Der Arbeitsbegriff in der Elastostatik:

Arbeitssatz:

$$\boxed{W = E^*}$$

Arbeit:

$$\boxed{W = \int \underline{F}(\underline{r}) \cdot d\underline{r} = \int F_x\,dx + \int F_y\,dy + \int F_z\,dz}$$

Potenzial:

$$\boxed{E_p = -\int \underline{F}(\underline{r}) \cdot d\underline{r}}$$

Ergänzungsenergie:

$$\boxed{E^* = \frac{1}{2}\int_0^l \frac{N^2}{EA}\,dx}$$

Verzerrungsenergie:

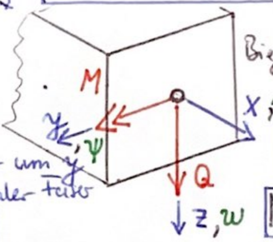
$$\boxed{E = \frac{1}{2}\int_0^l EA\,\varepsilon^2\,dx}$$

Biegebalken:

$$\boxed{\sigma = \frac{M_y}{I_y} z}$$

I = Trägheitsmoment um y
z = Abstand von neutraler Faser

$$\boxed{E^* = \frac{1}{2}\int_0^l \frac{M_y^2}{EI_y}\,dx}$$



$EI_y\ [N \cdot mm^2]$ Biegesteifigkeit

$w'(x) << 1$

$$\boxed{M_y = -w''(x) EI_y}$$

$$\boxed{M_z = EI_z v''(x)}$$

Für Stäbe und Balken:

Beanspruchung	Formänderungsenergie pro Längeneinheit	Formänderungsenergie
Zug / Druck	$E^* = \frac{1}{2} \frac{N^2}{EA}$	$E = \frac{1}{2} \int_l \frac{N^2}{EA} dx$
Biegung	$E^* = \frac{1}{2} \frac{M^2}{EI}$	$E = \frac{1}{2} \int_l \frac{M^2}{EI} dx$
Schub	$E = \frac{1}{2} \cdot \frac{Q^2}{GA_s}$	$E = \frac{1}{2} \int_l \frac{Q^2}{GA_s} dx$
Torsion	$E = \frac{1}{2} \cdot \frac{M_T^2}{GI_T}$	$E = \frac{1}{2} \int_l \frac{M_T^2}{GI_T} dx$

Gesamte Formänderungsenergie:
(Zug + Biegung + Schub + Torsion)

$$\boxed{E^* = \frac{1}{2}\int_0^l \frac{N^2}{EA}\,dx + \frac{1}{2}\int_0^l \frac{M_y^2}{EI_y}\,dx + \frac{1}{2}\int_0^l \frac{M_z^2}{EI_z}\,dx + \int_0^l \alpha_T \Delta T \ldots dx}$$

gilt für: 1. homogene Temperatur über dem Querschnitt
2. bei Vernachlässigung von Querkraft u. Torsion

1. Satz von Castigliano:

$$\boxed{\frac{\partial E^*}{\partial Q_K} = q_K}$$

~~Q_π = Verschiebung / Verdrehung~~ ~~$u = \frac{\partial E^*}{\partial X}$~~

q_K = Verschiebung / Verdrehung des Angriffspunktes der Kraft in Richtung von Q_K

$$W = E^* \Rightarrow \frac{\partial W}{\partial Q_K} = \frac{\partial E^*}{\partial Q_K}$$

Arbeitssatz für Energiesätze:

$$\boxed{q_j = \sum_i \delta_{ij} Q_i}$$

$[\delta_{ij}] = \frac{m}{N}$

Satz von Betti u. Maxwell

$$\boxed{W = \frac{1}{2}\sum_i \sum_j \delta_{ij} Q_i Q_j}$$

Satz von Menabrea:

$$\boxed{\frac{\partial E^*}{\partial X} = 0}$$

X = Auflagerkraft
→ Verformung des Lagers

Umkehrung →

$$\boxed{Q_j = \sum_i h_{ij} q_i}$$

2. Satz von Castigliano:

$$\boxed{\frac{\partial E^*}{\partial q_K} = Q_K}$$

$$\boxed{E^*(\text{Kreis}) = \frac{1}{2}\frac{1}{EI}\int_\varphi M^2 r\,d\varphi}$$

Dreiachsiger Spannungszustand:

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right] + \alpha_T \Delta T, \quad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx})\right] + \alpha_T \cdot \Delta T, \quad \gamma_{yz} = \frac{\tau_{yz}}{G}$$

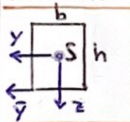
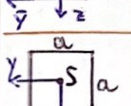
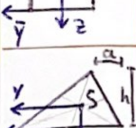
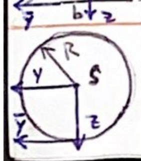
$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right] + \alpha_T \cdot \Delta T, \quad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

Balkentheorie:

Schwerpunktlage:

	$z_{s,i}$ $y_{s,i}$	A_i	$x_{s,i} \cdot A_i$	$y_{s,i} \cdot A_i$
1	... ...	... +	... +	... +
2	... ...	...	...	...
Σ_i	X	...	...	...

$$\boxed{\bar{z}_S = \frac{\sum_i z_{s,i} \cdot A_i}{\sum_i A_i}\ ; \quad \bar{y}_S = \frac{\sum_i y_{s,i} \cdot A_i}{\sum_i A_i}}$$

Fläche = A	I_y	I_z	I_{yz}
	$\frac{bh^3}{12}$	$\frac{hb^3}{12}$	0
	$\frac{a^4}{12}$	$\frac{a^4}{12}$	0
	$\frac{bh^3}{36}$	$\frac{hb}{36}(b^2-ba+a^2)$	$-\frac{bh^2}{72}(b-2a)$
	$\frac{\pi R^4}{4}$	$\frac{\pi R^4}{4}$	0

Flächenträgheitsmomente:

Satz von Steiner:

$$\boxed{\begin{aligned} I_y^* &= \sum_i^n \left[I_{yi} + z_{si}^{*2} \cdot A_i\right] \\ I_z^* &= \sum_i^n \left[I_{zi} + y_{si}^{*2} \cdot A_i\right] \end{aligned}}$$

Flächenmomententensor:

$$\underline{I} = \begin{pmatrix} I_y & -I_{yz} \\ -I_{yz} & I_z \end{pmatrix}$$

$$\boxed{I_{zy}^* = I_{yz}^* = \sum_i^n \left[I_{zyi} + y_{si}^* \cdot z_{si}^* \cdot A_i\right]}$$

Hauptachsensystem:

$$\tan(2\mathring{\varphi}) = -\frac{2 I_{yz}}{I_y - I_z}$$

$$\left.\begin{matrix} I_{\mathring{y}} \\ I_{\mathring{z}} \end{matrix}\right\} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$$

$$\underline{R} = \begin{pmatrix} \cos\mathring{\varphi} & -\sin\mathring{\varphi} \\ \sin\mathring{\varphi} & +\cos\mathring{\varphi} \end{pmatrix}$$

$$M_{\mathring{y}} = M_y \cos\mathring{\varphi}$$

$$M_{\mathring{z}} = M_y \sin\mathring{\varphi}$$

bsp:

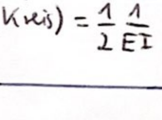
$\mathring{P} = \underline{R} \cdot \underline{P}$

$\mathring{Q} = \underline{R} \cdot \underline{Q}$

↳ $\sigma_{xx}(P_{\mathring{y}}, P_{\mathring{z}})$

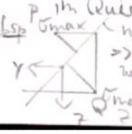
$\sigma_{xx}(\mathring{Q}_y, \mathring{Q}_z)$ maximale Zug- und Druckspannung im Querschnitt

bsp:



$M_{y_{max}} = -Fl$

bsp



neutrale Faser
=> Punkte, die am weitesten entfernt sind, dort σmax herrscht größte Zug/Druck σ

Biegung und Zug / Druck:

Superposition:

$$\boxed{\sigma_{xx}(\mathring{z}, \mathring{y}) = \underbrace{\frac{N}{A}}_{\text{reine Zug/Druck}} + \underbrace{\frac{M_{\mathring{y}}}{I_{\mathring{y}}} \cdot \mathring{z} - \frac{M_{\mathring{z}}}{I_{\mathring{z}}} \cdot \mathring{y}}_{\text{reine Biegung}}}$$

neutrale Faser:

$$\boxed{\sigma_{xx} = 0}$$

Für neutrale Faser nach $\mathring{z}$ auflösen

$\mathring{z} = z(\mathring{y}) = 1{,}4\,\mathring{y}$

Transformationsbeziehungen für Flächenträgheitsmomente:

$$\boxed{\begin{aligned} I_{\bar{y}} &= \frac{I_{yy} + I_{zz}}{2} + \frac{I_{yy} - I_{zz}}{2}\cos(2\varphi) + I_{yz}\sin(2\varphi) \\ I_{\bar{z}} &= \frac{I_{yy} + I_{zz}}{2} - \frac{I_{yy} - I_{zz}}{2}\cos(2\varphi) - I_{yz}\sin(2\varphi) \\ I_{\bar{y}\bar{z}} &= I_{\bar{z}\bar{y}} = -\frac{I_{yy} - I_{zz}}{2}\sin(2\varphi) + I_{yz}\cos(2\varphi) \end{aligned}}$$

TM II - Formelsammlung

Def. von Spannung: $\boxed{\sigma = \frac{F}{A}\left[\frac{N}{mm^2} = MPa\right]}$

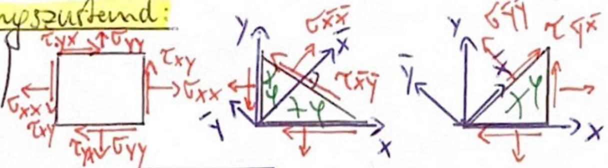
σ = Normalspannung (⊥ zur Fläche)
τ = Schubspannung (in der Fläche)

Spannungsvektor: $\boxed{\underline{t} = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}}$

Ebener Spannungszustand:

$$\underline{\sigma} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{xy} & \sigma_{yy} \end{pmatrix}$$

↳ symmetrisch!



Räumlicher Spannungszustand:

$$\underline{\sigma} = \begin{pmatrix} \sigma_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{pmatrix} = (\underline{t}_x \underline{t}_y \underline{t}_z$$

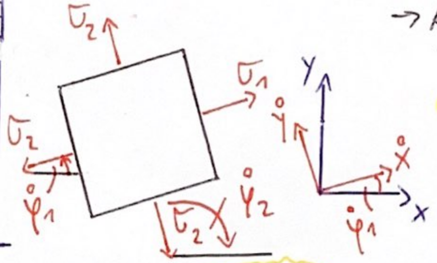
→ Aus Momentengleichgewicht gilt:
$\tau_{xy} = \tau_{yx}$; $\tau_{xz} = \tau_{zx}$; $\tau_{yz} = \tau_{z}$

Hookesches Gesetz: $\boxed{\sigma = \frac{F}{A} = \varepsilon \cdot E = \frac{\Delta L}{L_0} E}$

Hauptspannungen:

$$\sigma_{1,2} = \sigma_M \pm R$$
$$= \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$
$$\tan(2\mathring{\varphi}) = \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}$$

$\mathring{\varphi}$ = Hauptnormalspannungsrichtung



Beachte: - Schubspannung hier = 0
- Hauptspannungsrichtungen $(\mathring{\varphi}_1, \mathring{\varphi}_2)$ ⊥
$\mathring{\varphi}_2 = \mathring{\varphi}_1 \pm \frac{\pi}{2}$

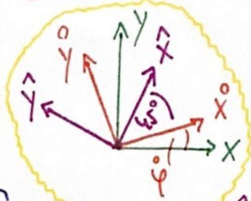
$$\mathring{\underline{\sigma}} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix}$$ Hauptnormalspannungszustand

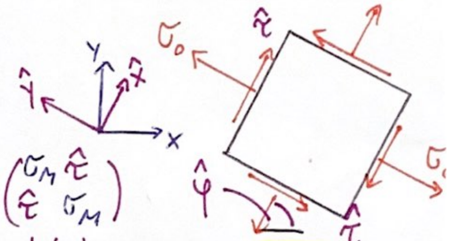
$\mathring{x}, \mathring{y}$ - KOS

Maximale Schubspannung

$$\hat{\tau} = \tau_{max} = R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2};$$
$$\hat{\varphi} = \mathring{\varphi} \pm \frac{\pi}{4}$$

Normalspannung hier größe $\sigma_0 = \frac{\sigma_{xx} + \sigma_{yy}}{2}$





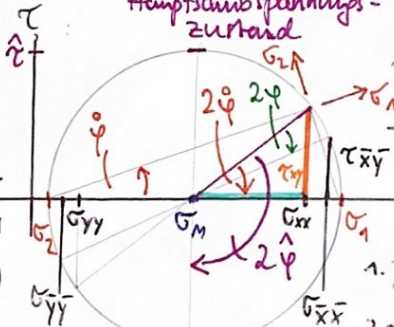
$\hat{x}, \hat{y}$-KOS: $\hat{\underline{\sigma}} = \begin{pmatrix} \sigma_M & \hat{\tau} \\ \hat{\tau} & \sigma_M \end{pmatrix}$

Kreisring: $A = 2r\pi s$
Kesselformel: $\underline{\sigma} = \begin{pmatrix} \Delta p \frac{r}{s} & 0 & 0 \\ 0 & \Delta p \frac{r}{2s} & 0 \\ 0 & 0 & 0 \end{pmatrix}$
$\sigma_{xx} = 2\sigma_{yy}$

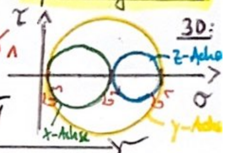
Transformationsbeziehungen:

$$\sigma_{\bar{x}\bar{x}} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2}\cos(2\varphi) + \tau_{xy}\sin(2\varphi)$$
$$\sigma_{\bar{y}\bar{y}} = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \frac{\sigma_{xx} - \sigma_{yy}}{2}\cos(2\varphi) - \tau_{xy}\sin(2\varphi)$$
$$\tau_{\bar{x}\bar{y}} = \tau_{\bar{y}\bar{x}} = -\frac{\sigma_{xx} - \sigma_{yy}}{2}\sin(2\varphi) + \tau_{xy}\cos(2\varphi)$$

Hauptschubspannungszustand



Mohrscher Spannungskreis:

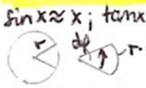


Regeln:
1. Jeder Durchmesser des Kreises ≙ KOS.
2. τ_{xy} wird über σ_{xx} positiv angetragen.
3. Transformationswinkel φ wird doppelt und in entgegengesetzte Richtung angetragen.

Verzerrung:

Verschiebungsvektor: $\underline{u} = \underline{u}(\underline{x}) = \begin{pmatrix} u \\ v \\ w \end{pmatrix}$

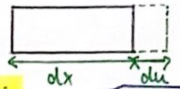
Kleinwinkeländerung:
$\sin x \approx x$; $\tan x \approx x$; $\cos x \approx 1$



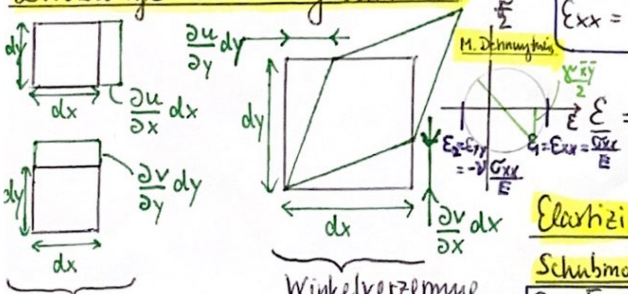
$r \cdot d\varphi$ (anstatt: $\sin\varphi \cdot r$)

Einachsiger Verzerrungszustand:

$\boxed{\text{Dehnung: } \varepsilon(x) = \frac{du}{dx} = u'[-]}$



Zweiachsiger Verzerrungszustand:



$\underbrace{}_{\text{Dehnungen}}$ $\underbrace{}_{\text{Winkelverzerrung}}$

$$\boxed{\varepsilon_{xx} = \frac{\partial u}{\partial x};\ \varepsilon_{yy} = \frac{\partial v}{\partial y};\ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}}$$

Dreiachsiger Verzerrungszustand:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x};\ \varepsilon_{yy} = \frac{\partial v}{\partial y};\ \varepsilon_{zz} = \frac{\partial w}{\partial z};\ \gamma_{xy} = \gamma_{yx} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$
$$\gamma_{yz} = \gamma_{zy} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$
$$\gamma_{zx} = \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

$$\underline{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_{yy} & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_{zz} \end{pmatrix}$$

$\boxed{\gamma_{xy} = 2\varepsilon_{xy} \quad \frac{1}{2}\gamma_{xy} = \varepsilon_{xy}}$

Elastizitätsgesetz: $\boxed{\varepsilon = \frac{\sigma}{E} + \alpha_T \cdot \Delta T}$ $\boxed{\varepsilon = \varepsilon_M + \varepsilon_T = \frac{\sigma}{E} + \alpha_T \cdot \Delta T}$

Schubmodul: $\boxed{G = \frac{E}{2(1+\nu)}}$

Querdehnzahl: $\boxed{\nu = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}}}$

Ebener Spannungszustand:

$$\varepsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{yy}) + \alpha_T \cdot \Delta T,$$
$$\varepsilon_{yy} = \frac{1}{E}(\sigma_{yy} - \nu\sigma_{xx}) + \alpha_T \cdot \Delta T,$$
$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$

Hauptrichtungen:

$$\tan(2\mathring{\varphi}) = \frac{\gamma_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}}$$
$$\varepsilon_{1,2} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} \pm \sqrt{\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

KO-Transformation:

$$\varepsilon_{\bar{x}\bar{x}} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\cos(2\varphi) + \frac{\gamma_{xy}}{2}\sin(2\varphi)$$
$$\varepsilon_{\bar{y}\bar{y}} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} - \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\cos(2\varphi) - \frac{\gamma_{xy}}{2}\sin(2\varphi)$$
$$\frac{1}{2}\gamma_{\bar{x}\bar{y}} = \varepsilon_{\bar{x}\bar{y}} = -\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\sin(2\varphi) + \frac{\gamma_{xy}}{2}\cos(2\varphi)$$

Umkehrung des H. Gesetzes:

$$\sigma_{xx} = \frac{E}{1+\nu}\left[\varepsilon_{xx} + \frac{\nu}{1-2\nu}(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})\right] + \frac{E}{1-\nu}\alpha_T\Delta T,\ \tau_{xy} = G\gamma_{xy}$$
$$\sigma_{yy} = \frac{E}{1+\nu}\left[\varepsilon_{yy} + \frac{\nu}{1-2\nu}(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})\right] + \frac{E}{1-\nu} \cdot \alpha_T \cdot \Delta T,\ \tau_{xz} = G\gamma_{xz}$$
$$\sigma_{zz} = \frac{E}{1+\nu}\left[\varepsilon_{zz} + \frac{\nu}{1-2\nu}(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})\right] + \frac{E}{1-\nu} \cdot \alpha_T \cdot \Delta T,\ \tau_{yz} = G\gamma_{yz}$$

* Dreiachsiger Spannungszustand auf S. 2

Formeln die ich in die Formelsammlung aufnehmen muss

$$\sigma_{\bar{x}\bar{x}} = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\varphi + \tau_{xy} \sin 2\varphi$$

$$\sigma_{\bar{y}\bar{y}} = \frac{\sigma_{xx} + \sigma_{yy}}{2} - \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\varphi - \tau_{xy} \sin 2\varphi$$

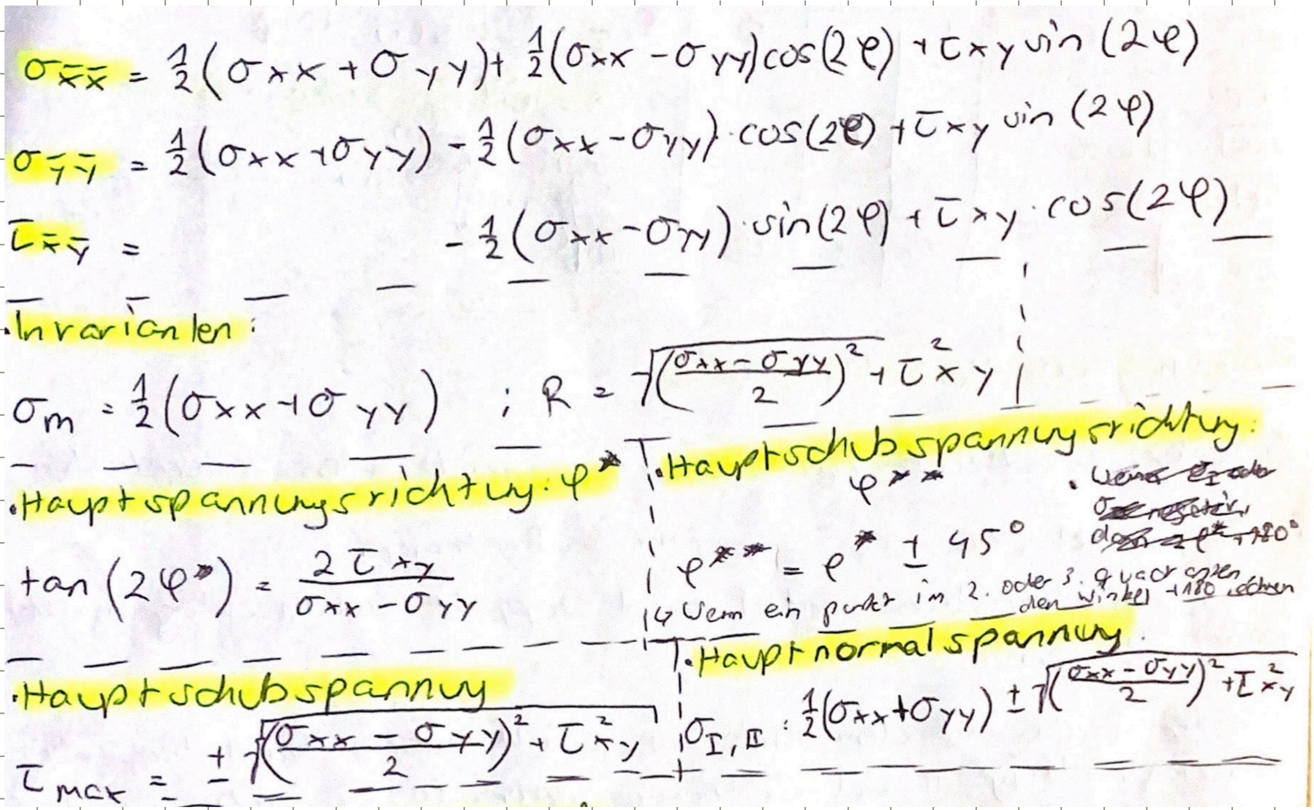
$$\tau_{\bar{x}\bar{y}} = - \frac{\sigma_{xx} - \sigma_{yy}}{2} \sin 2\varphi + \tau_{xy} \cos 2\varphi$$

$$\sigma_M = \frac{\sigma_{xx} + \sigma_{yy}}{2} =$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} =$$

$$\Downarrow$$

$$\sigma_{1,2} = \sigma_M \pm R$$



$$\sigma_{\bar{x}\bar{x}} = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) + \frac{1}{2}(\sigma_{xx} - \sigma_{yy})\cos(2\varphi) + \tau_{xy} \sin(2\varphi)$$

$$\sigma_{\bar{y}\bar{y}} = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) - \frac{1}{2}(\sigma_{xx} - \sigma_{yy}) \cdot \cos(2\varphi) + \tau_{xy} \sin(2\varphi)$$

$$\tau_{\bar{x}\bar{y}} = - \frac{1}{2}(\sigma_{xx} - \sigma_{yy}) \cdot \sin(2\varphi) + \tau_{xy} \cdot \cos(2\varphi)$$

• Invarianten:

$$\sigma_m = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) \quad ; \quad R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

• Hauptspannungsrichtung: φ^*

$$\tan(2\varphi^*) = \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}$$

• Hauptschubspannungsrichtung: φ^{**}

$$\varphi^{**} = \varphi^* \pm 45°$$

• ~~Wenn σ_I oder σ negativ, dann $2\varphi^*$ +180°~~

↳ Wenn ein Punkt im 2. oder 3. Quadranten den Winkel +180 rechnen

• Hauptschubspannung

$$\tau_{max} = \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

• Hauptnormalspannung

$$\sigma_{I,II} : \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

Formeln zur schiefen Biegung

$$\sigma_x(\overset{\circ}{x}, \overset{\circ}{y}) \quad \frac{N}{A} + \frac{M_{\overset{\circ}{y}}}{I_{\overset{\circ}{y}}} \overset{\circ}{z} - \frac{M_{\overset{\circ}{z}}}{I_{\overset{\circ}{z}}} \cdot \overset{\circ}{y}$$

(Die Koordinaten müssen erst auf ein Hauptsystem gebracht werden)

$$\tan(2\varphi^*) = \frac{2 I_{yz}}{I_y - I_z}$$ (Hauptwinkel

-> Haupt Trägheitsmoment : $I_{\overset{\circ}{y}, \overset{\circ}{z}}$

$$I_{\bar{y}, \bar{z}} = \frac{I_y + I_z}{2} \pm \sqrt{\left(\frac{I_y - I_z}{2}\right)^2 + I_{yz}^2}$$

Ergebnis 1 ≙ $I_{\overset{\circ}{y}}$, Ergebnis 2 ≙ $I_{\overset{\circ}{z}}$

Transformation von $M_{y,z,max}$ ins Hauptachsensystem

$M_y^0 = M_y \cdot \cos(\varphi^*)$

$M_z^0 = M_y \cdot \sin(\varphi^*)$

$I_{YZ} = I_{yz} - y_s \cdot z_s \cdot A$

↳ Bei Rechtecken ist es 0

Drehmatrix $= \begin{bmatrix} \cos & -\sin \\ \sin & \cdot\cos \end{bmatrix}$

Kompatibilität Stabkräfte

$\varepsilon = \frac{S}{EA} \rightarrow \Delta l = \frac{Sl}{EA}$

Federkraft: $F_F = -c \cdot x$