ɔlva la ecuación integral $y(t) - \int_0^t sen(t-\tau)\, y(\tau)\, d\tau = u(t-1)$

$\mathcal{L}\{y(t)\} = Y(s)$; $\mathcal{L}\{\int_0^t sen(t-\tau)\, y(\tau)\, d\tau\} = \mathcal{L}\{sen(t) * y(t)\} =$

$= \frac{1}{s^2+1} Y(s)$

$\mathcal{L}\{u(t-1)\} = \frac{e^{-s}}{s}$

$Y(s) - \left[\frac{Y(s)}{s^2+1}\right] = \frac{e^{-s}}{s}$

$Y(s)\left[1 - \frac{1}{s^2+1}\right] = \frac{e^{-s}}{s}$

$Y(s)\left[\frac{s^2\cancel{+1-1}}{s^2+1}\right] = \frac{e^{-s}}{s}$; $Y(s) = \frac{e^{-s}}{s}\left(\frac{s^2+1}{s^2}\right)$

$\frac{s^2+1}{s^3} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3}$; $s^2+1 = As^2 + Bs + C$

$A = 1$; $B = 0$; $C = 1$

$\therefore \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{e^{-s}\left[\frac{1}{s} + \frac{1}{s^3}\right]\right\}$

$y(t) = u(t-1) + \frac{1}{2}(t-1)^2\, u(t-1)$ //

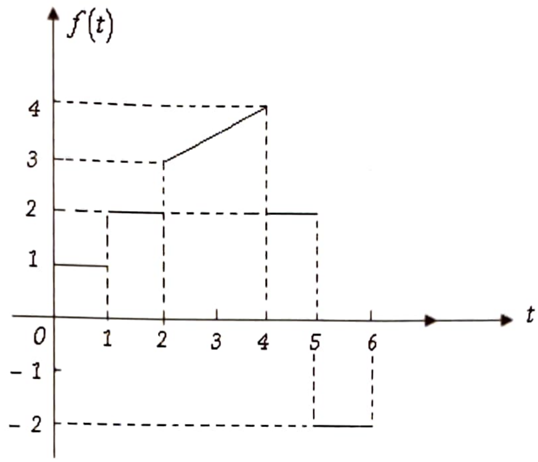
11) Obtenga la transformada de Laplace de la función

$f(t) = \int_0^t (t-\tau)\, e^{3\tau}\, d\tau$

$\mathcal{L}\{f(t)\} = \mathcal{L}\{(t) * e^{3t}\}$

$F(s) = \frac{1}{s^2} \cdot \frac{1}{s-3}$ //

12) Sea la función f cuya gráfica es



$$f(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 2 & 1 \le t < 2 \\ \frac{1}{2}t & 2 \le t \le 2 \\ 2 & 4 \le t < 5 \\ -2 & 5 \le t < 6 \\ 0 & t > 6 \end{cases}$$

a) Exprese f en términos de las funciones generalizadas escalón y rampa unitarios.

b) Obtenga la transformada de $f(t)$

$$f(t) = \cancel{u(t)}^{1/s} + \cancel{u(t-1)}^{e^{-s}/s} + \frac{1}{2}r(t-2)\,u(t-2) + u(t-2) - \frac{1}{2}r(t-2)\,u(t-4) - 2u(t-4)$$
$$-4u(t-5)$$

$$\mathcal{L}\{u(t-2)\} = \frac{e^{-2s}}{s} \qquad -2\mathcal{L}\{u\,t-4\} = -\frac{2e^{-4s}}{s}$$

$$+\frac{1}{2}\mathcal{L}\{r(t-2)\,u(t-2)\} = \frac{1}{2}\,\frac{e^{-2s}}{s^2}$$

$$-\frac{1}{2}\mathcal{L}\{r(t-2)\,u(t-4)\} = -\frac{1}{2}\mathcal{L}\{r(t-4)\,u(t-4) + 4u(t-4)\} =$$
$$= -\frac{1}{2}\left[\frac{e^{-4s}}{s^2} + \frac{4e^{-4s}}{s}\right]$$

// Reduciendo

$$\mathcal{L}\{f(t)\} = \frac{1}{s} + \frac{e^{-s}}{s} + \frac{1}{2}\frac{e^{-2s}}{s^2} + \frac{e^{-2s}}{s} - \frac{e^{-4s}}{2s^2} - \frac{2e^{-4s}}{s} -$$
$$- \frac{2e^{-4s}}{s} - \frac{4e^{-5s}}{s} + \frac{2e^{-6s}}{s}$$

3) Sea el sistema de ecuaciones diferenciales

$$\frac{dx}{dt} - 5x + 2y = 3e^{4t} \qquad x(0) = 3$$
$$; $$
$$\frac{dy}{dt} - 4x + y = 0 \qquad y(0) = 0$$

Utilice la transformada de Laplace para obtener $x(t)$ y $y(t)$

$\mathcal{L}\{x'(t)\} = sX(s) - \cancel{x(0)}^{\,3}$; $3\mathcal{L}\{e^{4t}\} = \frac{3}{s-4}$

$-5\mathcal{L}\{x(t)\} = -5X(s)$

$2\mathcal{L}\{y(t)\} = 2Y(s)$

$\therefore$ ① $sX(s) - 3 - 5X(s) + 2Y(s) = \frac{3}{s-4}$

② $sY(s) - 4X(s) + Y(s) = 0$

① $X(s)[s-5] - 3 + 2Y(s) = \frac{3}{s-4}$; $Y(s)[s+1] = 4X(s)$

$Y(s) = \frac{4X(s)}{s+1}$

$\therefore X(s)[s-5] - 3 + \frac{8X(s)}{s+1} = \frac{3}{s-4}$

$X(s)\left[\frac{(s-5)(s+1)+8}{s+1}\right] - 3 = \frac{3}{s-4}$

$X(s)\left[\frac{(s-5)(s+1)+8}{s+1}\right] = \frac{3+3(s-4)}{s-4}$; $X(s) = \left(\frac{3s-9}{s-4}\right)\frac{s+1}{(s-3)(s-1)} =$

$= \frac{3\cancel{(s-3)}\cancel{(s+1)}}{(s-4)\cancel{(s+1)}\cancel{(s-3)}} = 3\frac{s+1}{(s-4)(s-1)} =$

$\rightarrow 3s+3 = A(s-1) + B(s-4)$; $3s+3 = s(A+B) - A - 4B$

$A + B = 3$ $\quad -A - 4B = 3$

$A = 3 - B$ $\quad (B-3) - 4B = 3$; $-3B = 6$; $\underline{B = -2}$

$\underline{A = 5}$

$\therefore \mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\left\{\frac{5}{s-4} - \frac{2}{s-1}\right\} = 5e^{4t} - 2e^{t}$

$\rightarrow Y(s) = \left[\frac{4\,\frac{3(s+1)}{(s-4)(s-1)}}{s+1}\right] = \frac{12}{(s-4)(s-1)} = \frac{4}{s-4} - \frac{4}{s-1}$

* Sole en la hoja

$$\frac{12}{(s-4)(s-1)} = 12 = A(s-1) + B(s-4)$$
$$= s(A+B) - A - 4B$$
$$A + B = 0 \qquad -A - 4B = 12$$
$$-A = B \qquad B - 4B = 12 \;;\; B = -4$$
$$A = 4$$

$$\mathcal{L}\{y(s)\} = \mathcal{L}\left\{\frac{4}{s-4} - \frac{4}{s-1}\right\} = 4e^{4t} - 4e^{t}$$

14) La corriente en un circuito LC en serie queda expresada mediante el problema de valores iniciales

$$I''(t) + 4t = g(t) \;;\; I(0) = 0 \;,\; I'(0) = 0$$

donde

$$g(t) = \begin{cases} 1, & 0 < t < 1 \\ -1, & 1 < t < 2 \\ 0, & 2 < t \end{cases}$$

Determine la corriente $I(t)$

- $\mathcal{L}\{I''(t)\} = s^2 I(s) - s\cancel{I(0)}^{0} - \cancel{I'(0)}^{0} = s^2 I(s)$
- $4\mathcal{L}\{t\} = \frac{4}{s^2}$
- $\mathcal{L}\{u(t) - 2u(t-1) + u(t-2)\} = \frac{1}{s} - 2\frac{e^{-t}}{s} + \frac{e^{-2t}}{s}$

$$\therefore s^2 I(s) = -\frac{4}{s^2} + \frac{1}{s} - \frac{2e^{-t}}{s} + \frac{e^{-2t}}{s}$$

$$I(s) = -\frac{4}{s^4} + \frac{1}{s^3} - \frac{2e^{-t}}{s^3} + \frac{e^{-2t}}{s^3}$$

$$\mathcal{L}^{-1}\{I(s)\} = \mathcal{L}^{-1}\left\{-\frac{4}{s^4} + \frac{1}{s^3} - \frac{2e^{-t}}{s^3} + \frac{e^{-2t}}{s^3}\right\}$$

$$I(t) = \frac{1}{2}t^2 - (t-1)^2 u(t-1) + \frac{1}{2}(t-2)^2 u(t-2) + \frac{1}{27} + 3$$

5) Resuelva la ecuación integro – diferencial

$$y'(t) + y(t) - \int_0^t sen(t-v)\,y(v)\,dv = -sent\ ,\ y(0) = 1$$

$$\mathcal{L}\{y'(t)\} = s\,y(s) - y(0)$$

$$\mathcal{L}\{y(t)\} = y(s)$$

$$-\mathcal{L}\left\{\int_0^t sen(t-v)\,y(v)\,dv\right\} = \mathcal{L}\{sen(t) * y(t)\} = -y(s)\,\frac{1}{s^2+1}$$

$$-\mathcal{L}\{sen(t)\} = -\frac{1}{s^2+1}$$

$$\therefore\ s\,y(s) - 1 + y(s) - y(s)\,\frac{1}{s^2+1} = -\frac{1}{s^2+1}$$

$$y(s)\left[s+1-\frac{1}{s^2+1}\right] = 1 - \frac{1}{s^2+1}\ ;\ y(s)\left[\frac{(s+1)(s^2+1)-1}{s^2+1}\right] = \frac{s^2+1-1}{s^2+1}$$

$$\therefore\ y(s)\left[\frac{s(s^2+s+1)}{s^2+1}\right] = \frac{s^2}{s^2+1}\ ;\ y(s) = \frac{s^{\cancel{2}}}{\cancel{s^2+1}}\,\frac{\cancel{s^2+1}}{\cancel{s}(s^2+s+1)} =$$

$$= \frac{s}{s^2+s+1} = \frac{s}{(s+1/2)^2+3/4}$$

$$\rightarrow y(s) = \frac{s+1/2}{(s+1/2)^2+3/4} - \frac{1}{\cancel{2}}\,\frac{\cancel{2}}{\sqrt{3}}\,\frac{\frac{\sqrt{3}}{2}}{(s+1/2)^2+3/4}$$

$$\mathcal{L}^{-1}\{y(s)\} = \mathcal{L}^{-1}\left\{\frac{(s+1/2)}{(s+1/2)^2+3/4}\right\} - \frac{1}{\sqrt{3}}\,\mathcal{L}^{-1}\left\{\frac{\frac{\sqrt{3}}{2}}{(s^2+1/2)^2+3/4}\right\}$$

$$y(t) = e^{-1/2t}\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{1}{\sqrt{3}}\,e^{-\frac{1}{2}t}\,sen\left(\frac{\sqrt{3}}{2}t\right)$$

16) Exprese la función $f(t) = \begin{cases} t^2, & 0 \le t < 2 \\ 2t, & t \ge 2 \end{cases}$ en términos de la función escalón unitario y calcule su transformada de Laplace

$$f(t) \begin{cases} g(t) & 0 \le t < a \\ h(t) & a \ge t \end{cases} \qquad f(t) = g(t) + h(t)\,u(t-a) - g(t)\,u(t-a)$$

$$f(t) = t^2 + 2t\,u(t-2) - t^2\,u(t-2)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{t^2 + 2t\,u(t-2) - t^2\,u(t-2)\}$$

$$\mathcal{L}\{t^2\} = \frac{2}{s^3}$$

$$\mathcal{L}\{2t\,u(t-2)\} = 2\mathcal{L}\{(t-2)\,u(t-2)\} + 4\,\mathcal{L}\{u(t-2)\} = 2\frac{e^{-2t}}{s^2} + 4\frac{e^{-2t}}{s}$$

$$-\mathcal{L}\{t^2\,u(t-2)\} = -\mathcal{L}\{(t^2-4t+4)\,u(t-2)\} + \mathcal{L}\{-4t+4\,u(t-2)\} =$$

$$= -\mathcal{L}\{(t-2)^2\,u(t-2)\} - \underbrace{4\mathcal{L}\{(t-1)\,u(t-2)\}}_{-4\mathcal{L}\{(t-2)u(t-2)\} - 4\mathcal{L}\{u(t-2)\}}$$

$$= e^{-2t}\frac{2}{s^3} - 4\frac{e^{-2t}}{s^2} - \frac{4e^{-2t}}{s}$$

$$\therefore F(s) = \frac{2e^{-2t}}{s^3} - \frac{4e^{-2t}}{s^2} - \frac{4e^{-2t}}{s}$$

ASIGNATURA: Ecuaciones Diferenciales GRUPO: 6 Serie_núm.: 4

ALUMNO: Murrieta Villegas Alfonso FECHA: 23 / 11 / 2018

9/9

1) Obtenga una solución completa de la ecuación diferencial en derivadas parciales

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=u \qquad \underline{u(x,y) = F(x)\,G(y)}$$

Hipótesis

para una constante de separación negativa.

$\therefore \alpha < 0 \; ; \; \alpha = -k^2$

$$\frac{\partial u}{\partial x} = u - \frac{\partial u}{\partial y}$$

$$\frac{\partial u}{\partial x} = F'(x)G(y) \qquad \frac{\partial u}{\partial y} = F(x)\,G'(y)$$

$\therefore F'(x)G(y) = F(x)G(y) - F(x)G'(y)$

$\therefore F'(x)\,G(y) = F(x)G(y) - F(x)G'(y) \; ; \; F'(x)G(y) = F(x)\left[G(y) - G'(y)\right] \; ;$

$$\frac{F'(x)}{F(x)} = \frac{G(y) - G'(y)}{G(y)}$$

// Evaluando

$$\frac{F'(x)}{F(x)} = -k^2 \; ; \; F'(x) + k^2 F(x) = 0$$

$$\lambda + k^2 = 0$$

$$\lambda = -k^2$$

$$\therefore F(x) = c_1 e^{-k^2 x}$$

$$\frac{G(y) - G'(y)}{G(y)} = -k^2 \; ; \; G(y) - G'(y) + k^2 G(y) = 0$$

$$-G'(y) + (1+k^2)G(y) = 0$$

$$-\lambda + 1 + k^2 = 0$$

$$\lambda = 1 + k^2$$

$$\therefore G(y) = c_2 e^{(1+k^2)y}$$

$\therefore$ Final Expression

$$u(x,y) = \left[c_1 e^{-k^2 x}\right]\left[c_2 e^{1+k^2 y}\right]$$

2) Obtenga una solución completa de la ecuación diferencial en derivadas parciales

$$-y^2\frac{\partial u}{\partial x}+x^2\frac{\partial u}{\partial y}=0$$

Hipótesis

$$u(x,y) = F(y)\,G(y)$$

$$\frac{\partial u}{\partial x} = F'(x)G(y) \qquad \frac{\partial u}{\partial y} = F(x)G'(y)$$

para una constante de separación $\alpha = 3$

$\therefore -y^2 F'(x)\,G(y) + x^2 F(x)G'(y) = 0 \; ; \; x^2 F(x)\,G'(y) = y^2 F'(x)\,G(y) \; ;$

$$x^2 \frac{F(x)}{F'(x)} = y^2 \frac{G(y)}{G'(y)}$$

continua 2 →

$$x^2 \frac{F(x)}{F'(x)} = 3 \; ;$$

$$x^2 F(x) = 3 F'(x) ;$$

$$\int x^2 dx = \int 3 \frac{F'(x)}{F(x)} \; ; \int x^2 dx = \int 3 \frac{dF}{F}$$

$$\frac{x^3}{3} + C = 3 \ln(F(x)) ;$$

$$e^{\ln F(x)} = e^{\frac{x^3}{9} + C} ;$$

$$F(x) = C_1 e^{\frac{x^3}{9}}$$

$$y^2 \frac{G(y)}{G'(y)} = 3$$

// Lo mismo pero con $G(x)$ y y

$$\therefore \; G(x) = C_2 e^{\frac{y^3}{9}}$$

Final Expression | $u(x,y) = \left[C_1 e^{\frac{x^3}{9}}\right]\left[C_2 e^{\frac{y^3}{9}}\right]$ //

3) Determine una solución completa de la ecuación diferencial en derivadas parciales

$$x^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

considerando una constante de separación $\alpha = 0$

Hipótesis

$$u(x,y) = F(x) G(y)$$

- $\frac{\partial u}{\partial x} = F'(x) G(y)$

  $\frac{\partial^2 u}{\partial x^2} = F''(x) G(y) \leftarrow$

- $\frac{\partial u}{\partial y} = F(x) G'(y)$

  $\frac{\partial^2 u}{\partial y^2} = F(x) G''(y) \leftarrow$

$$\rightarrow x^2 F''(x) G(y) + F(x) G''(y) = 0 ;$$

$$x^2 F''(x) G(y) = -F(x) G''(y) ; \; -x^2 \frac{F''(x)}{F(x)} = \frac{G''(y)}{G(y)}$$

$$-x^2 \frac{F''(x)}{F(x)} = 0 \; ; \; -x^2 F''(x) = 0 \; ; \; F''(x) = 0$$

$$\therefore \int F''(x) = \int 0 \; ; \int F'(x) = \int C \; ; \; F(x) = Cx + C$$

$$\frac{G''(y)}{G(y)} = 0 \; ; \; G''(y) = 0$$

$$\int G''(y) = \int 0 \; ; \int G'(y) = \int C$$

$$G(y) = Cy + C$$

$$\therefore u(x,y) = [C_1 x + C_2][C_3 y + C_4]$$ //

4) Resuelva la EDDP

$$\frac{1}{a}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u \quad ; \; a = constante$$

considerando una constante de separación $\alpha = -2$

$$u(x,t) = F(x) G(t)$$

- $\frac{\partial^2 u}{\partial x^2} = F''(x) G(t)$
- $\frac{\partial u}{\partial t} = F(x) G'(t)$

$$\frac{1}{a} F(x) G'(t) = F''(x) G(t) - F(x) G(t) \; ; \; \frac{1}{a} F(x) G'(t) = [F''(x) - F(x)] G(t) ;$$

$$\frac{1}{a}\frac{G'(t)}{G(t)} = \frac{F''(x) - F(x)}{F(x)}$$

$$\therefore \frac{1}{a}\frac{G'(t)}{G(t)} = -2 ;$$

$$G'(t) + 2a\,G(t) = 0$$

$$\lambda + 2a = 0 ; \lambda = -2a$$

$$\therefore G(t) = C_1 e^{-2at}$$

$$\therefore F''(x) - F(x) = -2F(x) ;$$

$$F''(x) + \cancel{2F(x)} - \overset{F(x)}{\cancel{F x}} = 0$$

$$\lambda^2 + 1 = 0 ; \lambda^2 = -1$$

$$\therefore \lambda_{1,2} = \pm i$$

$$\therefore F(x) = C_2 \cos(x) + C_3 \operatorname{sen}(x)$$

Final Expression (English) | $u(x,t) = \left[C_1 e^{-2at}\right]\left[C_2\cos(x) + C_3 \operatorname{sen}(x)\right]$

5) Obtenga el desarrollo en términos de la serie de Fourier de la función

$$f(x) = \begin{cases} 2 & -2 \le x < 0 \\ -x+2 & 0 \le x \le 2 \end{cases}$$

$$-2 \le x \le 2 \qquad L = 2$$
$$-L \le x \le L$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + B_n \operatorname{sen}\frac{n\pi x}{L}\right)$$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx = \frac{1}{2}\left[\int_{-2}^{0} 2\,dx + \int_{0}^{2} -x+2\,dx\right] = \frac{1}{2}\left[(+4) + \left[-\frac{x^2}{2} + 2x\right]\Big|_0^2\right]$$

$$= \frac{1}{2}\left[4 + [\ 2\ ]\right] = 3$$

$$a_n = \frac{1}{2}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L} = \frac{1}{2}\left[\int_{-2}^{0} 2\cos\left(\frac{n\pi x}{2}\right) + \int_{0}^{2}(-x+2)\cos\frac{n\pi x}{2}\right] = \qquad \frac{d}{dx}\cos x = -\operatorname{se}$$

$$= \frac{1}{2}\left[\underbrace{2\frac{\operatorname{sen}\frac{n\pi x}{2}}{\frac{n\pi}{2}}\Big|_{-2}^{0}}_{0} + \left[(-x+2)\left(\frac{\operatorname{sen}\frac{n\pi x}{2}}{\frac{n\pi}{2}}\right) - \frac{\cos\frac{n\pi x}{2}}{n^2\frac{\pi^2}{4}}\right]\Big|_0^2\right]$$

$+$	$-x+2$	$\cos\frac{n\pi x}{2}$
$-$	-1	$\frac{\operatorname{sen}\frac{n\pi x}{2}}{n\frac{\pi}{2}}$
$+$	0	$-\frac{\cos\frac{n\pi x}{2}}{n^2\frac{\pi^2}{4}}$

$$= \left[0 - \frac{\overset{(-1)^n}{\cancel{\cos\frac{n\pi 2}{2}}}}{n^2\frac{\pi^2}{4}}\right] - \left[0 - \frac{\overset{1}{\cancel{\cos\frac{n\pi(0)}{2}}}}{n^2\frac{\pi^2}{4}}\right] =$$

$$= -\frac{(-1)^n}{n^2\frac{\pi^2}{4}} + \frac{1}{n^2\frac{\pi^2}{4}} = \frac{1}{2}\left[\frac{1-(-1)^n}{\left(\frac{n\pi}{2}\right)^2}\right]$$

$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\,\text{sen}\left(\frac{n\pi x}{L}\right)dx = \frac{1}{2}\left[\int_{-2}^{0} 2\,\text{sen}\left(n\frac{\pi}{2}x\right)dx + \int_{0}(-x+2)\,\text{sen}(n\right.$

$= \frac{1}{2}\left[\frac{-2}{n\frac{\pi}{2}}\cos\left(n\frac{\pi}{2}x\right)\Big|_{-2}^{0} + \left[(-x+2)\left(-\frac{\cos n\frac{\pi}{2}x}{n\frac{\pi}{2}}\right) - \frac{\text{sen}\, n\frac{\pi}{2}x}{\left(n\frac{\pi}{2}\right)^2}\right]\Big|_{0}^{2}\right.$

(+)	$-x+2$	$\text{sen}\left(n\frac{\pi}{2}x\right)$
(−)	-1	$-\frac{\cos\left(n\frac{\pi}{2}x\right)}{n\frac{\pi}{2}}$
	0	$-\frac{\text{sen}\left(n\frac{\pi}{2}x\right)}{\left(n\frac{\pi}{2}\right)^2}$

$= \frac{1}{2}\left[\left[\frac{-2}{n\frac{\pi}{2}} - \frac{-2}{n\frac{\pi}{2}}(-1)^n\right] + [0-0] - \left[2\left(-\frac{1}{n\frac{\pi}{2}}\right) - 0\right]\right] =$

$= \frac{1}{2}\left[\cancel{\frac{-2}{n\frac{\pi}{2}}} + \frac{2}{n\frac{\pi}{2}}(-1)^n + \cancel{\frac{2}{n\frac{\pi}{2}}}\right] = \frac{1}{n\frac{\pi}{2}}(-1)^n$

NOTES

$\text{sen}(n\pi) = 0$

$\cos(n\pi) = (-1)^n$

$\cancel{\cos[n\pi(0)]} \to 1$

$\cancel{\text{sen}[n\pi(0)]} \to 0$

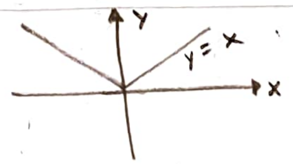
$\frac{d}{dx}\cos x = -\text{sen}\, x$

$\therefore$ Serie T. Fourier

$F(x) = \frac{3}{2} + \sum_{n=1}^{\infty}\left(\left[\left(\frac{1}{2}\right)\left(\frac{1-(-1)^n}{\left(n\frac{\pi}{2}\right)^2}\right)\cos\left(\frac{n\pi x}{2}\right)\right] + \left[\left(\frac{1}{n\frac{\pi}{2}}(-1)^n\right)\left(\text{sen}\,\frac{n\pi x}{2}\right)\right]\right)$

6) Obtenga la serie trigonométrica de Fourier de la función $f(x) = |x|$ en el intervalo $-\pi < x < \pi$

• $F(x) = |x|$



; $F(x) = |x| = \begin{cases} -x \;; & -\pi \le x < 0 \\ x \;; & 0 \le x \le \pi \end{cases}$

• Al ser par ; $b_n = 0$; Coseno ; Respecto a "y" (Simétrica)

$-L \le x \le L$

$-\pi < x < \pi$

$\therefore L = \pi$

▸ $a_0 = \frac{2}{\pi}\int_0^{\pi} x\,dx \;;\; \frac{2}{\pi}\frac{x^2}{2}\Big|_0^{\pi} = \frac{\pi^2}{\pi} = \pi$

▸ $a_n = \frac{2}{L}\int_0^{L} F(x)\cos\left(\frac{n\pi x}{L}\right)dx = \frac{2}{\pi}\int_0^{\pi} x\cos(nx)\,dx =$

(+)	x	$\cos(nx)$
(−)	1	$\frac{1}{n}\,\text{sen}(nx)$
	0	$-\frac{1}{n^2}\cos(nx)$

$= \frac{2}{\pi}\left[(x)\left(\frac{1}{n}\,\text{sen}(nx)\right) + \frac{1}{n^2}\cos(nx)\right]\Big|_0^{\pi} =$

$= \frac{2}{\pi}\left[(\pi)(\;\to 0) + \frac{1}{n^2}(-1)^n\right] - \left[0 + \frac{1}{n^2}(1)\right] = \frac{2}{\pi}\left[\frac{1}{n^2}(-1)^n - \frac{1}{n^2}\right] =$

$= \frac{2}{\pi n^2}\left((-1)^n - 1\right)$

$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) \right) \leftarrow \text{Par}$$

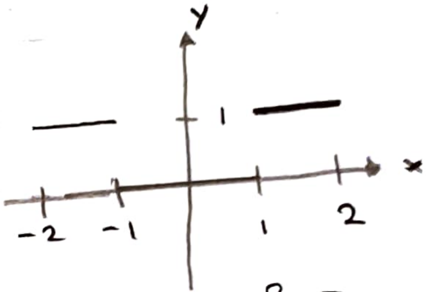
$$= \frac{\pi}{2} + \sum_{n=1}^{\infty} \left[\left(\frac{2}{\pi n^2} \left((-1)^n - 1 \right) \right) \cos(nx) \right] //$$

7) Desarrolle la función

$$f(x) = \begin{cases} 1, & -2 < x < -1 \\ 0, & -1 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}$$

en términos de la serie trigonométrica de Fourier y aproxime la función considerando los cinco primeros términos no nulos.

Bosquejo



Función Par

∴ coseno

$b_n = 0$ //

$$a_0 = \frac{2}{L} \int_0^L f(x)\,dx = \left[\int_0^1 0\,dx + \int_1^2 1\,dx \right] = 1 //$$

$$a_n = \left[\frac{2}{L} \int_0^L f(x) \cos\left(\frac{n\pi x}{L}\right) dx \right] = \cancel{\int_0^1 (0)\left(\cos\left(\frac{n\pi x}{2}\right)\right) dx} \to 0 \; +$$

$$\int_1^2 \cos \frac{n\pi x}{2}\,dx = \frac{1}{n\frac{\pi}{2}} \operatorname{sen}\left(\frac{n\pi x}{2}\right)\Big|_1^2 =$$

$$= \frac{1}{n\frac{\pi}{2}} \cancel{\operatorname{sen}\left(n\frac{2\pi}{2}\right)} \to 0 \; - \frac{1}{n\frac{\pi}{2}} \operatorname{sen}\left(n\frac{\pi}{2}\right) /$$

$-2 \le x \le 2$

$-L \le x \le L$

$L = 2$ /

¿?

$\operatorname{sen}\left(n\frac{\pi}{2}\right) \to$ impar $(-1)^n$

$\to$ par 0

∴ Serie T. Fourier

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \left(-\cancel{\frac{1}{n\frac{\pi}{2}}}^{\,-\frac{2}{n\pi}} \operatorname{sen} n\frac{\pi}{2} \right) \left(\cos n\frac{\pi}{2}x \right) //$$

// Aproximación de la Función (No nulos) $n=1$, $n=3$, $n=5$, $n=7$, $n=9$
$-$ $+$ $-$ $+$ $-$
senos

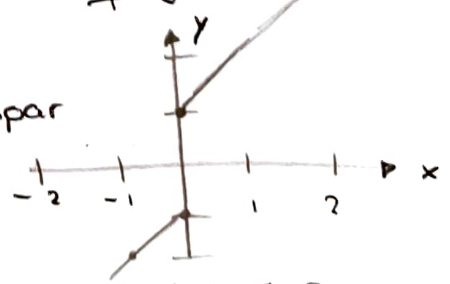
$$\frac{1}{2} + \left(-\frac{2}{\pi} \cos\left(\frac{\pi}{2}x\right) \right) + \left(\frac{2}{3\pi} \cos\left(\frac{3\pi}{2}x\right) \right) +$$

$$\left(-\frac{2}{5\pi} \cos\left(\frac{5\pi}{2}x\right) \right) + \left(\frac{2}{7\pi} \cos\left(\frac{7\pi}{2}x\right) \right) + \left(-\frac{2}{9\pi} \cos\left(\frac{9\pi}{2}x\right) \right) //$$

8) Obtenga el desarrollo en serie de Fourier de la función

$$f(x) = \begin{cases} x - 1, & -2 < x < 0 \\ x + 1, & 0 \le x < 2 \end{cases}$$

Bosquejo



∴ Función = Impar

∴ Seno

∴ $a_0 = 0$

$a_n = 0$

$-2 \le x \le 2$

$L = 2$ /

$$b_n = \frac{2}{L} \int_0^L f(x) \operatorname{sen}\left(\frac{n\pi x}{L}\right) dx =$$

$$= \int_0^2 (x+1) \operatorname{sen}\left(\frac{n\pi x}{2}\right) dx =$$

$$= \int_0^2 (x+1)\,\text{sen}\left(\frac{n\pi x}{2}\right)dx =$$

(+) $x+1$ $\quad$ $\text{sen}\,\frac{n\pi x}{2}$

(−) 1 $\quad$ $-\frac{1}{(n\frac{\pi}{2})}\cos\frac{n\pi x}{2}$

(+) 0 $\quad$ $-\frac{1}{(n\frac{\pi}{2})^2}\text{sen}\,\frac{n\pi x}{2}$

$$= (x+1)\left(-\frac{1}{(n\frac{\pi}{2})}\cos\left(\frac{n\pi x}{2}\right)\right) + \frac{1}{(n\frac{\pi}{2})^2}\,\text{sen}\left(\frac{n\pi x}{2}\right)\Big|_0^2 =$$

$$= \left[(3)\left(-\frac{1}{n\frac{\pi}{2}}\cancel{\cos(n\pi)}^{(-1)^n}\right) + \frac{1}{(n\frac{\pi}{2})^2}\cancel{\text{sen}(n\pi)}^{0}\right]$$

$$- \left[(1)\left(-\frac{1}{(n\frac{\pi}{2})}\cancel{\cos(n\pi 0)}^{1}\right) + 0\right] =$$

$$= (3)\left(-\frac{1}{n\frac{\pi}{2}}(-1)^n\right) + \frac{1}{n\frac{\pi}{2}} = \frac{1-3(-1)^n}{n\frac{\pi}{2}} \;//$$

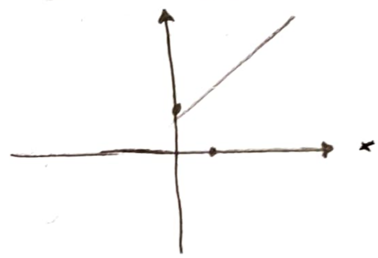
∴ Serie T. Fourier | $f(x) = \left(\frac{1-3(-1)^n}{n\frac{\pi}{2}}\right)\left(\text{seno}\left(\frac{\pi}{2}nx\right)\right)$ //

9) Calcule la serie de senos de Fourier de la función

$$f(x) = 1 + x \;,\; 0 \le x \le 1$$

Además, grafique la función aproximada en términos de la serie indicada, considerando los 7 primeros términos no nulos. (Emplear algún programa de cómputo).

Bosquejo



Serie Senos

∴ Impar (simétrica respecto x)

∴ $a_0 = 0 \quad a_n = 0$

$$\cancel{a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx \;;\; a_0 = 2\int_0^2 x+1\,dx = 2\left[\frac{x^2}{2}+x\right]\Big|_0^2 =}$$

$$\cancel{= 2[(4) - 0] = 8 \;//}$$

$f_e(x) \quad -1 \le x \le 1 \;;\; L = 1$ /

$$\therefore\; b_n = \frac{2}{L}\int_0^L f(x)\,\text{sen}\left(\frac{n\pi x}{L}\right) = 2\int_0^1 (x+1)\,\text{sen}(n\pi x) =$$

(+) $x+1$ $\quad$ $\text{sen}\,n\pi x$

(−) 1 $\quad$ $-\frac{1}{n\pi}\cos n\pi x$

(+) 0 $\quad$ $-\frac{1}{(n\pi)^2}\text{sen}\,n\pi x$

$$= 2\left[(x+1)\left(-\frac{1}{n\pi}\cos n\pi x\right) + \frac{1}{(n\pi)^2}\text{sen}\,n\pi x\right]\Big|_0^1 =$$

$$= 2\left[(2)\left(-\frac{1}{n\pi}\cancel{\cos n\pi}^{(-1)^n}\right) + \frac{1}{(n\pi)^2}\cancel{\text{sen}(n\pi)}^{0}\right] -$$

$$\left[-\frac{1}{n\pi}\cancel{\cos(n\pi 0)}^{1} + 0\right] = -\frac{2}{n\pi}(-1)^n + \frac{1}{n\pi} =$$

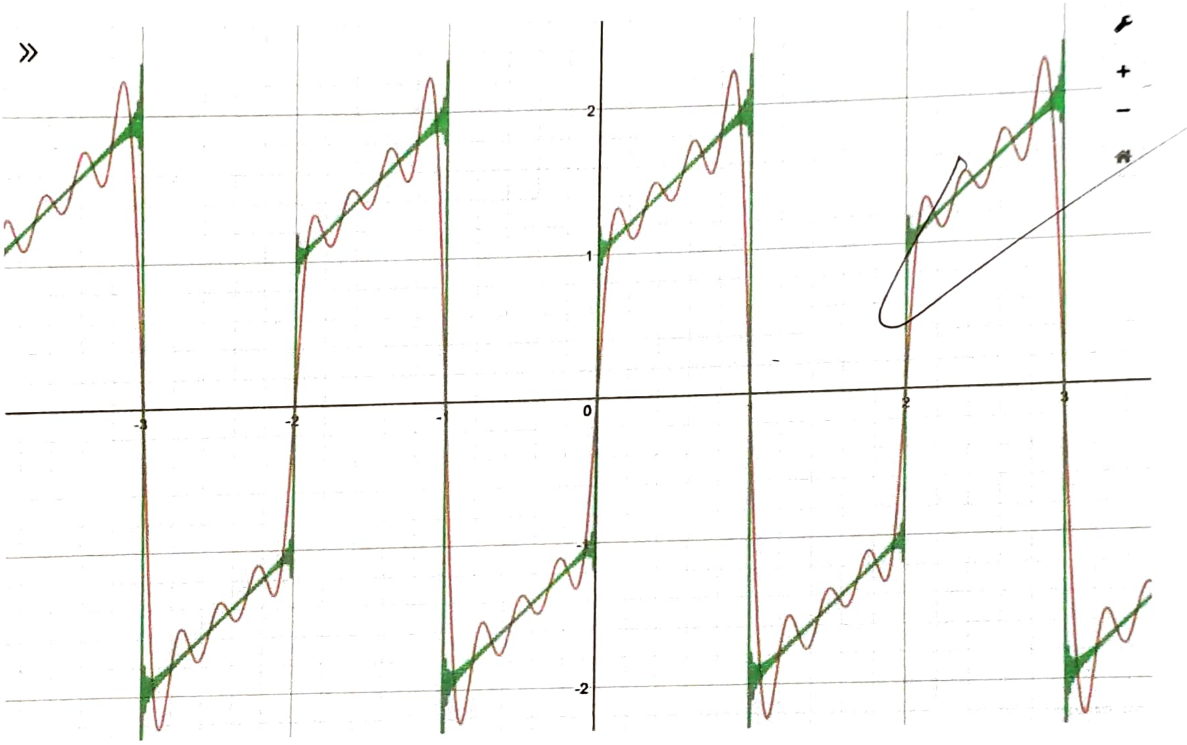
$$= 2\left[\frac{1-2(-1)^n}{n\pi}\right] //$$

∴ S.F Fourier | $f(x) = \sum_{n=1}^{\infty} 2\left(\frac{1-2(-1)^n}{n\pi}\right)(\text{sen}\,n\pi x)$

Alumno: Murrieta Villegas Alfonso

Serie de SENOS DE FOURIER :

F(x)=$\sum_{x=1}^{7}\left(\frac{1-2(-1)^{n}}{\pi n}\right)(\sin \pi n x)$



$$f(x) = \left(\frac{1-2(-1) \to 3}{\pi}\right)(\operatorname{sen} \pi x) + \left(\frac{-1}{2\pi}\right)(\operatorname{sen} 2\pi x) + \left(\frac{3}{\pi 3}\right)(\operatorname{sen} 3\pi x) + \left(\frac{-1}{\pi 4}\right)(\operatorname{sen} 4\pi x)$$

$$+ \left(\frac{3}{\pi 5}\right)(\operatorname{sen} 5\pi x) + \left(\frac{-1}{\pi 6}\right)(\operatorname{sen} 6\pi x) + \left(\frac{3}{7\pi}\right)(\operatorname{sen} 7\pi x)$$