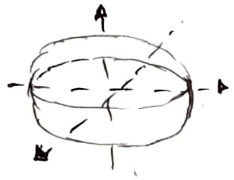
Formulario

- Elipsoide

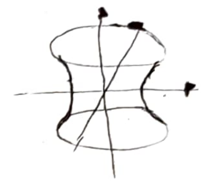
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}+\frac{(z-L)^2}{c^2}=1$$



- Hiperboloide

O eje

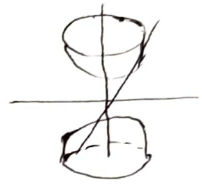
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}-\frac{(z-L)^2}{c^2}=1$$



- Hiperboloide (2 Hojas)

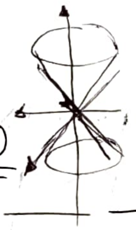
$$+\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}-\frac{(z-L)^2}{c^2}=1$$

eje



- Cono Elíptico

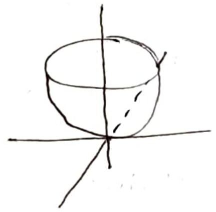
$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}-\frac{(z-L)^2}{c^2}=0$$



- Paraboloide Elíptico

$$\frac{(x-h)^2}{a^2}+\frac{(y-h)^2}{b^2}=c(z-L)$$

Eje 1° potencia



- Paraboloide Hiperbólico

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=c(z-L)$$

jinete → término lineal
cuadrático (+) → Piernas
(−) → Caballo

- Esfera

$$(x-h)^2+(y-k)^2+(z-L)^2=r^2$$

Al revés el desplazamiento

▶ Volumen y Superficie

▶ Esfera

$$V=\frac{4\pi}{3}r^3$$

$$A=(\text{Supeficie})=4\pi r^2$$

▶ Cónicas

- Elipse $\left\{ \frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=r^2 \right.$

circun a=b

- Parábola $(x-k)^2=4p(y-k)$

Abr en y — sentido

| DERIVADAS

- $f'(x)=uv'+u'v$
- $f'(x)=\frac{vu'-v'u}{v^2}$
- $\frac{d}{dx}[F(x)]^n=n[F(x)]^{n-1}[F'(x)]$
- $f(v)=e^v;\ f'(v)=e^v\cdot v'$
- $f(v)=a^v;\ f'(v)=a^v\cdot v'\cdot\ln a$
- $f(v)=\ln v,\ f'(v)=\frac{1}{v}\cdot v'$ | $f(x)=\log_a u=\frac{1}{u\ln a}\cdot u'$

$\frac{d}{dx}\operatorname{sen}(u)=\cos(u)\frac{du}{dx}$

$\frac{d}{dx}\cos(u)=-\operatorname{sen}u\frac{du}{dx}$

$\frac{d}{dx}\tan(u)=\sec^2u\cdot\frac{du}{dx}$

$\cot u=-\csc^2 u$

$\csc u=-\csc u\cdot\cot u$

▶ Max/Min → 2 variables

① Derivadas parciales
② Igualar a 0 → Encontrar puntos críticos | • Todas sus combinaciones

② Determinante Hessiano | $\Delta = (f_{xx})(f_{yy}) - (f_{xy})^2$ ← No olvidar

③ Evaluar puntos | $\Delta > 0$; se evalua en "f_{xx}" o "f_{yy}" → <0; Máximo relativo; >0; Mínimo relativo
$\Delta = 0$; Criterio no decide
$\Delta < 0$; Punto Silla

▶ Ma/Min → 3 variables

① Derivadas Parciales
② Igualar a 0 (Puntos Críticos)
③ Matriz Hessiana; ECUACION con (λ)

$$H = \begin{bmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{bmatrix}$$

④ Evaluar punto por punto en Matriz

Casos |
λ; Signos diferente = Punto Silla
λ; Signos Positivos = Mínimo
λ; Signos Negativos = Máximo
λ; 0 = se indetermina

• Dentro {
• Frontera { Condición { Dentro intervalo {, Extremos { ; Función Restricción {

④ Resumir

▶ Lagrange

① $\mathcal{L}(\quad) = \text{Función Objetivo} + \lambda(\text{Función Restricción})$

Función Objetivo ← La que se óptimizará (Leer bien)
Función Restricción ← Tiene forma de ecuación

② Si se tiene más de una restricción $\mathcal{L}(\quad) = \text{Función objetivo} + \lambda_1(\text{Función } R_1) + \lambda_2(F.R_2)$

▶ Max y Min Absolutos

① Demostrar y representar | ① Cerrada ② Acotada ③ Continua

② Dentro de R
• Derivadas parciales función
• Igualar 0 | Puntos Críticos
• Evaluar en función (Valor)

③ En la frontera
// Condición n (Restricción) → (De la Faltante)
- Dentro intervalo (Parciales y evaluar)
- Extremos intervalo (Solo evaluar)
// Función Restricción
- Hacer Lagrange
↳ Puntos y evaluar [Función Objetivo]

④ Resumir y Concluir

• Murrieta Villegas Alfonso
• Cálculo Vectorial

▶ Ecuación Vectorial Curva

• Curva $\mathbb{R}^3$ | $\bar{f}(t) = f_1(t)\hat{i} + f_2(t)\hat{j} + f_3(t)\hat{k}$

• Paramétrica | $x = f_1(t)$, $y = f_2(t)$, $z = f_3(t)$

• Cartesianas | $F(x,y,z)$, $G(x,y,z)$

▶ Longitud arco

$$S = \int_a^b \sqrt{[f_1'(t)]^2 + [f_2'(t)]^2 + [f_3'(t)]^2}$$

$$= \int_a^b \|\bar{f}'(t)\|\, dt$$

* $\bar{f}'(s)$ es unitario y tangente

▶ $\bar{T} = \dfrac{\bar{f}'(t)}{\|\bar{f}'(t)\|}$; $\dfrac{ds}{dt} = \|\bar{f}'(t)\| = \text{rapidez} = \|\bar{v}(t)\| = v(t)$

$\hookrightarrow \bar{v}(t) = v(t)\,\bar{T}$

▶ $\bar{N} = \dfrac{[\bar{f}'(t) \times \bar{f}''(t)] \times [\bar{f}'(t)]}{\|[\bar{f}'(t) \times \bar{f}''(t)] \times [\bar{f}'(t)]\|}$

▶ $\bar{N} = \dfrac{\frac{d\bar{T}}{dt}}{\left\|\frac{d\bar{T}}{dt}\right\|}$

▶ $\bar{B} = \bar{T} \times \bar{N}$

▶ $\bar{N} = \bar{B} \times \bar{T}$

▶ $\bar{B} = \dfrac{\bar{f}'(t) \times \bar{f}''(t)}{\|\bar{f}'(t) \times \bar{f}''(t)\|}$

• $\bar{a} = \bar{v}'(t) = \bar{f}''(t)$; $\bar{a} = \dfrac{dv}{dt}\bar{T} + k v^2 \bar{N}$

$\rightarrow \bar{a}_T = \dfrac{dv}{dt}\bar{T}$ $\rightarrow \bar{a}_N = k v^2 \bar{N}$

$\hookrightarrow a_T = \dfrac{dv}{dt}$ $\hookrightarrow a_N = k v^2$

$\hookrightarrow a_T = \dfrac{\bar{v} \cdot \bar{a}}{v}$ $\hookrightarrow a_N = \dfrac{\|\bar{v} \times \bar{a}\|}{v}$

$\left[\dfrac{\text{longitud}}{(\text{Tiempo})^2}\right]$

$K = \dfrac{\left\|\frac{d\bar{T}}{dt}\right\|}{v(t)}$ $\left[\dfrac{1}{\text{longitud}}\right]$

$K = \dfrac{\|\bar{v} \times \bar{a}\|}{v^3}$

$K = \dfrac{1}{\rho}$

* ρ = radio de curvatura

$\rightarrow \bar{a}_N = \bar{a} - \left[\dfrac{\bar{v} \cdot \bar{a}}{v}\,\dfrac{\bar{v}}{v}\right]$; $\bar{f}''(t) - \dfrac{\bar{f}'(t) \cdot \bar{f}''(t)}{\|\bar{f}'(t)\|}\,\dfrac{\bar{f}'(t)}{\|\bar{f}'(t)\|}$

• $\frac{d\bar{T}}{ds} = k\bar{N}$ Frenet Serret

• $\frac{d\bar{B}}{ds} = \tau\bar{N}$

• $\frac{d\bar{N}}{ds} = -(k\bar{T}+\tau\bar{B})$

$$\tau = -\frac{\bar{f}'(t)\times\bar{f}''(t)\cdot\bar{f}'''(t)}{\|\bar{f}'(t)\times\bar{f}''(t)\|}$$

$\sigma = \frac{1}{\tau}$ | Radio de Torsión

$$k(t) = \frac{f_1'(t)\,f_2''(t) - f_1''(t)\,f_2'(t)}{\left([f_1'(t)]^2 + [f_2'(t)]^2\right)^{3/2}}$$

$\tau =$ curva plana $= 0$

$\tau > 0$ T.R.D

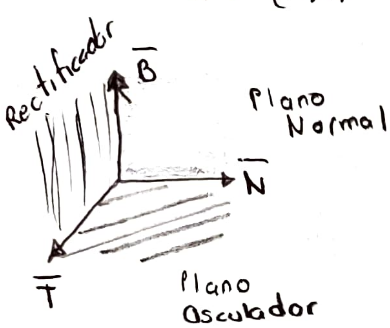
$\tau < 0$ T.R.I

▶ Longitud de Arco

• $\bar{f}'(s) = \bar{T}$

• $\bar{B} = \bar{T}\times\bar{N}$

• $\bar{N} = \frac{\bar{f}''(s)}{\|\bar{f}''(s)\|}$



▶ Frenet – Serret

• $k\bar{N} = \bar{f}''(s)$; $k\bar{N} = \frac{d\bar{T}}{ds}$ • $k = \|\bar{f}''(s)\|$

• $\bar{N} = \frac{\bar{f}''(s)}{\|\bar{f}''(s)\|}$

• $\tau = -\frac{\bar{f}'(s)\times\bar{f}''(s)\cdot\bar{f}'''(s)}{\|\bar{f}''(s)\|^2}$

▶ Función vectorial

$\bar{f}(x,y) = \bar{f}_1\hat{i} + \bar{f}_2\hat{j}$

$\bar{f}_x = \frac{\partial\bar{f}}{\partial x} =$ | Parciales

$\bar{f}_y = \frac{\partial\bar{f}}{\partial y} =$

▶ $\frac{d}{dt}\left[(\bar{\phi}t)(\bar{F}(t))\right] = \bar{\phi}'(t)\bar{F}(t) + \bar{\phi}\,t\,\bar{F}'(t)$

▶ $\frac{d}{dt}\left[\bar{f}(t)\times\bar{g}(t)\right] = \bar{f}(t)\times\bar{g}'(t) + \bar{f}'(t)\times\bar{g}(t)$

$* \ \bar{C}(t) = \bar{r}(t) + \varphi(t)\,\bar{N}(t)$

• $\bar{a}_T(t) = \frac{\bar{r}'(t)\cdot\bar{r}''(t)}{\|\bar{r}'(t)\|}\bar{T}(t)$

• $\bar{a}_N(t) = \bar{a}(t) - \bar{a}_T(t)$

• $\cos\theta = \frac{\bar{r}\ \ \bar{r}'(t)}{\|\bar{r}\|\|\bar{r}'(t)\|}$

▶ Formas Expresión

- $\bar{f}(u,v) = f_1(u,v)\hat{i} + f_2(u,v)\hat{j} + f_3(u,v)\hat{k}$ ← $\bar{F}(u,v) = f_1(u,v)\hat{i} \dots$

- $S: \begin{cases} x = f_1(u,v) \\ y = f_2(u,v) \\ z = f_3(u,v) \end{cases}$ - $F(x,y,z) = 0$

$\|\bar{N}\| = \sqrt{(\ )^2 + (\ )^2 + (\ )^2}$

▶ Vector Normal

$\bar{N}_s = \dfrac{\partial \bar{F}}{\partial u} \times \dfrac{\partial \bar{F}}{\partial v} = \bar{\nabla}$ | vector gradiente

▶ Ecuación Normal

$[\bar{P} - \bar{P}_0] \cdot \bar{N}_s = 0$

$df(\bar{x}) : \begin{bmatrix} \ & \ & \ \end{bmatrix} \begin{bmatrix} dx \\ dy \\ \vdots \\ dm \end{bmatrix} = \left[\dfrac{\partial f}{\partial x}\right] dx + \dots +$

$\uparrow \dfrac{\partial \bar{F}}{\partial x} \quad \uparrow \dfrac{\partial \bar{F}}{\partial y} \dots \uparrow \dfrac{\partial \bar{F}}{\partial m}$

// Campo Vectorial

$d f(t) = \dfrac{d \bar{F}}{d t} dt$ | curva espacio

$d\bar{f}(u,v) = \begin{bmatrix} \dfrac{\partial f_1}{\partial u} & \dfrac{\partial f_1}{\partial v} \\ \dfrac{\partial f_2}{\partial u} & \vdots \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} du \\ dv \end{bmatrix} = \dfrac{\partial \bar{F}}{\partial u} du + \dfrac{\partial \bar{F}}{\partial v} dv$

// Superficie en el espacio

$T: \begin{cases} x = x(u,v) \\ y = y(u,v) \end{cases}$ • $\bar{F}(u,v) = x(u,v)\hat{i} + y(u,v)\hat{j}$

• $\bar{x} = (u,v)$

$\dfrac{d F}{d \bar{x}} = \begin{bmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{bmatrix}$; $J\left(\dfrac{x,y}{u,v}\right) = \dfrac{\partial(x,y)}{\partial(u,v)}$

// Puntos singulares $\triangleq$ Jacobiano se anula

▶ Propiedades Jacobiano

• $\dfrac{\partial(x,y)}{\partial(u,v)} \cdot \dfrac{\partial(u,v)}{\partial(x,y)} = 1$

• $\dfrac{A.R}{A.R'} = \left|\dfrac{\partial(x,y)}{\partial(u,v)}\right|$

• $J\left(\dfrac{x,y}{u,v}\right) \neq 0 \therefore \exists\, T^{-1}$ existe

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$T: \begin{cases} x = x(u,v,w) \\ y = y(u,v,w) \\ z = z(u,v,w) \end{cases}$ $T^{-1}: \begin{cases} u = u(x,y,z) \\ v = v(x,y,z) \\ w = w(x,y,z) \end{cases}$

• V. Posición | $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k} = x(u,v,w) + y(u,v,w) + \dots$

• Diferencial | $d\bar{r} = \dfrac{\partial \bar{r}}{\partial u} du + \dfrac{\partial \bar{r}}{\partial v} dv + \dfrac{\partial \bar{r}}{\partial w} dw$

• F. Escala | $h_u = \left\|\dfrac{\partial \bar{r}}{\partial u}\right\|$ $h_v = \left\|\dfrac{\partial \bar{r}}{\partial v}\right\|$ $h_w = \dots$

• V.N. (unitarios) | $\bar{e}_u = \dfrac{\frac{\partial \bar{r}}{\partial u}}{h_u}$ $\bar{e}_v = \dfrac{\frac{\partial \bar{r}}{\partial v}}{\left\|\frac{\partial \bar{r}}{\partial v}\right\|}$ $\bar{e}_w = \dots$

• Ortogonal | $\bar{e}_u \cdot \bar{e}_v \cdot \bar{e}_w = 0$ | $\dfrac{\partial \bar{r}}{\partial u} \cdot \dfrac{\partial \bar{r}}{\partial v} = 0$

Ortogonal

$\bar{e}_u \cdot \bar{e}_v = 0$; $\dfrac{\bar{\nabla} u}{\|\bar{\nabla} u\|} \cdot \dfrac{\bar{\nabla} v}{\|\bar{\nabla} v\|} = 0$

$\bar{\nabla} u = \left(\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y} \dots\right)$

$\bar{\nabla} v = \left(\dfrac{\partial v}{\partial x}, \dfrac{\partial v}{\partial y} \dots\right)$

→ $\bar{\nabla} u \cdot \bar{\nabla} v = 0$

$h_u = \dfrac{1}{\|\bar{\nabla} u\|}$ $h_v = \dfrac{1}{\|\bar{\nabla} v\|}$ $h_w =$

// Ángulo

$\cos\theta = \dfrac{\bar{N}_1 \cdot \bar{N}_2}{\|\bar{N}_1\| \|\bar{N}_2\|}$

C. Polares

1] $T: \begin{cases} x = \rho \cos\theta \\ y = \rho \operatorname{sen}\theta \end{cases}$ $\quad T^{-1}: \begin{cases} \rho = \sqrt{x^2+y^2} \\ \theta = \operatorname{angtan}\left(\frac{y}{x}\right) \end{cases}$

2] Factores Escala | $h_\rho = 1 \quad h_\theta = \rho$

3] V. Base | $\bar{e}_\rho = \cos\theta\, \hat{i} + \operatorname{sen}\theta\, \hat{j}$
$\bar{e}_\theta = -\operatorname{sen}\theta\, \hat{i} + \cos\theta\, \hat{j}$

// Relación V. Bases | $\hat{i} = \cos\theta\, \bar{e}_\rho - \operatorname{sen}\theta\, \bar{e}_\theta$
$\hat{j} = \operatorname{sen}\theta\, \bar{e}_\rho + \cos\theta\, \bar{e}_\theta$

4] V. Posición | $\bar{r} = \rho\, \bar{e}_\rho$

5] $d\bar{r} = d\rho\, \bar{e}_\rho + \rho\, d\theta\, \bar{e}_\theta$

6] Arco | $ds^2 = (d\rho)^2 + (\rho\, d\theta)^2$

7] Jaco | $J\left(\frac{x,y}{\rho,\theta}\right) = \rho$

8] $dA = \rho\, d\rho\, d\theta$

Cilíndricas

$T: \begin{cases} x = \rho\cos\theta \\ y = \rho \operatorname{sen}\theta \\ z = z \end{cases}$ $\quad T^{-1}: \begin{cases} \rho = \sqrt{x^2+y^2} \\ \theta = \operatorname{angtan}\left(\frac{x}{y}\right) \\ z = z \end{cases}$

$$\begin{bmatrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\operatorname{sen}\theta & 0 \\ \operatorname{sen}\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{e}_\rho \\ \bar{e}_\theta \\ \bar{e}_z \end{bmatrix} \qquad \bar{e}_\rho, \bar{e}_\theta, \bar{e}_z$$

7] $J\left(\frac{x,y,z}{\rho,\theta,z}\right) = \begin{vmatrix} \cos\theta & -\rho\operatorname{sen}\theta & 0 \\ \operatorname{sen}\theta & \rho\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = \rho$

2] $h_\rho = 1 \quad h_\theta = \rho \quad h_z = 1$

3] $\bar{e}_\rho = \cos\theta\, \hat{i} + \operatorname{sen}\theta\, \hat{j}$
$\bar{e}_\theta = -\operatorname{sen}\theta\, \hat{i} + \cos\theta\, \hat{j}$
$\bar{e}_z = \hat{k}$

4] $\bar{r} = \rho\, \bar{e}_\rho + z\, \bar{e}_z$

5] $d\bar{r} = d\rho\, \bar{e}_\rho + \rho\, d\theta\, \bar{e}_\theta + dz\, \bar{e}_z$

6] $ds^2 = (d\rho)^2 + (\rho\, d\theta)^2 + (dz)^2$

8] $dV = \rho\, d\rho\, d\theta\, dz$

Esféricas

$T: \begin{cases} x = \rho \operatorname{sen}\phi \cos\theta \\ y = \rho \operatorname{sen}\phi \operatorname{sen}\theta \\ z = \rho\cos\phi \end{cases}$ $\quad T^{-1}: \begin{cases} \rho = \sqrt{x^2+y^2+z^2} \\ \phi = \operatorname{angcos} \frac{z}{\sqrt{x^2+y^2+z^2}} \\ \theta = \operatorname{angtan}\left(\frac{y}{x}\right) \end{cases}$

7] $J\left(\frac{x,y,z}{\rho,\phi,\theta}\right) = \begin{vmatrix} \operatorname{sen}\phi\cos\theta & \rho\cos\phi\cos\theta & -\rho\operatorname{sen}\phi\operatorname{sen}\theta \\ \operatorname{sen}\phi \operatorname{sen} & \rho\cos\phi\operatorname{sen}\theta & \rho\operatorname{sen}\phi\cos\theta \\ \cos\phi & -\rho\operatorname{sen}\phi & 0 \end{vmatrix} = \rho^2 \operatorname{sen}\phi$

2] $h_\rho = 1 \quad h_\phi = \rho \quad h_\theta = \rho\operatorname{sen}\phi$

3] $\bar{e}_\rho = \operatorname{sen}\phi\cos\theta\, \hat{i} + \operatorname{sen}\phi\operatorname{sen}\theta\, \hat{j} + \cos\phi\, \hat{k}$
$\bar{e}_\phi = \cos\phi\cos\theta + \cos\phi\operatorname{sen}\theta\, \hat{j} - \operatorname{sen}\phi\, \hat{k}$
$\bar{e}_\theta = -\operatorname{sen}\theta\, \hat{i} + \cos\theta\, \hat{j}$

4] $\bar{r} = \rho\, \bar{e}_\rho$ $\qquad$ 5] $d\bar{r} = d\rho\, \bar{e}_\rho + \rho\, d\phi\, \bar{e}_\phi + \rho\operatorname{sen}\phi\, d\theta\, \bar{e}_\theta$

6] $ds^2 = (d\rho)^2 + (\rho\, d\phi)^2 + (\rho\operatorname{sen}\phi\, d\theta)^2$

8] $dV = \rho^2 \operatorname{sen}\phi\, d\rho\, d\phi\, d\theta$

- $\bar{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$
- $\bar{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$

▸ $\operatorname{div}\bar{F} = \bar{\nabla}\cdot\bar{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$

>0 Flujo saliente, Punto fuente

<0 entrante, sumidero

$\bar{\nabla}\cdot\bar{F} = 0$ // campo solenoidal

▸ $\operatorname{rot}\bar{F} = \bar{\nabla}\times\bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix} = \left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right)\hat{i} - \left(\frac{\partial f_3}{\partial x} - \frac{\partial f_2}{\partial z}\right)\hat{j} + \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)\hat{k}$

// $\bar{\nabla}\times\bar{F} = 0$ C. Irrotac

▸ Laplaciano $\nabla\cdot\nabla f = \nabla^2 f = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial z}\right)$

$\nabla^2 f = 0$ // Función armónica

- $\bar{\nabla} f = \frac{1}{h_u}\frac{\partial f}{\partial u}\bar{e}_u + \frac{1}{h_v}\frac{\partial f}{\partial v}\bar{e}_v + \frac{1}{h_w}\frac{\partial f}{\partial w}\bar{e}_w$
- $\bar{F} = F_u\bar{e}_u + F_v\bar{e}_v + F_w\bar{e}_w$

▸ $\operatorname{div} = \bar{\nabla}\cdot\bar{F} = \frac{1}{h_u h_v h_w}\left[\frac{\partial}{\partial u}(h_v h_w f_u) + \frac{\partial}{\partial v}(h_u h_w f_v) + \frac{\partial}{\partial w}(h_u h_v f_w)\right]$

▸ $\operatorname{Rot} = \bar{\nabla}\times\bar{F} = \frac{1}{h_u h_v h_w}\begin{vmatrix} h_u\bar{e}_u & h_v\bar{e}_v & h_w\bar{e}_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_u f_u & h_v f_v & h_w f_w \end{vmatrix}$

▸ Laplaciano $= \frac{1}{h_u h_v h_w}\left[\frac{\partial}{\partial u}\left(\frac{h_v h_w}{h_u}\frac{\partial f}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_u h_w}{h_v}\frac{\partial f}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_u h_v}{h_w}\frac{\partial f}{\partial w}\right)\right]$

L.A

▶ $\int_C f(x,y)\,dS \quad C:\begin{cases} x = x(t) \\ y = y(t) \end{cases} ; \ a \le t \le b$; $\bar{r} = x(t)\hat{i} + y(t)\hat{j}$

$S = \int_a^b \|\bar{r}'(t)\|\,dt$; $dS = \|\bar{r}'(t)\|\,dt$ $\therefore \int_C f(x,y)\,dS = \int_a^b f(x(t),y(t))\|\bar{r}'(t)\|\,dt$

$\int f(x,y,z)\,dx \quad C:\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases} \quad \begin{matrix} dx = x'(t)\,dt \\ dy = y'(t)\,dt \\ dz = z'(t)\,dt \end{matrix} \quad = \int_a^b f(x(t),y(t),z(t))\,x'(t)\,dt =$

what

* Direc ↻ $\quad W = \int_C \bar{F}\cdot d\bar{r} = \int_a^b (\sim,\sim,\sim)\cdot(dx,dy,\sim)$

Área Curva cerrada Interna

$A = \oint_C -\frac{y}{2}dx + \frac{x}{2}dy$

- Campo Conservativo | 1] Conexa 2] Irrotacional

$\int_C \bar{F}\cdot d\bar{r} = \phi(B) - \phi(A) \quad A(x(a),y(a),z(a)) \quad B(x(b),y(b),z(b))$

- Rotacional $\quad \bar{\nabla}\times\bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sim & \sim & \sim \end{vmatrix}$

$\frac{d}{dx}\operatorname{sen} x = \cos x$

$\frac{d}{dx}\cos x = -\operatorname{sen} x$

- Diferencial Exacta $\quad \int du = u$; $u = u(x,y,z)$

$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$; $\int \frac{\partial u}{\partial x}dx \cup \int \frac{\partial u}{\partial y}dy \cup \int \frac{\partial u}{\partial z}dz$

$\bar{F} \to$ conservativo $\bar{F} = \bar{\nabla}\phi$; $\bar{F}\cdot d\bar{r} = \bar{\nabla}\phi\cdot d\bar{r} = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz$

- $\frac{\partial\phi}{\partial x} = \int \sim\,dx \quad \cdot \frac{\partial\phi}{\partial y} = \int \sim\,dy$

$\phi(x,y,z) = \cup \quad \cup \quad \cup + C$

$I = \underbrace{\phi(\sim,\sim)}_{B} - \underbrace{\phi(\sim,\sim)}_{A} =$

$=$ [unidades]

Coordenadas

- $\bar{\nabla}\times\bar{F} = \frac{1}{h_u h_v h_w}\begin{vmatrix} h_u\bar{e}_u & h_v\bar{e}_v & h_w\bar{e}_w \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ h_u f_u & h_v f_v & h_w f_w \end{vmatrix}$

- $\bar{F} = \bar{\nabla}\phi =$

$= \underbrace{\frac{1}{h_u}}_{\int}\frac{\partial\phi}{\partial u}\bar{e}_u + \frac{1}{h_v}\frac{\partial\phi}{\partial v}\bar{e}_v + \ldots$

Teorema Green

$\oint_C P\,dx + Q\,dy = \iint\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA_{xy}$

* Curva ○

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▶ C. Cilíndricas

$$T = \begin{cases} x = \rho\cos\theta \\ y = \rho \operatorname{sen}\theta \\ z = z \end{cases} \qquad T^{-1} = \begin{cases} \rho = \sqrt{x^2+y^2} \\ \theta = \arg\tan\left(\frac{x}{y}\right) \\ z = z \end{cases}$$

Jaco $= \rho$

$h_\rho = 1 \quad h_\theta = \rho \quad h_z = 1$

▶ C. Esféricos

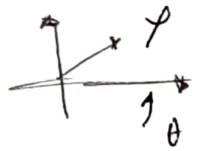
$$T = \begin{cases} x = \rho \operatorname{sen}\theta \cos\phi \\ y = \rho \operatorname{sen}\theta \operatorname{sen}\phi \\ z = \rho\cos\phi \end{cases} \qquad T^{-1} = \begin{cases} \rho = \sqrt{x^2+y^2+z^2} \\ \phi = \arg\cos\frac{z}{\sqrt{x^2+y^2+z^2}} \\ \theta = \arg\tan\left(\frac{y}{x}\right) \end{cases}$$

Jaco $= \rho^2 \operatorname{sen}\phi$

$h_\rho = 1 \quad h_\phi = \rho$

$h_\theta = \rho \operatorname{sen}\phi$

Inter Curvas | Puntos



ϕ max π

- Elipsoide: $Ax^2 + By^2 + Cz - J = 0$
- Hiperboloide; $x^2+y^2-z^2=1$
- Cono; $x^2+y^2-z^2=0$
- Paraboide; → Elíptico $x^2+y^2-z=0$
  ↳ Hiper $x^2-y^2-z=0$
- Cilindro; $x^2+y^2=r^2$
- Plano $x+y+z=J$

▶ $A(S) = \iint_S \underset{\text{Jaco}}{\uparrow} dS$ - $dS = \left\|\frac{\partial \bar{F}}{\partial u} \times \frac{\partial \bar{F}}{\partial v}\right\| du\, dv$

- $\iint_S f(x,y,z)\, dS$, $f(x,y,z)=1$; $\iint dS = \text{Área}_S$

▶ $dV = dx\, dy\, dz$

▶ $\int_V f(x,y,z)\, dV = \iiint_V f(x,y,z)$

$= \int_{q}^{t}\int_{\varphi_1}^{\varphi_2}\int_{f_1}^{f_2} dx\, dy\, dz$ ← Jaco

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▶ Stokes (Circulación)

$$\oint_C \bar{F} \cdot d\bar{r} = \iint_S (\bar{\nabla} \times \bar{F}) \cdot \hat{n}\, dS$$

* Curva Cerrada

- $\hat{n} = \frac{N_s}{\|\bar{N}_s\|}$

$N_S = \frac{\partial \bar{F}}{\partial u} \times \frac{\partial \bar{F}}{\partial v}$ ← Vector Normal $\|S$

$dS = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\; dx\, dy$

$\bar{\nabla} \cdot \bar{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$

▶ Gauss

$$\iint_S \bar{F} \cdot \hat{n}\, dS = \iiint_V \bar{\nabla} \cdot \bar{F}\, dV \stackrel{\Delta}{=} \text{Flujo} =$$

$\bar{\nabla} \cdot \bar{F}\big|_{P_0} > 0$; Punto fuente
Flujo Saliente

$\bar{\nabla} \cdot \bar{F}\big|_{P_0} < 0$; sumidero
entrante

$= 0$; incompresible