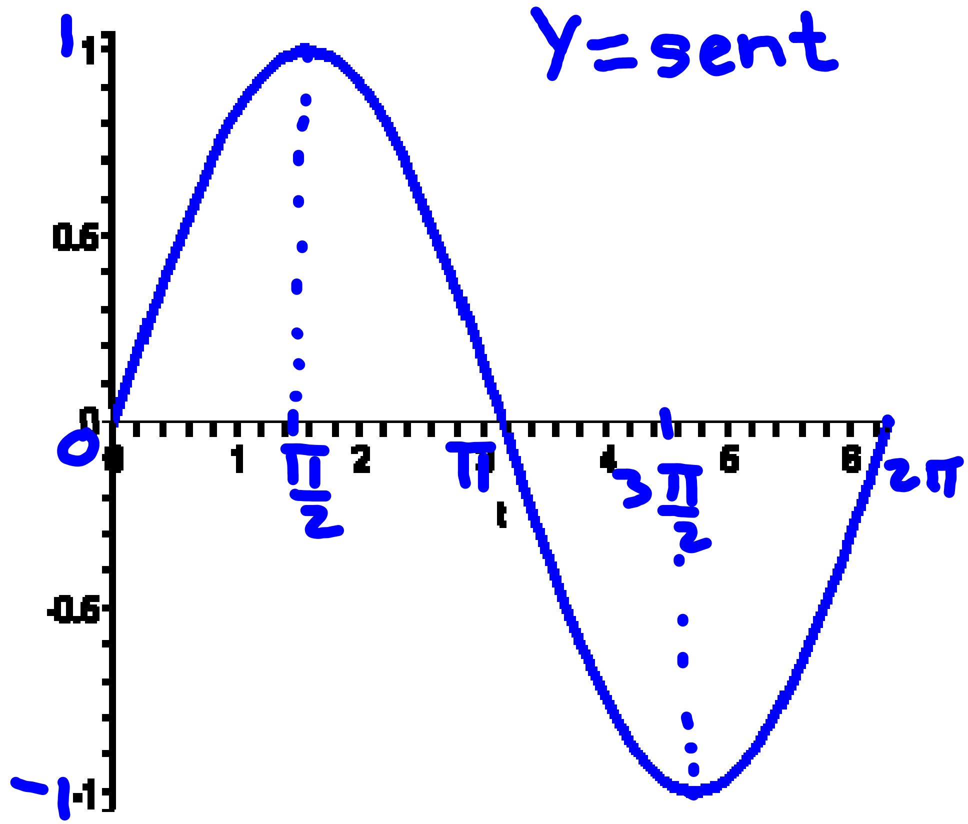
plot(sin(t),t,color=blue,thickness=2);

Y= sen t

Es impar



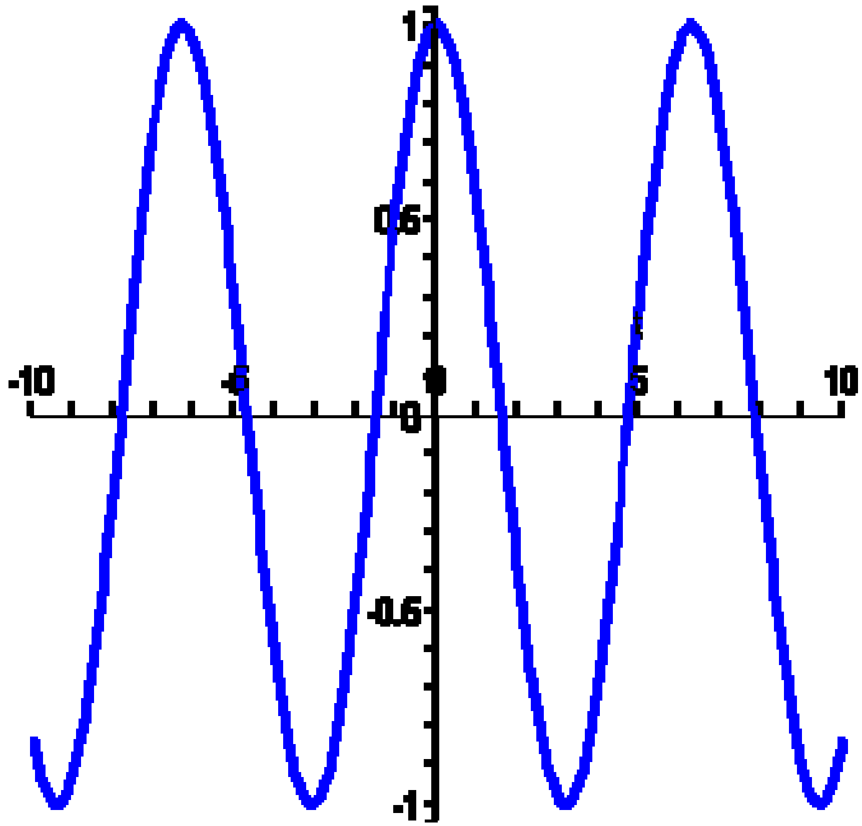
```
plot(sin(t),t=0..2*Pi,color=blue,thickness=2);
```



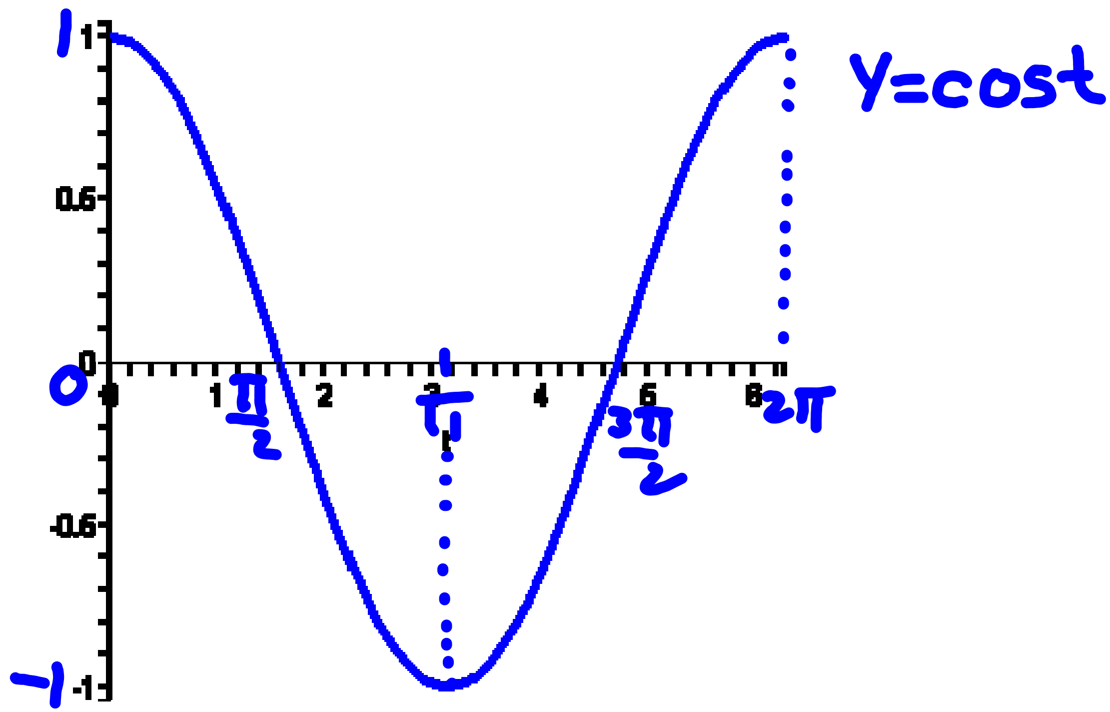
plot(cos(t),t,color=blue,thickness=2);

$y = \cos t$

Es par



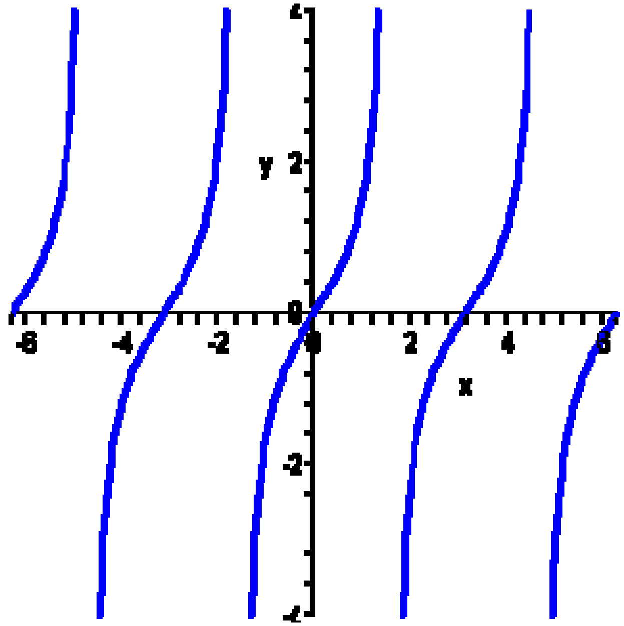
```
plot(cos(t),t=0..2*Pi,color=blue,thickness=2);
```



```
> restart;
  plot(tan(x),x=-2*Pi..2*Pi,y=-4..4,discont=true,color=blue,thi
  ckness=2);
```

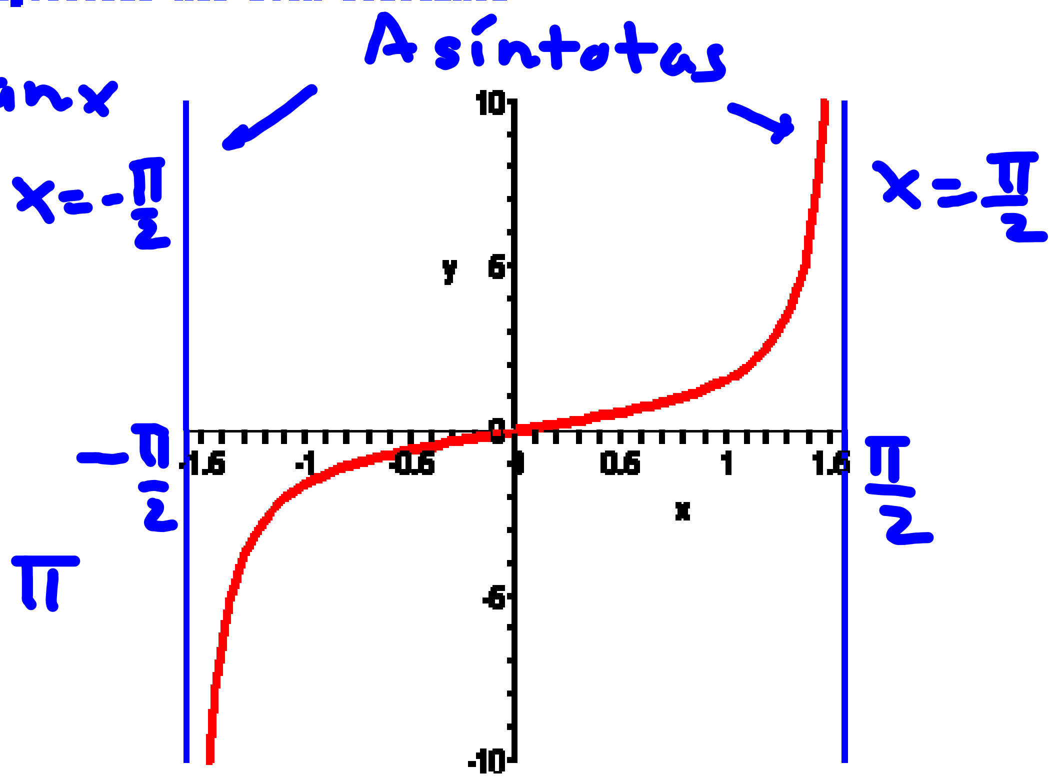
$y = \tan x$

Es impar



```
> restart;
  with(plots):
  G1:=implicitplot([x=Pi/2,x=-Pi/2],x=-2..2,y=-10..10,color=[blue,blue]):
  G2:=plot(tan(x),x=-Pi/2..Pi/2,y=-10..10,thickness=2,discont=true):
  display(G1,G2);
Warning, the name changecoords has been redefined
```

$y = \tan x$



Periodo $= \pi$

```
> restart;
  plot(cot(x),x=-2*Pi..2*Pi,y=-4..4,discont=true,color=blue,thickness=2);
```

$y = \cot x$

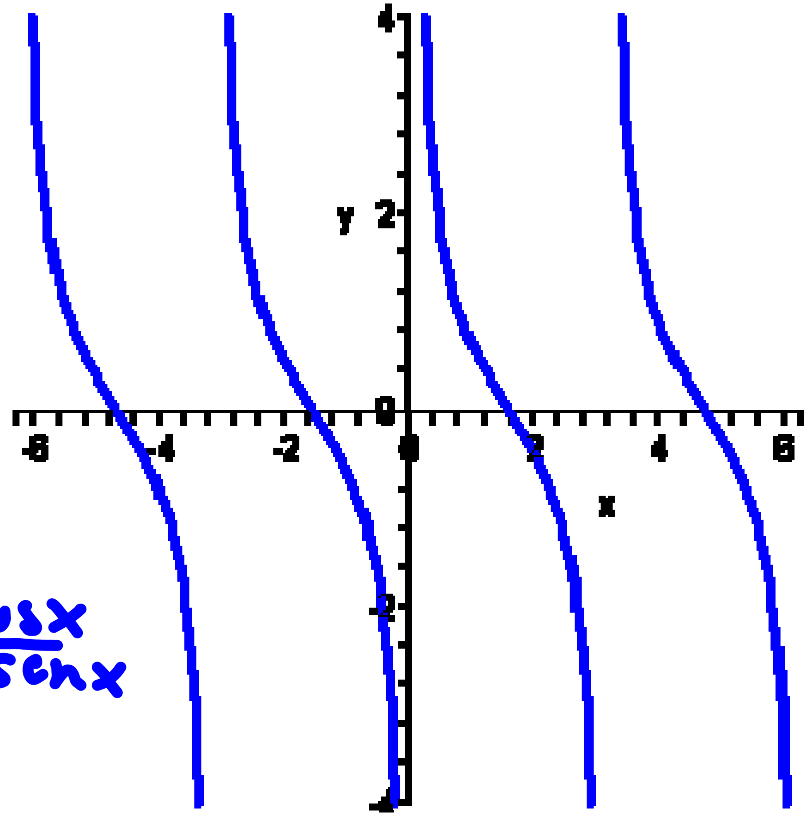
es impar

$f(x) = \dfrac{\cos x}{\operatorname{sen} x}$

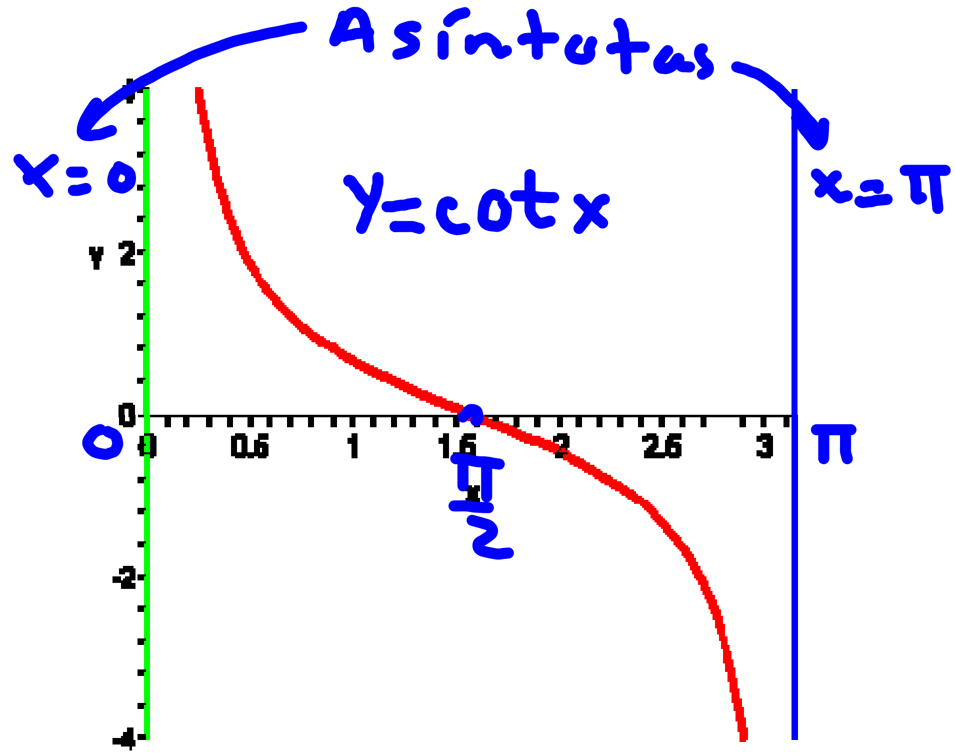
$f(-x) = \dfrac{\cos(-x)}{\operatorname{sen}(-x)} =$

$= \dfrac{\cos x}{-\operatorname{sen} x} = -\dfrac{\cos x}{\operatorname{sen} x}$

$\cot(-x) = -\cot x$



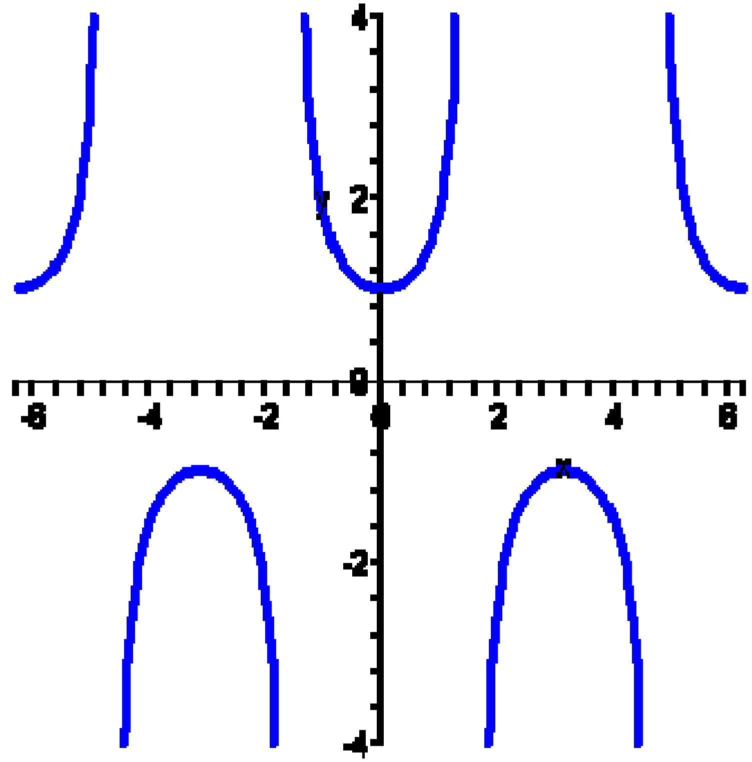
```
> restart;
  with(plots):
  G1:=implicitplot([x=0,x=Pi],x=-0.1..4,y=-5..5,color=[green,blue]):
  G2:=plot(cot(x),x=0..Pi,y=-4..4,thickness=2,discont=true):
  display(G1,G2);
Warning, the name changecoords has been redefined
```



```
> restart;
  plot(sec(x),x=-2*Pi..2*Pi,y=-4..4,discont=true,color=blue,thickness=2);
```

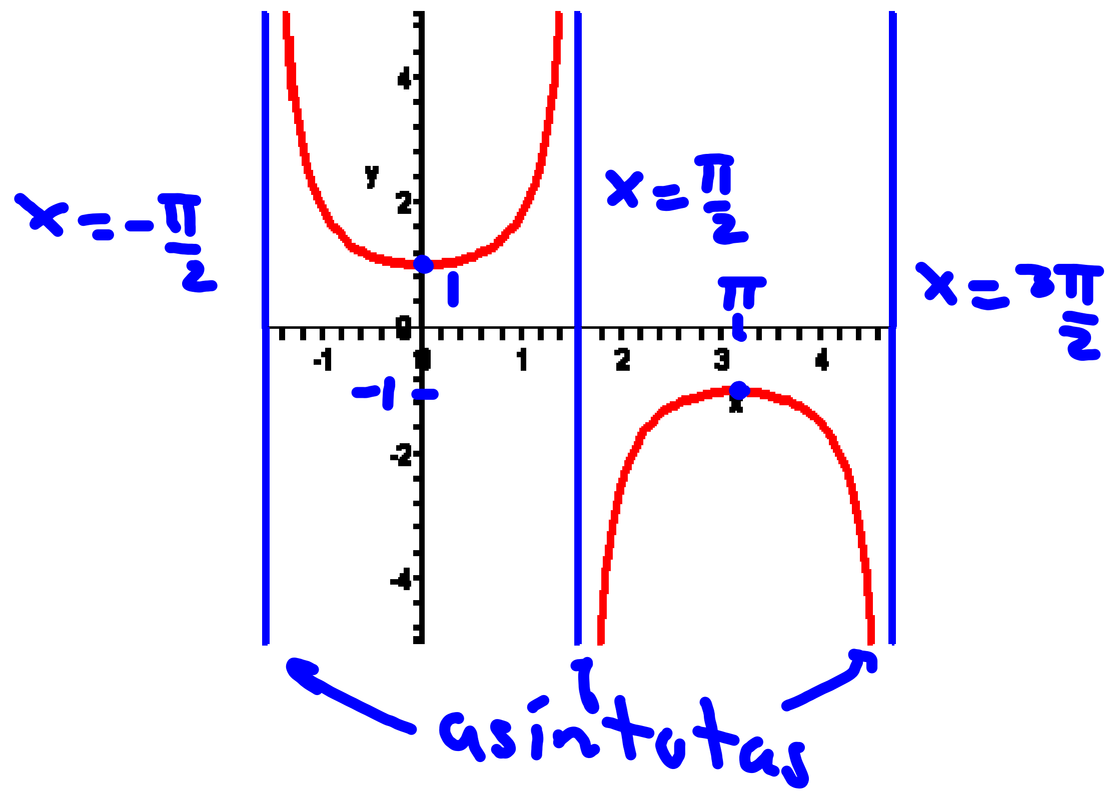
Y = sec x

Es par



```
> restart;
  with(plots):
  G1:=implicitplot([x=3*Pi/2,x=Pi/2,x=-Pi/2],x=-2..5,y=-5..5,color=[blue,blue,blue],thickness=2):
  G2:=plot(sec(x),x=-Pi/2..3*Pi/2,y=-5..5,thickness=2,discont=true):
  display(G1,G2);
Warning, the name changecoords has been redefined
```

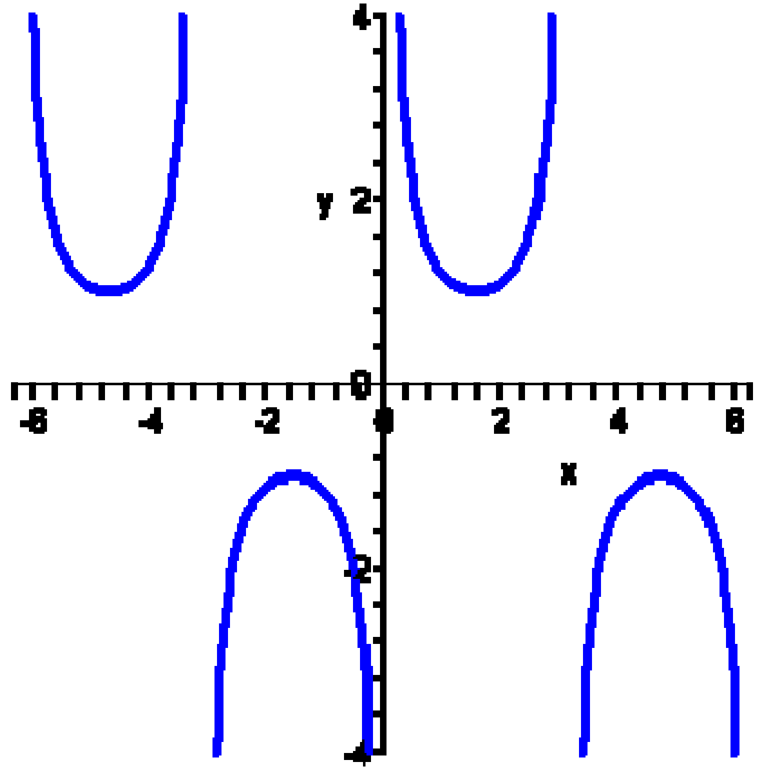
$y = \sec x$



```
> restart;
  plot(csc(x),x=-2*Pi..2*Pi,y=-4..4,discont=true,color=blue,thickness=2);
```

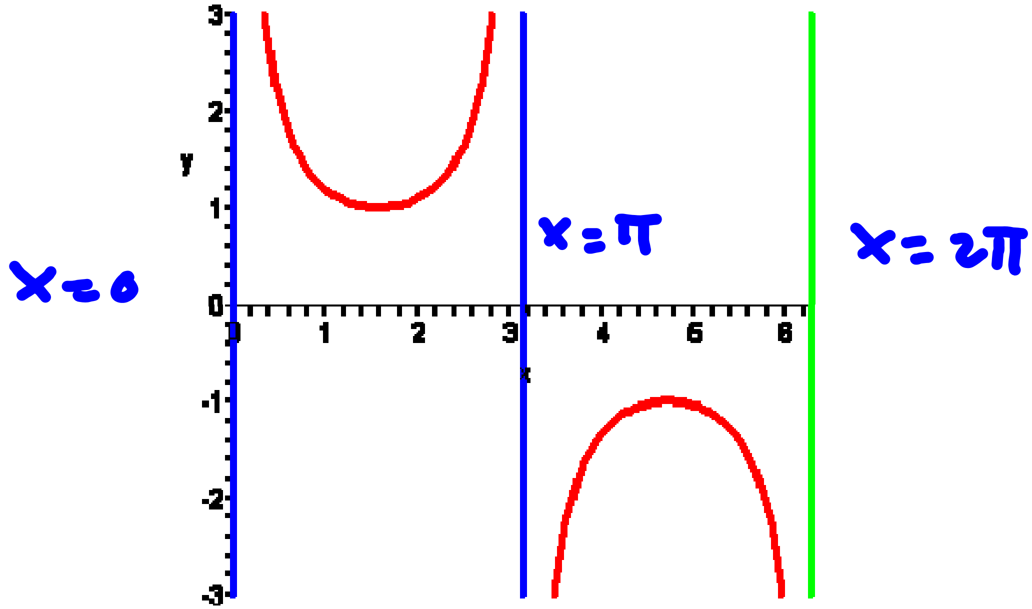
$y = \csc x$

es impar



```
> restart;
  with(plots):
  e1:=implicitplot([x=0,x=Pi,x=2*Pi],x=-0.1..8,y=-3..3,color=[blue,blue,green],thickness=2):
  e2:=plot(csc(x),x=0..2*Pi,y=-3..3,thickness=2,discont=true):
  display(e1,e2);
Warning, the name changecoords has been redefined
```

$y = \csc x$



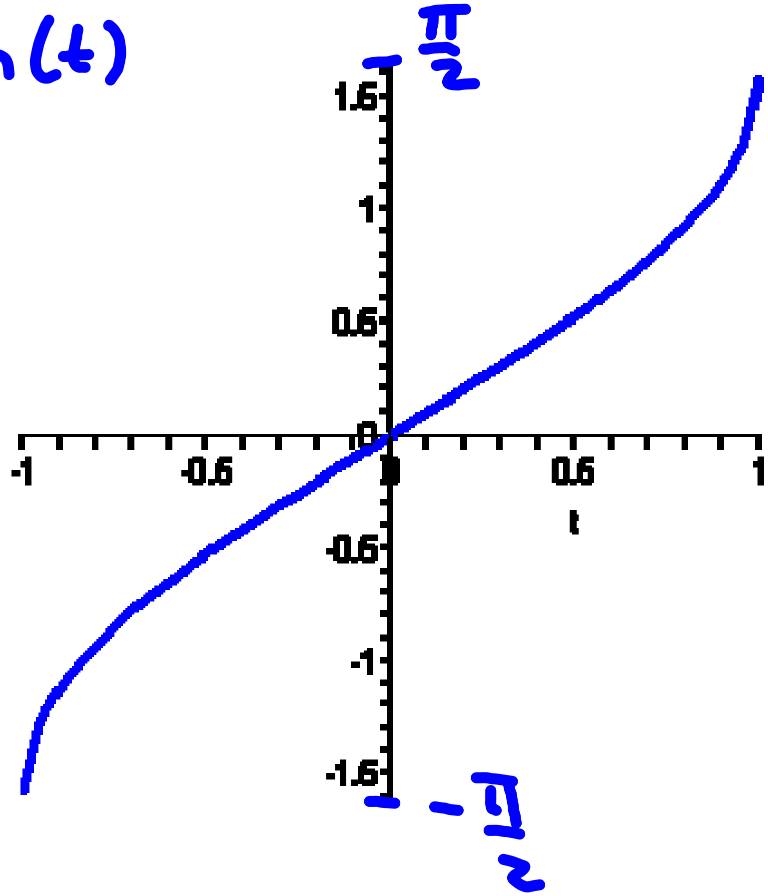
Ninguna de las seis funciones trigonométricas es inyectiva.

```
> restart;
  plot(arcsin(t),t=-1..1,color=blue,thickness=2);
```

$y = \text{angsen}(t)$

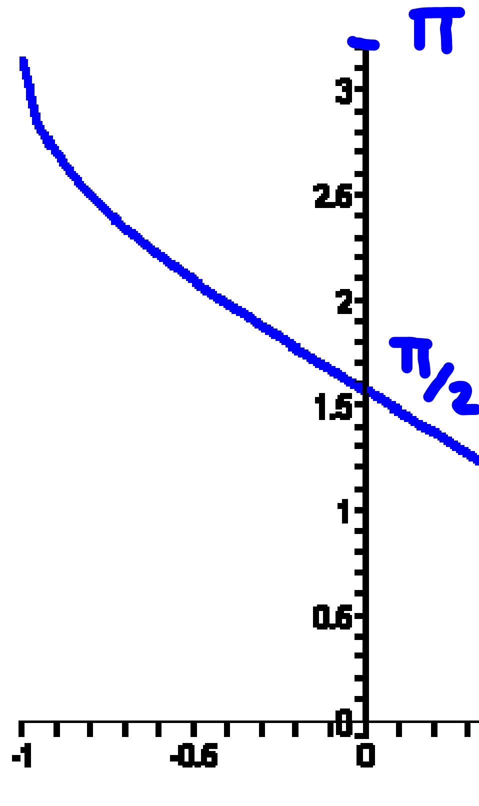
$\text{angsen}\,x = \text{sen}^{-1}x$

$\text{sen}^{-1}x \neq \frac{1}{\text{sen}\,x}$



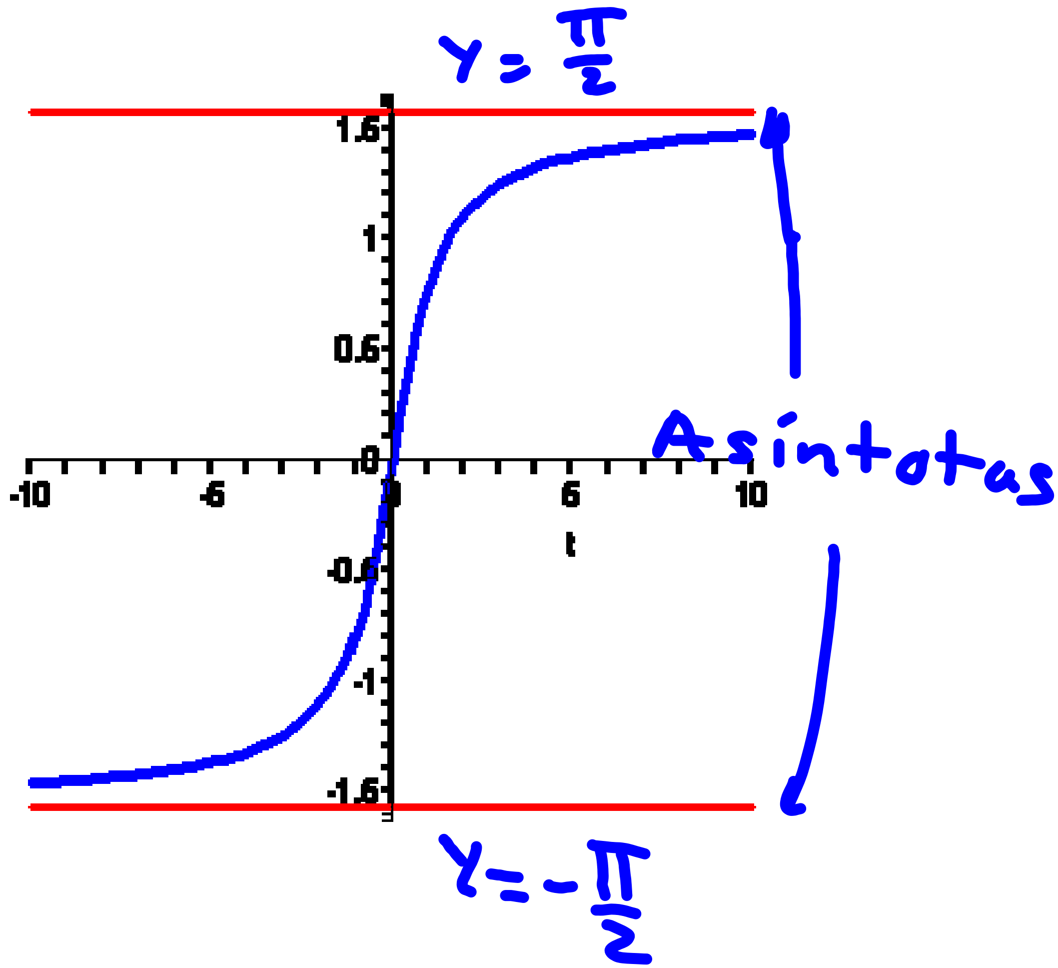
```
> restart;
  plot(arccos(t),t=-1..1,color=blue,thickness=2);
```

$y = \arccos t$



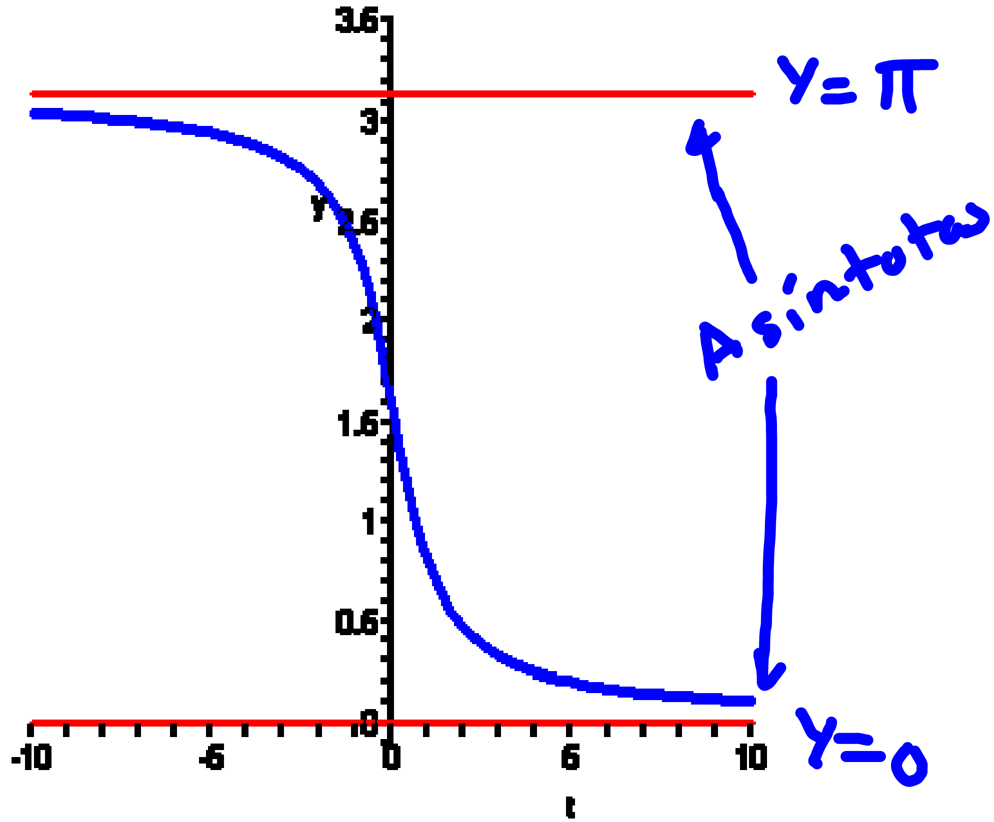
```
> restart;
  plot([arctan(t),Pi/2,-Pi/2],t,color=[blue,red,red],thickness=2);
```

$Y = \text{angtan}\, t$



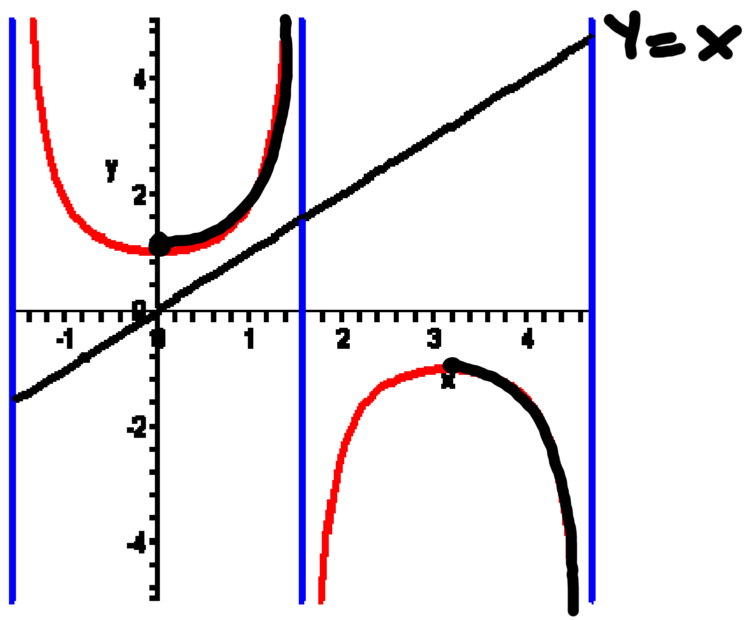
```
> restart;
  plot([arccot(t),0,Pi],t=-10..10,y=-0.1..3.5,color=[blue,red,red],thickness=2);
```

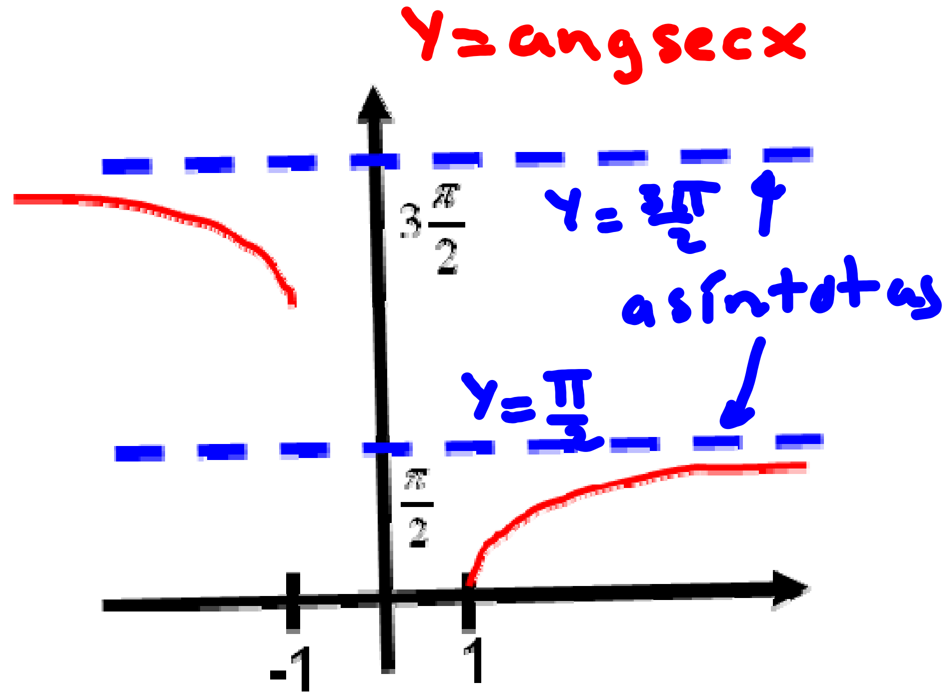
Y=angcot(t)



Intervalo para obtener la secante inversa

$$\left[0,\frac{\pi}{2}\right)\cup\left[\pi,\frac{3\pi}{2}\right)$$





Intervalo para obtener la cosecante inversa

$$\left(-\pi,-\frac{\pi}{2}\right]\cup\left(0,\frac{\pi}{2}\right]$$