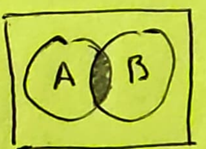
- $_nP_r = \frac{n!}{(n-r)!}$
- $_nPR_r = n^r$
- $_nC_r = \frac{n!}{r!(n-r)!}$

- $n \times n = A.L$
- $n \times (n-1) = A.L*$
- $\underbrace{_{M_{T_1}}C_{M_{P_1}} \times {}_{M_{T_2}}}_{T}\underbrace{C_{M_{P_2}}}_{P} =$

- $\left(\frac{M_{B_1}}{T}\right)\left(\frac{M_{B_2}}{T}\right) + \dots + \left(\frac{M_{B_1}}{T-1}\right)$
- $\left(\frac{C_1}{T}\right)\left(\frac{C_2}{T}\right)$
- "y" $\rightarrow M(x)$
- "o" $\rightarrow S(+)$

$A \cup B$: "O"



$A \cap B$: "y"

E.M.E

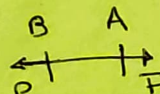
E.C.E

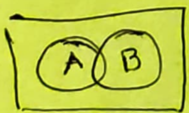
- $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$
- $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

- $P(\emptyset) = 0$
- $P(\bar{A}) = 1 - P(A)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- $P(M_p = x) = \frac{_{M_{T_1}}C_{x_1} \times {}_{M_{T_2}}C_{x_2}}{_{M_T}C_x}$
- $P(M_p \geq x) = P(M_p = x_1) + \dots + P(M_p = x_n)$

- $P(A|B) = \frac{P(A \cup B)}{P(B)}$





- $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\cancel{P(B)}}{\cancel{P(B)}} \leftarrow [I]$

S • $P(G) = P(G_1 \cap G_2 \cap \dots G_n) = \sqcup \times \sqcup \times \sqcup$

↳ $P(W) = 1 - P(G)$

P • $P(C_1 | C_2) = C_1 \longleftarrow [I]$

S $\rightarrow$ G P $\rightarrow$ W

- $P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) \dots$

- $P(A|B_1) = \lessgtr$
- $P(\bar{A}|B_1) = 1 - P(A|B_1)$

* $P(A = x) = 1 - P(A)^x$

- $P(B_x|A) = \frac{\cancel{P(B_x \cap A)}}{P(A)} =$

↳ $= \frac{P(B_x)P(A|B_x)}{P(A)}$

* $A|B \cap C = A \cap B \cap C$

- E.P — C → $\frac{n(M_P)}{n(M_T)}$
  - F.R → A.P → $P(A) = \lim_{n \to \infty} \frac{n(A)}{n}$
  - S - ¿?

1. $P(A) \geq 0$
1. $P(S) = 1$
- $P(A_1 \cup A_2 \cup \dots \cup A_n) = \sum_{i=1}^{n} P(A_i) \mid A_1 \dots \epsilon$

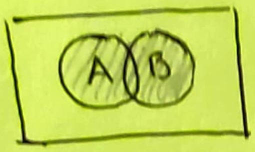
E.M.E

N

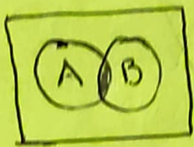
- $_nP_r = \dfrac{n!}{(n-r)!}$
- $_nPR_r = n^r$
- $_nC_r = \dfrac{n!}{r!(n-r)!}$
  - $\to$ x-o

- $n \times n = A.L$
- $n \times (n-1) = A.L^*$
- $\underbrace{{}_{M_T}C_{M_P} \times {}_{M_{T'}}C_{M_{P'}}}_{T} =$ P

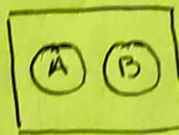
- $\left(\frac{M_B}{T}\right)\left(\frac{M_G}{T}\right) + \dots + \left(\frac{M_B}{(T-1)}\right)$ S.R
- $\left(\frac{C_1}{T}\right)\left(\frac{C_2}{1}\right)$
- "y" $\to$ M (x)
- "o" $\to$ S (+)



$A \cup B$ | "o"



$A \cap B$ | "y"



E.M.E

E.C.E

- $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$
- $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

- E.P
  - C $\to$ $P(A) = \dfrac{n(M_P)}{n(M_T)}$
  - F.R $\to$ A.P $\to$ $P(A) = \lim_{n \to \infty} \dfrac{n(A)}{n}$
  - S $\to$ d?

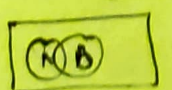
3 - A

- $P(A) \geq 0$
- $P(S) = 1$
- $P(A_1 \cup A_2 \dots) = \sum_{i=1}^{n} P(A_i)$ | $A_1 \dots \in$ E.M.E

- $P(\emptyset) = 0$
- $P(\bar{A}) = 1 - P(A)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- $P(M_P = x) = \dfrac{{}_{M_T}C_x \times {}_{M_{T'}}C_{x'}}{{}_{H_T}C_x} + \dots$
- $P(M_P \geq x) = P(M_P = 1) + P(M_P = 2) \dots + P(M_P = x)$

- $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ B A P F

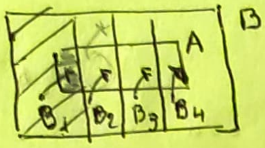


- $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)\cancel{P(B)}}{\cancel{P(B)}} = P(A)$

S • $P(G) = (P\ G_1 \cap G_2 \dots \cap G_n)$
  - $\to P(W) = 1 - P(G)$

P • $P(C_1|C_2) = C_1$ I

S $\to$ G / P $\to$ W

- $P(A) = P(B_1) \cdot P(A|B_1) + P(B_2) \cdot P(A|B_2) \ldots + P(B_N) P(A|B_N)$
- 

- $P(A|B_1) =$
- $P(\overline{A}|B_1) = 1 - P(A|B_1)$
- $P(A = x) = 1 - P(A)^x$

---

* $A|B \cap C = A \cap B \cap C$

- $P(B_x|A) = \dfrac{P(B_x \cap A)}{P(A)} = \dfrac{P(B_x)\, P(A|B_x)}{P(A)}$

.a	.02	.03

▶ v.a.d

1) $0 \le f_x(x) \ \forall x$

2) $\sum_{\forall x} f_x(x) = 1$

3) $P(a \le x \le b) = \sum_{x=a}^{b} f_x(x)$

* d.e.n = m.a = $f_x(x)$

▶ v.a.c

1) $\int_{-\infty}^{\infty} f_x(x)\,dx$

2) $P(a \le x \le b) = \int_a^b f_x(x)$

▶ v.a.d

• $F_x(x) = P(X = x) = \sum_{i=-\infty}^{x} f_x(x_i)$

• $0 \le F_x(x) \le 1$

• $\sum_{x=-\infty}^{x} f_x(x) = F_x(x)$

• $F_x(b) \ge F_x(a)$

▶ v.a.c

$P(a < x \le b) = F_x(b) - F_x(a)$

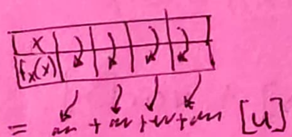
* a.c.u = $F_x(x)$

* "<" ≠ "≤"

$M_r(v) = \sum (x_i - v)^r f(x_i)$ $\quad \int_{-\infty}^{\infty} (x - v)^r f(x)\,dx$ $\quad v = 0$ $\quad \cancel{v = \mu}$

▶ $M_x(\theta) = E[e^{\theta x}] = \int (e^{\theta x}) f_x(x)$

▶ $M'_x(\theta) = E[x] = \mu = \sum_{\forall x} x_i f_x(x_i)$ $\quad \int_{-\infty}^{\infty} x f_x(x)\,dx$ | $E[x] = M'_x(0)$



▶ X_{mo} =

↳ max

▶ $P(x \le \tilde{x}) = .5$; $F_x(\tilde{x}) = .5$

// m.e.d

▶ DM = $E[|X - \mu|]$; $\sum_{i=1}^{\infty} |x_i - \mu| f_x(x_i)$

$\longrightarrow \int_{-\infty}^{\infty} |x_i - \mu| f_x(x)\,dx$

▶ $var[x] = \sigma^2 = E[(X - \mu)^2] = E[x^2] - \mu$

$\int_{-\infty}^{\infty} (x_i - \mu)^2 f_x(x)\,dx$

$\sum_{\forall x} (x_i - \mu)^2 f_x(x)$

▶ $\sigma = \sqrt{\sigma^2} = \sqrt{var[x]}$ | d. e.

▶ $CV = \frac{\sigma}{\tilde{x}}$

• $var[c] = 0$

• $var[cX] = c^2 var[x]$

• $var[ax + by] = a^2 var[x] + 2ab\,cov[x, y] + b^2 var[y]$

• $E[x^2] = M''_x(0)$

▶ $a_3(x) = \frac{\mu_3}{\sigma^3_x}$ $\quad$ s.e $\rightarrow 0$

▶ $a_4(x) = \frac{\mu_4}{(\sigma_x)^4}$ $\quad$ c.u $\rightarrow 3$

▶ $0 \le f_{xy}(x,y) \le 1$ vv. a. d

• $\sum_{\forall x} \sum_{\forall y} f_{xy}(x,y) = 1$

• $P(x_0 \le X \le X_1, y_0 \le y \le y_1) = \sum_{x=0}^{x_1} \sum_{y=0}^{y_1} f_{xy}(x,y)$

• $f_x(x) = \sum_{\forall y} f_{xy}(x,y)$

• $f_y(y) = \sum_{\forall x} f_{xy}(x,y)$

* $P(X = m, Y = m) = \frac{{}_{\square}C_m \times {}_{\square}C_m}{C_k}$

• $f_{X|Y_0}(X|Y_0) = \left\{ \frac{f_{xy}(x, y_0)}{f_y(y_0)} \right.$

▶ vv. a. c

▶ $P((x,y)) = \iint_A f_{xy}(x,y)\,\underline{dx\,dy}$

$f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x,y)\,dy \quad -\infty < x < \infty$ ← y

$f_y(y) = \int_{-\infty}^{\infty} f_{xy}(x,y)\,dx \quad -\infty < y < \infty$ ← x

▶ $E[g(x,y)] = \sum_{\forall R_x} \sum_{\forall R_y} g(x,y)\,f_{xy}(x,y)$

$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y)\,f_{xy}(x,y)\,\underline{dx\,dy}$

// $Var[x+y] = var[x] - 2cov[x,y] + var[y]$

$0 \overset{*}{\triangleq}$ a. I

• $E[c] = c$

• $E[x+y] = E[x] + E[y]$

• $E[g_1(x)\,g_2(x)] = E[g_1 x]\,E[g_2 x]$

\# a. I

▶ $E[Y | x = x_0] = \left\{ \begin{array}{l} \int_{-\infty}^{\infty} y\,f_{y|x_0}(y|x_0)\,dy \\ \sum_{\forall R_y} y\,f_{y|x_0}(y|x_0) \end{array} \right.$

▶ $cov(x,y) = \sigma_{xy} = E[(x - \mu_x)(y - \mu_y)]$

▶ C. $\rho = \frac{Cov(x,y)}{\sqrt{var(x)}\sqrt{var(y)}} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$

▶ D $\rho^2 = \frac{Cov(x,y)^2}{var\,x \; var\,y} = \frac{(\sigma_{xy})^2}{(\sigma_x)^2 (\sigma_y)^2}$

▶ a. I

• $f_{xy}(x,y) = f_x(x)\,f_y(y)$

1] D.D.U ; $x \sim U(K= \ )$

$f_x(x) = \frac{1}{K}$ | $\mu_x = \sum_{x} x_i \cdot f_x(x)$ | $\sigma_x^2 = \sum_{\forall x} (x_i - \mu)^2 f_x(x) = \frac{1}{K} \sum_{\forall x} (x_i - \mu)^2$

~~2]~~ D.B ; $x \sim Be(n=1, p= \ )$

$f_x(x) = {}_nC_x \, p^x \underbrace{(1-p)}_{q}^{n-x}$ | $\mu_x = np$ | $\sigma_x^2 = npq \to \sigma_x = \sqrt{pq}$ | $M_x(\theta) = (q + e^\theta p)^n$

$p = ✓$
$q = ✗$

---

[3] D.B1 ; $x \sim B_1(n= \_, p= \_)$ | $\mu_x = np = E(x)$

$P(x \geq \_) = \sum_{x=\_}^{n} ({}_nC_x)(p)^x (1-p)^{n-x}$ | $\therefore f(x) = (\_)(E(x)) + (\_)(n - E(x))$

•É ; n

~~4]~~ D.G ; $x \sim G(P= \_)$

$f_x(x) = q^{x-1} p$ | $\mu_x = \frac{1}{p}$ | $\sigma_x^2 = \frac{q}{p^2}$ | $M_x(\theta) = \frac{pe^\theta}{1 - qe^\theta}$ | $q = ✗ = 1-p$, $p = ✓$

• $P(x = \_) = (q)^{x-1} p$ • $P(x \geq \_) = 1 - P(x < \_) = 1 - \sum_{x=1}^{\_-1} (q)^{x-1} (p)$

#.e. → r.É

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[5] D.P ; $x \sim P(r = \_, p = \_)$

$f_x(x) = ({}_{x-1}C_{r-1})(p^r)(q^{x-r})$ | $\mu_x = \frac{r}{p}$ | $\sigma_x^2 = \frac{rq}{p^2}$ | $M_x(\theta) = \left(\frac{pe^\theta}{1 - qe^\theta}\right)^r$

• $P(x = \_) = ({}_{x-1}C_{r-1})(p^r)((1-p)^{x-r})$ • $P(x > \_) = 1 - P(x \leq \_) =$
$= 1 - \sum_{x=r}^{\_} {}_{x-1}C_{r-1} (q)^{x-r} (p)^r$

6] D.H. $x \sim H(N= \_, n= \_, r= \_)$

$f_x(x) = \frac{({}_rC_x)({}_{N-r}C_{n-x})}{{}_NC_n}$ | $\mu_x = n\frac{r}{N} = np$ | $\sigma_x^2 = n\frac{r}{N}\left(1 - \frac{r}{N}\right)\left(\frac{N-n}{N-1}\right) =$
$= npq\left(\frac{N-n}{N-1}\right)$

• $P(x = \_) = \frac{({}_rC_x)({}_{N-r}C_{n-x})}{{}_NC_n}$ • $P(x \leq \_) = \sum_{n=0}^{\_} \frac{({}_rC_x)({}_{N-r}C_{n-x})}{{}_NC_n}$

7] $P \sim (\lambda = w\,[u])$

$f_x(x) = \frac{\lambda^x}{x!} e^{-\lambda}$ | $\mu_x = \lambda$, $\sigma^2_x = \lambda$

- $P(x < w) = P(x \le w-1)$

- $P(w \le x \le \Omega) = \sum_{w}^{\Omega} \frac{\lambda^x}{x!} e^{-\lambda} =$
- $P(w < x < \Omega) = \sum_{w+1}^{\Omega-1} \frac{\lambda^x}{x!} e^{-\lambda} =$
- $P(x \ge w) = 1 - P(x < w) =$

* $n \to \infty$ $n > 30$ $\therefore x \sim B$
$p \to 0$

- $P(x > w \mid x > \Omega) = \frac{P(x > w)}{P(x > \Omega)} = \frac{\int_w^b f_x(x)}{\int_\Omega^b f_x(x)}$

8] D.C.U.; $x \sim U(a= \ ,b= \ )$ $\mu_x = \frac{b+a}{2}$

$f_x(x) = \frac{1}{b-a}$ $a \le x \le b$ | $\sigma^2_x = \frac{(b-a)^2}{12}$

~~9~~] D.E; $x \sim E(\lambda = \quad)$

$f_T(t) = \begin{cases} \lambda e^{-\lambda t} \\ 0 \end{cases}$ | $F_T(t) = 1 - e^{-\lambda t}$ | $\mu_T = \frac{1}{\lambda}$, $\sigma^2_x = \frac{1}{\lambda^2}$

- $P(x > w) = 1 - P(T < w) = 1 - [1 - e^{-\lambda w}]$
- $P(T < t) = w$; $1 - e^{-\lambda t} = w$; $e^{-\lambda t} = 1 - w$

10] D.G; δ $x \sim G(r = w, \lambda = \ )$

- $\mu_T = \frac{r}{\lambda} = \alpha\beta$ • $\sigma^2_T = \frac{r}{\lambda^2} = \alpha\beta^2$ • $M_T(\theta) = \left(\frac{\lambda}{\lambda - \theta}\right)^r$ $\alpha = r$, $\beta = \frac{1}{\lambda}$

- $P(x > w) = 1 - P(x < w) = 1 - \left[1 - \sum_{k=0}^{\alpha-1} \frac{(\lambda x)^k}{k!} e^{-\lambda x}\right] =$

11] D.N; $x \sim N(\mu_x = w, \sigma = \Omega)$

- $P(x > \Omega) = P\left(z > \left(\frac{\Omega - \mu_x}{\sigma}\right)\right) = P(z > w) = P(z < -w)$
- $P(w < x < \Omega) = P(x < \Omega) - P(x < w)$
- $P(w < x < \Omega) = P(w < z < \Omega) = P(z < \Omega) - P(z < w)$
- $P(x < x_1) = .25$; $P\Big(x < \underbrace{\frac{x_1 - \mu_x}{\sigma_x}}_{z_1}\Big) = w \xrightarrow{T}$; $x_1 = z\,\sigma_x + \mu_x$; $(\ )(\sigma_x) + \mu_x = w$

11] D.N → D.B $n > 30$

- $P(x \quad x_i) = \frac{x - np \pm \frac{1}{2}}{\sqrt{npq}}$ | $> \ -\frac{1}{2}$, $< \ +\frac{1}{2}$ | $\sigma_x = \sqrt{npq}$, $\mu_x = (n)(p)$

$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$

12] D. χ^2

$P(S^2_{n-1} \ge w) = P\left(S^2_{n-1}\left(\frac{n}{\sigma^2}\right) \ge w\left(\frac{n}{\sigma^2}\right)\right) = P(\chi^2_{n-1} \ge w) = w$

$P(\chi^2(\alpha, v) > \chi^2) = w \to \alpha$; $\therefore \chi^2 = w$

13] D.T $x \sim (\mu = \mu_x, \sigma = \frac{s}{\sqrt{n}})$

- $P(\bar{x} < \bar{x}_{,v}) = P\left(T_n < \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}\right) = P(T > \pm w)$

14] D.F; $P\left(\frac{s_1^2}{s_2^2} < w\right) = P(F(n_1, n_2) < w) = 1 - \alpha$