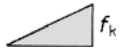
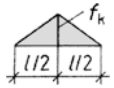
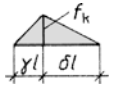
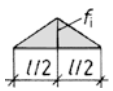
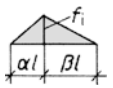
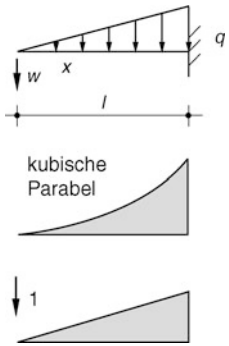
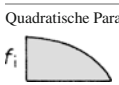
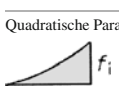
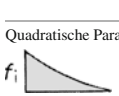
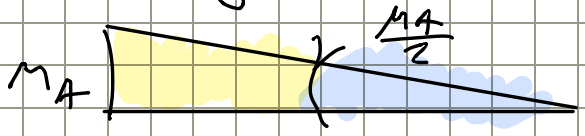
5.11 Integrationstafeln

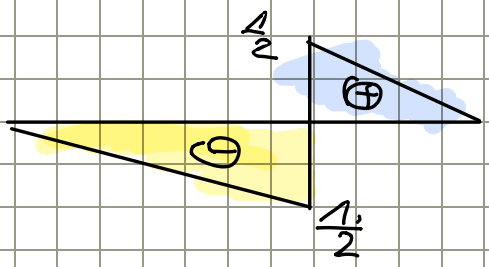
Tafel 5.11 Wert der Integrale $\int f_\mathrm{i} \cdot f_\mathrm{k}\,\mathrm{d}x = l \cdot$ Tafelwert

l	Rechteck 	Dreieck 	Dreieck 	Dreieck 		Kubische Parabel 	Kubische Parabel 	$\int f_\mathrm{i} \cdot f_\mathrm{i}\,\mathrm{d}x$
Rechteck 	$f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{2} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{2} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{2} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{2} f_\mathrm{i}(f_{\mathrm{k}1} + f_{\mathrm{k}2})$	$\frac{1}{4} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{4} f_\mathrm{i} f_\mathrm{k}$	f_i^2
Dreieck 	$\frac{1}{2} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{3} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{4} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{6}(1+\gamma) f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{6} f_\mathrm{i}(f_{\mathrm{k}1} + f_{\mathrm{k}2})$	$\frac{2}{15} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{5} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{3} f_\mathrm{i}^2$
Dreieck 	$\frac{1}{2} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{6} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{4} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{6}(1+\delta) f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{6} f_\mathrm{i}(2f_{\mathrm{k}1} + f_{\mathrm{k}2})$	$\frac{7}{60} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{20} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{3} f_\mathrm{i}^2$
Dreieck 	$\frac{1}{2} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{4} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{3} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{12}\frac{3-4\gamma^2}{\delta} f_\mathrm{i} f_\mathrm{k}$ für $\gamma \le \delta$	$\frac{1}{4} f_\mathrm{i}(f_{\mathrm{k}1} + f_{\mathrm{k}2})$	$\frac{5}{32} f_\mathrm{i} f_\mathrm{k}$	$\frac{3}{32} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{3} f_\mathrm{i}^2$
Dreieck 	$\frac{1}{2} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{6}(1+\alpha) f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{12}\frac{3-4\alpha^2}{\beta} f_\mathrm{i} f_\mathrm{k}$ für $\alpha \le \beta$	$\frac{1}{6}\frac{2\alpha-\alpha^2-\gamma^2}{\alpha\cdot\delta} f_\mathrm{i} f_\mathrm{k}$ für $\alpha \ge \gamma$	$\frac{1}{6} f_\mathrm{i}[(1+\beta) f_{\mathrm{k}1} + (1+\alpha) f_{\mathrm{k}2}]$	$\frac{1}{20}(1+\alpha)\cdot\left(\frac{7}{3}-\alpha^2\right)\cdot f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{20}(1+\alpha)\cdot(1+\alpha^2)\cdot f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{3} f_\mathrm{i}^2$
Trapez 	$\frac{1}{2}(f_{\mathrm{i}1} + f_{\mathrm{i}2}) f_\mathrm{k}$	$\frac{1}{6}(f_{\mathrm{i}1} + 2f_{\mathrm{i}2}) f_\mathrm{k}$	$\frac{1}{4} f_\mathrm{k}(f_{\mathrm{i}1} + f_{\mathrm{i}2})$	$\frac{1}{6} f_\mathrm{k}[(1+\delta) f_{\mathrm{i}1} + (1+\gamma) f_{\mathrm{i}2}]$	$\frac{1}{6}[(2f_{\mathrm{k}1} + f_{\mathrm{k}2}) f_{\mathrm{i}1} + (f_{\mathrm{k}1} + 2f_{\mathrm{k}2}) f_{\mathrm{i}2}]$	$\frac{1}{60}(7f_{\mathrm{i}1} + 8f_{\mathrm{i}2})\cdot f_\mathrm{k}$	$\frac{1}{20}(f_{\mathrm{i}1} + 4f_{\mathrm{i}2})\cdot f_\mathrm{k}$	$\frac{1}{3}(f_{\mathrm{i}1}^2 + f_{\mathrm{i}2}^2 + f_{\mathrm{i}1} f_{\mathrm{i}2})$
Quadratische Parabel 	$\frac{2}{3} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{3} f_\mathrm{i} f_\mathrm{k}$	$\frac{5}{12} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{3}(1+\gamma\delta)\cdot f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{3}(f_{\mathrm{k}1} + f_{\mathrm{k}2}) f_\mathrm{i}$	**Beispiel:** 	gesucht: w $M = -q\frac{l^2}{6} = f_\mathrm{k}$ $\overline{M} = -1 \cdot l = f_\mathrm{i}$	
Quadratische Parabel 	$\frac{2}{3} f_\mathrm{i} f_\mathrm{k}$	$\frac{5}{12} f_\mathrm{i} f_\mathrm{k}$	$\frac{17}{48} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{12}(5-\delta-\delta^2) f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{12} f_\mathrm{i}(3f_{\mathrm{k}1} + 5f_{\mathrm{k}2})$			
Quadratische Parabel 	$\frac{2}{3} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{4} f_\mathrm{i} f_\mathrm{k}$	$\frac{17}{48} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{12}(5-\gamma-\gamma^2) f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{12} f_\mathrm{i}(5f_{\mathrm{k}1} + 3f_{\mathrm{k}2})$			
Quadratische Parabel 	$\frac{1}{3} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{4} f_\mathrm{i} f_\mathrm{k}$	$\frac{7}{48} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{12}(1+\gamma+\gamma^2) f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{12} f_\mathrm{i}(f_{\mathrm{k}1} + 3f_{\mathrm{k}2})$			
Quadratische Parabel 	$\frac{1}{3} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{12} f_\mathrm{i} f_\mathrm{k}$	$\frac{7}{48} f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{12}(1+\delta+\delta^2) f_\mathrm{i} f_\mathrm{k}$	$\frac{1}{12} f_\mathrm{i}(3f_{\mathrm{k}1} + f_{\mathrm{k}2})$	$E \cdot J \cdot w = \text{Tafelwert} \cdot l \cdot f_\mathrm{i} \cdot f_\mathrm{k}$ $E \cdot J \cdot w = \frac{1}{5} \cdot l \cdot \left(-q\frac{l^2}{6}\right)\cdot(-l) = q\frac{l^4}{30}$ (siehe auch Tafel 5.7, 3. Fall)		

0-System



1-System



$$\left[\frac{1}{6}\left(2\cdot\frac{M_A}{2}+M_A\right)\cdot\left(-\frac{1}{2}\right)\right]+\left[\frac{1}{3}\cdot\frac{M_A}{2}\cdot\left(\frac{1}{2}\right)\right]$$

$$\Rightarrow \quad -\frac{1}{6}M_A \qquad +\frac{1}{12}M_A \quad = -\frac{1}{12}M_A$$