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Modeling and Control of a Single Axis Tilting Quadcopter

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Abstract— **In this paper, the dynamic model of a tilting rotor quadcopter, i.e., a quad-rotor aerial vehicle with rotors that can tilt along one of its axes, is presented. The tilting rotor quadcopter provides the added advantage in terms of additional stable configurations, made possible by additional actuated controls, as compared to a traditional quadcopter without titling rotors. The tilting rotor quadcopter design is accomplished by using an additional motor for each rotor that enables the rotor to rotate along the axis of the quadcopter arm. This turns the traditional quadcopter into an over-actuated flying vehicle allowing us to have complete control over its position and the orientation. In this paper, a dynamic model of the tilting rotor quadcopter vehicle is derived for flying and hovering modes. The model includes the relationship between vehicle orientation angle and rotor tilt-angle. Furthermore, a PD controller is designed to achieve the hovering and navigation capability at any desired pitch or roll angle . The dynamic model and the control design is verified with the help of numerical studies.**

I. INTRODUCTION

Quadcopters are one of the most popular designs for miniature aerial vehicles (MAVs) due to their vertical take-off and landing capability, simplicity of construction, maneuverability, and ability to negotiate tight spaces making it possible for use in cluttered indoor areas. Due to these capabilities, quadcopters have recently been considered for a variety of applications both in military and civilian domains. In particular, quadcopter MAVs have been explored for applications such as surveillance and exploration of disasters (such as fire, earthquake, and flood), search and rescue operations, monitoring of hazmat spills, and mobile sensor networks [1, 2].

Blimps, fixed-wing planes, single rotor helicopters, bird-like prototypes, coaxial dual rotor helicopters, quad-rotors, tilting rotor quadcopters are examples of different configurations and propulsion mechanisms that have been developed to allow 3D movements in aerial platforms [3]. Each of these has advantages and drawbacks. This paper focuses on quadcopters or quad-rotors which consist of four rotors in total, with two pairs of counter-rotating, fixed-pitch blades located at the four corners of the aircraft. This kind of design has two main advantages over the comparable vertical takeoff and landing (VTOL) Unmanned Aerial Vehicles (UAVs) such as single rotor helicopters. Firstly, quad-rotors do not require complicated mechanical linkage for rotor actuation. Quad-rotors utilize four fixed pitch rotors the variations of whose speeds form the basis of the control. It results in simplified design and maintenance of the quad-rotors. Secondly, the use of four individual rotors results in their smaller diameters as compared to the similar main rotor of a helicopter. The smaller the rotors the less is stored kinetic energy associated with each rotor. This diminishes the risk posed by the rotors if it comes in contact with any external object. Furthermore, by securing the rotors inside a frame, the protection of rotors during collisions is achieved. It allows indoor flights in obstacle-dense environments with lower risk of quad-rotor damage, and higher operator and surrounding safety. These benefits have resulted in safe test flight by inexperienced pilots in indoor environments and recovery time in case of collisions [4]. In particular, vertical, low speed, and stationary flight are well-known characteristics of a quad-rotors. Structurally, quad-rotors can be made in a small size, with a simple mechanics and control. Though, as a main drawback, the high energy consumption can be mentioned. However, the trade-off results are very positive. This configuration can be attractive in particular for surveillance, for imaging dangerous environments, and for outdoor navigation and mapping [5].

Conventionally, the quad-rotor attitude is controlled by changing the rotational speed of each motor. The front rotor and back rotor pair rotates in a clockwise direction, while the right rotor and left rotor pair rotates in a counter-clockwise direction. This configuration is devised in order to balance the moment created by each of the spinning rotor pairs. There are basically four maneuvers that can be accomplished by changing the speeds of the four rotors. By changing the relative speed of the right and left rotors, the roll angle of the quad-rotor is controlled. Similarly, the pitch angle is controlled by varying the relative speeds of the front and back rotors, and the yaw angle by varying the speeds of clockwise rotating pair and counter-clockwise rotating pair. Increasing or decreasing the speeds of all four rotors simultaneously controls the collective thrust generated by the robot [6, 7].

One of the basic limitations of the classical quad-rotor design is that by having only 4 independent control inputs, i.e., the 4 propeller spinning velocities, the independent control of the six-dimensional position and orientation of the quad-rotor is not possible. For instance, a quad-rotor can hover in place only and if only when being horizontal to the ground plane or it needs to tilt along the desired direction of motion to be able to move [8]. Tilting rotor quadcopter concept has evolved to solve these basic limitations of a quad-rotor. For example, in [9], a novel actuation concept for a quad-rotor in which the propellers are allowed to tilt about the axes connecting them to the main body frame, thus

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realizing a quad(tilt-)rotor [11].

Tilt-design makes the dynamics of the quadcopter more complex, and introduces additional challenges in the control design. However, tilting rotor quadcopter, designed by using additional four servo motors that allow the rotors to tilt, is an over-actuated system that potentially can track an arbitrary trajectory over time [10]. It gives the full controllability over the quad-rotor position and orientation providing possibility of hovering in a tilted configuration. The paper presents a mathematical dynamic modeling of the tilting rotor quadcopter which provides a description of the dynamical behavior of the quadcopter as a function of the rotational speeds of each of the four rotors, and their respective tilt-angles. The developed mathematical representation of the tilting rotor quadcopter can be used to predict the position and orientation of the quad-rotor. The same model can further be used to develop a PD control strategy, whereby manipulating the speeds of individual motors and tilt-angle results in achieving the desired motion and configuration.

II. DYNAMIC MODELING

Unlike traditional quad-rotor models, which have only four rotatory propellers as the vehicle's inputs, in tilting rotor quadcopters, there are four more servo motors attached to the each arm that adds one degree of freedom to each of the propellers, resulting in the tilting motions along their axes. In this section, at first the dynamic model of a traditional quad-rotor is described. Then, the equations of motion of a tilting rotor quadcopter are presented.

A. Traditional Quad-rotor

Fig. 1 schematically shows the co-ordinate system and forces acting on a traditional quad-rotor. In the 3 dimensional space of the quad-rotor, the world-frame (E) denotes the fixed reference frame with respect to which all motion can be referred to and the body-frame (B) is a frame attached to the center of mass of the vehicle. There is a vertical force for each rotor that comes from the rotation of the rotor. In addition to the forces, each rotor produces a moment perpendicular to the plane of propeller rotation. The moment produced by a propeller on the vehicle is directed opposite to the direction of rotation of the propeller, and therefore to cancel out rotation along the Z-axis, the moments for rotor 1 and 3 are set in clockwise ($-Z_B$) direction and for rotor 2 and 4 are set in counter clockwise (Z_B) direction.

Based on NASA Standard Airplane [12], Euler angle transformations are defined by ψ, θ and ϕ which respectively represents the heading, attitude and bank angles also referred to as yaw, pitch and roll angles. Combined transformation matrix from body coordinate to the world coordinate is obtained by three successive rotations. The first rotation is about X axis, followed by another rotation about Y axis and the last rotation is about Z axis.

$$R_{EB} = \begin{bmatrix} c\psi c\theta & c\psi s\theta s\phi - s\psi c\phi & c\psi s\theta c\phi + s\psi s\phi \\ s\psi c\theta & s\psi s\theta s\phi + c\psi c\phi & s\psi s\theta c\phi - c\psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \quad (1)$$

where $c\psi$ and $s\psi$ denote $cos(\psi)$ and $sin(\psi)$ respectively, and similarly for other angles.

By obtaining vehicle's vertical forces in the world frame and writing the equations of motion based on the Newton's second law along the X, Y and Z axes, we can write:

$$\begin{aligned} m\ddot{x} &= \sum F_i(s\psi s\phi + c\psi s\theta c\phi) - C_1\dot{x} \\ m\ddot{y} &= \sum F_i(s\psi s\theta c\phi - c\psi s\phi) - C_2\dot{y} \\ m\ddot{z} &= \sum F_i(c\theta c\phi) - mg - C_3\dot{z} \end{aligned} \quad (2)$$

where m is the total mass of quad-rotor, g is the acceleration due to gravity, x, y and z are quadcopter position in world frame coordinate, C_1, C_2 and C_3 are drag coefficients. note that these coefficients are negligible at low speed. F_i ,$(i = 1, 2, 3, 4)$ are forces produced by the four rotors as given by the following equation:

$$F_i = K_f \omega_i^2 \quad (3)$$

where ω_i is the angular velocity of i^{th} rotor and K_f is a constant. In addition, Euler equations are written in order to obtain angular accelerations of the vehicle given by:

$$\begin{aligned} I_x\ddot{\phi} &= l(F_3 - F_1 - C_1'\dot{\phi}) \\ I_y\ddot{\theta} &= l(F_4 - F_2 - C_2'\dot{\theta}) \\ I_z\ddot{\psi} &= M_1 - M_2 + M_3 - M_4 - C_3'\dot{\psi} \end{aligned} \quad (4)$$

where l is distance of each rotor from the vehicle's center of gravity. I_x, I_y and I_z are moment of inertia along x, y and z directions respectively. C_1' , C_2' and C_3' are rotational drag coefficients. M_i, $(i = 1, 2, 3, 4)$ are rotors moment produced by angular velocity of rotors and given by:

$$M_i = K_m \omega_i^2 \quad (5)$$

where ω_i is the angular velocity of i^{th} rotor and K_m is the constant.

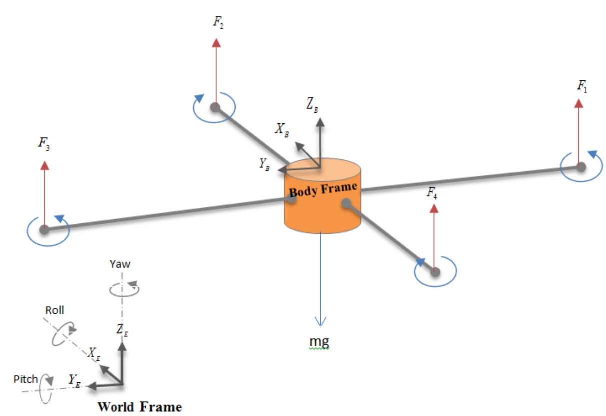

Fig. 1: Schematic diagram showing the coordinate systems and forces acting on the quad-rotor

During a hovering flight, the quad-rotor not only has zero acceleration but also velocities and angles of pitch and roll must be zero,i.e. $r = r_0, \theta = \phi = 0, \psi = \psi_0, \dot{r} = 0, \dot{\theta} =$

$\dot{\phi} = \dot{\psi} = 0$. At this nominal hover state, the produced force from each propellers must satisfy:

$$F_i = \frac{1}{4}(mg) \tag{6}$$

and hence motor speeds are given by:

$$\omega_i = \omega_h = \sqrt{\frac{mg}{4k_f}} \tag{7}$$

B. Tilting Rotor Quadcopters

For a tilting rotor quadcopter, four other variables are added representing the angles of quad-rotor arms. Adjustment of these angles results into improved vehicle maneuverability and capability for hovering at a tilted angle.

To illustrate the motion of the tilting rotors quadcopter, a schematic diagram showing the forces/moments acting and coordinate frames used in the modeling is provided in Fig 2. As it can be seen from this figure, the propellers are free to tilt along their axes. The planes shown with dashed lines are the original planes of rotation with zero tilt angles for the respective propellers. Similarly, planes shown with the rigid lines are the tilted planes of rotation for respective propellers. θ_i, $(i = 1, 2, 3, 4)$is the tilted angle of corresponding propellers. It may be noted that the forces generated by the propellers are perpendicular to these respective planes of rotations. Using the rotational matrix in

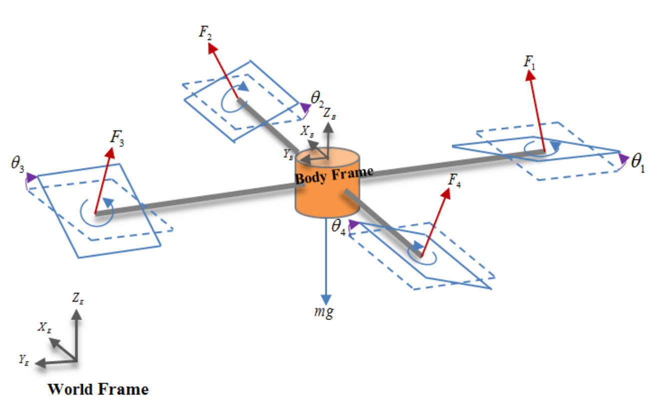

Fig. 2: Coordinate frames and free body diagram of tilting quadcopter

(1), equations of motion in world-frame can be written as:

$$\begin{aligned}
m\ddot{x} &= F_1 s\theta_1 c\psi c\theta - F_3 s\theta_3 c\psi c\theta - F_4 s\theta_4 c\psi s\theta s\phi \\
&+ F_4 s\theta_4 s\psi c\phi + F_2 s\theta_2 c\psi s\theta s\phi - F_2 s\theta_2 s\psi c\phi \\
&+ F_1 c\theta_1 c\psi s\theta c\phi + F_2 c\theta_2 c\psi s\theta c\phi \\
&+ F_3 c\theta_3 c\psi s\theta c\phi + F_4 c\theta_4 c\psi s\theta c\phi + F_1 c\theta_1 s\psi s\phi \\
&+ F_2 c\theta_2 s\psi s\phi + F_3 c\theta_3 s\psi s\phi + F_4 c\theta_4 s\psi s\phi - C_1\dot{x} \\
m\ddot{y} &= F_1 s\theta_1 c\psi c\theta - F_3 s\theta_3 s\psi c\theta - F_4 s\theta_4 s\psi s\theta s\phi \\
&+ F_2 s\theta_2 s\psi s\theta_2 s\phi - F_4 s\theta_4 c\psi c\phi + F_2 s\theta_2 c\psi c\phi \\
&+ F_1 c\theta_1 s\psi s\theta c\phi + F_2 c\theta_2 s\psi s\theta c\phi \\
&+ F_3 c\theta_3 s\psi s\theta c\phi + F_4 c\theta_4 s\psi s\theta c\phi - F_1 c\theta_1 c\psi s\phi \\
&- F_2 c\theta_2 c\psi s\phi - F_3 c\theta_3 c\psi c\phi - F_4 c\theta_4 c\psi s\phi - C_2\dot{y} \\
m\ddot{z} &= -F_1 s\theta_1 s\theta + F_3 s\theta_3 s\theta - F_4 s\theta_4 c\theta s\phi \\
&+ F_2 s\theta_2 c\theta s\phi + F_1 c\theta_1 c\theta c\phi + F_2 c\theta_2 c\theta c\phi \\
&+ F_3 c\theta_3 c\theta c\phi + F_4 c\theta_4 c\theta c\phi - mg - C_3\dot{z}
\end{aligned} \tag{8}$$

Similarly, the angular accelerations are determined by Euler equations:

$$\begin{aligned}
I_x\ddot{\phi} &= l(F_3 c\theta_3 - F_1 c\theta_1 - C_1'\dot{\phi}) \\
&+ (M_1 s\theta_1 - M_3 s\theta_3) + ({M_2}' + {M_4}') \\
I_y\ddot{\theta} &= l(F_4 c\theta_4 - F_2 c\theta_2 - C_2'\dot{\theta}) \\
&+ (M_4 s\theta_4 - M_2 s\theta_2) + ({M_1}' + {M_3}') \\
I_z\ddot{\psi} &= l(F_1 s\theta_1 + F_2 s\theta_2 + F_3 s\theta_3 + F_4 s\theta_4 - C_3'\dot{\psi}) \\
&+ (M_1 c\theta_1 - M_2 c\theta_2 + M_3 c\theta_3 - M_4 c\theta_4)
\end{aligned} \tag{9}$$

where $M_i', (i = 1, 2, 3, 4)$ are the tilting moments which are created by the four servo motors attached to the end of each arm to cause a tilt angle. By the assumption of $\theta_1 = -\theta_3$ and $\theta_2 = -\theta_4$, all of these moments will be canceled out. Based on the dynamic model presented above, we propose the following two Theorems.

Theorem 1: *Considering the dynamics of the tilting rotor quadcopter given by Equations (8) and (9), and assuming the relationship between the tilting angles of the four rotors $\theta_1 = -\theta_3$ and $\theta_2 = -\theta_4$ and all rotors having equal rotational speeds, the quadcopter, at an equilibrium hovering state, achieves a roll angle ϕ given by $\phi = \theta_1/2$ when pitch angle is zero, and a pitch angle θ given by $\theta = \theta_2/2$ when the roll angle is zero.*

Proof: In tilt-hovering the arm angles of the first and third propellers are tilted by θ_1 and $-\theta_3$, respectively. This produces a roll angle ϕ of the vehicle, and, the equations for linear motion of the quadcopter is given by:

$$\begin{bmatrix} m\ddot{x} \\ m\ddot{y} \\ m\ddot{z} \end{bmatrix} = \begin{bmatrix} F_1 s(\theta_1 - \phi) + F_3 s(-\theta_3 - \phi) - F_2 s\phi - F_4 s\phi \\ 0 \\ F_1 c(\theta_1 - \phi) + F_2 c\phi + F_3 c(-\theta_3 - \phi) - F_4 c\phi \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \tag{10}$$

For hovering, the accelerations $\ddot{x}$, $\ddot{y}$, and $\ddot{z}$ should all be equal to zero. Using the equation corresponding to the acceleration in X direction, and noting that $F_1 = F_2 =$

$F_3 = F_4$ since rotational speeds of all rotors are the same, the angle ϕ can be obtained as:

$$\phi = \frac{\theta_1}{2} \tag{11}$$

Similar to the equation (10), if the second and fourth arms are tilted, the equations of motion cane be written as:

$$\begin{bmatrix} m\ddot{x} \\ m\ddot{y} \\ m\ddot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ -F_1 s\theta_1 - F_3 s\theta + F_2 s(\theta_2 - \theta) + F_4 s(-\theta_4 - \theta) \\ F_1 c\theta_1 + F_2 c(\theta_2 - \theta) + F_3 c\theta + F_4 c(-\theta_4 - \theta) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \tag{12}$$

Similar to above, the angle θ resulted from tilting of the second and forth arms, is given by:

$$\theta = \frac{\theta_2}{2} \tag{13}$$

Theorem 2: *Considering the dynamics of the tilting rotor quadcopter given by Equations (8) and (9), and assuming the relationship between the tilting angles of the four rotors $\theta_1 = -\theta_3$ and $\theta_2 = -\theta_4$, the motor speed needed for vehicle for hovering with a pitch angle is given by:*

$$\omega_i = \omega_h = \sqrt{\frac{mg}{4k_f c\frac{\theta_1}{2}}} \qquad when \quad \theta = 0$$

and

$$\omega_i = \omega_h = \sqrt{\frac{mg}{4k_f c\frac{\theta_2}{2}}} \qquad when \quad \phi = 0 \tag{14}$$

Proof: In hovering with roll angle and zero pitch angle, the acceleration along z axis is zero, $\ddot{z} = 0$. Therefore, using the third row of Equation (10), we get:

$$cos(\frac{\theta_1}{2}) \sum F_i = mg \tag{15}$$

Based on Equation (3), and noting that each rotor's angular speed is the same (i.e., $F_1 = F_2 = F_3 = F_4$), the angular speed is given by:

$$\omega_i = \omega_h = \sqrt{\frac{mg}{4k_f c\frac{\theta_1}{2}}} \tag{16}$$

Similar to above, considering hovering with θ and zero roll angle, the Equation (12) gives:

$$cos(\frac{\theta_2}{2}) \sum F_i = mg \tag{17}$$

Now similar to above, the angular speeds of the rotors are given by:

$$\omega_i = \omega_h = \sqrt{\frac{mg}{4k_f c\frac{\theta_2}{2}}} \tag{18}$$

III. CONTROLLER DESIGN

In this section, the control strategy of the tilting rotor quadcopter is presented. The aim of the control strategy is not only control of the position of the vehicle to follow an arbitrary trajectory in 3 dimensions, but also to have control over the orientation of the vehicle in hovering as well as during trajectory tracking.

The controller inputs are four independent speeds of propellers and their rotations about the axes of quadcopter arms. Referring to Fig. 2 and the two Theorems, it is assumed that $\theta_1 = -\theta_3$ and $\theta_2 = -\theta_4$. It may be noted that these constraints, in fact, make the over-actuated system into fully actuated system (two inputs to tilt the rotors another four inputs for their rotational speeds make total number of independent control inputs to be six). For 6 DoF quadcopter, this results into complete control over its position and orientation. The dynamic model of the tilting rotor quadcopter, described in (8) and (9), is used to design the PD controllers for orientation adjustment and trajectory tracking.

To start tracking a specific trajectory, first, hovering from the initial starting point is necessary. Then, the orientation of the vehicle to a specific pitch or roll angle is obtained. In [13], the relationship between the rotational speeds of the motors and the deviation of the orientations from nominal vectors and hovering speeds is described in details. In conventional quad-rotors, the hovering speed for maintaining a specific height is constant. However, in the current design, when the quadcopter tilts, the hovering speed is changed according to Equation (14) as follows:

$$\begin{bmatrix} \omega_1^{des} \\ \omega_2^{des} \\ \omega_3^{des} \\ \omega_4^{des} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \omega_h + \Delta\omega_f \\ \Delta\omega_\phi \\ \Delta\omega_\theta \\ \Delta\omega_\psi \end{bmatrix} \tag{19}$$

where ω_i^{des}, $(i = 1, 2, 3, 4)$ are the desired angular velocities of the respective rotors. The hovering speed, ω_h, is calculated from (14). The proportional- derivative control laws are used to control $\Delta\omega_\phi, \Delta\omega_\theta, \Delta\omega_\psi$ and $\Delta\omega_f$ which are deviations that result into forces/moments causing roll, pitch, yaw, and a net force along the z_B axis, respectively, and are calculated as:

$$\begin{aligned} \Delta\omega_\phi &= k_{p,\phi}(\phi^{des} - \phi) + k_{d,\phi}(p^{des} - p) \\ \Delta\omega_\theta &= k_{p,\theta}(\theta^{des} - \theta) + k_{d,\theta}(q^{des} - q) \\ \Delta\omega_\psi &= k_{p,\psi}(\psi^{des} - \psi) + k_{d,\psi}(r^{des} - t) \end{aligned} \tag{20}$$

where p, q and t are the component of angular velocities of the vehicle in the body frame. The relationship between these components and the pitch, roll, and yaw are provided in [14].

The relationship between the tilt angles of individual rotors, given by $\theta_i^{des}, i = 1, 2..4$, and the reference pitch and roll angles is given by :

$$\begin{bmatrix} \theta_1^{des} \\ \theta_2^{des} \\ \theta_3^{des} \\ \theta_4^{des} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2\phi_h^{des} \\ 2\theta_h^{des} \\ \Delta\phi_h \\ \Delta\theta_h \end{bmatrix} \tag{21}$$

where ϕ_h^{des} and θ_h^{des} are reference roll and pitch angles and $\Delta\phi_h$ and $\Delta\theta_h$ orientation deviations. Fig. 3 shows the orientation of the vehicle with respect to the tilted propellers. A proportional-derivative controller is used to control the orientation deviation using the reference orientation values as:

$$\begin{aligned} \Delta\phi_h &= k_{p,\phi_h}(\phi_h^{des} - \phi) - k_{d,\phi_h} p \\ \Delta\theta_h &= k_{p,\theta_h}(\theta_h^{des} - \theta) - k_{d,\theta_h} q \end{aligned} \tag{22}$$

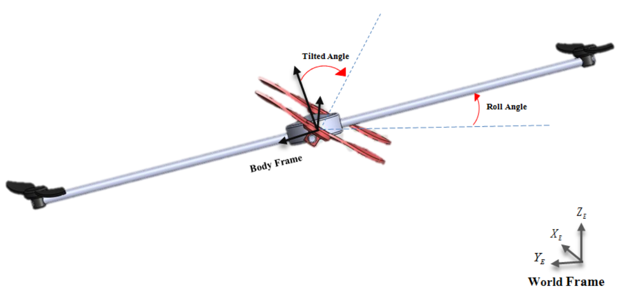

Fig. 3: Hovering with tilted arms

In order to have the quad-rotor track a desired trajectory $r_{i,T}$, the command acceleration, $\ddot{r}_i^{des}$ is calculated from proportional-derivative controller based on position error, as [13]:

$$(\ddot{r}_{i,T} - \ddot{r}_i^{des}) + k_{d,i}(\dot{r}_{i,T} - \dot{r}_i) + k_{p,i}(r_{i,T} - r_i) = 0 \tag{23}$$

where r_i and $r_{i,T}$ $(i = 1, 2, 3)$ are the 3-dimensional position of the quad-rotor and desired trajectory respectively. It may be noted that $\dot{r}_{i,T} = \ddot{r}_{i,T} = 0$ for hover.

During the flight of a tilting quadcopter, the orientation of the vehicle needs be set at specific pitch or roll. This can be obtained by linearizing the equation of motion that correspond to the nominal hover states. The nominal hover state $(\phi = \phi_h^{des} = \theta_1/2, \theta = \theta_2 = 0, \psi = \psi_T, \dot{\theta} = \dot{\psi} = \dot{\phi} = 0)$ corresponds to equilibrium hovering configuration with the reference pitch angle. The change of the pitch and roll angles are supposed to be small during flight. By linearizing Equation (8) about these nominal hovering states, desired pitch and roll angles to cause the motion can be derived as given by the following equations :

$$\begin{aligned} \ddot{r_1}^{des} &= \frac{2g}{cos(\frac{\theta_1}{2})}(A\widetilde{\theta}^{des} + B\widetilde{\phi}^{des} + C) \\ \ddot{r_2}^{des} &= \frac{2g}{cos(\frac{\theta_1}{2})}(D\widetilde{\theta}^{des} - E\widetilde{\phi}^{des} + F) \end{aligned} \tag{24}$$

where

$$\begin{aligned} A &= c(2\theta_h^{des})c(\psi_T)c(\theta_h^{des}) + c(\psi_T)c(\theta_h^{des}) \\ B &= c(2\theta_h^{des})s(\psi_T)c(\theta_h^{des}) + s(\psi_T)c(\theta_h^{des}) \\ C &= s(2\theta_h^{des})c(\psi_T) + c(2\theta_h^{des})s(\psi_T)s(\theta_h^{des}) \\ &+ s(\psi_T)s(\theta_h^{des}) \\ D &= c(2\theta_h^{des})s(\psi_T)c(\theta_h^{des}) + s(\psi_T) \\ E &= c(2\theta_h^{des})c(\psi_T)c(\theta_h^{des}) + c(\psi_T)c(\theta_h^{des}) \\ F &= s(2\theta_h^{des})s(\psi_T) - c(2\theta_h^{des})c(\psi_T)s(\theta_h^{des}) \\ &- c(\psi_T)s(\theta_h^{des}) \end{aligned}$$

$\widetilde{\phi}^{des}$ and $\widetilde{\theta}^{des}$ are also respectively the desired roll and pitch angles that are needed for position control when the orientation is set to be given by the nominal hovering values. It may be noted that a similar equation can be derived to obtained a reference orientation along the roll direction. Equation (24) represents a pair of two coupled linear equations which can be easily solved to obtain the $\widetilde{\phi}^{des}$ and $\widetilde{\theta}^{des}$. Finally, the desired pitch or roll angles are obtained by:

$$\begin{aligned} \phi^{des} &= \widetilde{\phi}^{des} + \phi_h^{des} \\ \theta^{des} &= \widetilde{\theta}^{des} + \theta_h^{des} \end{aligned} \tag{25}$$

The above equation is used in Equation (20) and subsequently in Equation (19) to obtain the desired speeds of the individual rotors.

IV. Numerical Simulation Results

A. Simulation set up

To validate the presented dynamic model and control method, a numerical simulation of the tilting rotor quadcopter was developed using the MATLAB. The vehicle's initial position was $(0.1, 0.05, 0)$. The final position was set to $(6.5, 0.3, 1)$. The desired pitch angle was set to 18^0 and the desired roll angle was set to 0^0. The discretized versions of the dynamic and the controller equations are solved by Euler method.

B. Simulation results

The procedure to accomplish the flight simulation is to have the vehicle take-off from an initial point vertically till the desired height, and then steer to the destination point with the horizontal flight. During flight, the orientation of the vehicle is supposed to change according to reference inputs without losing the height. The quadcopter trajectory in the three dimensional space from the initial point to the desired destination is shown in Fig 4.

Fig. 5 shows the reference pitch and roll angle. It can be seen that for time $t = 0$ sec to $t = 1$ sec, the reference pitch and roll angles are zero. At $t = 1$ sec, the reference pitch angle increases from 0^0 and reaches the value of 18^0 at $t = 4$ sec. Fig. 6 shows the actual change in the pitch, roll, and yaw angles of the tilting rotor quadcopter during the flight. It can be seen that the change in the roll and yaw angle is close to zero while the actual pitch follows the reference pitch value.

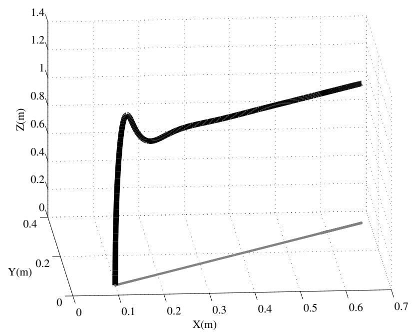

Fig. 4: Vehicle trajectory in three dimensional environment

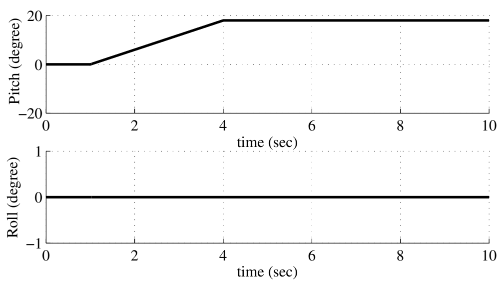

Fig. 5: The reference pitch and roll angle

Fig. 7 shows how the speed of four motors changes during the flight to track the trajectory and maintain the height of the vehicle. The increase in motor speed can be explained by Equation (14). Comparing Equation (14) (for tilted configuration) to Equation (7) (for non-tilted configuration), the theory predicts the need of more rotor speed in tilted configuration so that the vertical component of force still balances the weight in the tilted configuration.

V. CONCLUSION

In this paper, the dynamic modeling and control of a tilting rotor quadcopter was presented. The relationship between the tilting-rotor angles and the quadcopter orientation was derived using the dynamic model. It was shown that such this design makes the quadcopter a fully-controlled system which can track any arbitrary trajectory. Hovering with controlled pitch and roll angle, and motion with desired orientation are

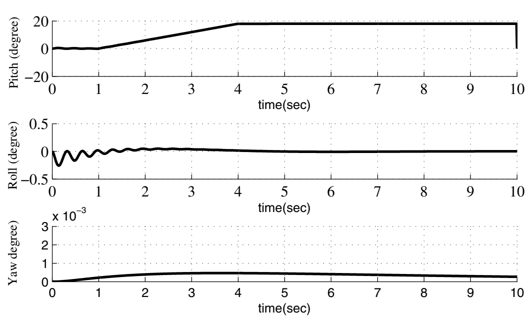

Fig. 6: The actual orientation

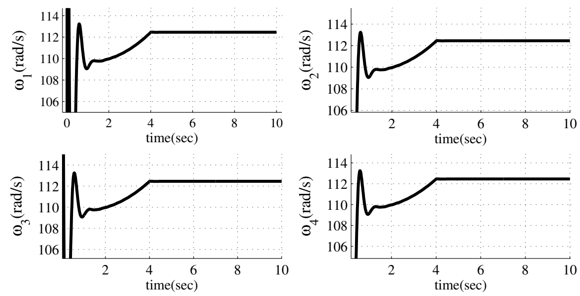

Fig. 7: The speed of each rotor

some of the features of the presented quadcopter system. The paper presents the dynamic model, and suggests a simple PD based control to achieve the desired motion. The model and the controller are verified with the help of numerical simulations. The paper presents the method for achievement of desired orientation of the quadcopter in only one direction (pitch or roll at a time). Future work would involve solving the dynamic model to allow the desired orientation to be achieved in any arbitrary combination of pitch and roll angles.

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