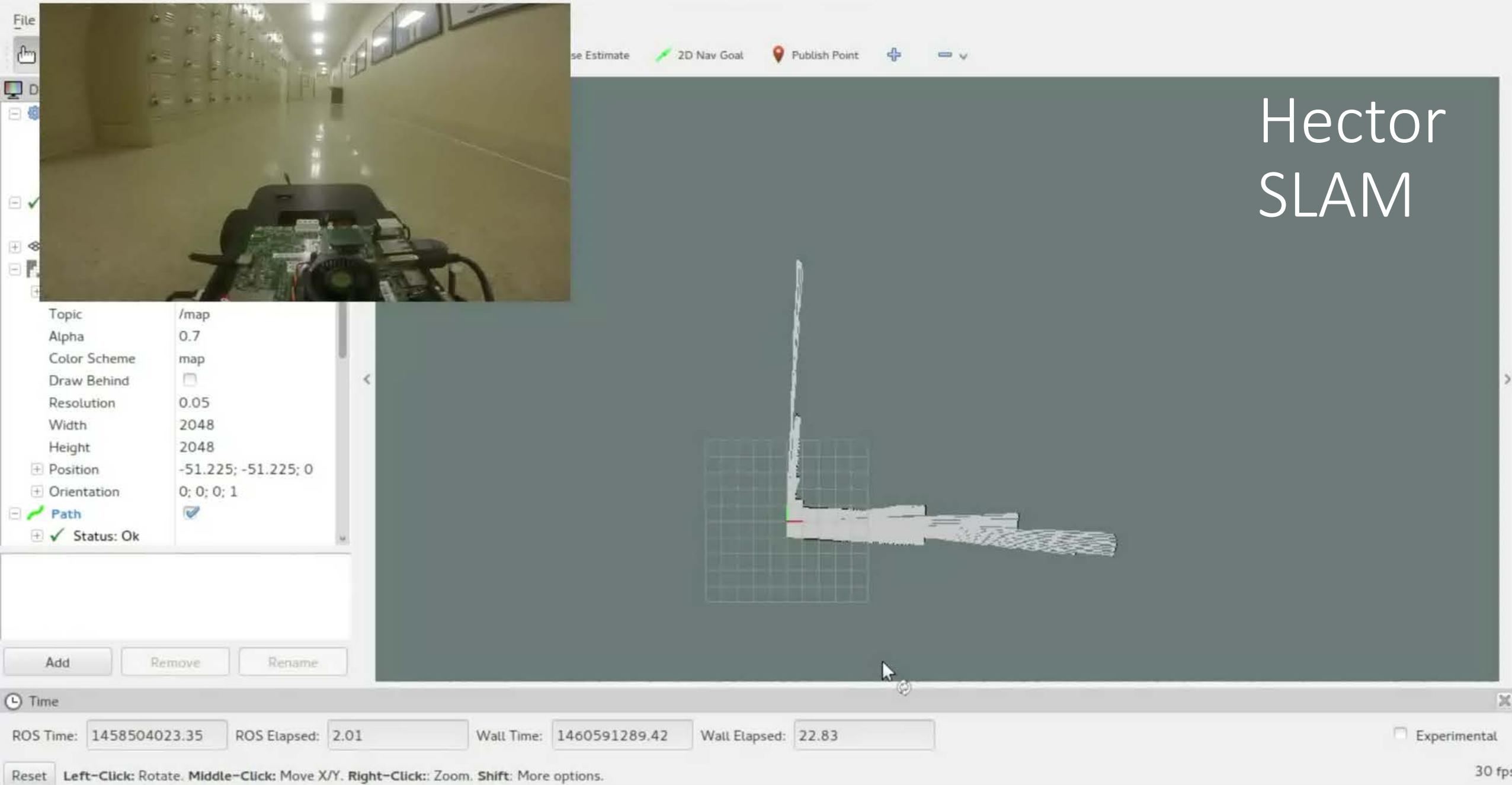


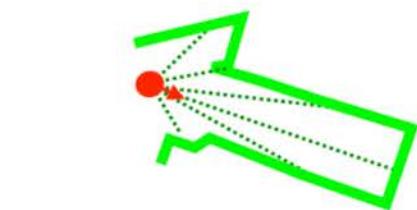
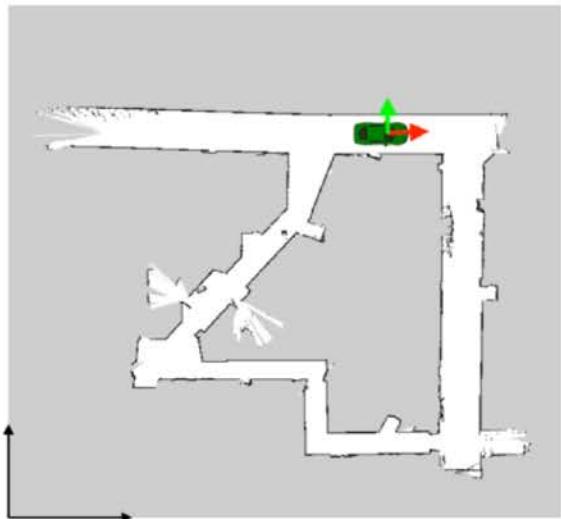
mapping_demo.rviz* - RViz



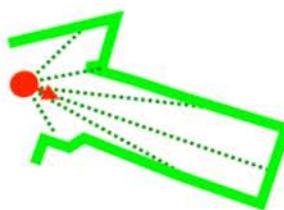
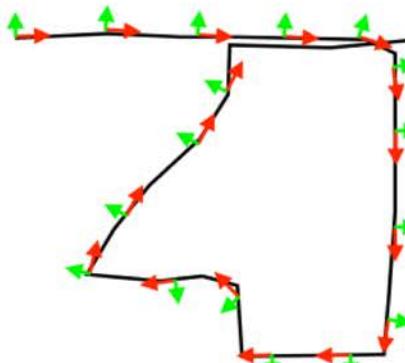
Objectives

- What is Simultaneous Localization and Mapping ?
- What are Occupancy Maps ?
- How are SLAM and Scan Matching related ?
- SLAM in ROS – Hector Mapping
- Google Cartographer SLAM Overview

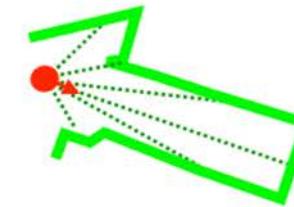
Problem Setting



Localization: given a **map**, use sensor data to estimate the current pose of the robot



Mapping: given robot pose at each time **(trajectory)**, use sensor data to build map



Simultaneous Localization and Mapping (SLAM): use sensor data to build map and estimate robot trajectory

A brief history of SLAM

Why do we need a map?

- In order to support path planning
- Limiting the error in state estimates, by providing the opportunity to ‘reset’
- Later... do we really need a map?

Historical Development (1986-2004): *Probabilistic Foundations*

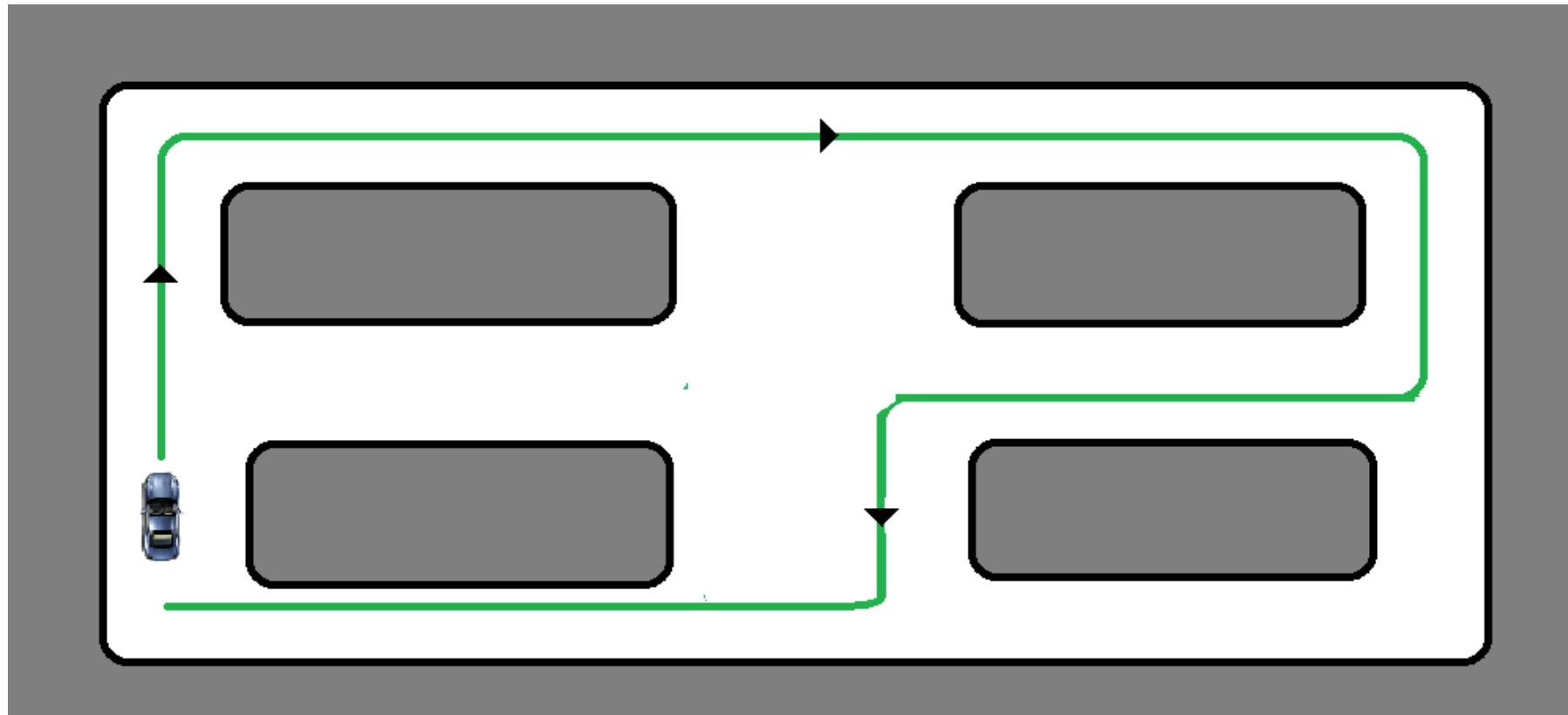
- EKF (you will still find this in visual inertial odometry)
- Particle Filter (very efficient localization)
- We will cover these methods next class in the context of localization

Modern Era (2004-Now): *Algorithmic Improvements*

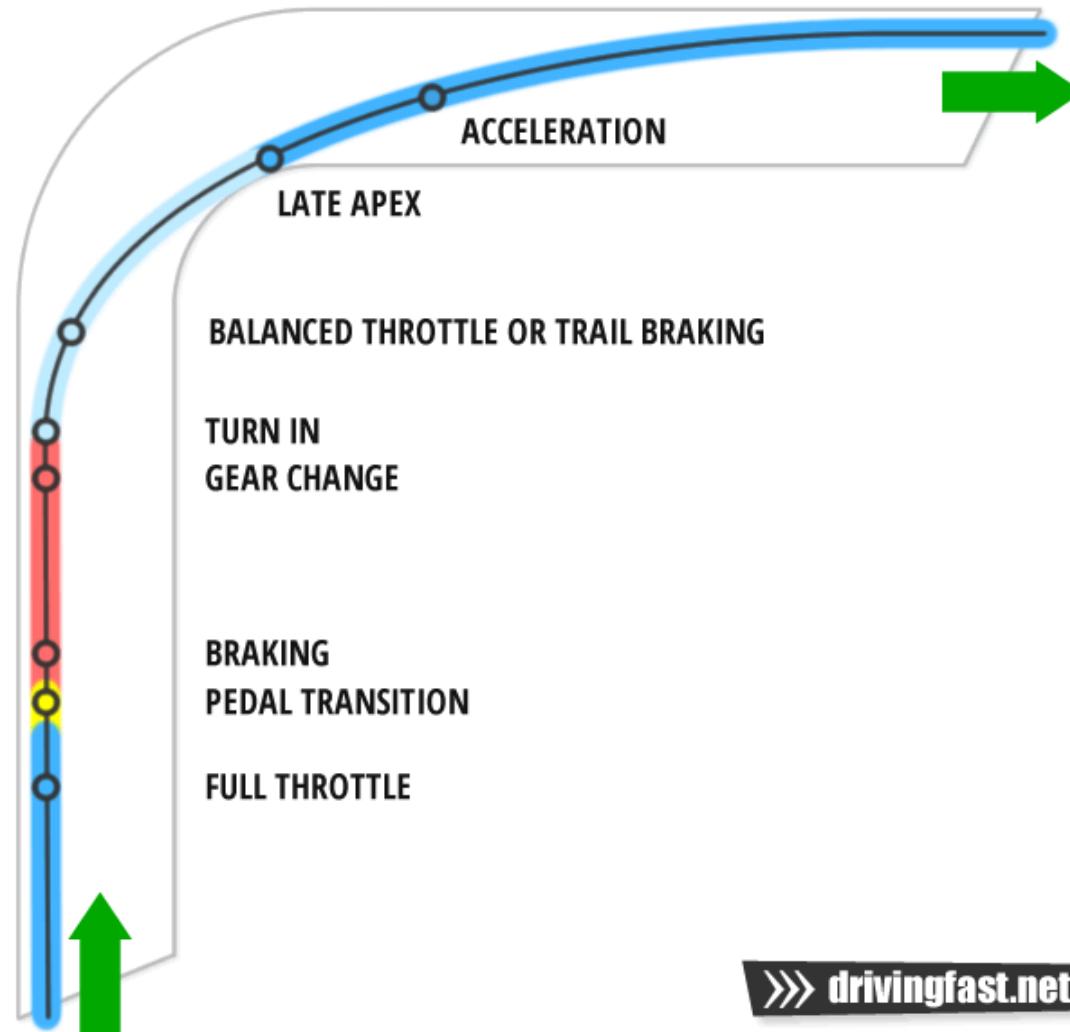
- Maximum a-posteriori Estimation
- Other names: factor graph optimization, graph-SLAM, smoothing and mapping (SAM), bundle adjustment

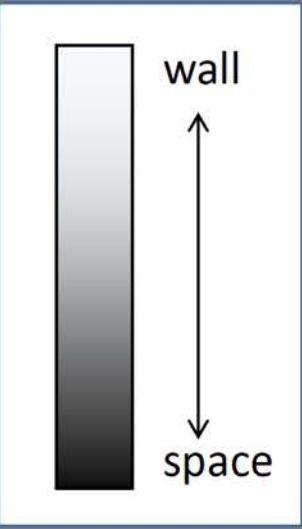
Limitations : Basic Path Planning

- High Level Path Assignments
 - 2nd right, 2nd right, 1st right, 1st left, 1st right



Race Lines





Occupancy Grid Mapping

- Occupancy: binary R.V.

$$m_{x,y}: \{free, occupied\} \rightarrow \{0, 1\}$$

[Review – Into Probability]

Given some probability space (Ω, P) ,
a **random variable** $X: \Omega \rightarrow \mathbb{R}$ is a *function* that
maps the sample space to the reals.

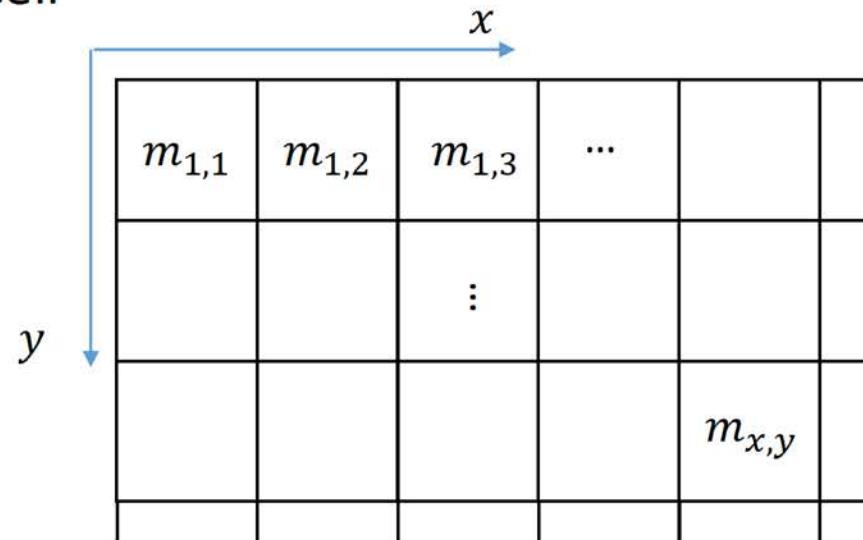
Occupancy Grid Mapping

- Occupancy: binary R.V.

$$m_{x,y}: \{free, occupied\} \rightarrow \{0, 1\}$$

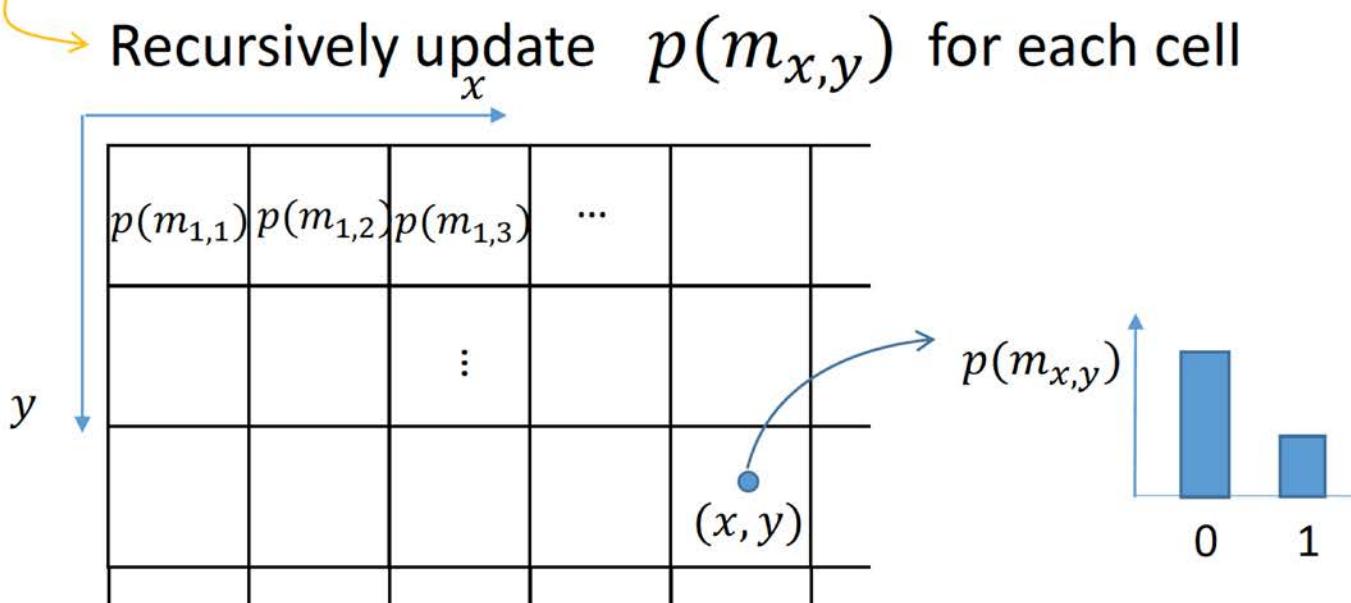
- Occupancy grid map

: fine-grained grid map where an occupancy variable associated with each cell



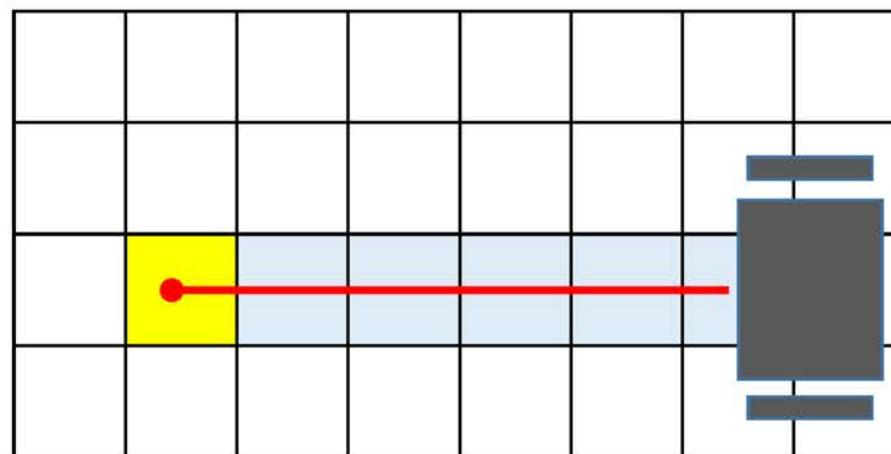
Occupancy Grid Mapping

- Occupancy grid mapping
: A Bayesian filtering to maintain a occupancy grid map.



Occupancy Grid Mapping

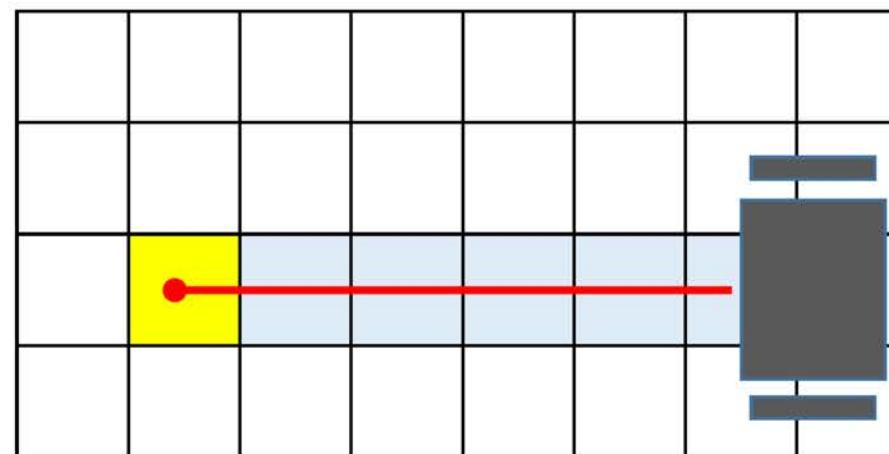
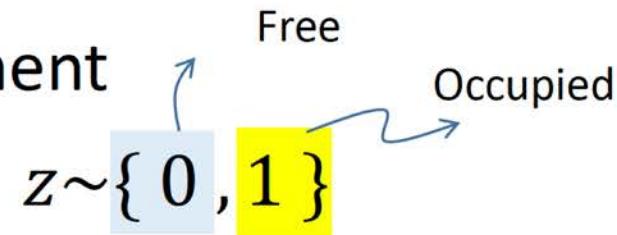
- Measurement



a range sensor

Occupancy Grid Mapping

- Measurement



a range sensor

Occupancy Grid Mapping

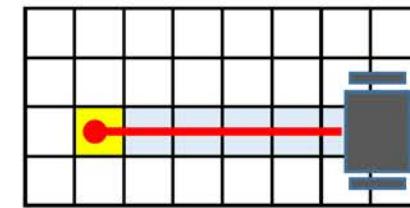
- Measurement

$z \sim \{ 0, 1 \}$

Free
↑
 $z \sim \{ 0, 1 \}$
Occupied

- Measurement model

$$p(z|m_{x,y})$$



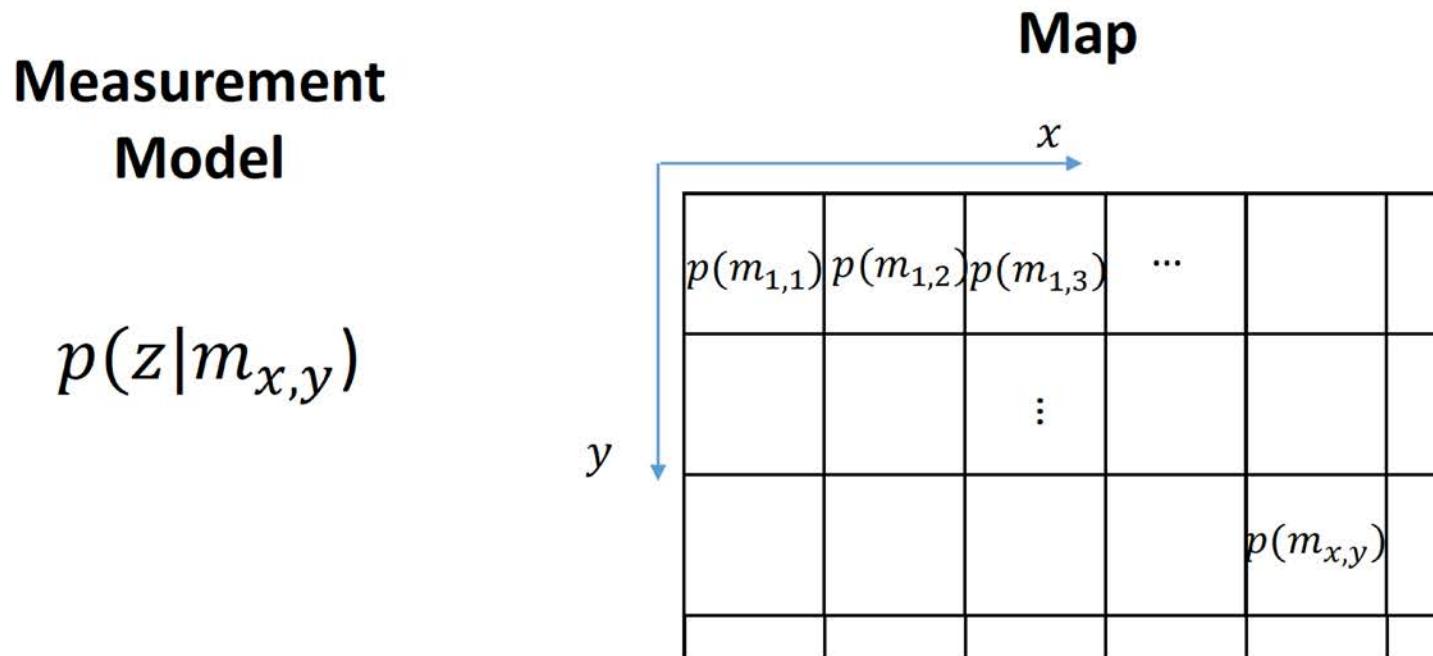
$p(z = 1|m_{x,y} = 1)$: True **occupied** measurement

$p(z = 0|m_{x,y} = 1)$: False **free** measurement

$p(z = 1|m_{x,y} = 0)$: False **occupied** measurement

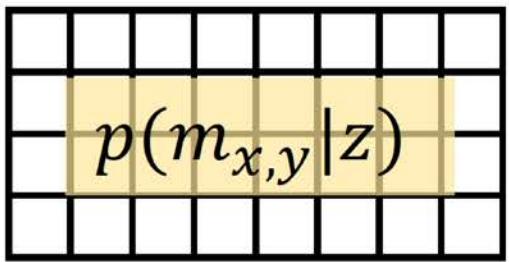
$p(z = 0|m_{x,y} = 0)$: True **free** measurement

Occupancy Grid Mapping



Occupancy Grid Mapping

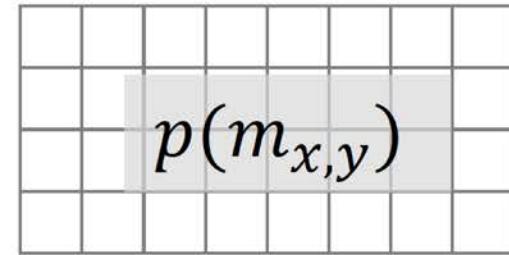
Posterior Map



Measurement Model

$$p(z|m_{x,y})$$

Prior Map



$$p(m_{x,y}|z) = \frac{p(z|m_{x,y})p(m_{x,y})}{p(z)}$$

Likelihood Prior
Posterior Evidence

Occupancy Grid Mapping

$$Odd := \frac{(X \text{ happens})}{(X \text{ not happens})} = \frac{p(X)}{p(X^c)}$$

- More convenient when we use “Odd”

$$Odd((m_{x,y} = 1) \text{ given } z) = \frac{p(m_{x,y} = 1 | z)}{p(m_{x,y} = 0 | z)}$$

Occupancy Grid Mapping

- Odd

(Bayes' Rule)

$$p(m_{x,y} = 1|z) = \frac{p(z|m_{x,y} = 1)p(m_{x,y} = 1)}{p(z)}$$

$$Odd = \frac{p(m_{x,y} = 1|z)}{p(m_{x,y} = 0|z)} = \frac{p(z|m_{x,y} = 1)p(m_{x,y} = 1)/p(z)}{p(m_{x,y} = 0|z)}$$

Occupancy Grid Mapping

- Odd

$$Odd = \frac{p(m_{x,y} = 1|z)}{p(m_{x,y} = 0|z)} = \frac{p(z|m_{x,y} = 1)p(m_{x,y} = 1)/p(z)}{p(z|m_{x,y} = 0)p(m_{x,y} = 0)/p(z)}$$


$$p(m_{x,y} = 0|z) = \frac{p(z|m_{x,y} = 0)p(m_{x,y} = 0)}{p(z)}$$

(Bayes' Rule)

Occupancy Grid Mapping

- Take the log!

Odd:

$$\frac{p(m_{x,y} = 1|z)}{p(m_{x,y} = 0|z)} = \frac{p(z|m_{x,y} = 1)p(m_{x,y} = 1)}{p(z|m_{x,y} = 0)p(m_{x,y} = 0)}$$

Log-Odd:

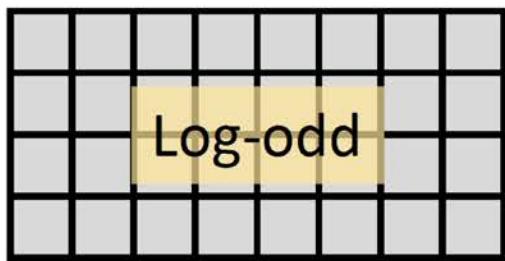
$$\begin{aligned}\log \frac{p(m_{x,y} = 1|z)}{p(m_{x,y} = 0|z)} &= \log \frac{p(z|m_{x,y} = 1)p(m_{x,y} = 1)}{p(z|m_{x,y} = 0)p(m_{x,y} = 0)} \\ &= \log \frac{p(z|m_{x,y} = 1)}{p(z|m_{x,y} = 0)} + \log \frac{p(m_{x,y} = 1)}{p(m_{x,y} = 0)}\end{aligned}$$

$$\boxed{\log odd^+ = \log odd\ meas + \log odd^-}$$

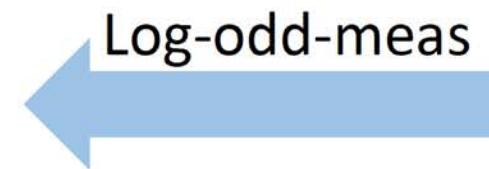
Occupancy Grid Mapping

- Log-odd update

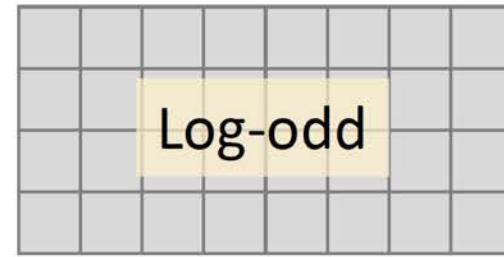
Posterior Map



Measurement Model



Prior Map

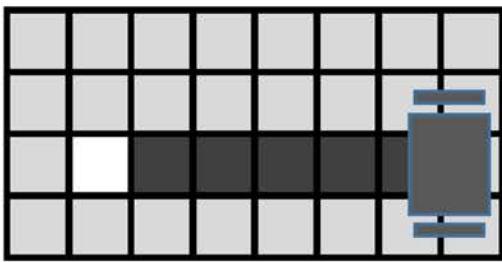


$$\log odd^+ = \log odd\ meas + \log odd^-$$

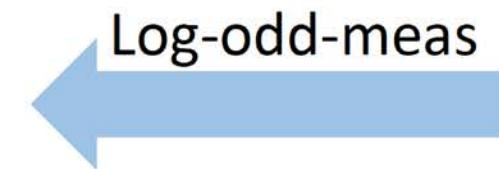
Occupancy Grid Mapping

- Log-odd update

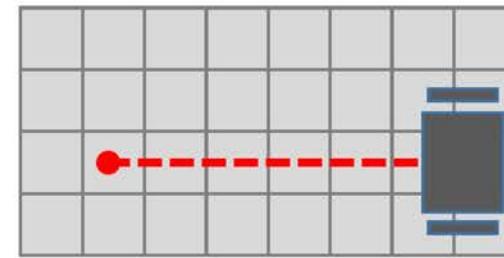
Posterior Map



Measurement Model



Prior Map



$$\log odd^+ = \log odd\ meas + \log odd^-$$

Occupancy Grid Mapping

- Measurement model in log-odd form

$$\log \frac{p(z|m_{x,y} = 1)}{p(z|m_{x,y} = 0)}$$

- **Two possible measurement:**

Case I : cells with z=1

$$\text{log odd_occ} := \log \frac{p(z = 1|m_{x,y} = 1)}{p(z = 1|m_{x,y} = 0)}$$

Case II : cells with z=0

$$\text{log odd_free} := \log \frac{p(z = 0|m_{x,y} = 0)}{p(z = 0|m_{x,y} = 1)}$$

(Trivial Case : cells not measured)

Occupancy Grid Mapping

- Example

Constant Measurement Model

$$\log odd_{occ} := 0.9$$

$$\log odd_{free} := 0.7$$

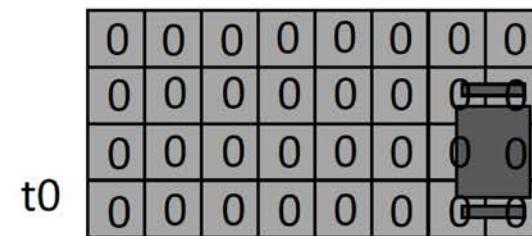
Initial Map:

$\log odd = 0$ for all (x,y)

$$p(m_{x,y} = 1) = p(m_{x,y} = 0) = 0.5$$

Update Rule:

$$\log odd += \log odd_meas$$



Occupancy Grid Mapping

- Example

Constant Measurement Model

$$\log \text{odd}_{\text{occ}} := 0.9$$

$$\log \text{odd}_{\text{free}} := 0.7$$

Update

- Case I : cells with $z=1$

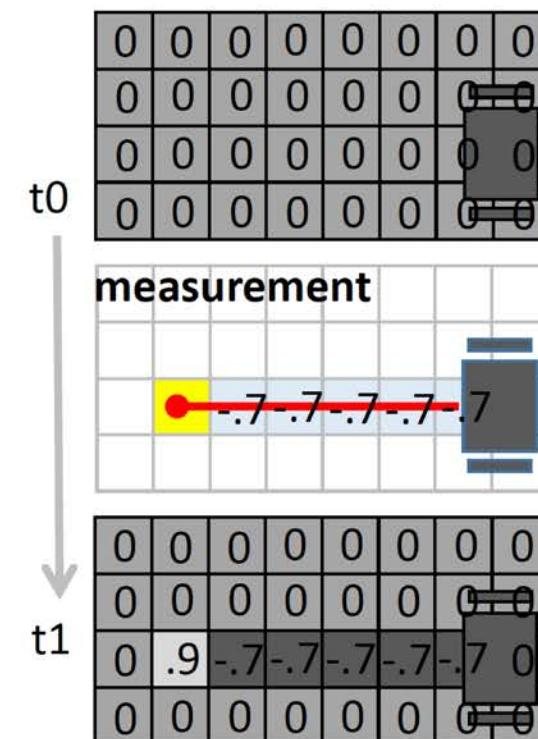
$$\log \text{odd} \leftarrow 0 + \log \text{odd}_{\text{occ}}$$

- Case II : cells with $z=0$

$$\log \text{odd} \leftarrow 0 - \log \text{odd}_{\text{free}}$$

Update Rule:

$$\log \text{odd} += \log \text{odd}_{\text{meas}}$$



Occupancy Grid Mapping

- Example

Constant Measurement Model

$$\log \text{odd}_{\text{occ}} := 0.9$$

$$\log \text{odd}_{\text{free}} := 0.7$$

Update

- Case I : cells with $z=1$

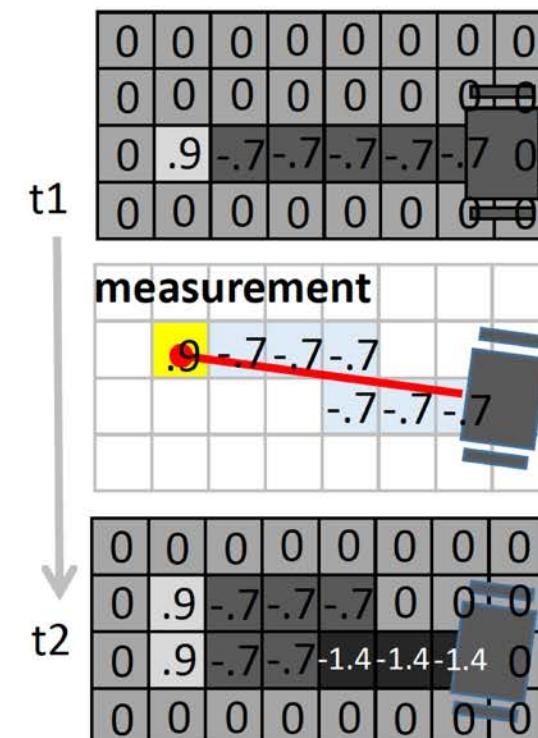
$$\log \text{odd} \leftarrow 0 + \log \text{odd}_{\text{occ}}$$

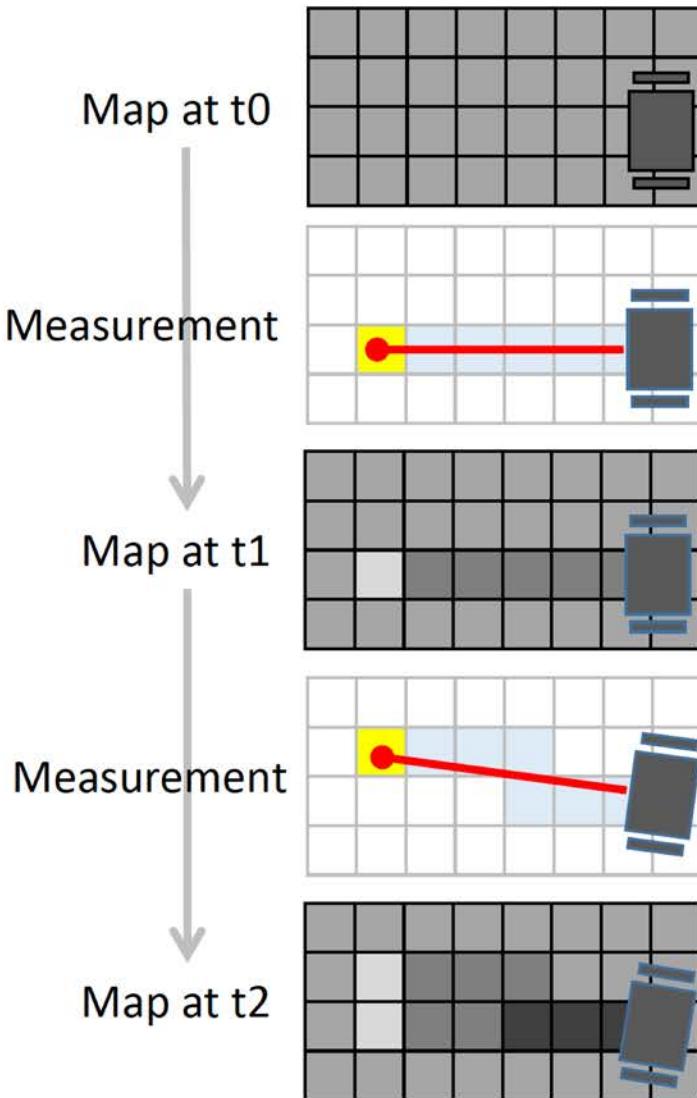
- Case II : cells with $z=0$

$$\log \text{odd} \leftarrow 0 - \log \text{odd}_{\text{free}}$$

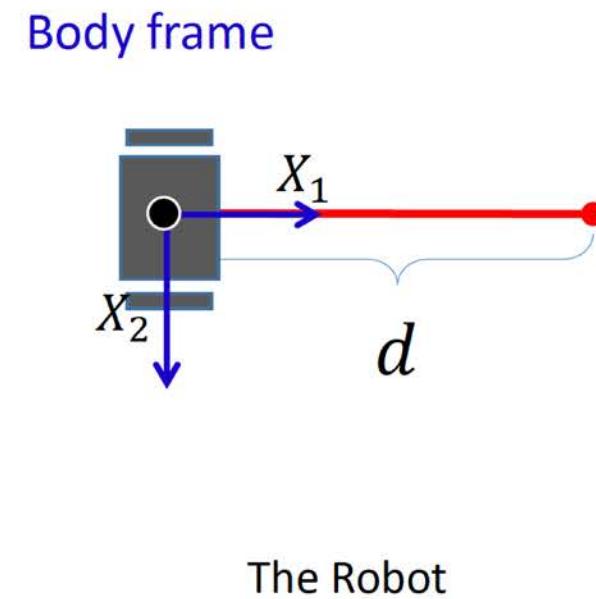
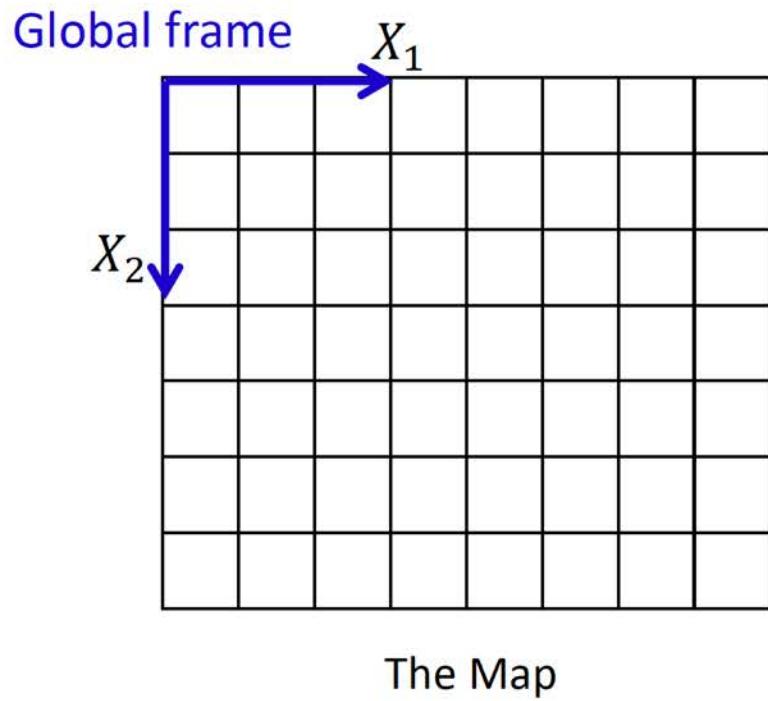
Update Rule:

$$\log \text{odd} += \log \text{odd}_{\text{meas}}$$

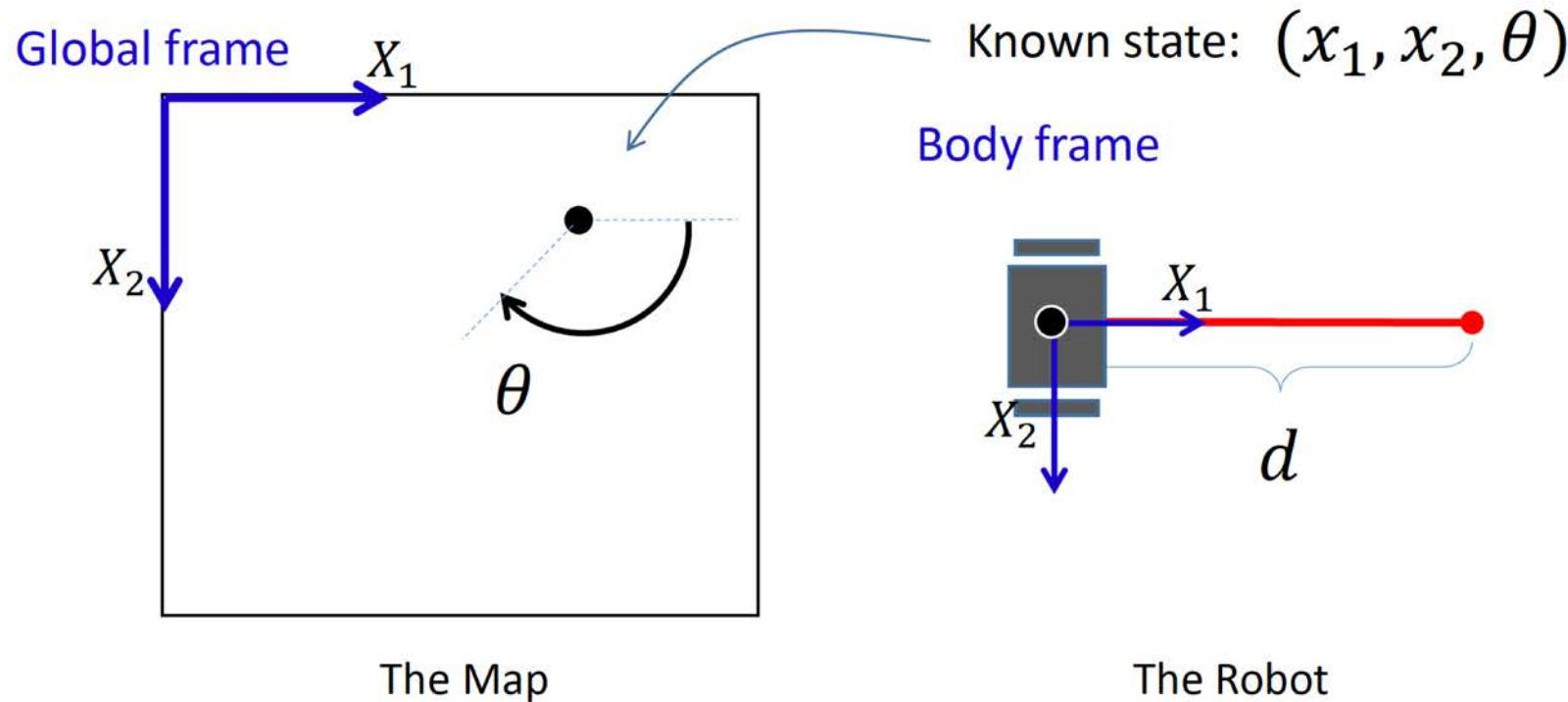




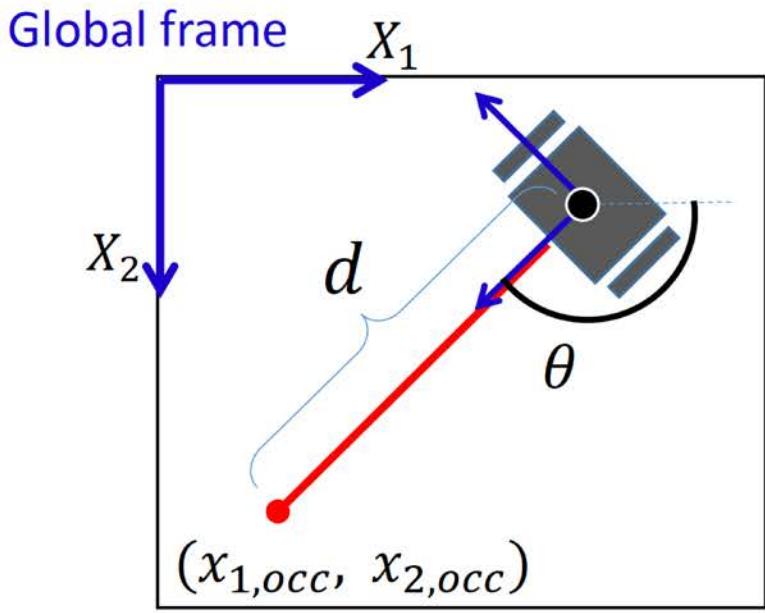
Handling Range Measurement on Grid



Handling Range Measurement on Grid



Handling Range Measurement on Grid



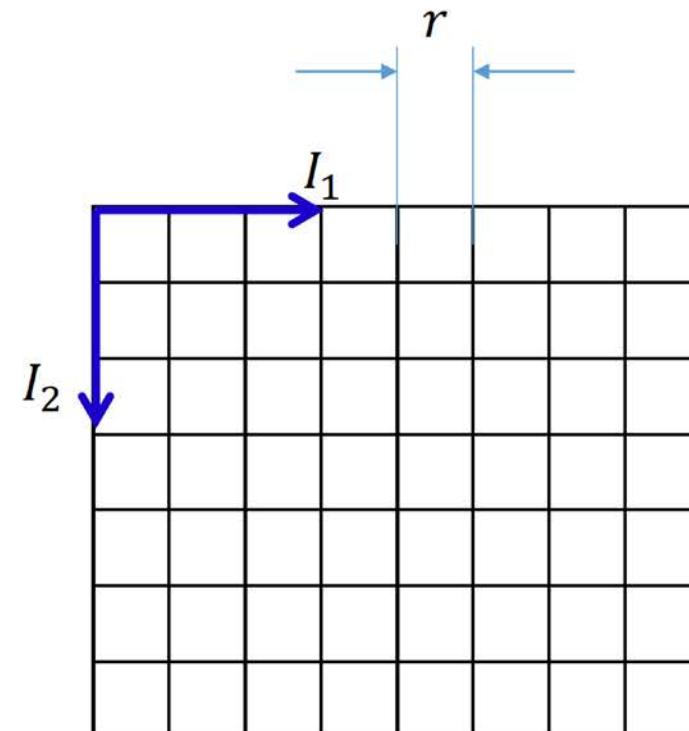
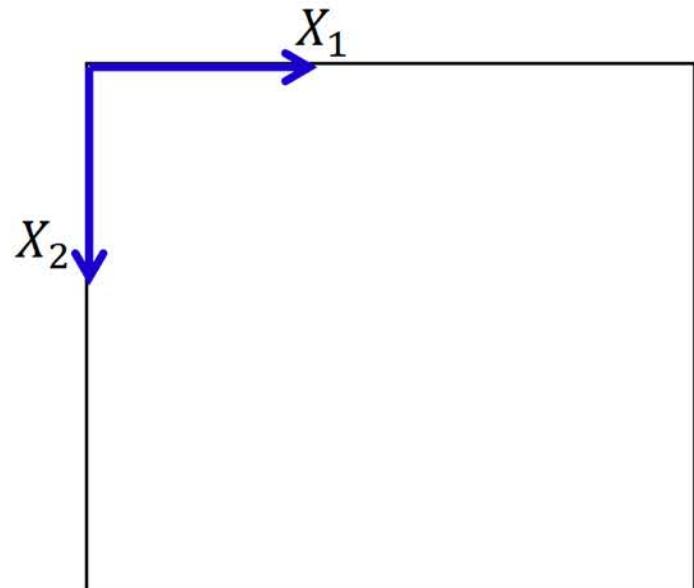
The Map

Distance measurement: d

Known state: (x_1, x_2, θ)

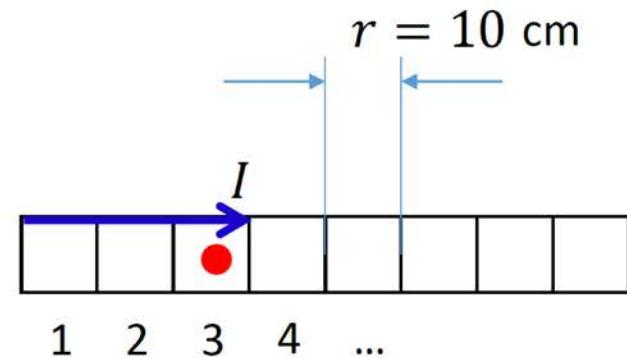
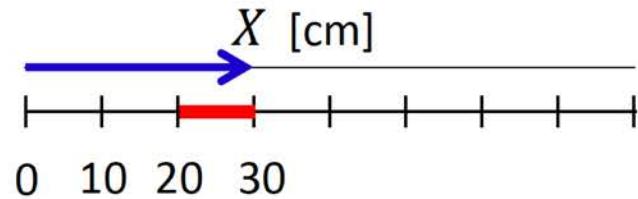
$$\begin{bmatrix} x_{1,occ} \\ x_{2,occ} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} d \\ 0 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Handling Range Measurement on Grid



Handling Range Measurement on Grid

- Example



[cm] $0 < x \leq 10$ $\rightarrow i = 1$ [index]

$10 < x \leq 20$ $\rightarrow i = 2$

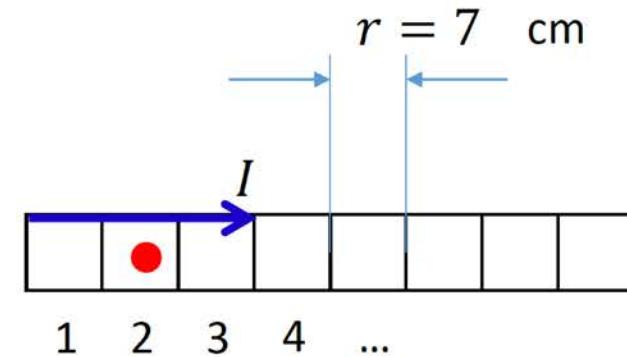
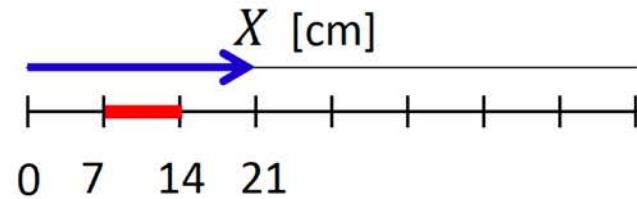
$20 < x \leq 30$ $\rightarrow i = 3$

:

:

Handling Range Measurement on Grid

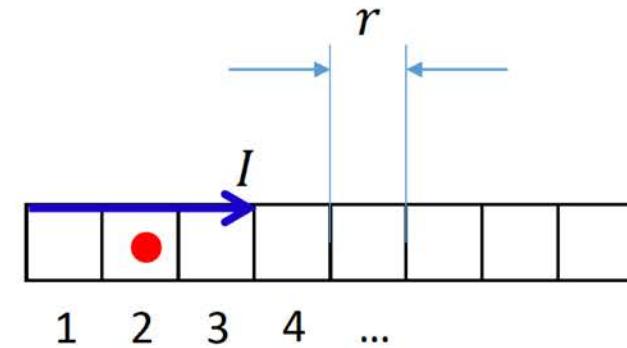
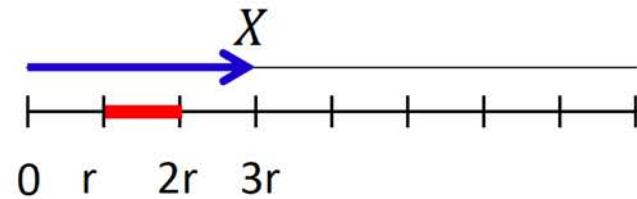
- Example



$$\begin{array}{lll} [\text{cm}] & 0 < x \leq 7 & \xrightarrow{\hspace{1cm}} i = 1 \quad [\text{index}] \\ & 7 < x \leq 14 & \xrightarrow{\hspace{1cm}} i = 2 \end{array}$$

Handling Range Measurement on Grid

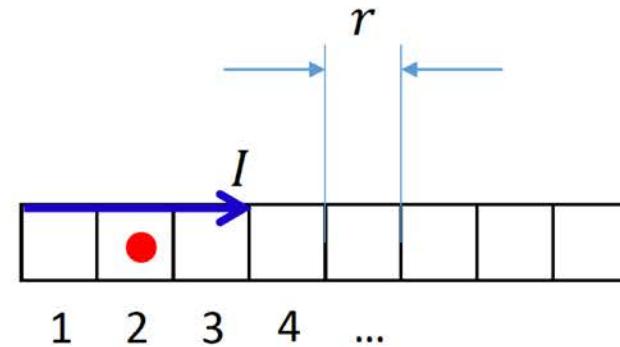
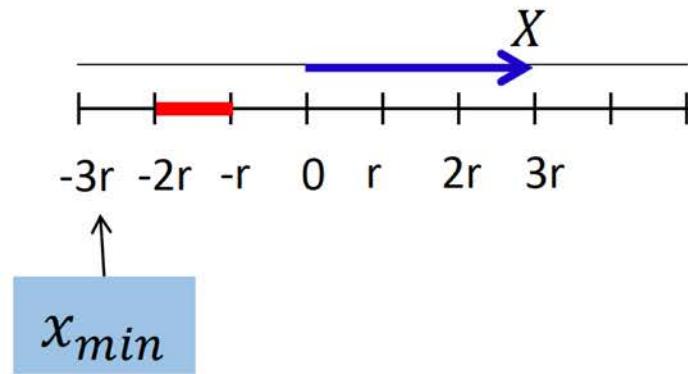
- Example



$$i = \text{ceil}(x/r)$$

Handling Range Measurement on Grid

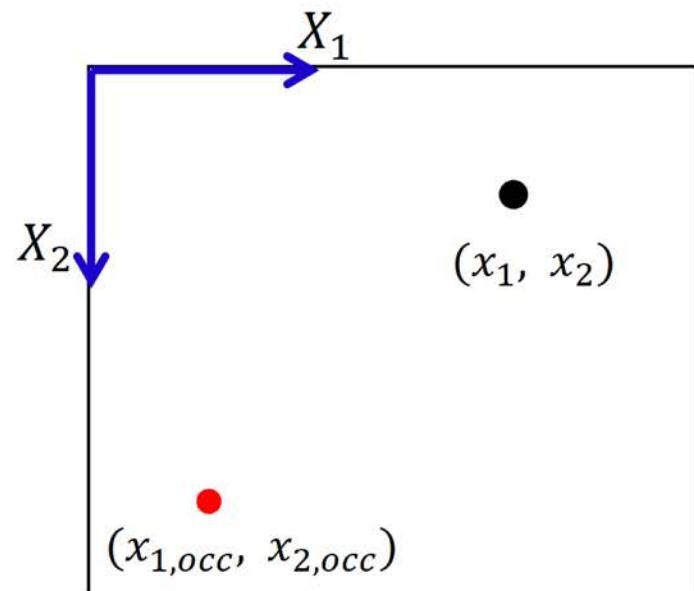
- Example



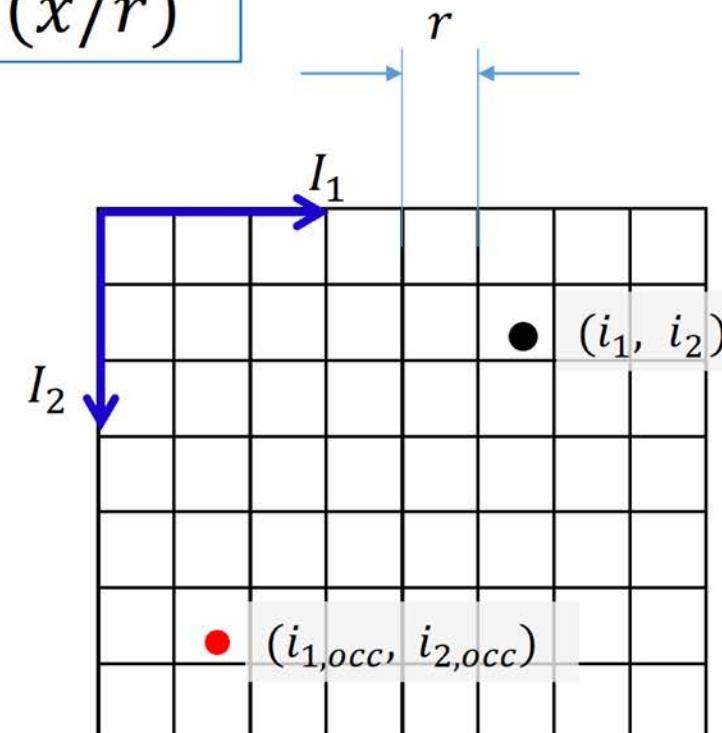
$$i = \text{ceil}((x - x_{min})/r)$$

Handling Range Measurement on Grid

$$i = \text{ceil}(x/r)$$

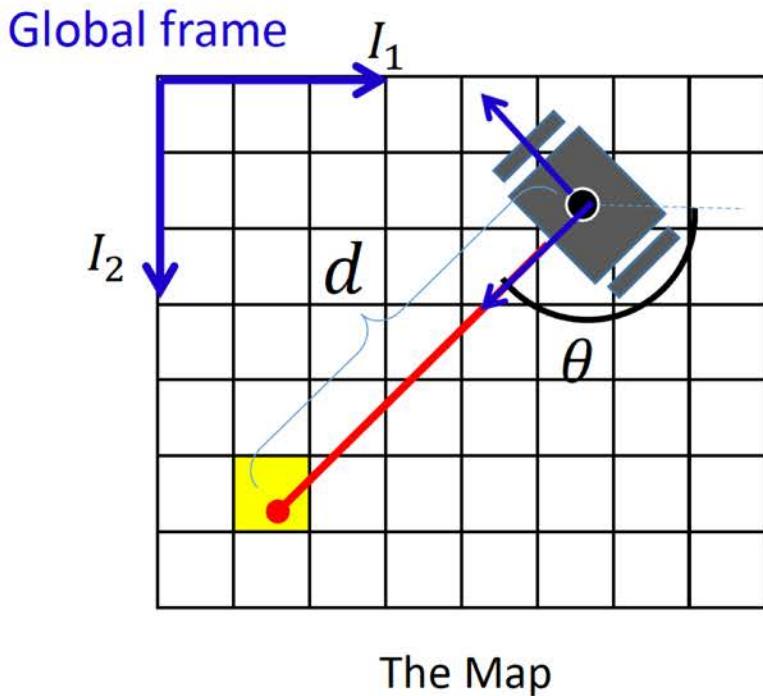


The (Continuous) Map



The Discretized Map

Handling Range Measurement on Grid



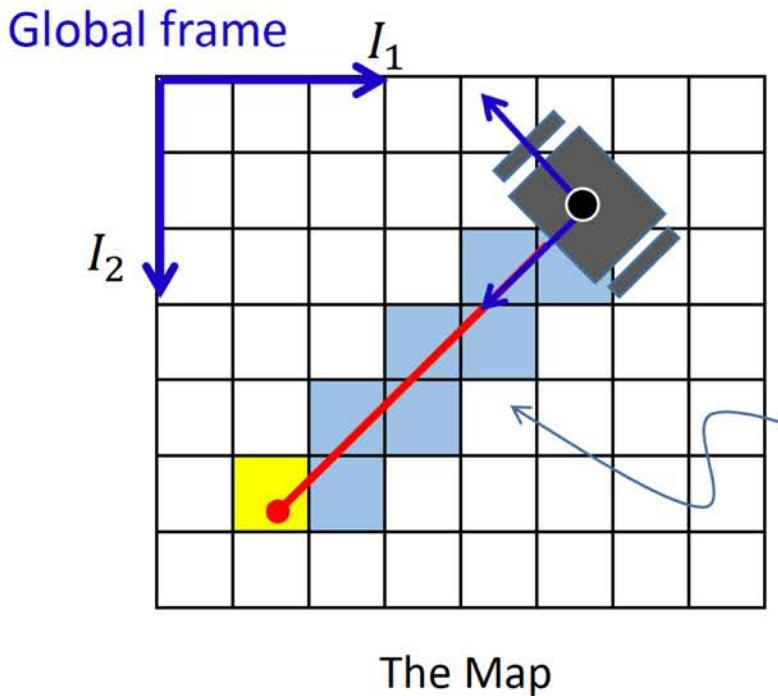
Distance measurement: d

Known state: (x_1, x_2, θ)

$$\begin{bmatrix} x_{1,occ} \\ x_{2,occ} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} d \\ 0 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} i_{1,occ} \\ i_{2,occ} \end{bmatrix} = ceil \left(\frac{1}{r} \begin{bmatrix} x_{1,occ} \\ x_{2,occ} \end{bmatrix} \right)$$

Handling Range Measurement on Grid



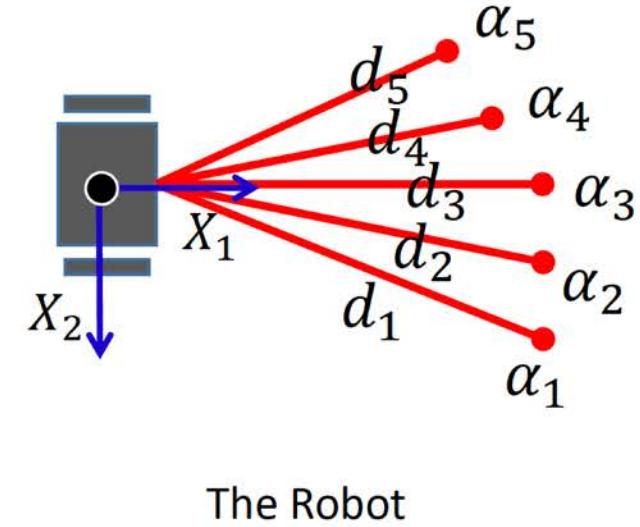
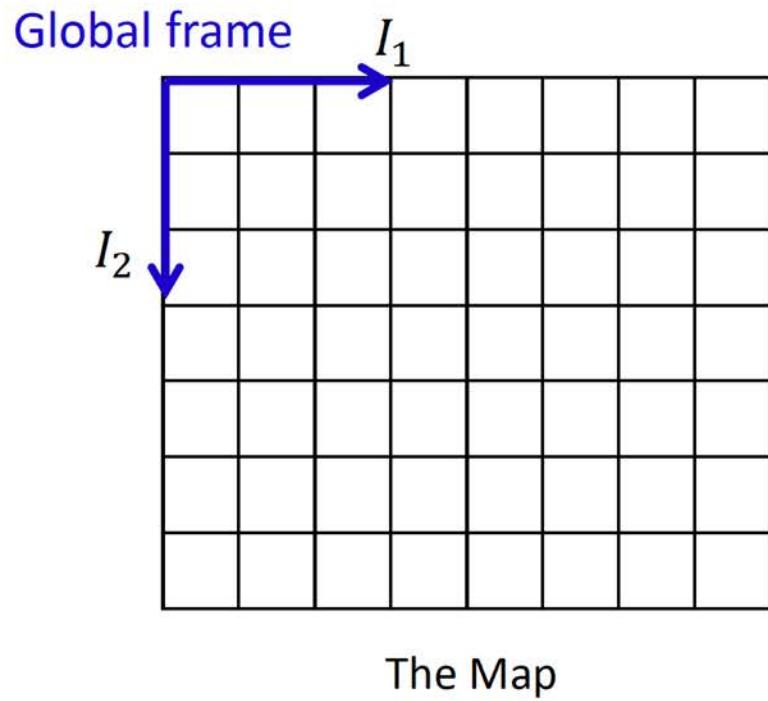
Distance measurement: d

Known state: (x_1, x_2, θ)

Occupied cell: $(x_{1,occ}, x_{2,occ})$

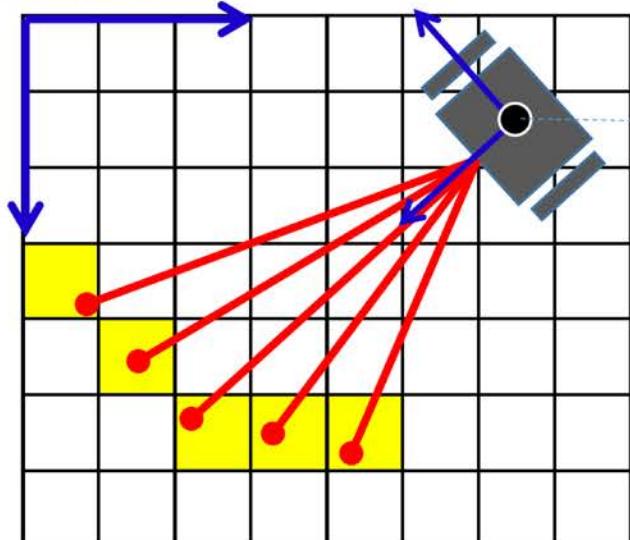
*Bresenham's line algorithm

Handling Range Measurement on Grid



Handling Range Measurement on Grid

Global frame



The Map

Distance measurement:

$$(d_1, d_2, d_3, d_4, d_5)$$

Directions of rays:

$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)$$

Known state: (x_1, x_2, θ)

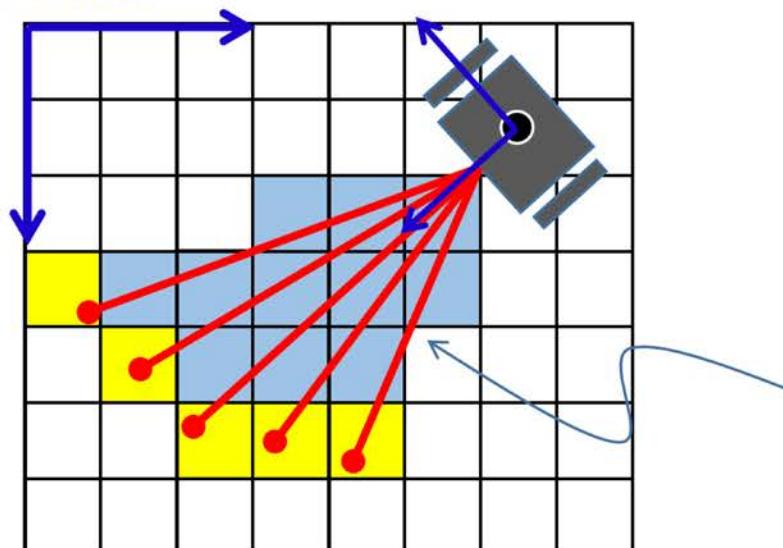
For k -th occupied cell:

$$\begin{bmatrix} x_{1k} \\ x_{2k} \end{bmatrix} = \begin{bmatrix} d_k \cos(\theta + \alpha_k) \\ -d_k \sin(\theta + \alpha_k) \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} i_{1k} \\ i_{2k} \end{bmatrix} = \text{ceil} \left(\frac{1}{r} \begin{bmatrix} x_{1k} \\ x_{2k} \end{bmatrix} \right)$$

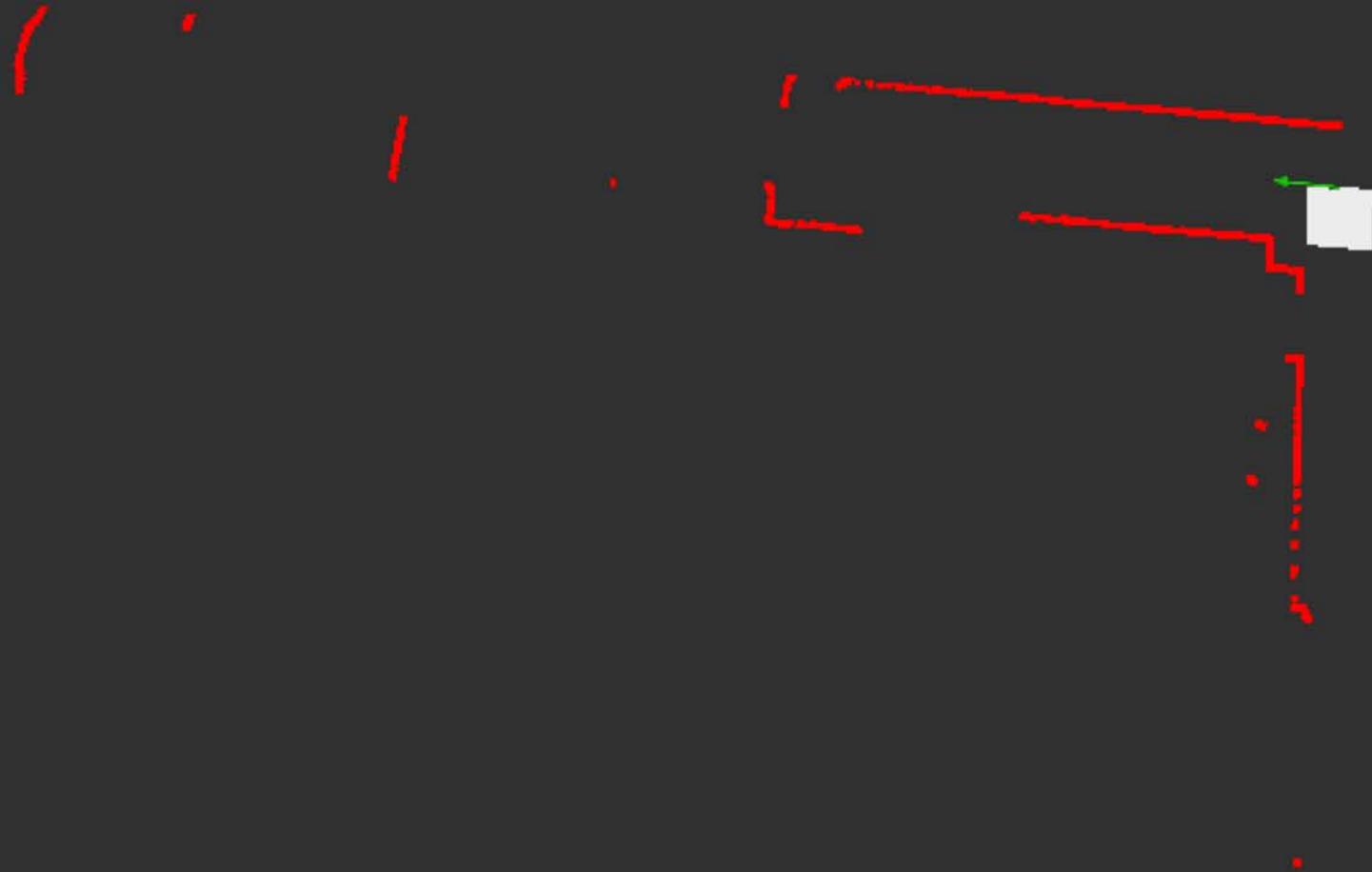
Handling Range Measurement on Grid

Global frame



*Bresenham's line algorithm

Registering the first Scan

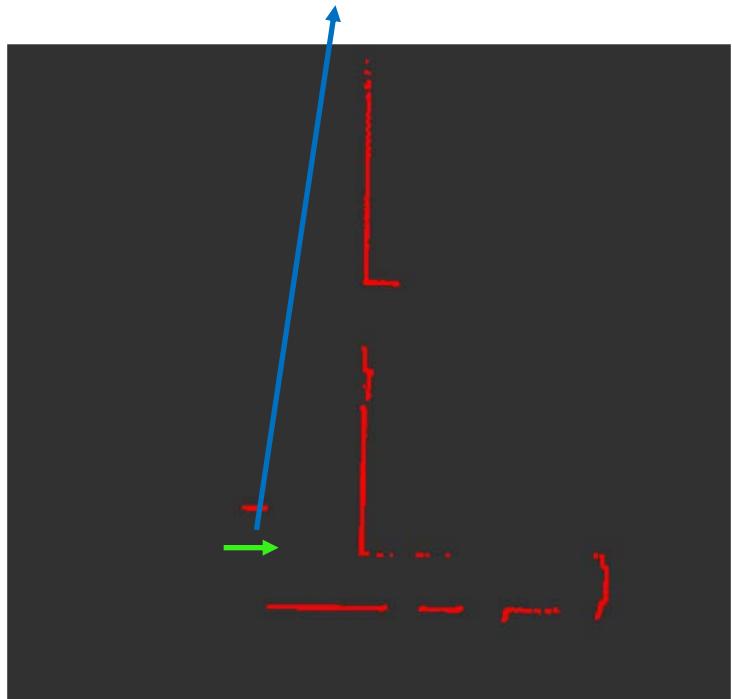


Registering the first Scan



Scan Matching

Pose of the Car at $t = t_1$



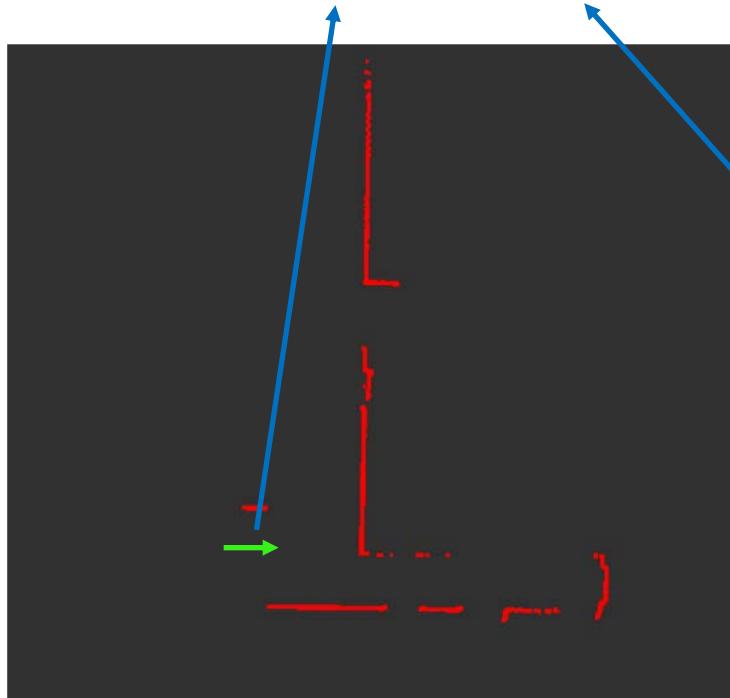
Laser Scans w.r.t car at Time $t = t_1$



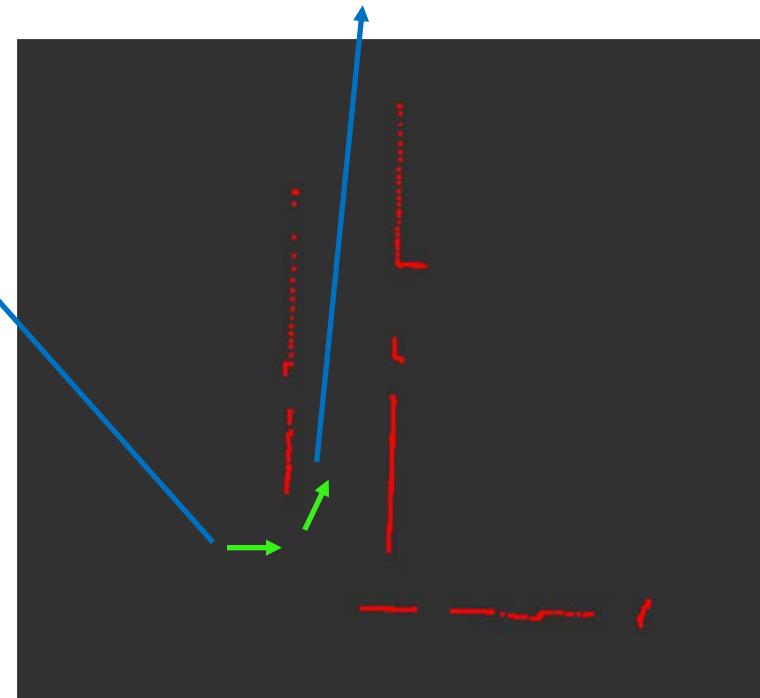
Laser Scans w.r.t car at Time $t = t_2$

Scan Matching

Pose of the Car at $t = t_1$

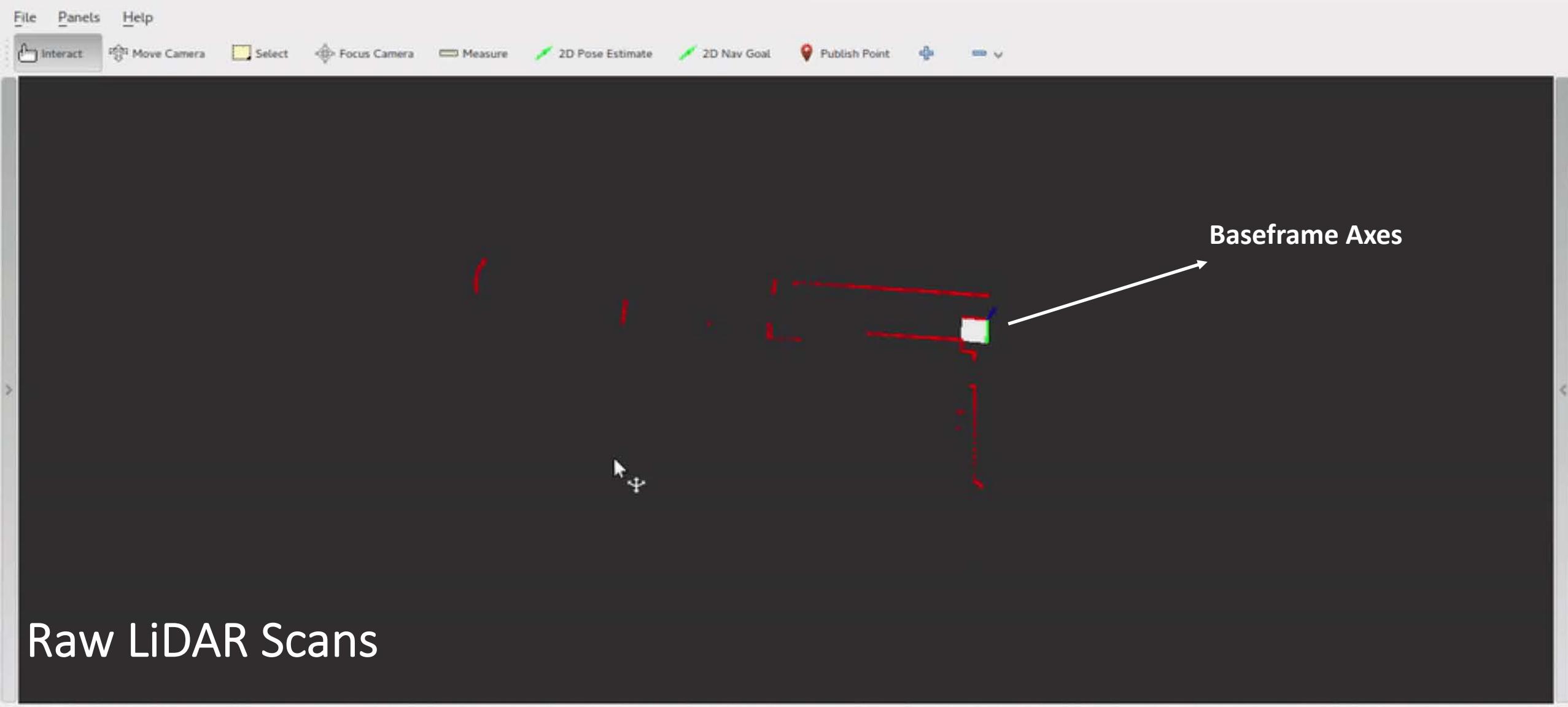
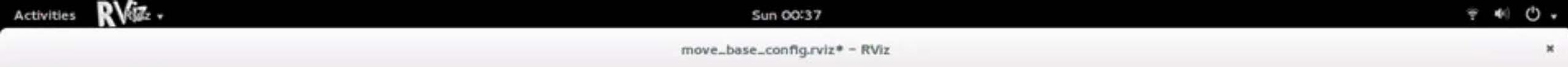


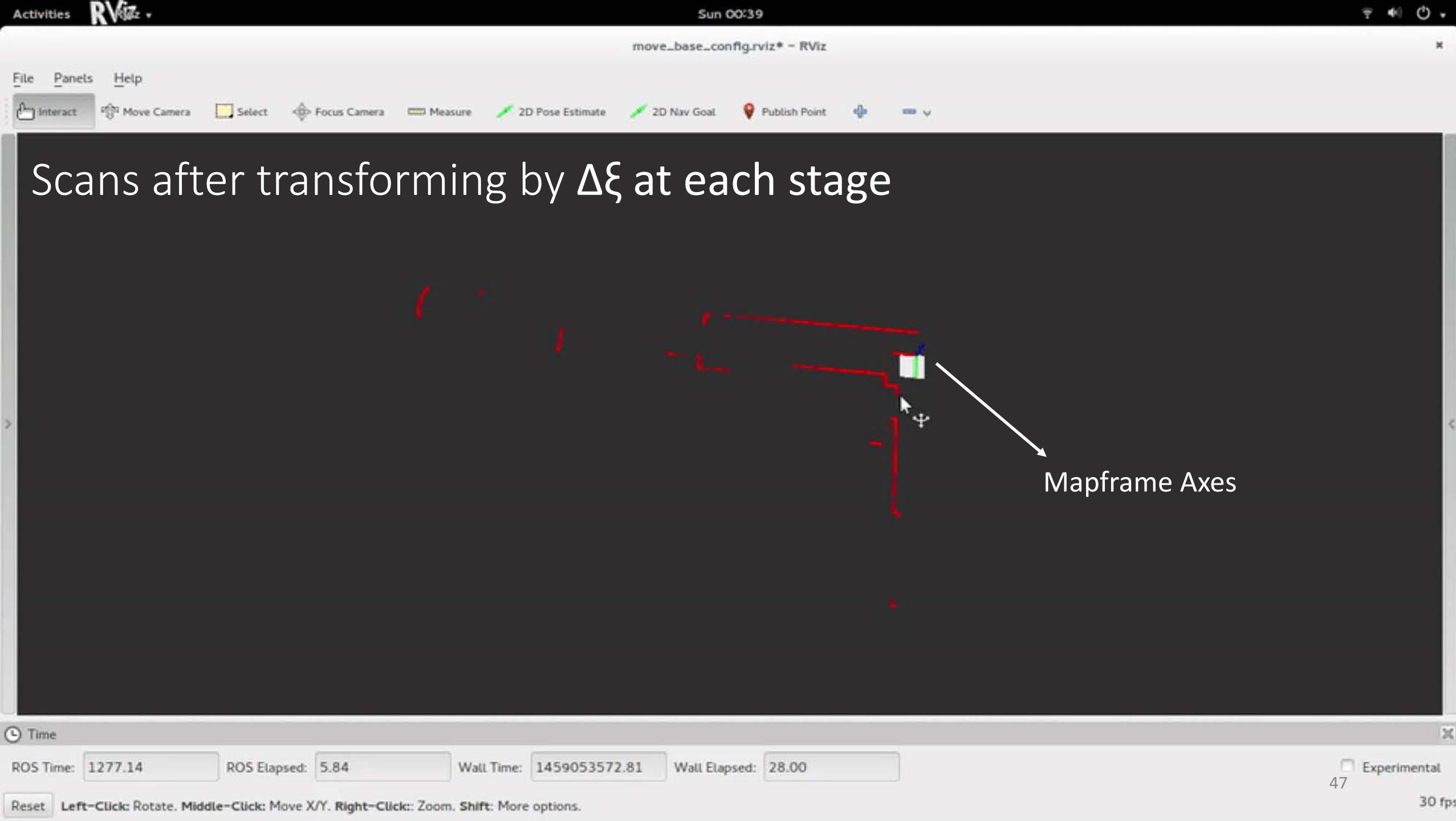
Pose of the Car at $t=t_2$



Laser Scans w.r.t car at Time $t = t_1$

Laser Scans w.r.t car at Time $t = t_2$





move_base_config.rviz* - RViz



File Panels Help

Interact

Move Camera

Select

Focus Camera

Measure

2D Pose Estimate

2D Nav Goal

Publish Point

+

v



Map Update

Time

ROS Time: 1288.50

ROS Elapsed: 17.33

Wall Time: 1459056336.31

Wall Elapsed: 57.23

 Experimental
48

Reset

Left-Click: Rotate. Middle-Click: Move X/Y. Right-Click: Zoom. Shift: More options.

30 fps

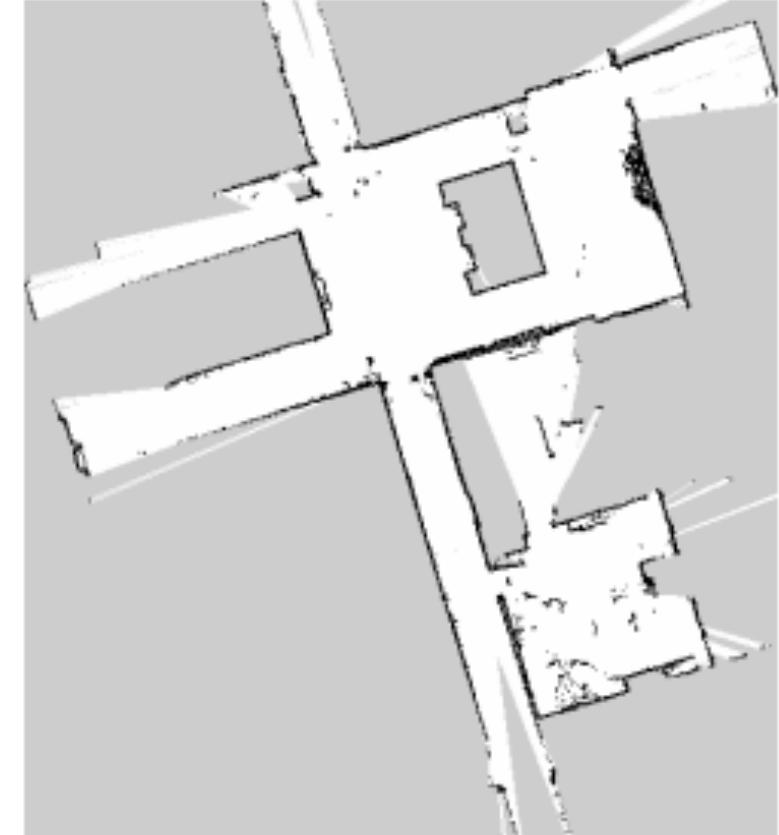
Multi-Resolution Map Representation



20 cm Grid Cell



10 cm Grid Cell

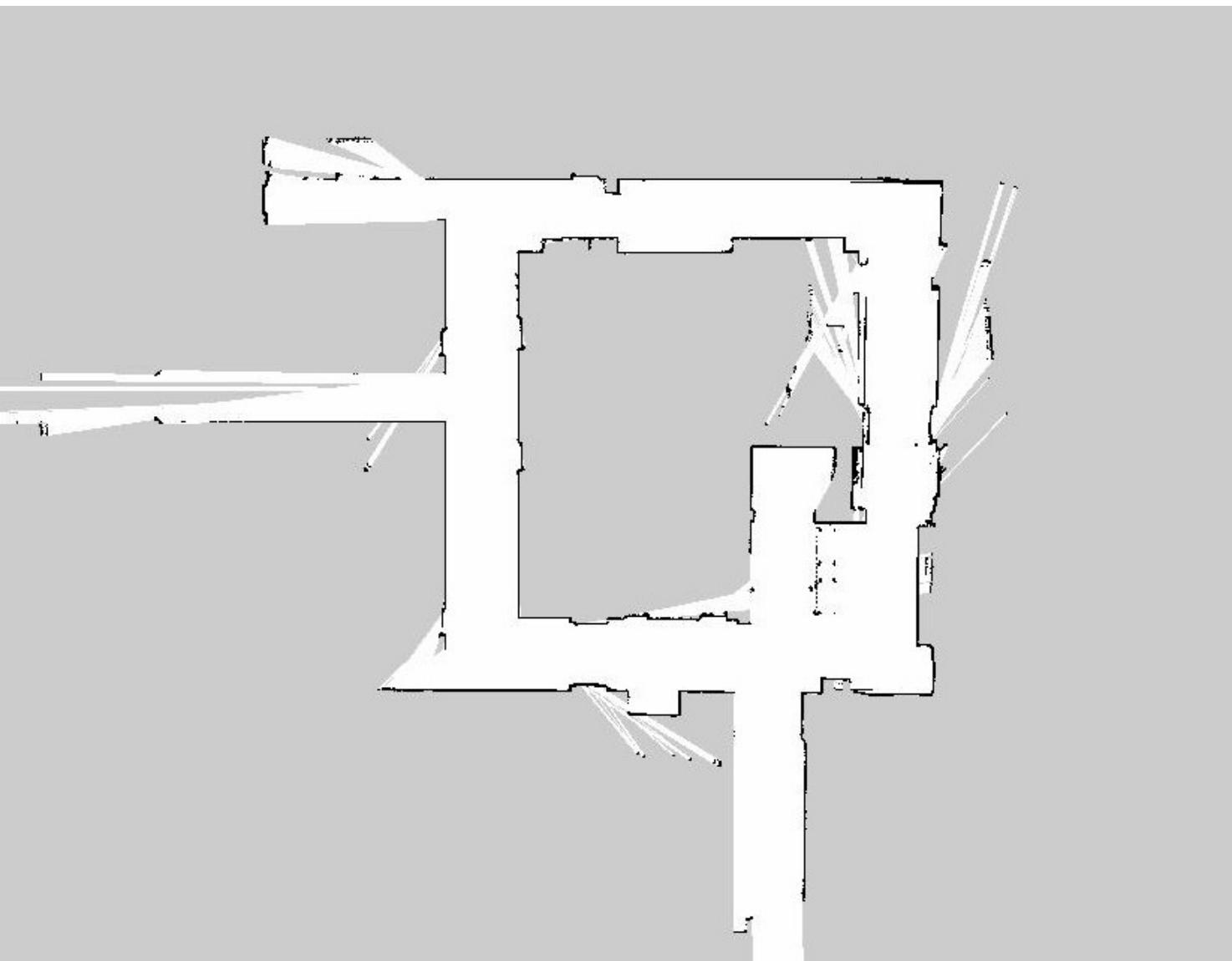


5 cm Grid Cell

Saving the map

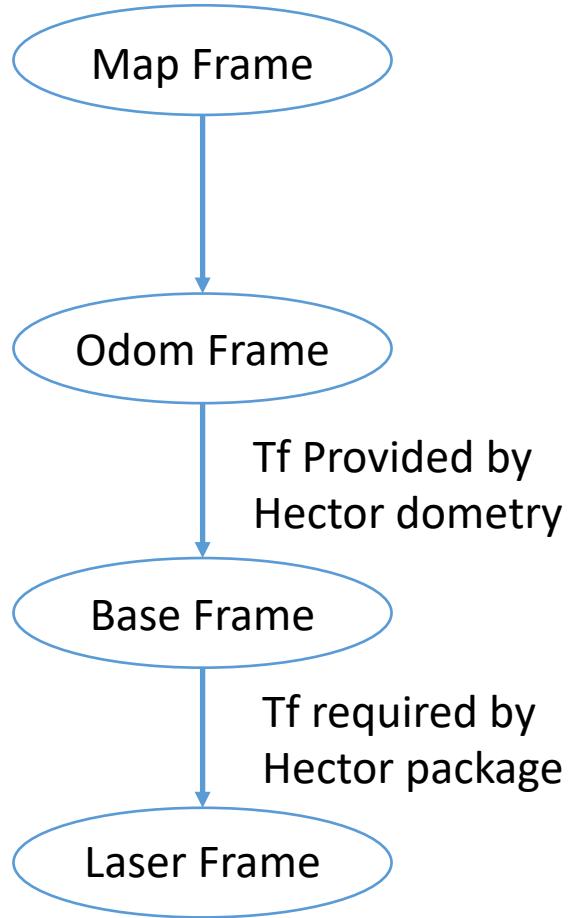


Saving the map



- ROS Package called **MAP Server**
- Allows saving a map currently being published over /map topic
- The saved map can be loaded for future tasks.

System Tf tree



Parameters for Hector SLAM : ROS

- map_resolution
- map_update_distance_thresh
- map_update_angle_thresh
- laser_max_dist
- update_factor_free
- update_factor_occupied

Google Cartographer Overview

The Problem

Why did Google (not Waymo) make a SLAM package?



What's different about Cartographer

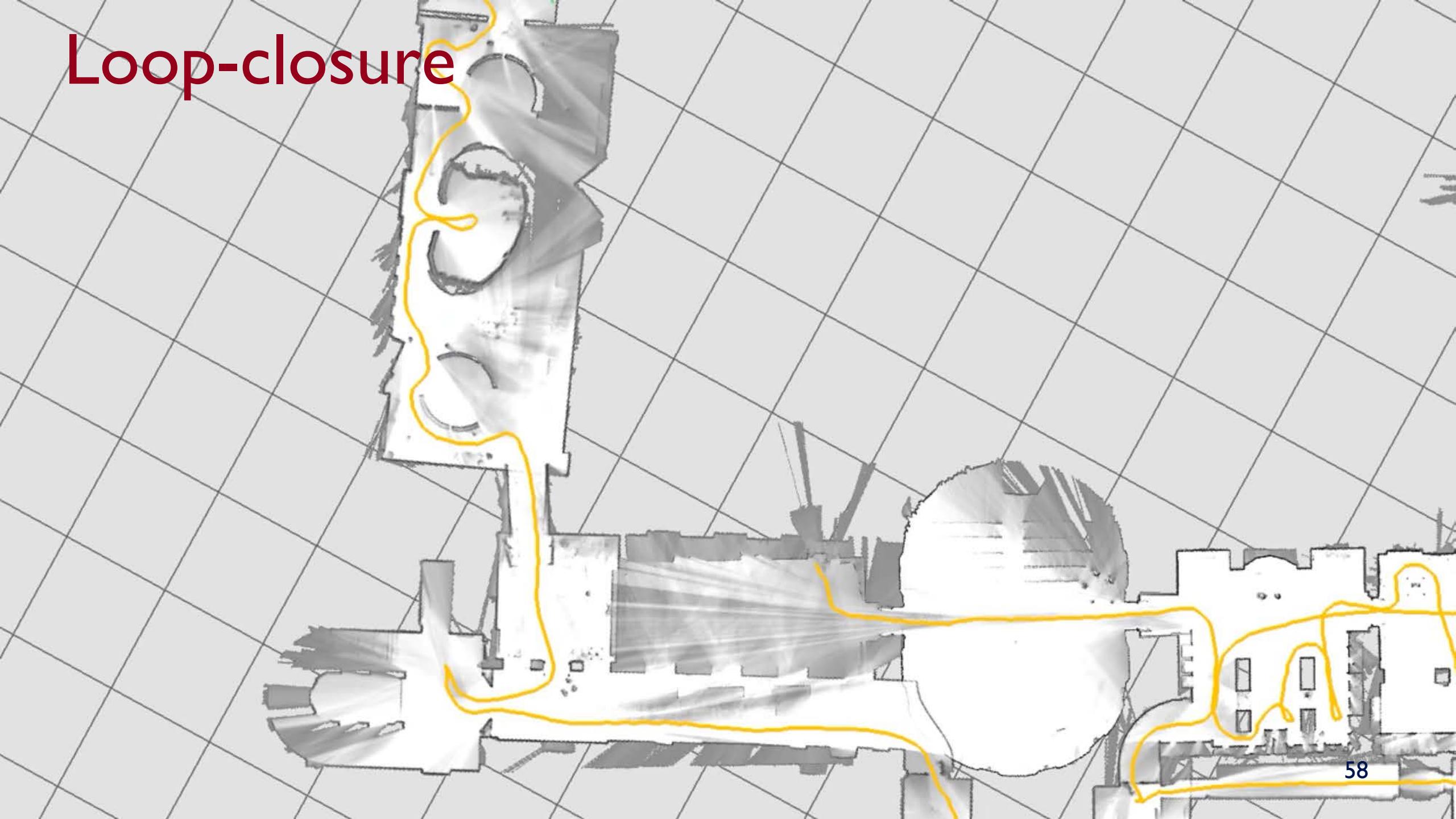
The contribution of this paper is a novel method for ***reducing the computational requirements of computing loop closure constraints*** from laser range data.

Loop-closure

Two regions in the map are found to be the same region in the world even though their position is incompatible given the uncertainty estimate in the map — the classic loop closure problem.

The system must then be able to calculate the transformation needed to align these two regions to ‘close the loop’.

Loop-closure



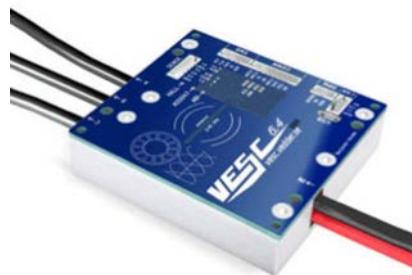
System Overview: Sensor Inputs

Up to four types of sensor measurements: (I) Range (II) Odometry (III) IMU (IV) Fixed Frame Pose

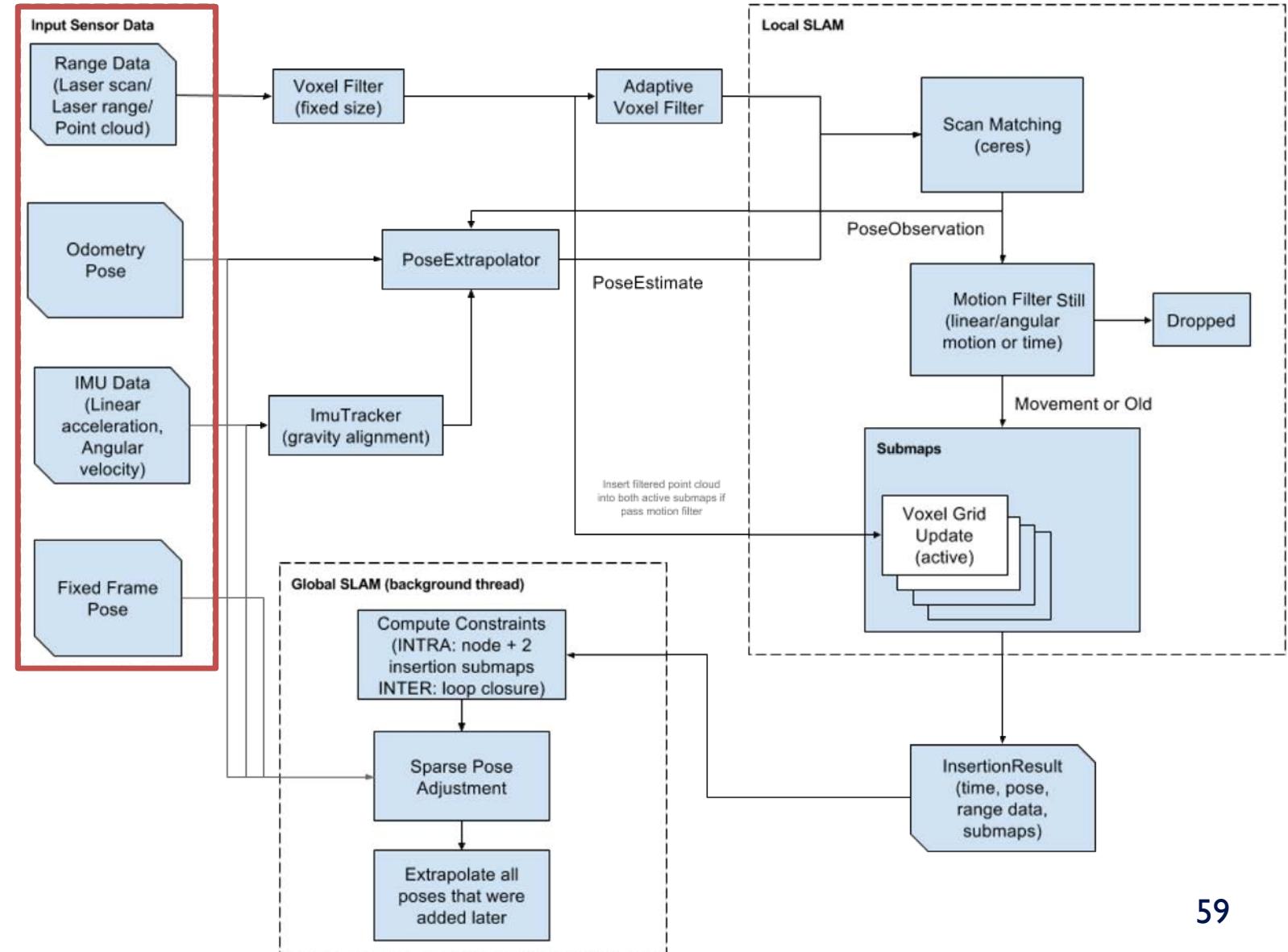
The FI/I0 vehicle supplies two sensors:



2D LIDAR
(Range)



VESC
(Odometry)

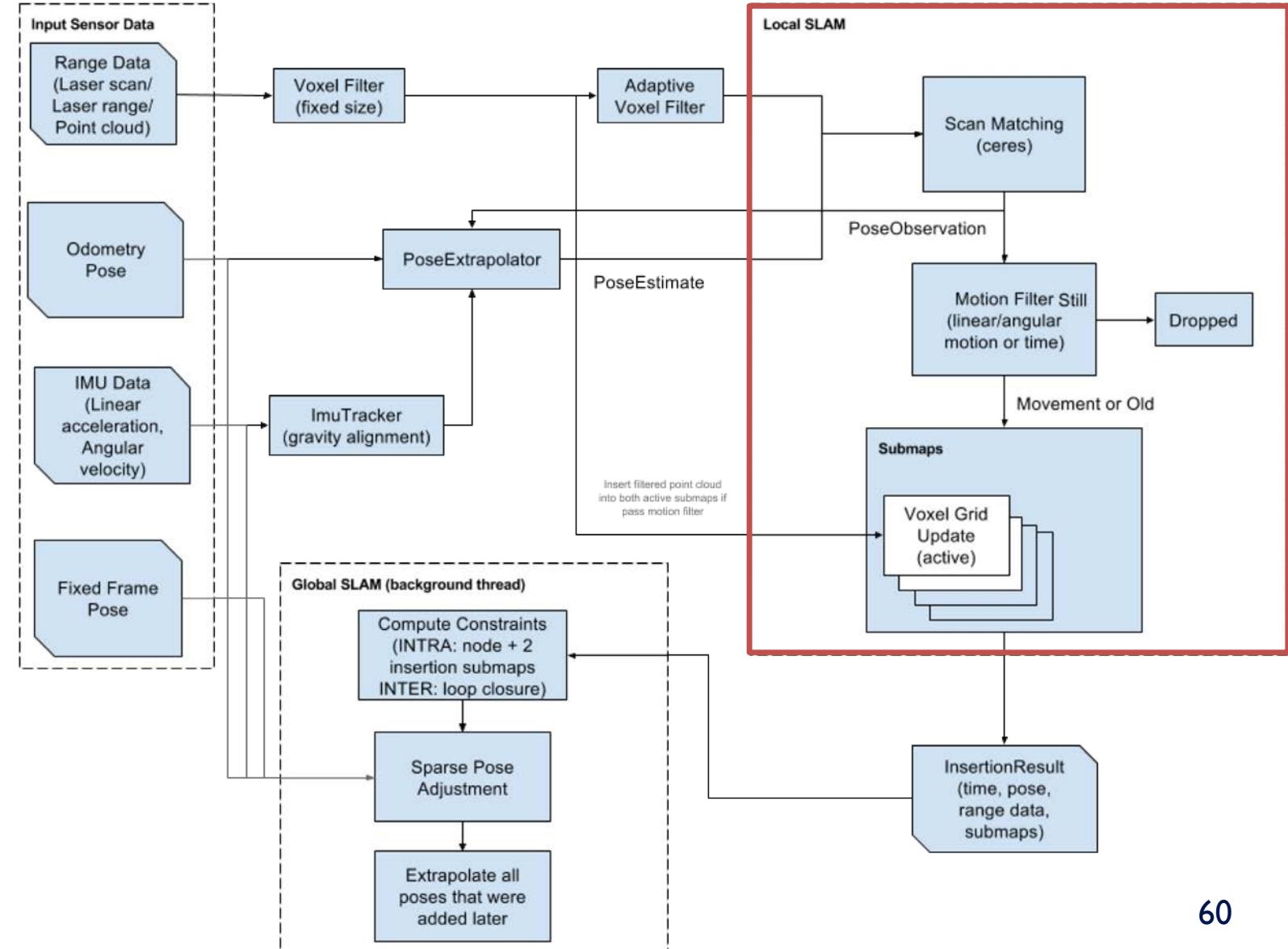


System Overview: Frontend

Cartographer consists of two separate subsystems: local SLAM (frontend or trajectory builder) and global SLAM (backend).

The job of local SLAM is to generate good submaps.

The job of global SLAM is to tie the submaps consistently together.

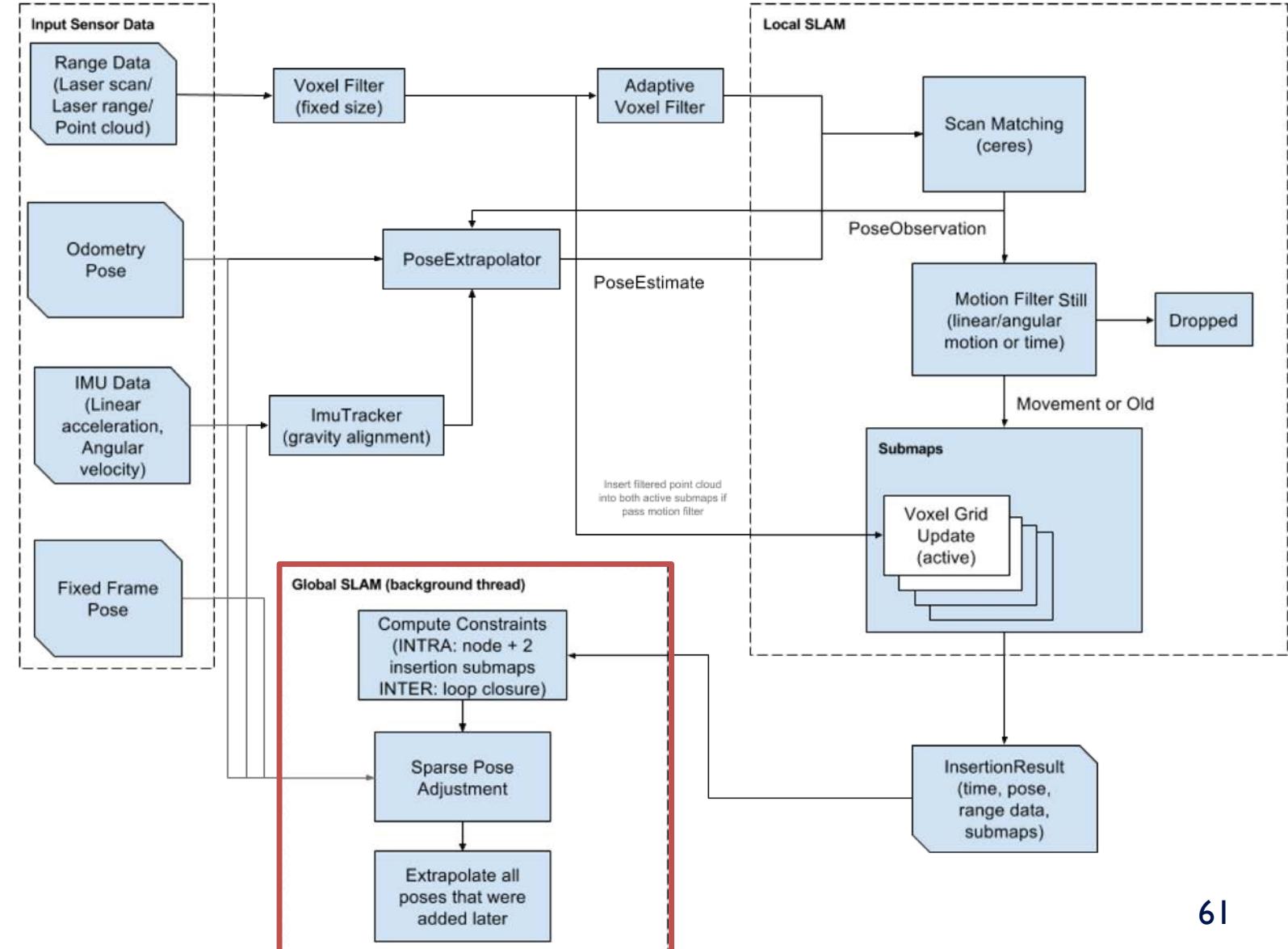


System Overview: Backend

Cartographer consists of two separate subsystems: local SLAM (frontend or trajectory builder) and global SLAM (backend).

The job of local SLAM is to generate good submaps.

The job of global SLAM is to tie the submaps consistently together.



Frontend: Local SLAM

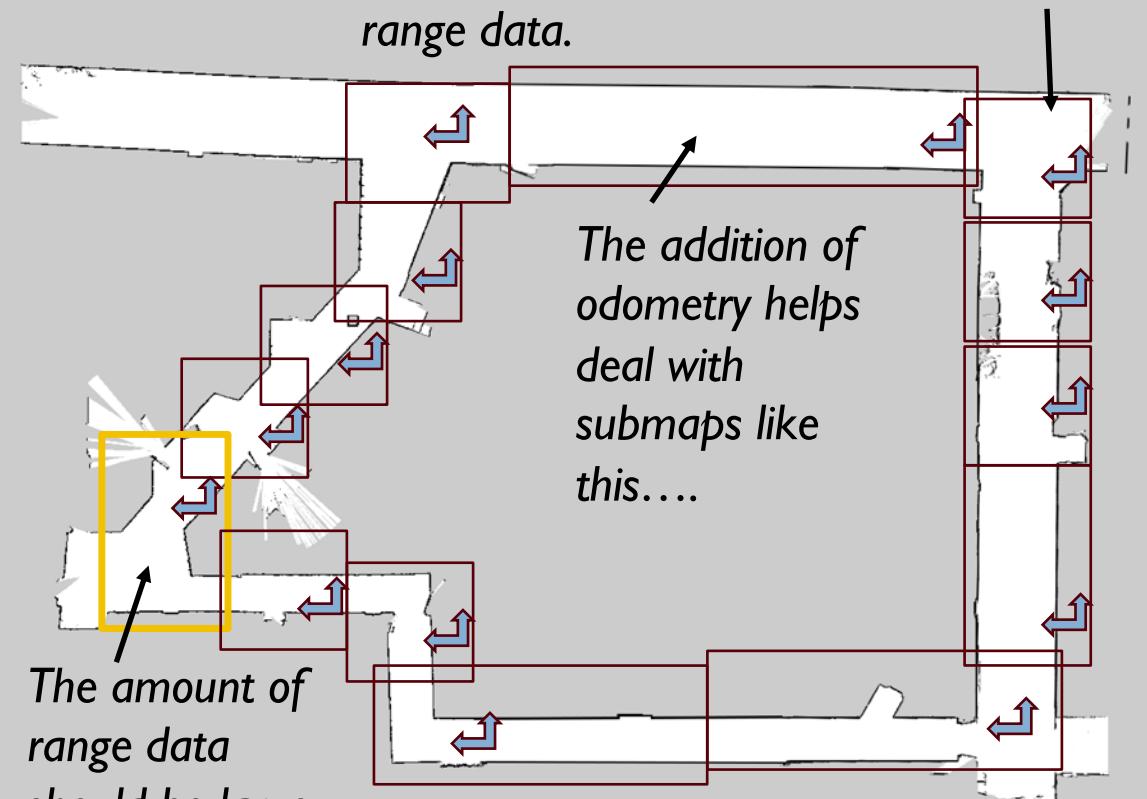
What is a submap?

A submap is considered as complete when the local SLAM has received a given amount of range data.

Submaps must be small enough so that the drift inside them is below the resolution of the occupancy grid, so that they are locally correct.

On the other hand, they should be large enough to be distinct for loop closure to work properly. More on this later

Submaps are small chunks of the world filled with a fixed amount of registered range data.



The amount of range data should be large enough that submaps are distinguishable...

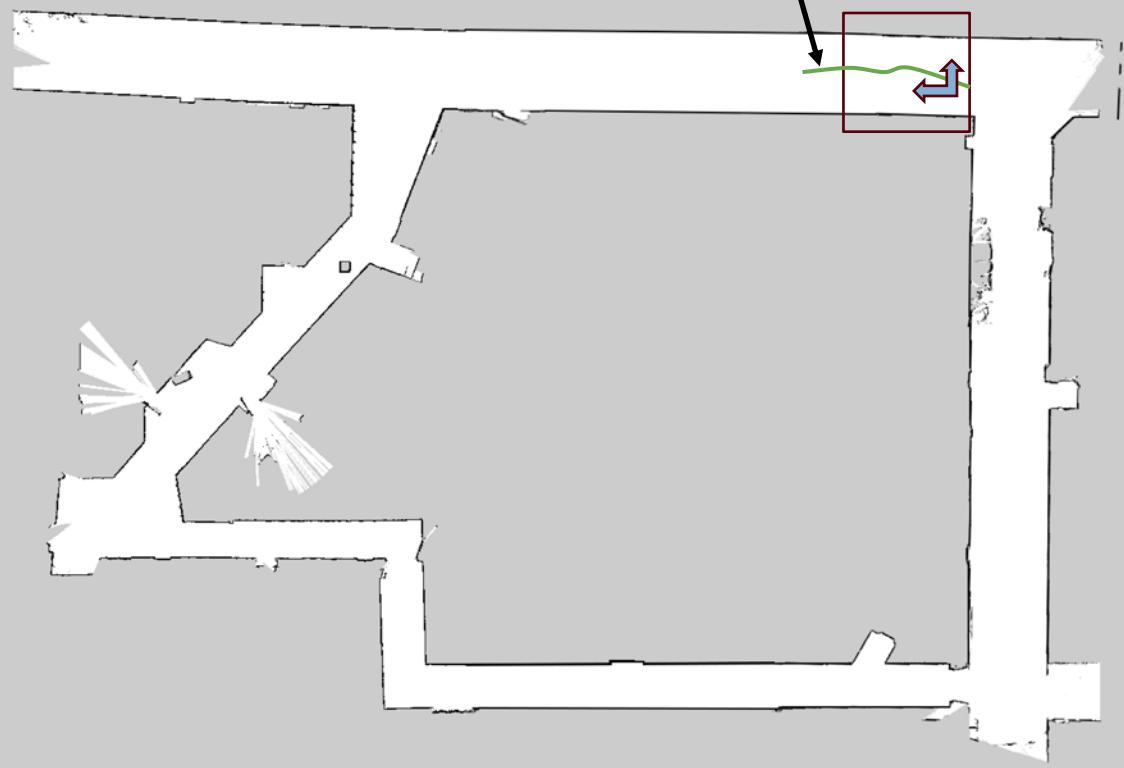
What is a submap?

Submaps each have their own static transform and contain a collection of registered range-measurements.

Consecutive measurements are connected by constraints which are ‘local’.

Here local means derived from odometry or recent scan overlaps and resultant scan matches.

Localization: the robot's trajectory in a submap is computed via scan matching.



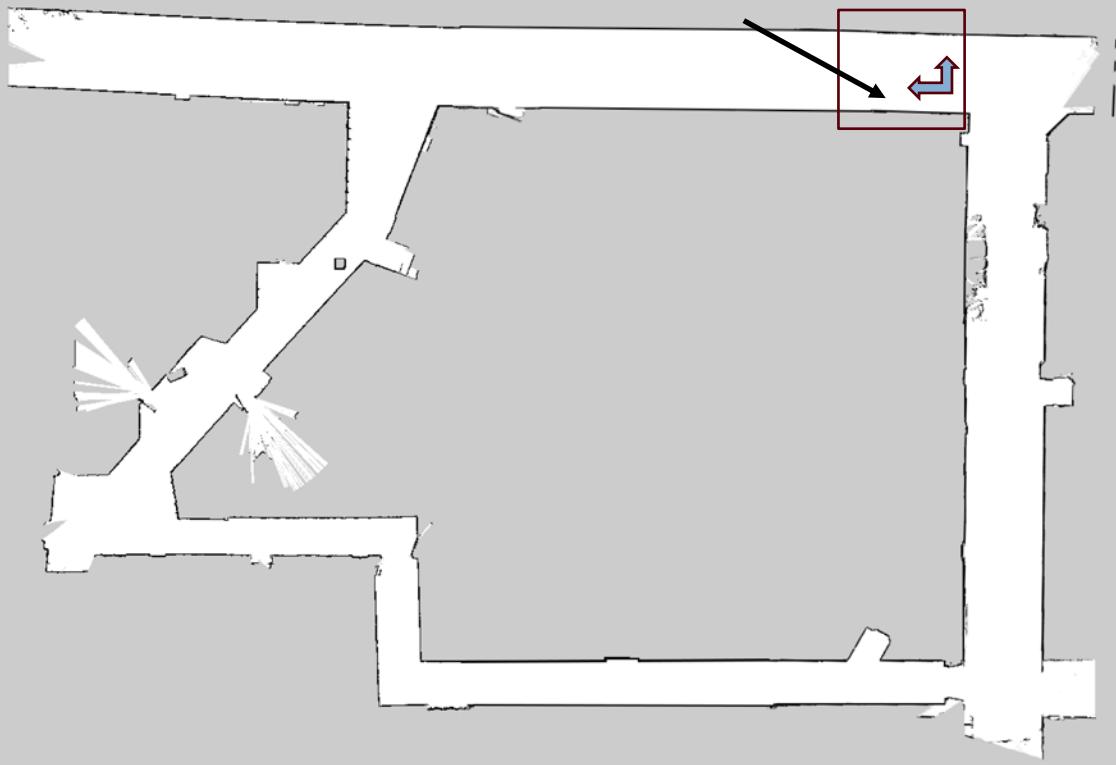
What is a submap?

Submaps each have their own static transform and contain a collection of registered range-measurements.

Consecutive measurements are connected by constraints which are ‘local’.

Here local means derived from odometry or recent scan overlaps and resultant scan matches.

Once a motion between two scans is found by the scan matcher, it goes through a motion filter. A scan is inserted into the current submap only if its motion is above a certain distance, angle or time threshold.

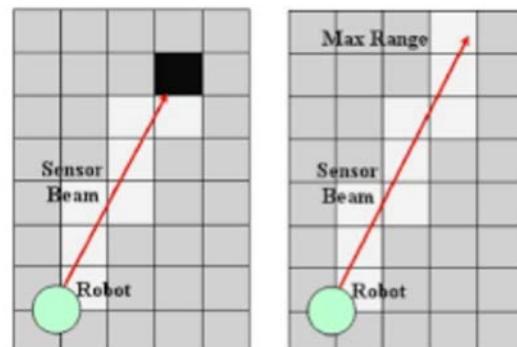


Submap Representation

Submaps can **store their range data in a probability grid**. For 2D a signed distance field representation is also possible.

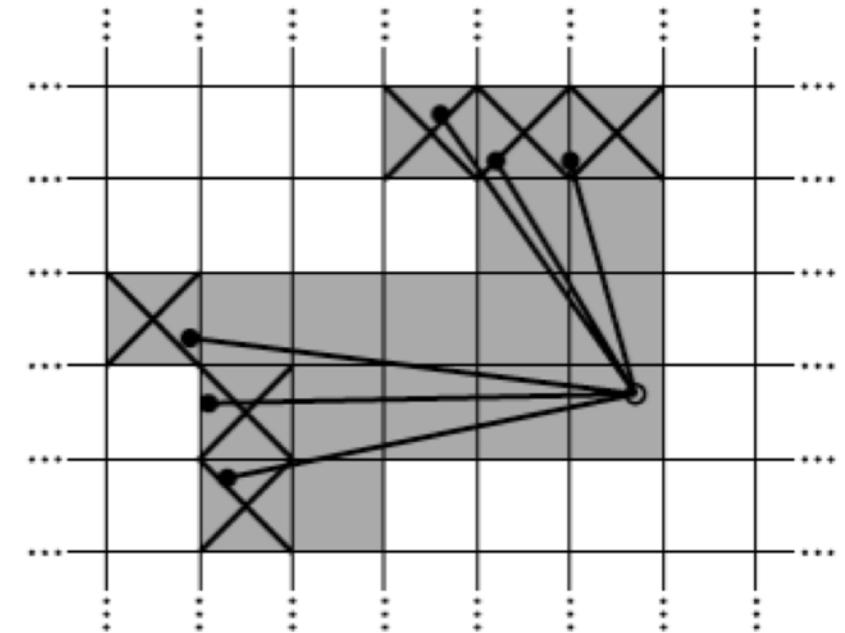
Probability grids are a 2D table where each **cell has a fixed size and contains the odds** of being obstructed.

Odds are updated according to “**hits**” (where the range data is measured) and “**misses**” (the free space between the sensor and the measured points).



Updating the submap

1. For every hit, we **insert the closest grid point** into the hit set.
2. For every miss, **we insert the grid point associated with each pixel that intersects one of the rays between the scan origin and each scan point**, excluding grid points which are already in the hit set.
3. If a grid point **has yet to be observed it is assigned a value p_{hit} or p_{miss}** depending on which set it is in.



Define the submap as:

$$M : r\mathbf{Z} \times r\mathbf{Z} \rightarrow [p_{min}, p_{max}]$$

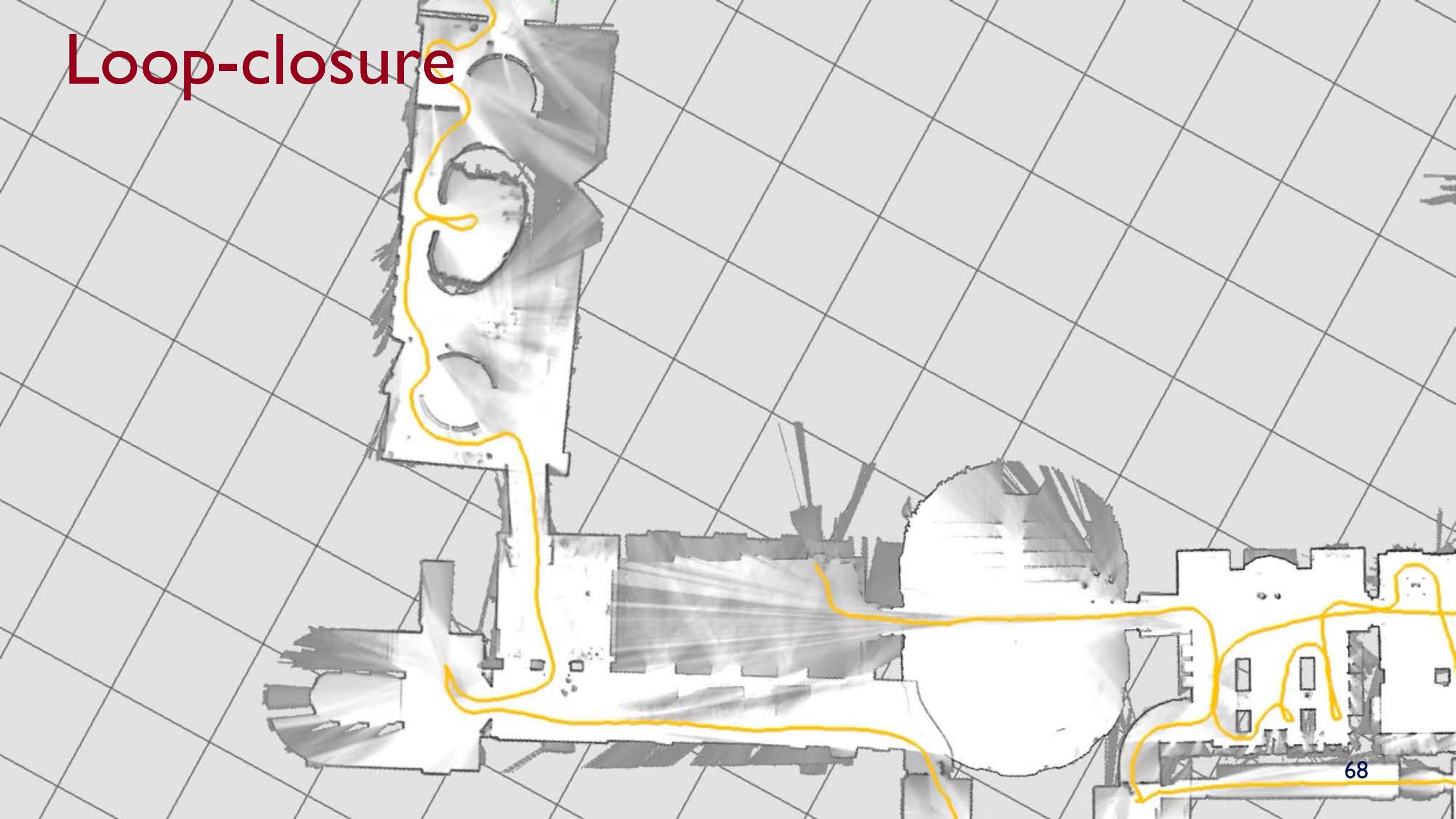
Then the map is updated according to the following operation:

$$M_{new}(x) = \text{clamp}(\overbrace{\text{odds}^{-1}(\text{odds}(M_{old}(x)) \text{ odds}(p_{hit}))}^{\text{Compute new } p \text{ from odds...}})$$

Recall the definition of the odds function:

$$\text{odds}(p) = \frac{p}{1 - p}$$

Loop-closure



Scan Matching:

How do we know that ‘hits’ and misses map to a particular cell in the probability grid?

- The collection of scan points relative to a moving reference frame attached to the robot are, $H = \{h_k\}$ for $k=1\dots K$ where h_k is a point in \mathbb{R}^2 .
- These points are placed in a submap at pose ξ in the global reference frame by applying a rigid body transform to each (rotation and translation).
- Submap construction is the iterative process of repeatedly aligning scan and submap coordinate frames. *Sound familiar?*

So we *can understand why scan matching alone is insufficient for robust localization and explore the basics.*

Does Cartographer use ICP?

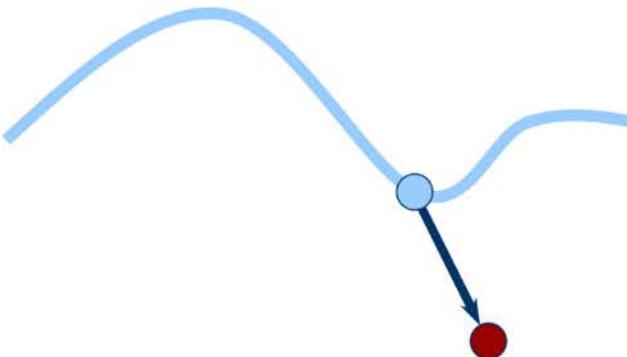
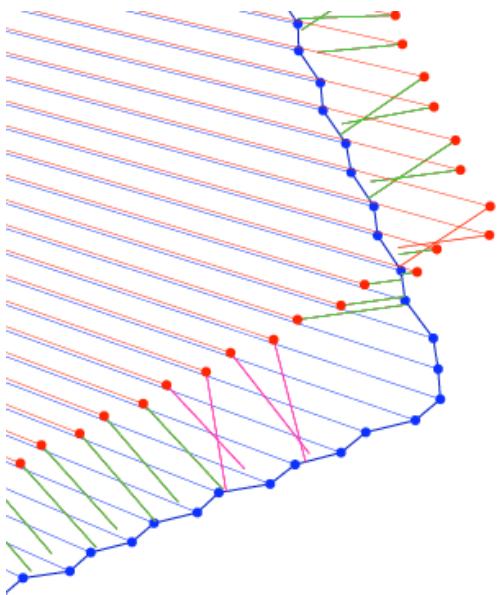
No. Cartographer needs to support 3D LIDARs and the correspondence problem is insidious.

- Scan matching variants:
 - Iterative closest point (ICP)
 - Scan-to-scan
 - Scan-to-map
 - Map-to-map
 - Feature-based
 - RANSAC for outlier rejection
 - Correlative matching

Cartographer uses the Ceres-solver to formulate a nonlinear least-squares correlative scan matching problem.

Closest-Point Matching

- Find closest point in other the point set



Closest-point matching generally stable,
but slow and requires preprocessing

Correlation-based Scan Matching

- In ICP, correspondences between two scans are explicitly computed, allowing a rigid-body transformation to be computed.
- Susceptible to local minima; poor initial estimates lead to incorrect data associations and divergence.
- **Correlation based methods search for a rigidbody transformation (without computing correspondences) that projects one set of LIDAR points (the query scan) on top of a reference map.**
- The reference map is generally implemented as a look-up table that assigns a cost for every projected query point.

Correlation-based Scan Matching

Sum over each scan point

$$\operatorname{argmin}_{\xi} \sum_{k=1}^K (1 - M_{smooth}(T_{\xi} h_k))^2$$

Pose in local submap reference frame

Transform scan point from scan frame to local submap frame

Bicubic interpolation of probability grid

Coordinate of scan return

Backend: Global SLAM

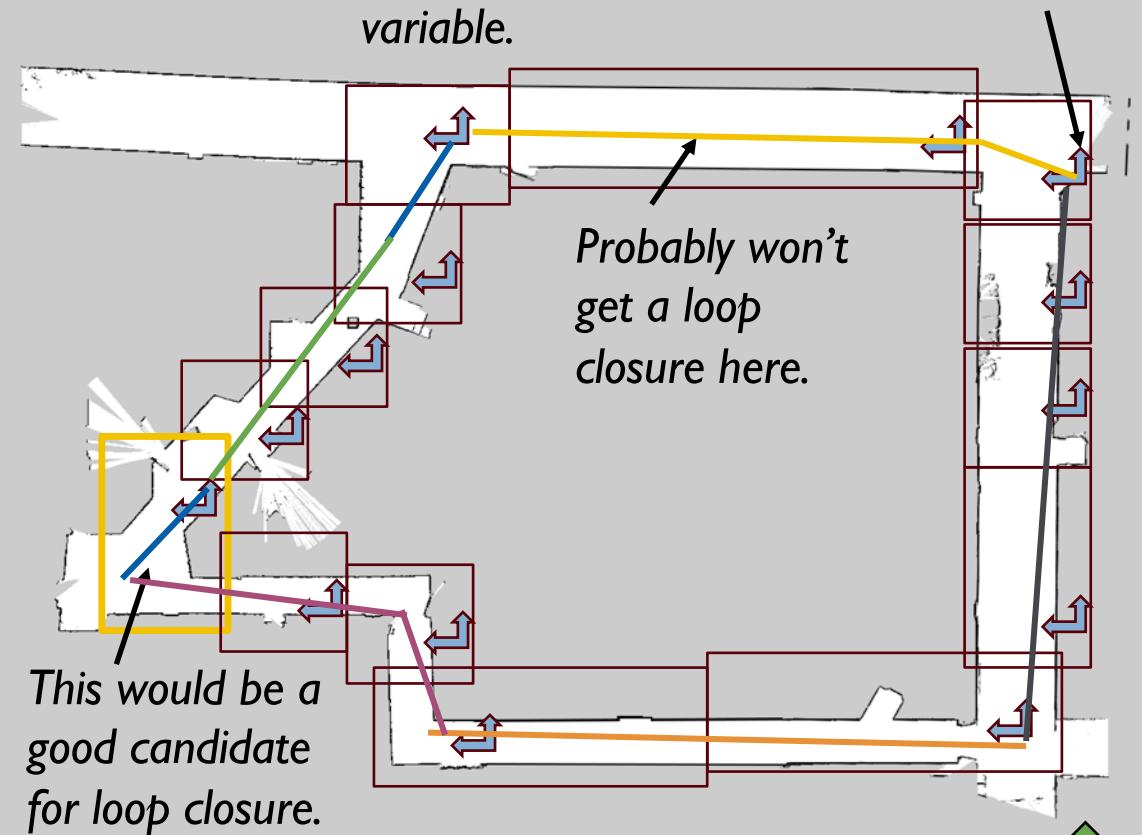
Closing Loops

Goal: is our current scan in one of the submaps we have already seen?

Constraints take the form of relative poses ξ_{ij} and associated covariance matrices Σ_{ij} . **Relative poses now include both submap and scan poses.**

For a pair of submap i and scan j , the pose ξ_{ij} describes where in the submap coordinate frame the scan was matched.

Pose of the submap is now a decision variable.



Loop Closure

Sum over each scan,
submap combination

$$\operatorname{argmin}_{\Xi^m, \Xi^s} \frac{1}{2} \sum_{i,j} \rho(E^2(\xi_i^m, \xi_j^s; \Sigma_{i,j}, \xi_{ij}))$$

Submap poses Scan poses

Residual,
see paper

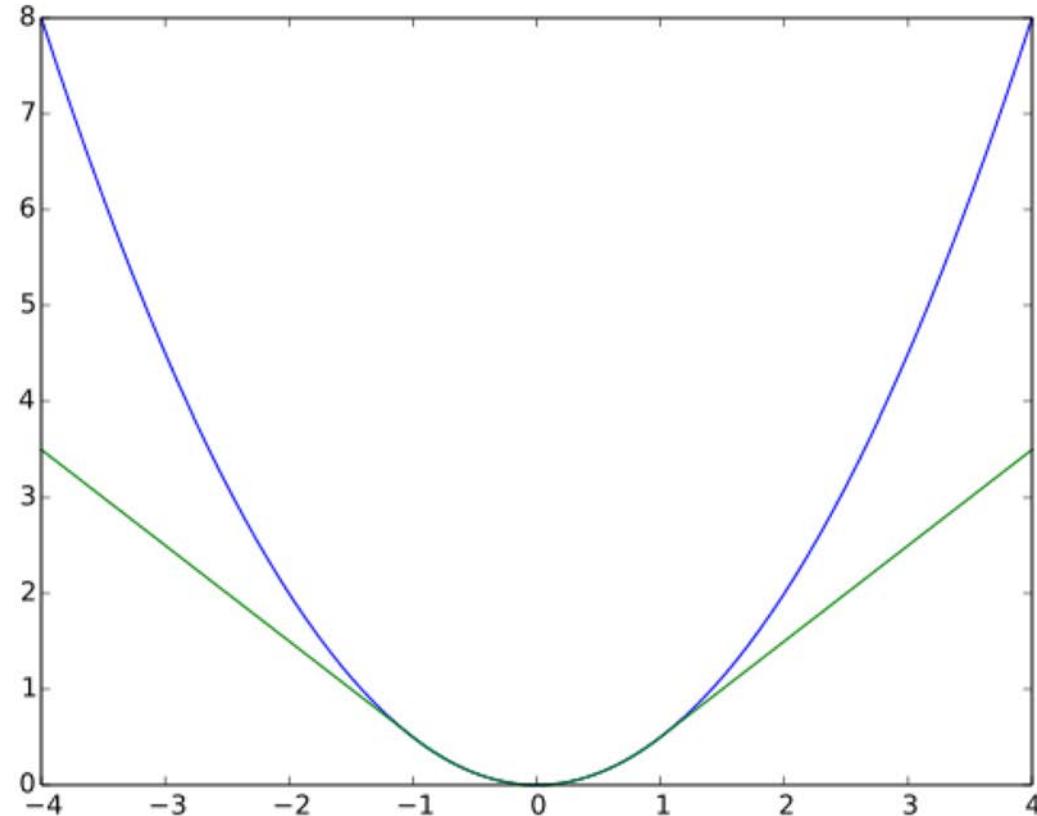
Huber loss,
outlier
rejection

Covariance
matrix

Relative pose
between submap
and scan

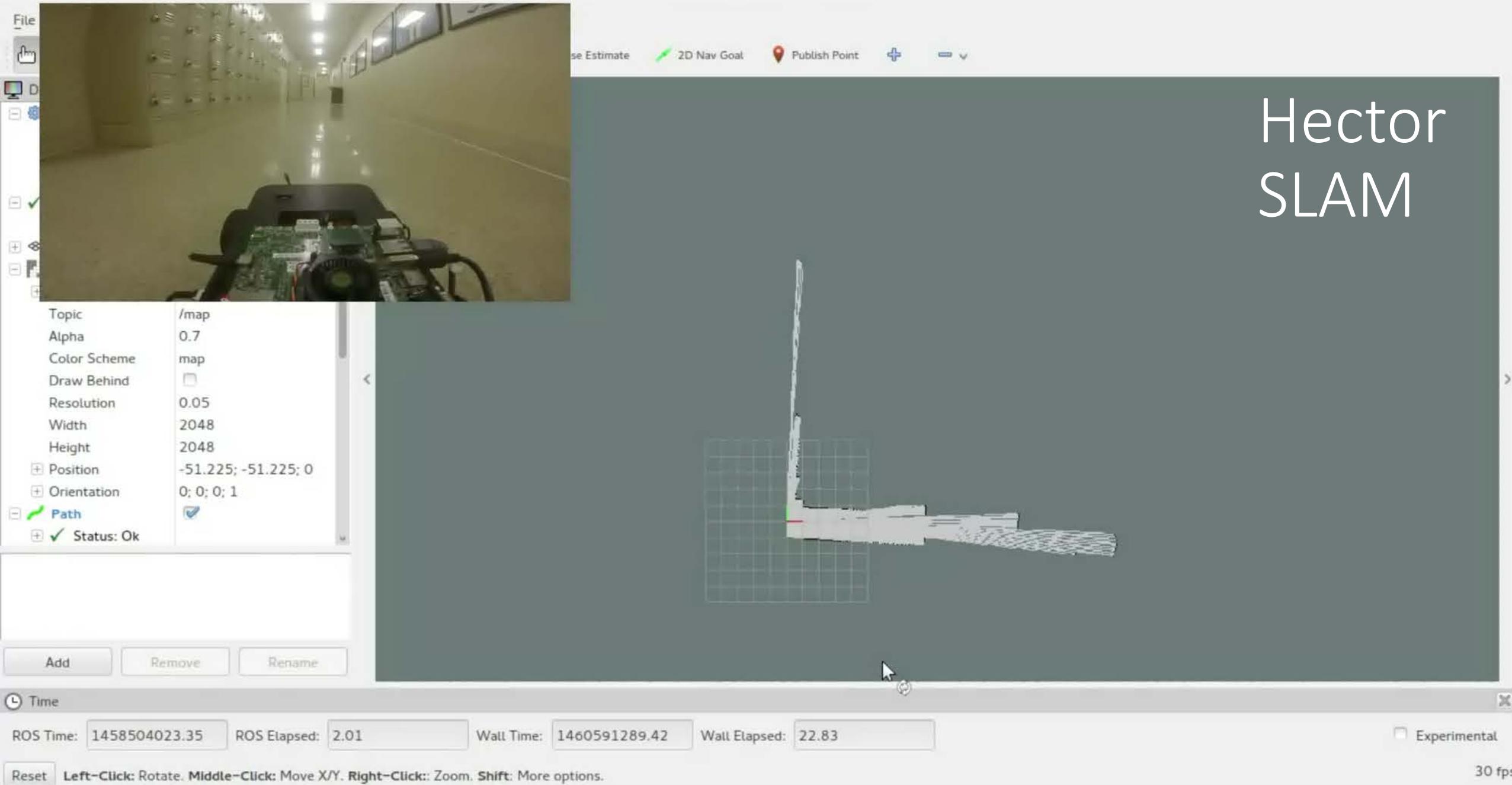
Huber Loss

Uses a Huber loss to make the objective less susceptible to spurious constraints which are often added in symmetric environments... False positive is a disaster

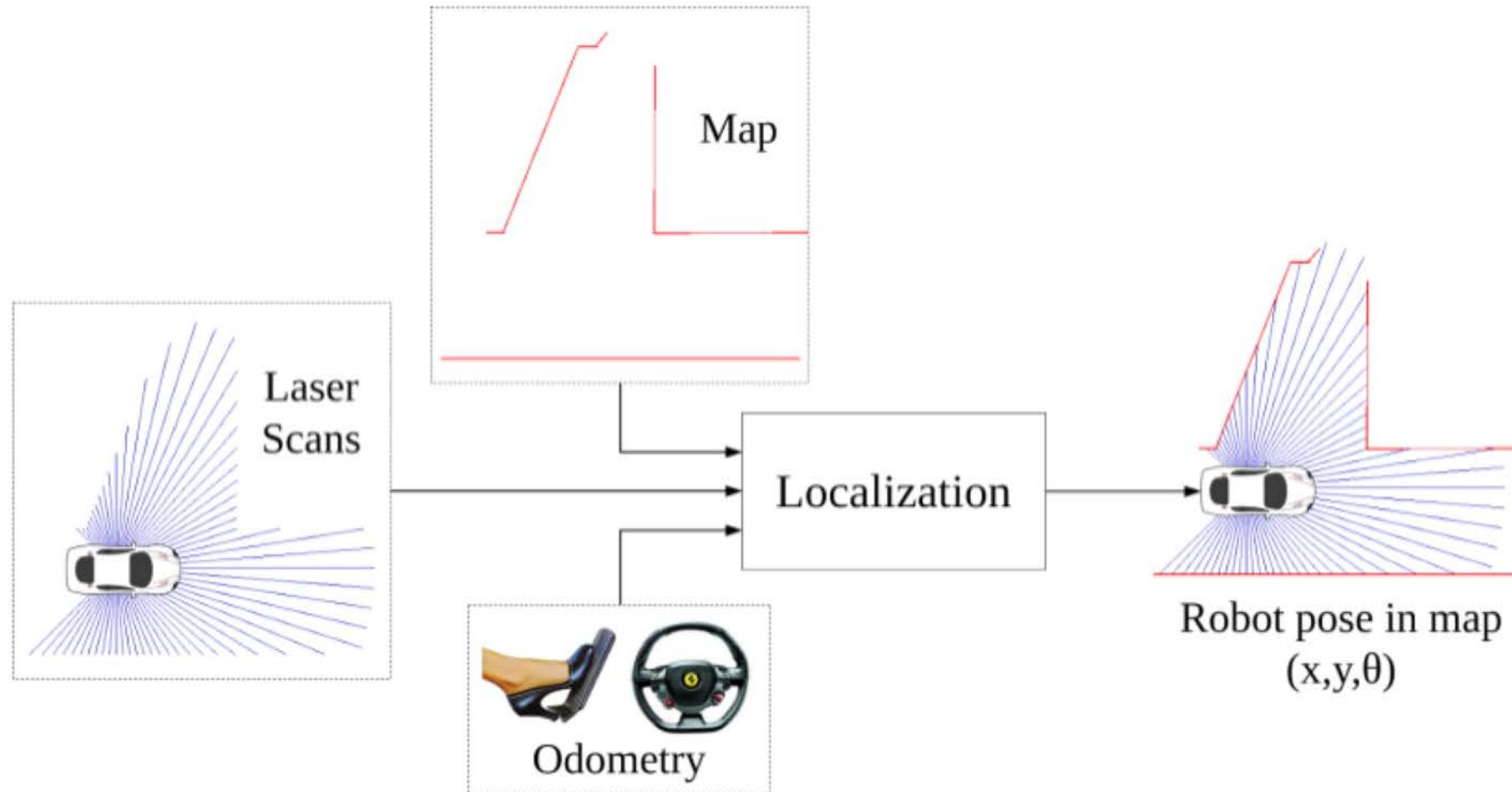


$$L_\delta(a) = \begin{cases} \frac{1}{2}a^2 & \text{for } |a| \leq \delta, \\ \delta(|a| - \frac{1}{2}\delta), & \text{otherwise.} \end{cases}$$

mapping_demo.rviz* - RViz



Problem Setting



Overview

Input

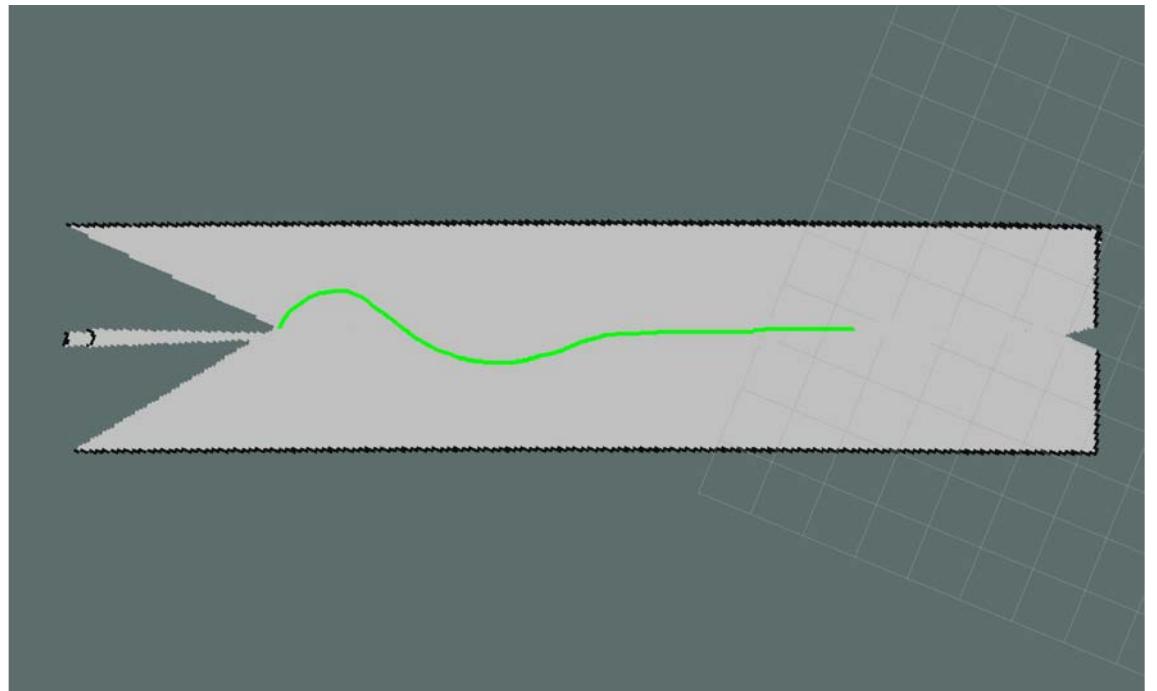
I. Laser Scan



Overview

Input

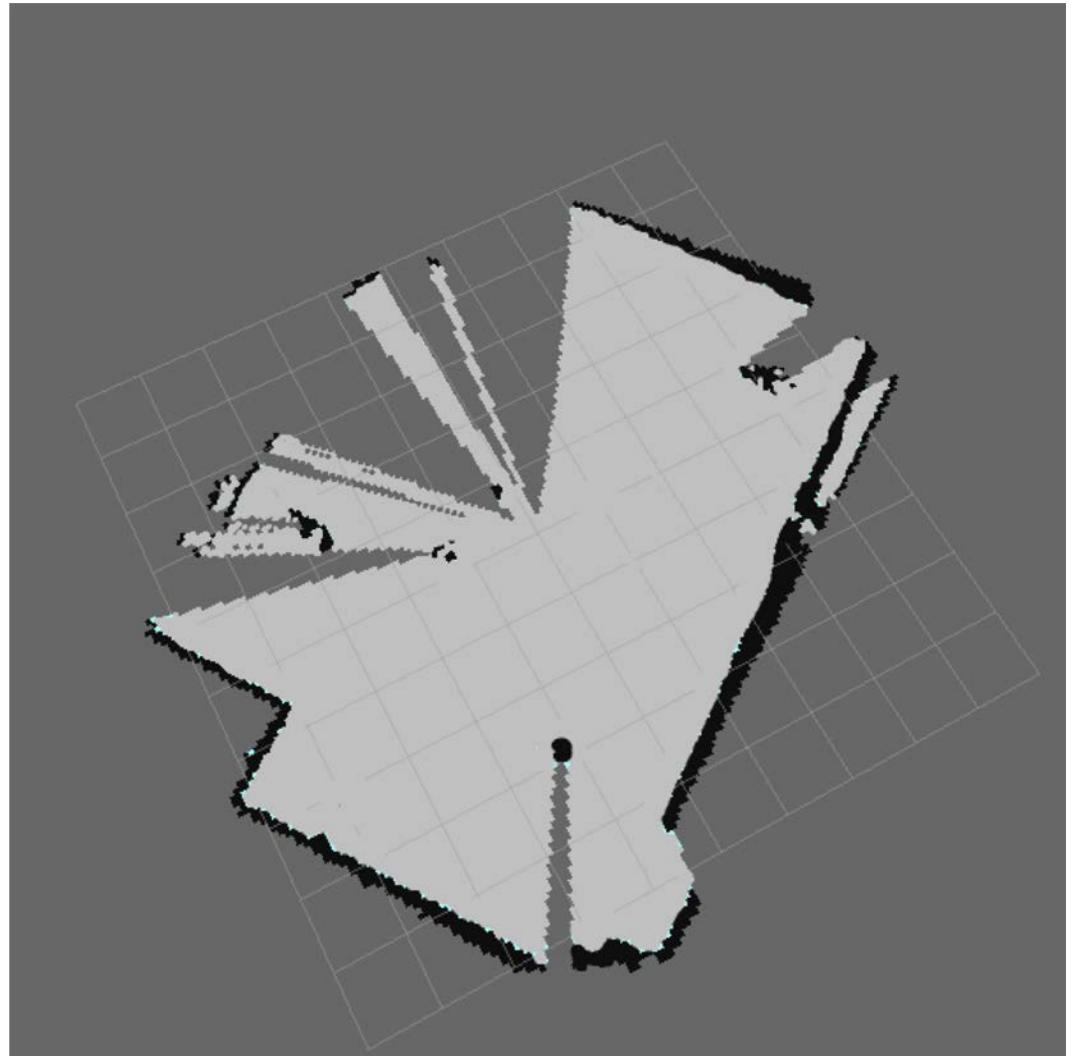
- I. Laser Scan
2. Control Input



Overview

Input

1. Laser Scan
2. Control Input
3. Map



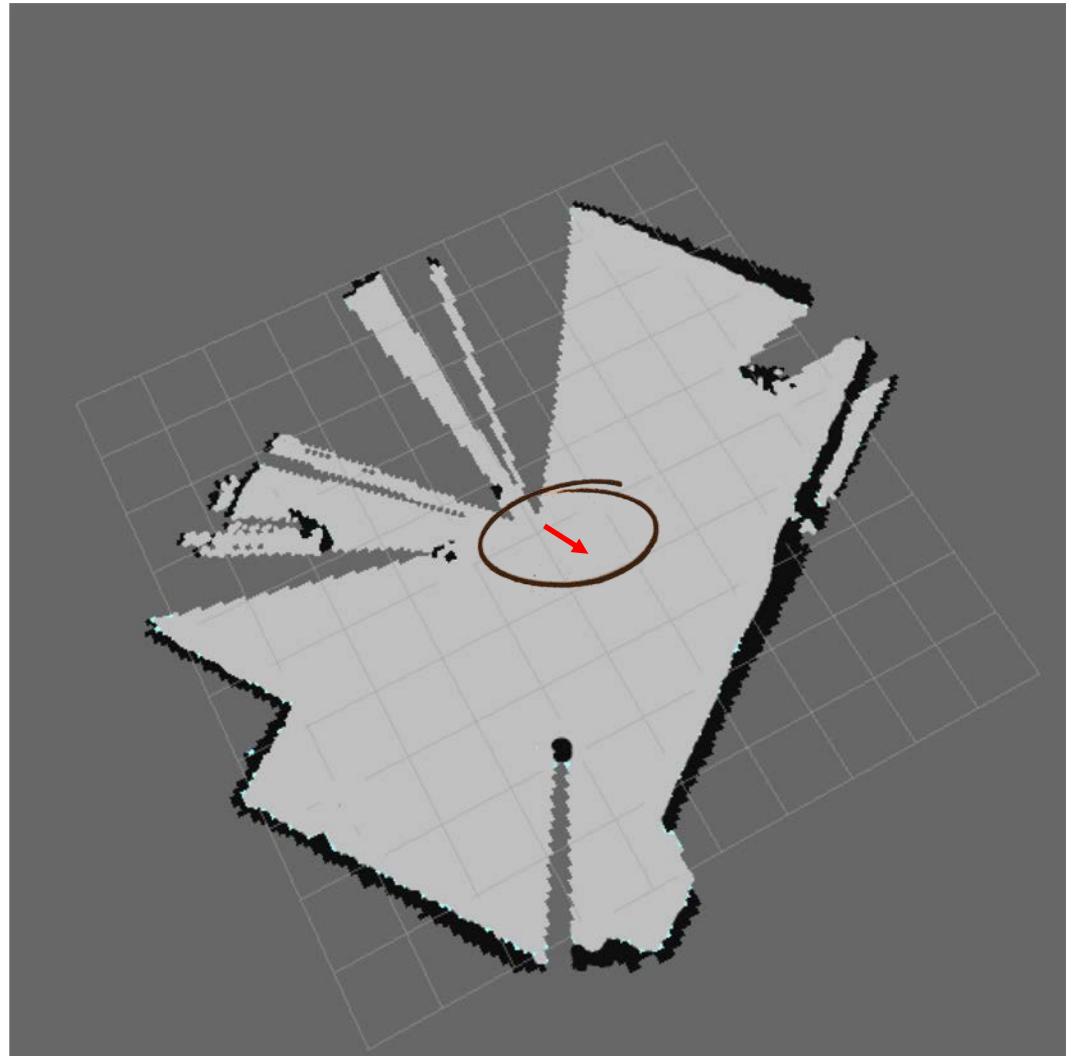
Overview

Input

- I. Laser Scan
2. Control Input
3. Map

Output

- I. Pose



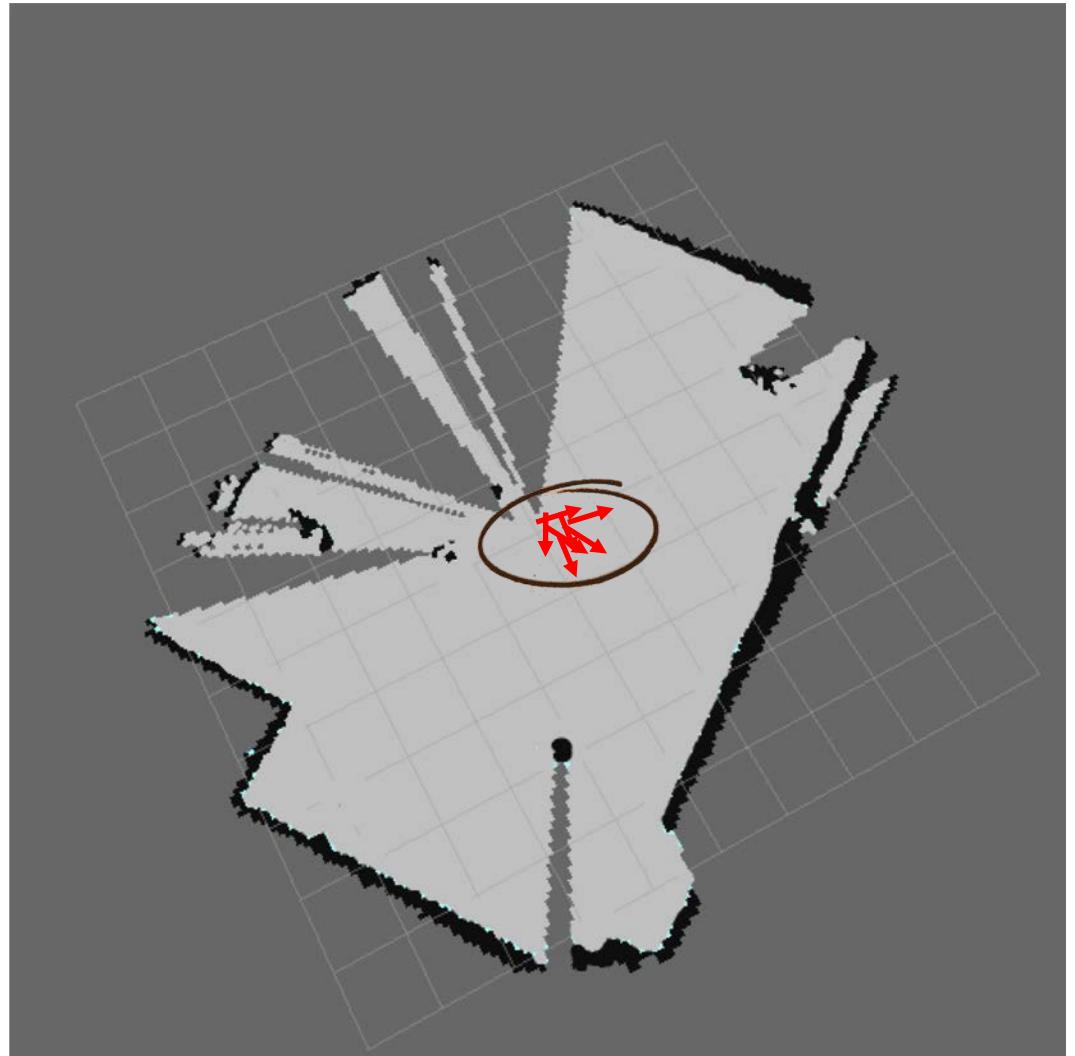
Overview

Input

1. Laser Scan
2. Control Input
3. Map

Output

1. Pose
2. Particles



Particle Filter Localization

Key Idea

The key idea of Bayes filtering is to estimate a probability density over the state-space conditioned on the data. This posterior is typically called the belief and is denoted:

$$Bel(x_t) = p(x_t \mid d_{0\dots t})$$

The diagram shows the equation $Bel(x_t) = p(x_t \mid d_{0\dots t})$. Four arrows point from labels below the equation to specific parts of the expression:

- An arrow points from "Belief or posterior" to the word Bel .
- An arrow points from "Robot state" to the variable x_t .
- An arrow points from "At time t" to the vertical bar separating x_t from $d_{0\dots t}$.
- An arrow points from "The data from time 0 to t." to the term $d_{0\dots t}$.

Belief or posterior

Robot state At time t

The data from time 0 to t.

Questions

1. How would you describe the robot state in the localization problem?
2. Is the belief a probability distribution, what kind, how do you write it down?
3. How would you describe the data from the Fltenth car?

$$Bel(x_t) = p(x_t \mid d_{0\dots t})$$

Belief or posterior Robot state At time t The data from time 0 to t.

```
graph LR; A[Belief or posterior] --> B["Bel(x_t)"]; C[Robot state] --> D["p(x_t | d_{0\dots t})"]; E[At time t] --> D; F["The data from time 0 to t."] --> D;
```

Getting more specific...

For mobile robots, we distinguish two types of data: perceptual data such as laser range measurements, and odometry data, which carries information about robot motion. Denoting the former by o (for observation) and the latter by a (for action), we have:

$$Bel(x_t) = p(x_t \mid o_t, a_{t-1}, o_{t-1}, \dots)$$

The diagram illustrates the components of the belief equation. Four arrows point from labels below the equation to specific terms:

- An arrow points from "Belief or posterior" to the first x_t .
- An arrow points from "Robot state" to the second x_t .
- An arrow points from "Perceptual Observation" to the first o_t .
- An arrow points from "Odometry" to the term a_{t-1} .
- An arrow points from "History..." to the term o_{t-1} .

Recursion

After some simplification (see additional resources) we have:

$$Bel(x_t) = \eta p(o_t | x_t) \int p(x_t | x_{t-1}, a_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

Normalization
Constant Make sure everything adds up to 1!

Sensor Model
Compute how likely your measurements were given updated particles.

Motion Model
Simulate noisy dynamics of particles based on control input.

Integrate out over previous beliefs. In practice finite approximation

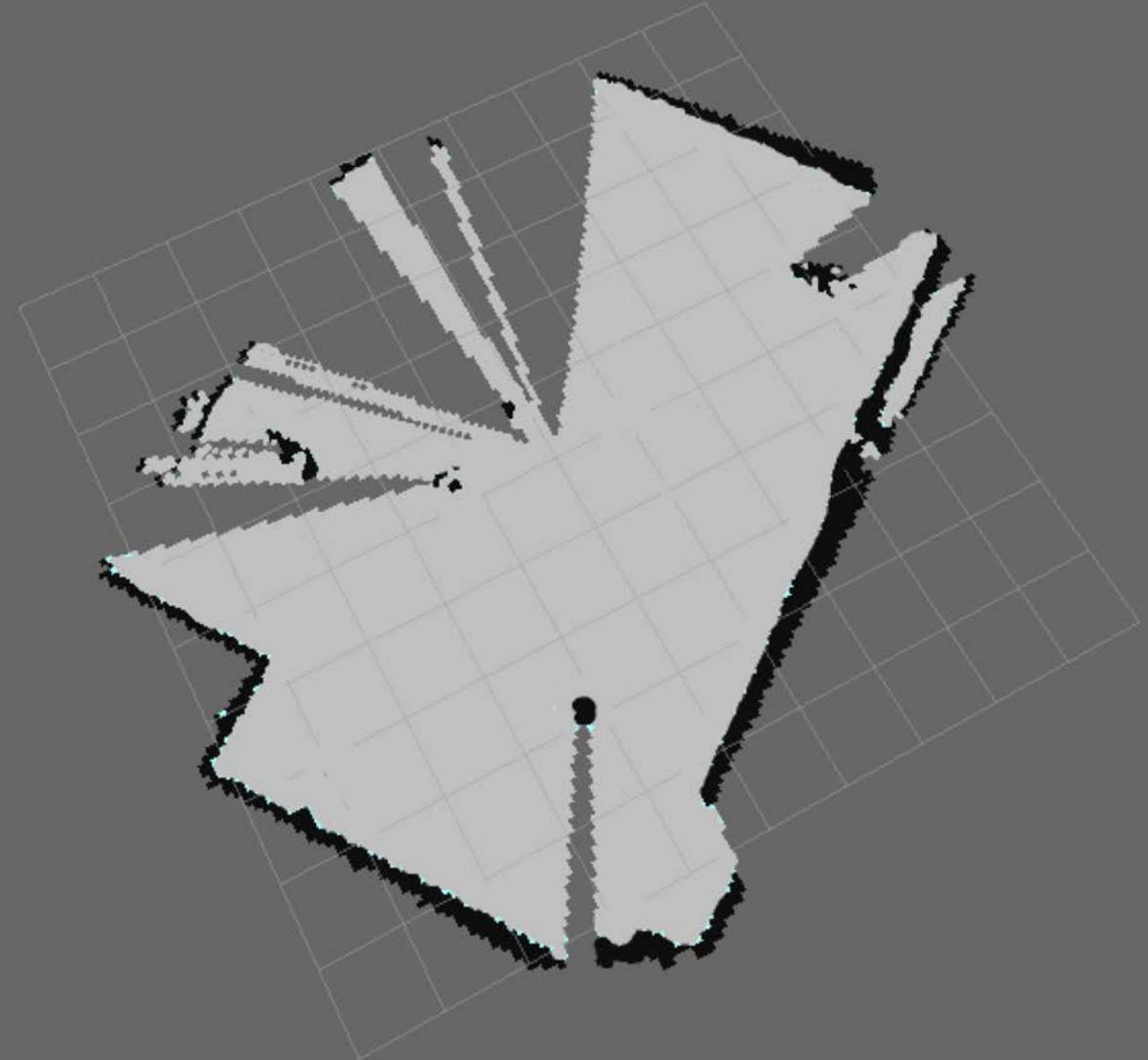
Previous Belief
Draw your particles with replacement according to importance weight.

Read left to right

In practice we represent the distributions non-parametrically using particles (a finite set of samples). More in the next slides.

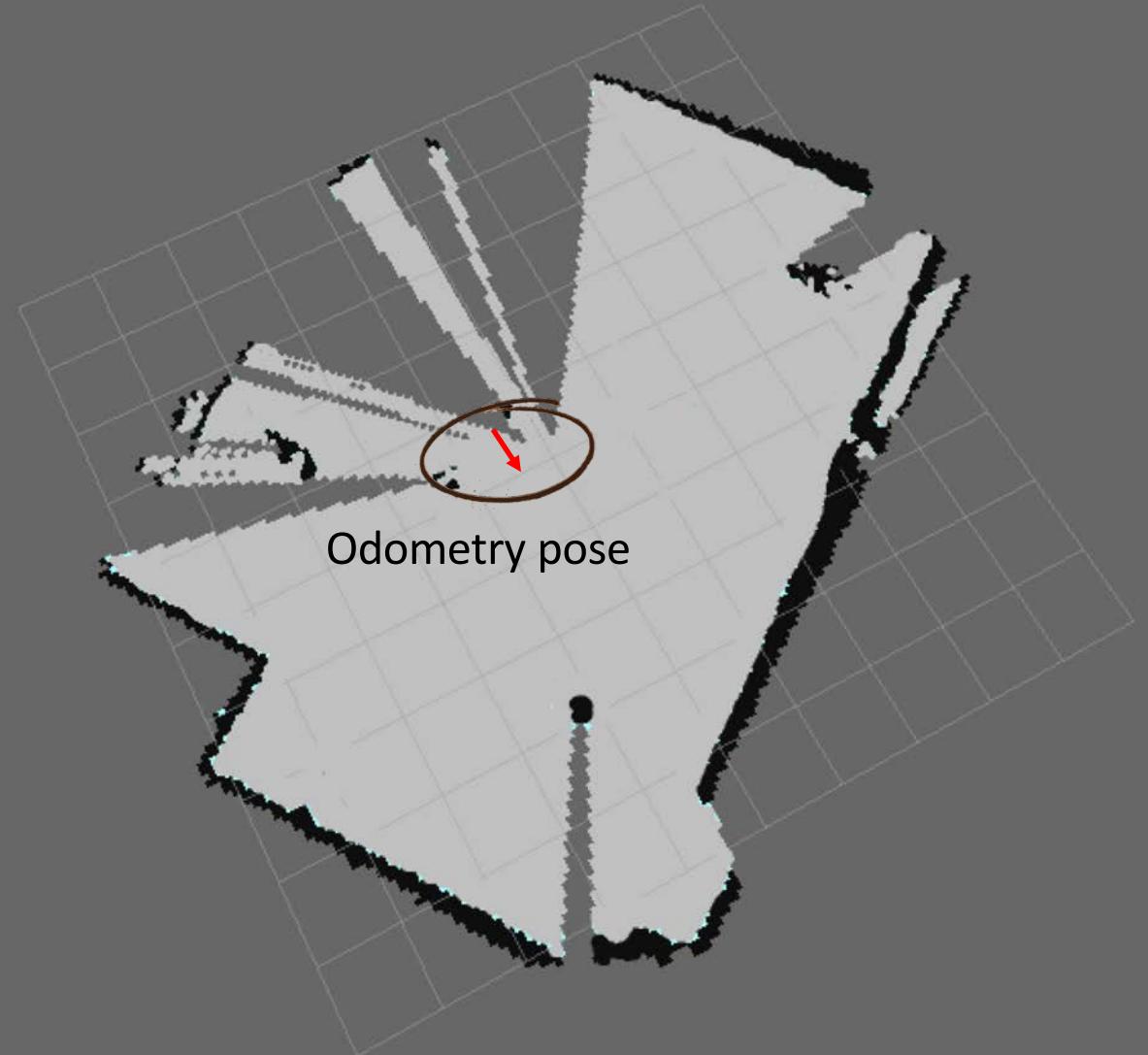
Initialization

Now let's look at particle filters in three dimensions and the process of using them in our car. Here we see a part of a map previously generated using Cartographer. The black pixels on the map denote the walls and the grey pixels denote free space.



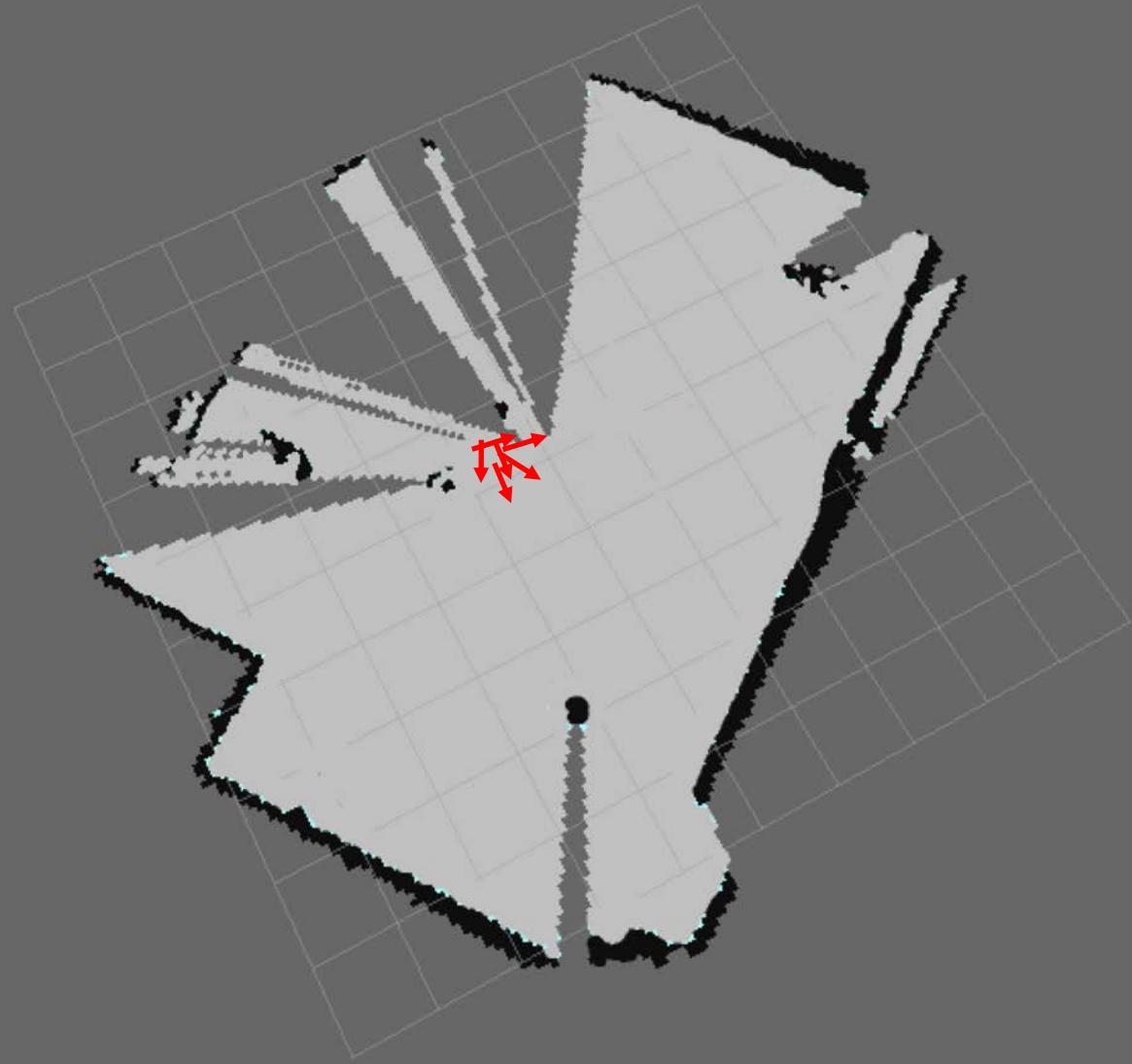
Initialization

The red arrow indicates our initial pose obtained from user input. Lets use particle filters to solve the pose tracking problem and localize more accurately in the map.



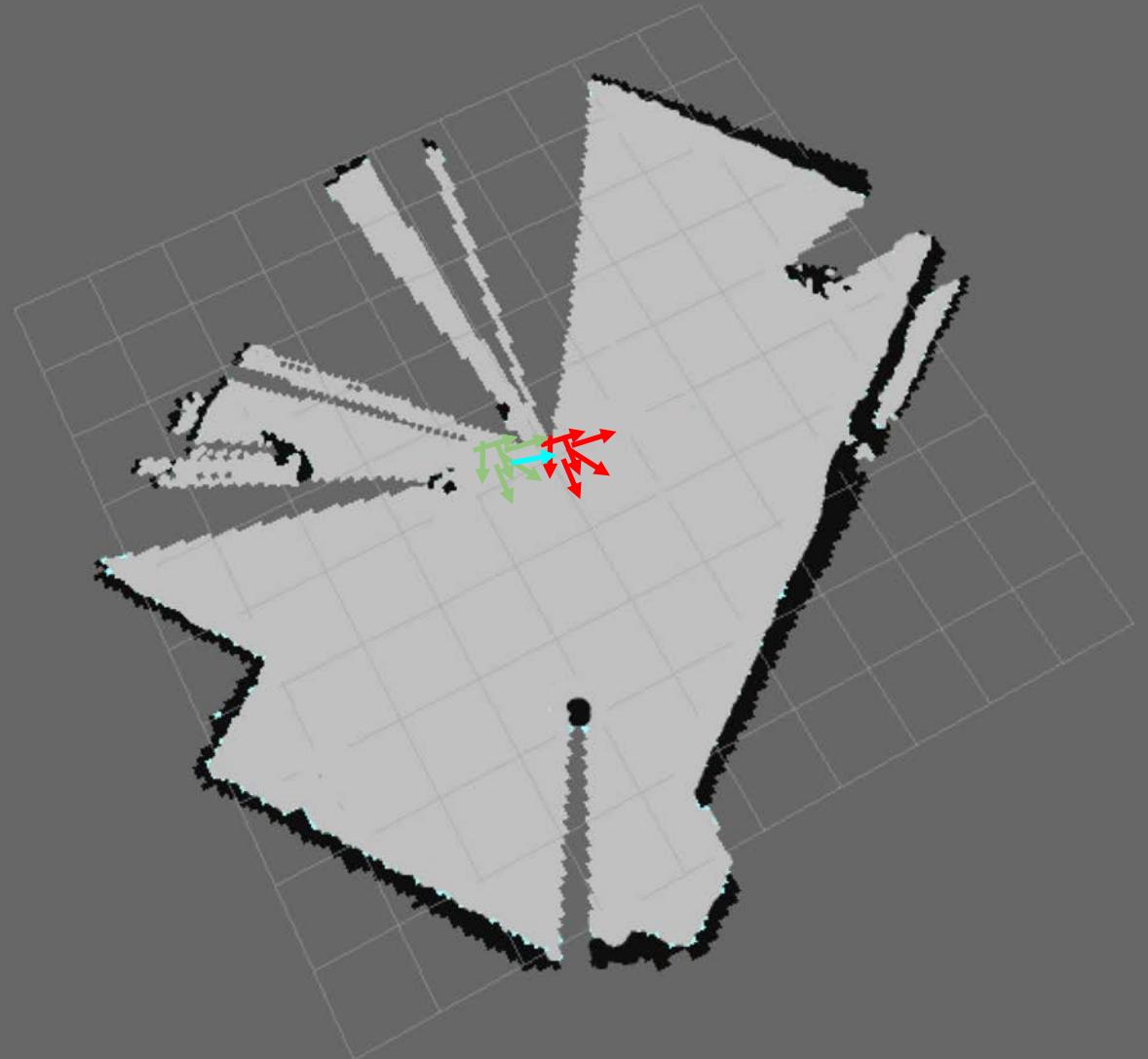
Initialization

First we need to generate a set of hypothesis for our first position. These the discrete particles drawn from a Gaussian distribution of mean being the initial guess and with a small covariance.



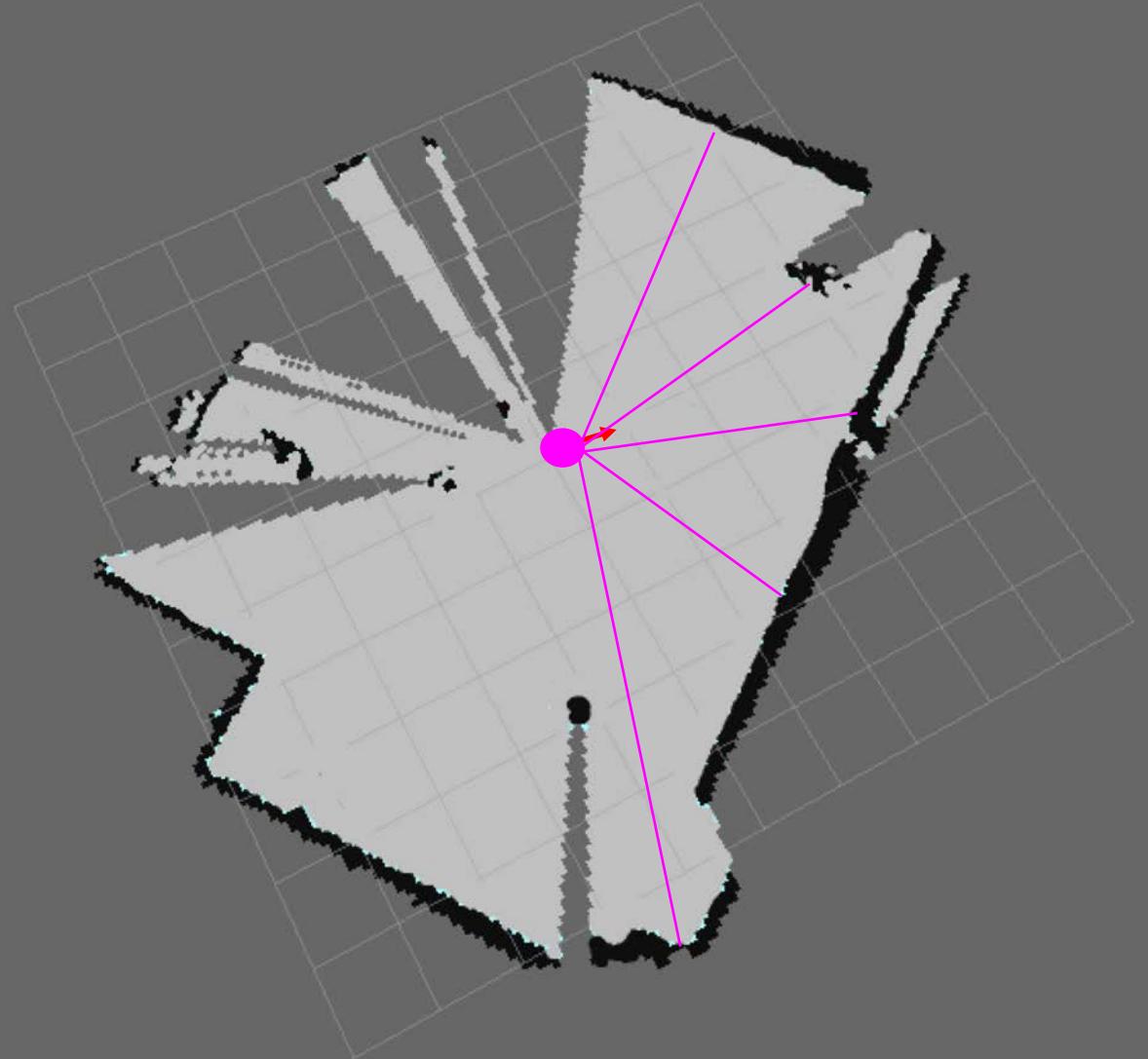
Motion Model

Applying the measured control input to the motion model (with noise), yields an updated set of possible poses.



Sensor Model

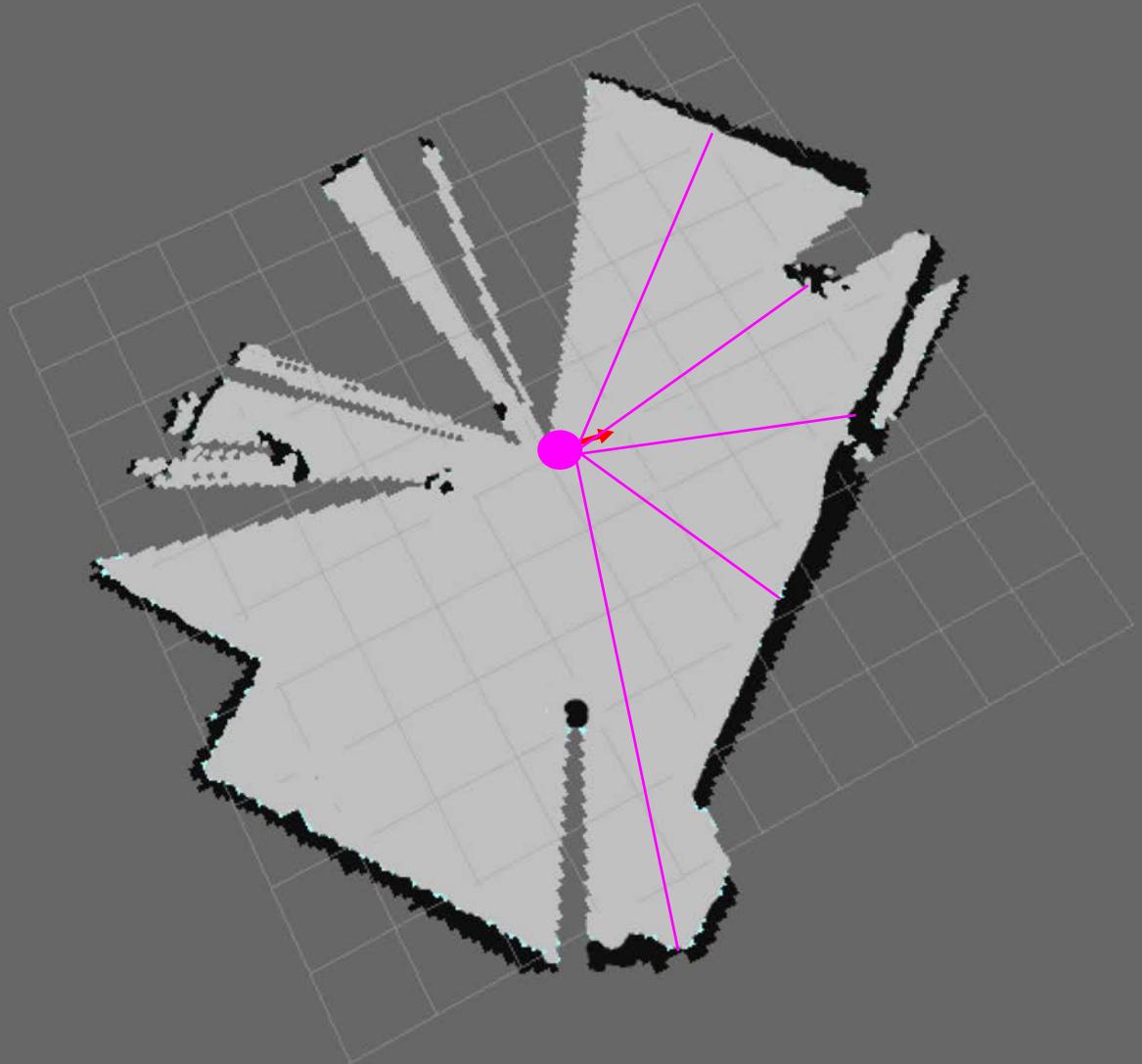
For each particle we can create a ‘fake’ laser scan by raycasting against the map at the particles pose.



Questions

How hard is creating the fake laser scan relative to computing the motion model update?

What is special about the sensor model?



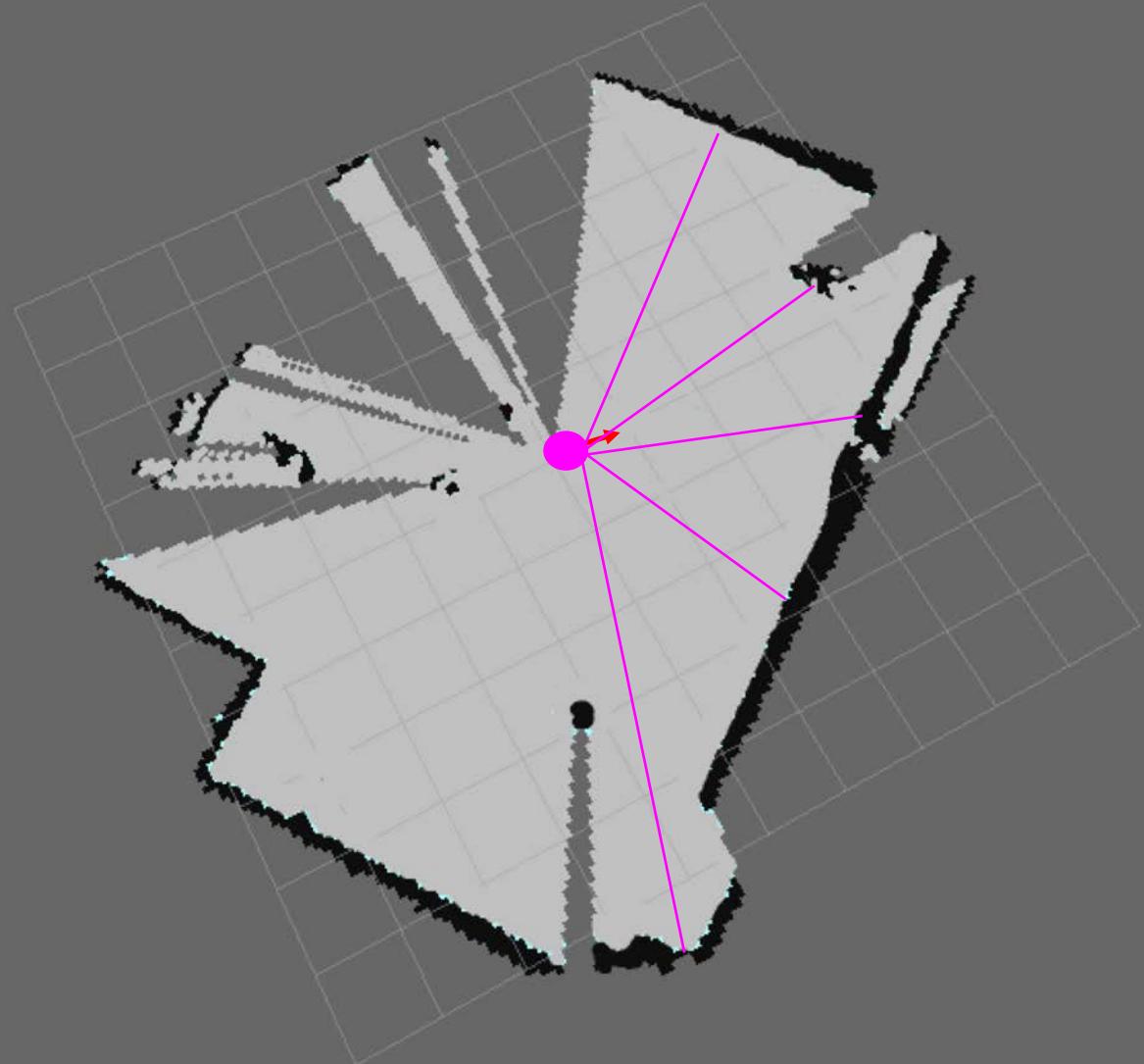
Questions

**How hard is creating
the fake laser scan
relative to
computing the
motion model
update?**

Way harder.

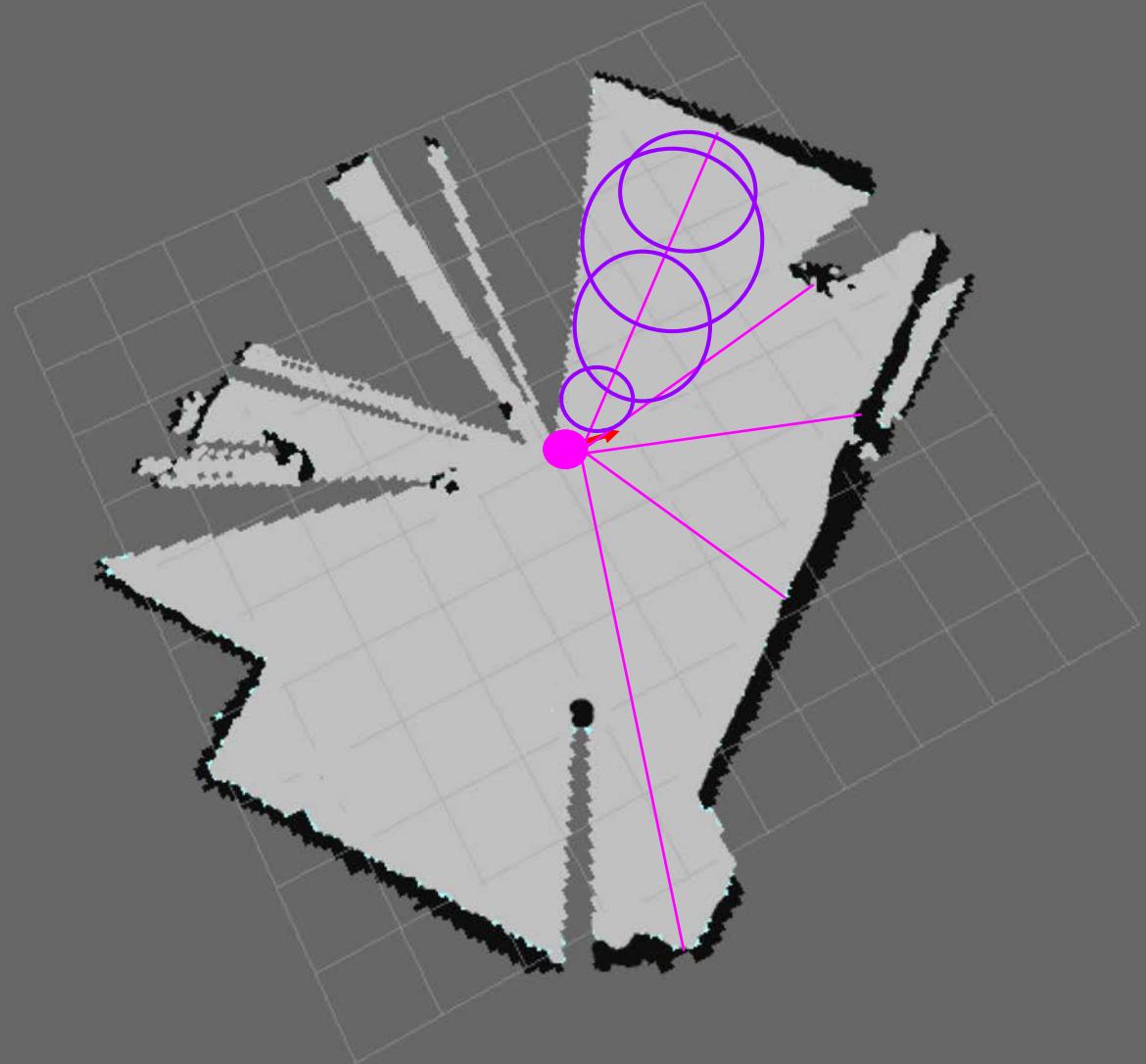
**What is special
about the sensor
model?**

It's embarrassingly



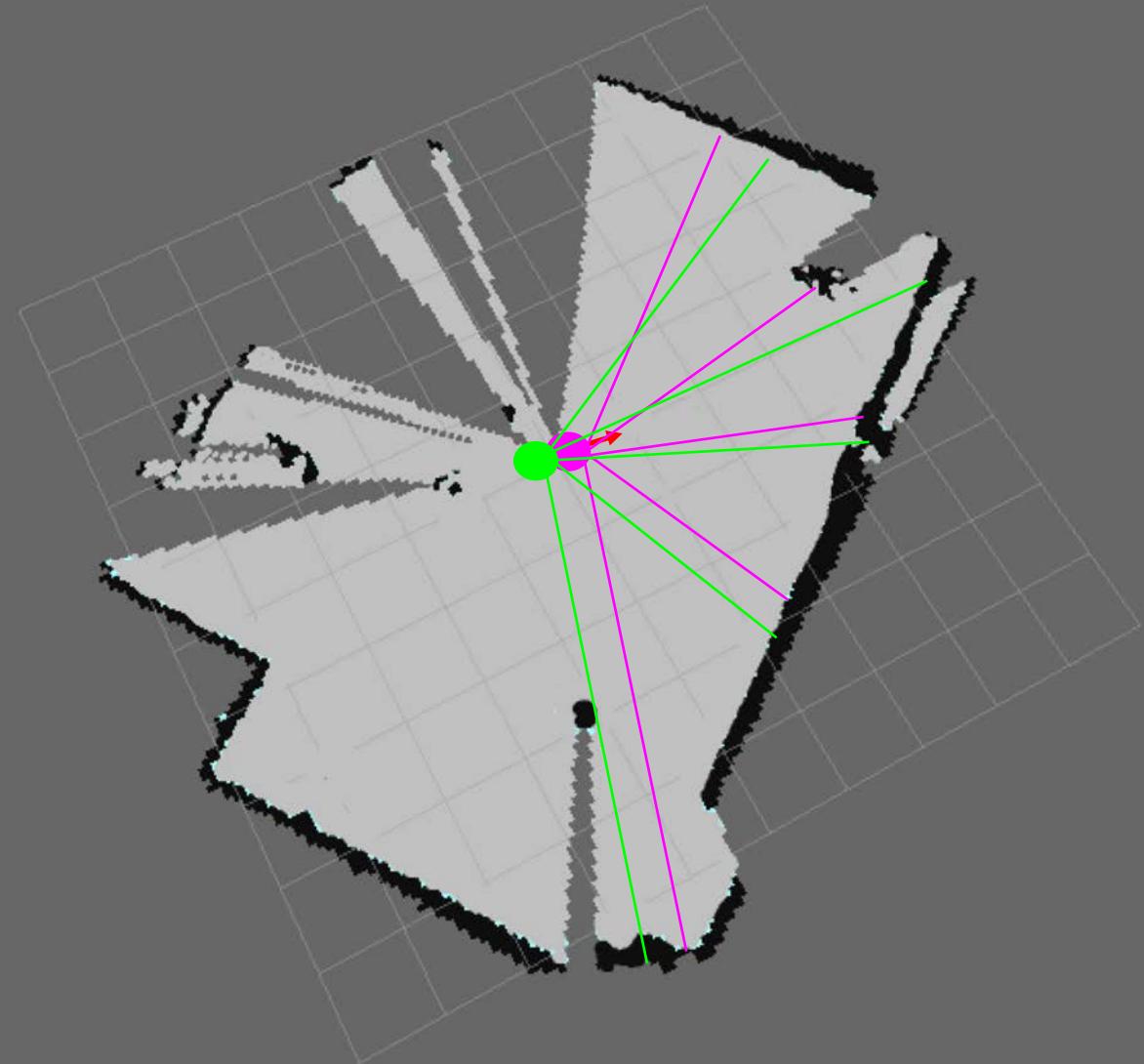
Raycasting

Efficient methods for raycasting such as Bresenham's Line method can be deployed. Alternately a signed distance field can be pre-computed.



Computing Particle Weights

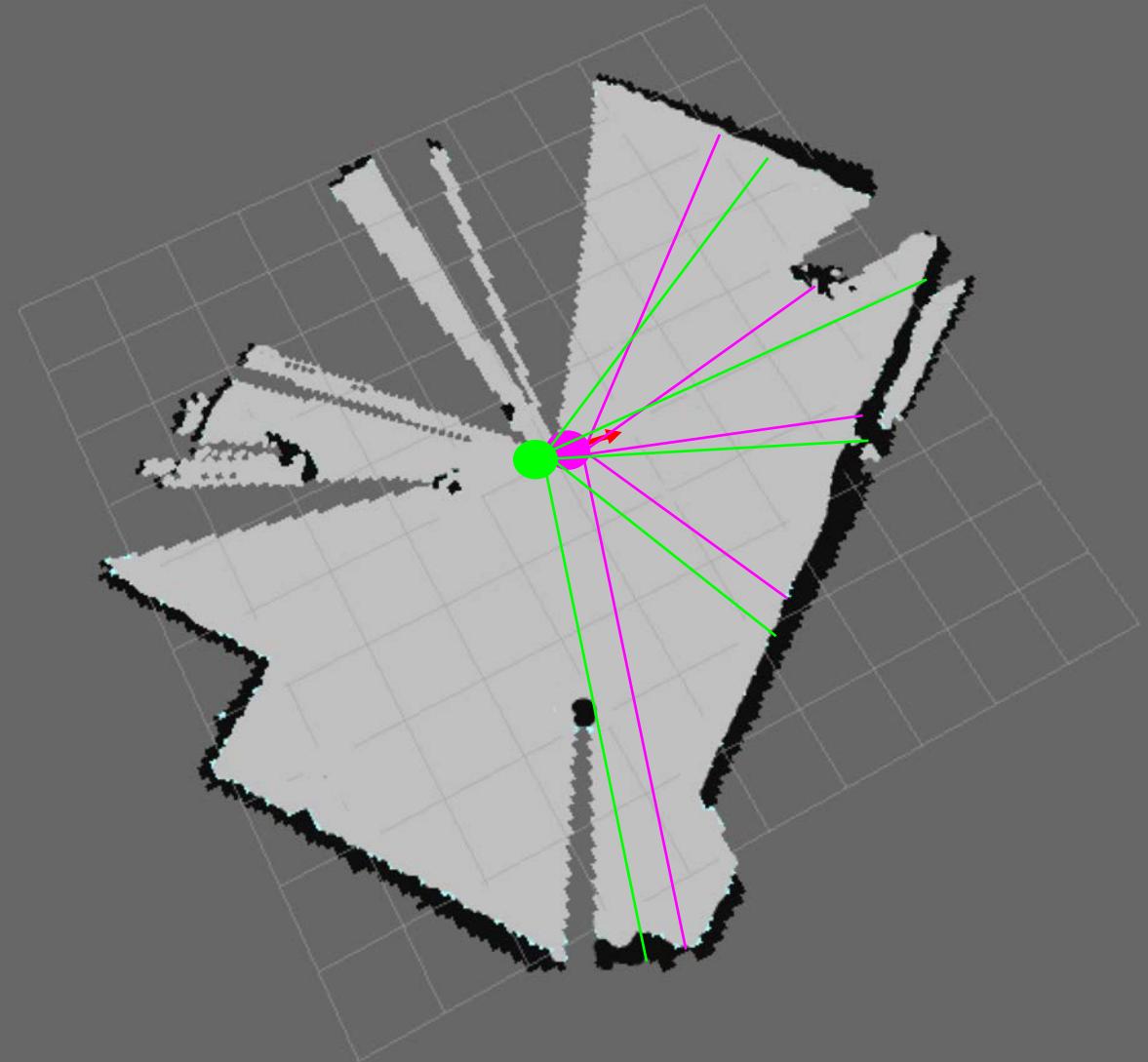
Recall, we have the ‘real’ laser scan which the car observed (shown in green). Note we don’t know where the green dot is but we do know the range measurements.



Computing Particle Weights

We can compute a score
for the fake laser scan:

$$p(o_t \mid x_t^i) = \frac{1}{\sigma \sqrt{2\pi}} e^{\frac{(o_t - r^i)^2}{2\sigma^2}}$$

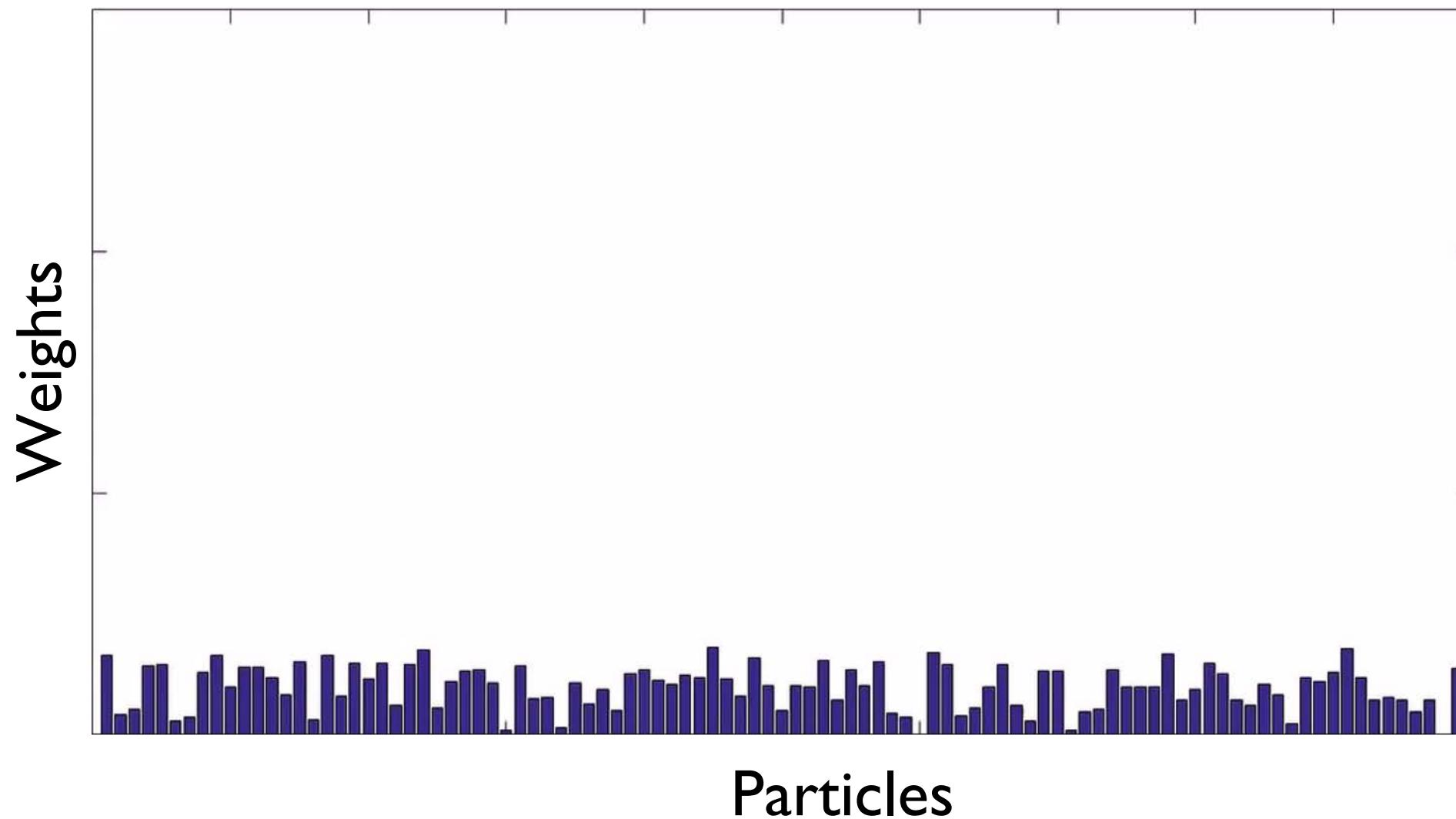


The Algorithm

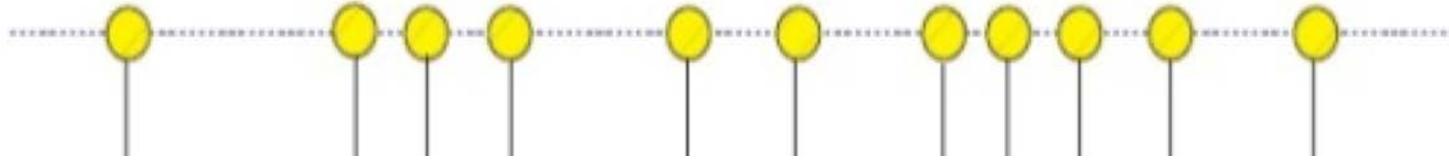
```
# a single recursive update step of the MCL algorithm
def MCL(Xt-1, at-1, o):
    Xt = {}
    for i in range(m):
        # sample a pose from the old particles according to old weights. Samples
        # implicitly represent the prior prob. dist. Bel(x_{t-1}) in eqn. (3)
        xit-1 ~ Xt-1
        # update the sampled pose according to the motion model
        xit ~ p(xt | xt-1, at-1)
        # weight the updated pose according to the sensor model
        wit = p(o | xit)
        # add the new pose and weight to the new distribution
        Xt = Xt ∪ {(xit, wit)}
    # normalize weights, should sum to 1
    Xt = normalize(Xt)
    return Xt

# iterative application of the MCL algorithm
def particle_filter():
    X = Bel(x0) ← initial particles
    while true:
        a = get_last_odometry()
        o = get_last_sensor_readings()
        X = MCL(X, a, o)
        # inferred pose ← expected value over particle distribution
        pose = Ex[X]
```

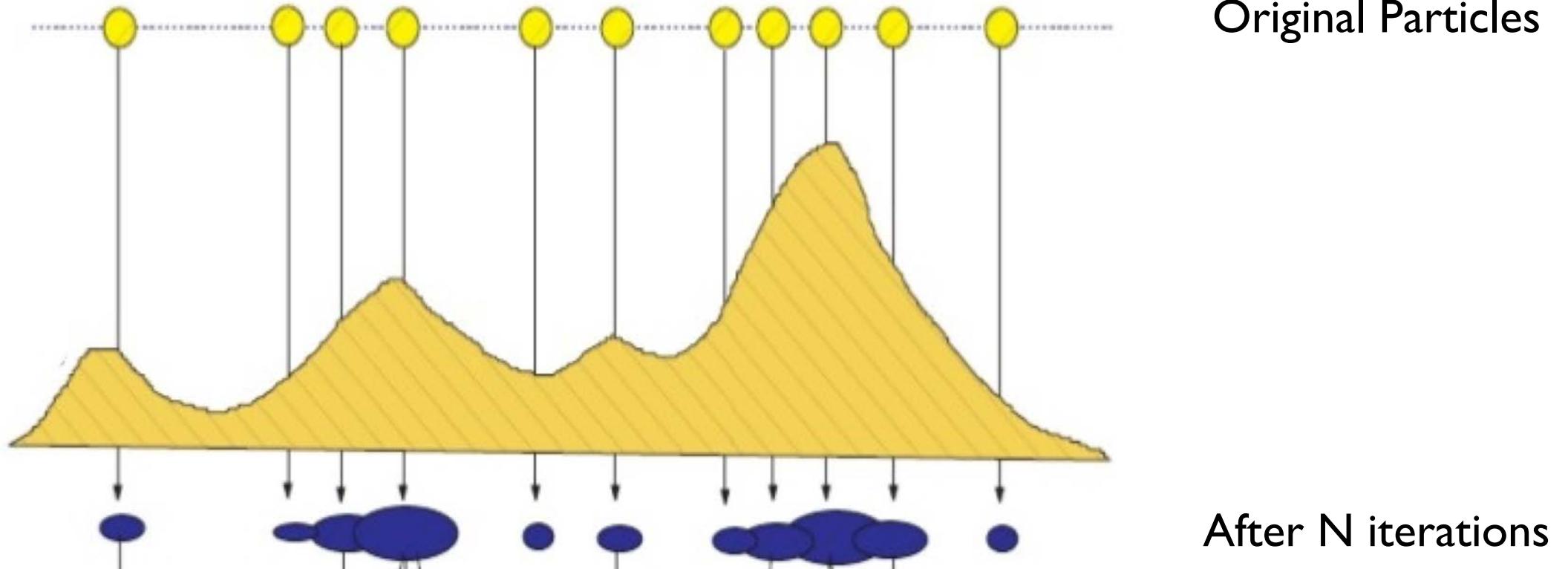
Particle Filter without Resampling



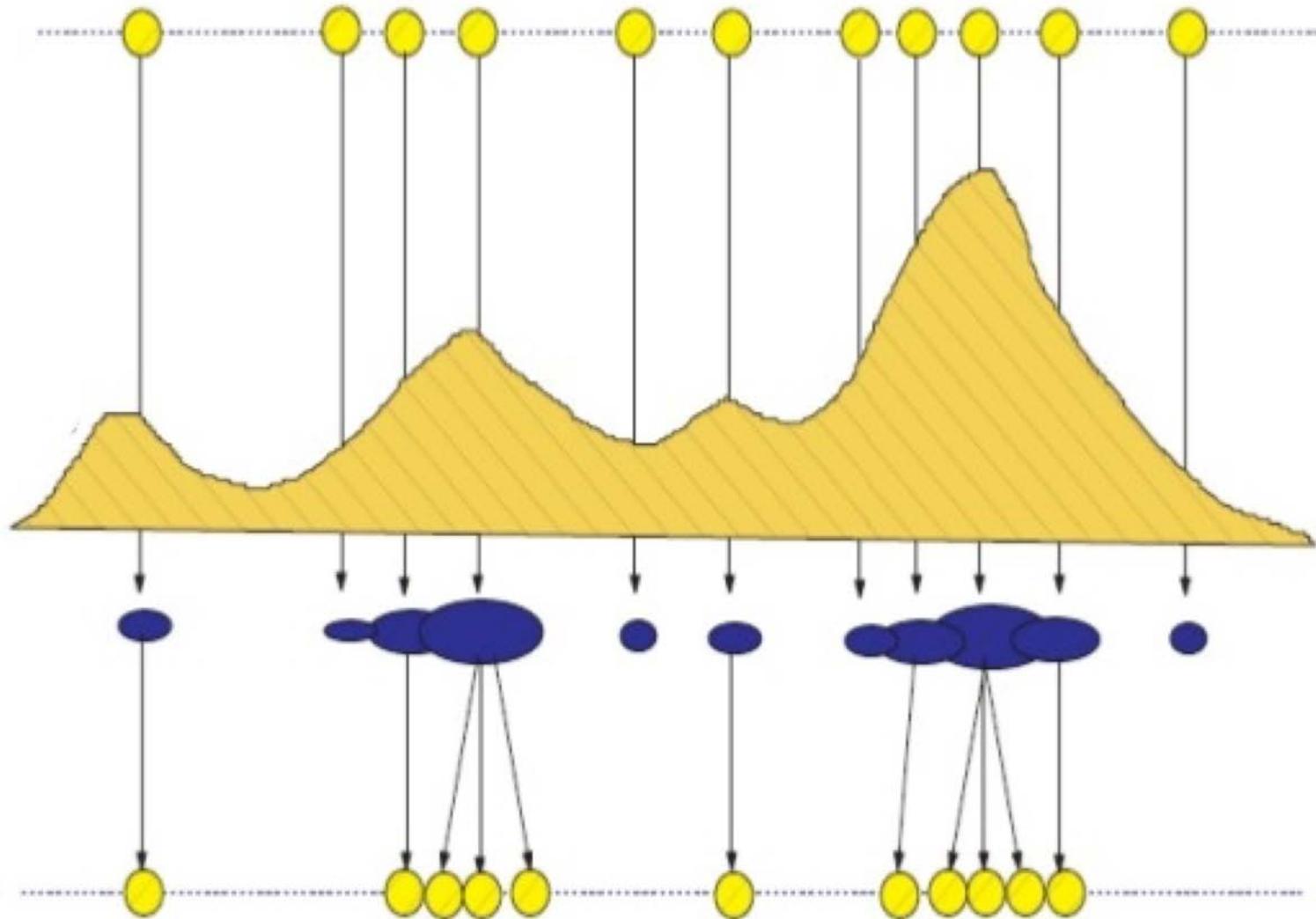
Resampling



Resampling



Resampling

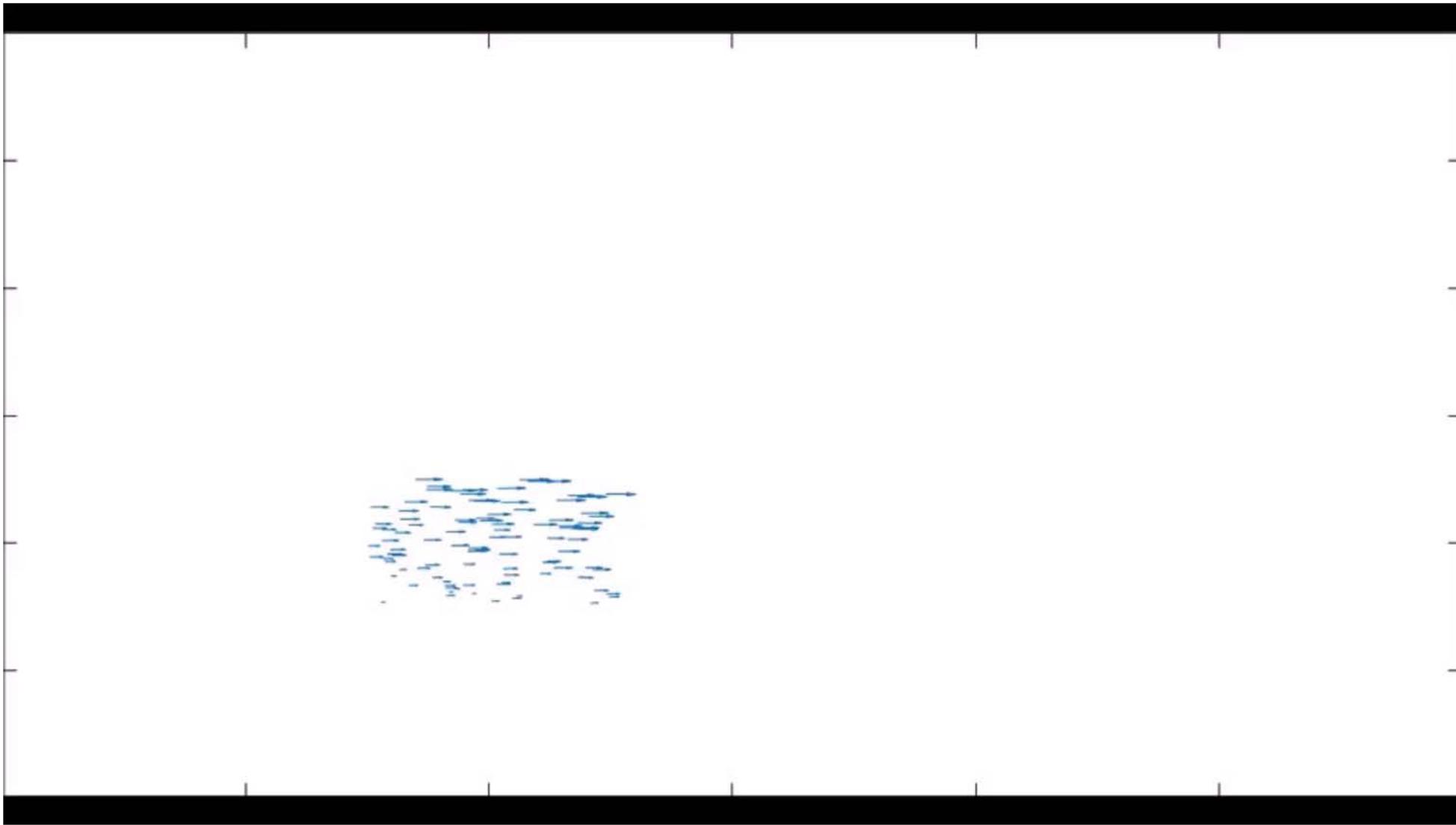


Original Particles

After N iterations

Resampling

Particles



Particle Filters in ROS

- Adaptive Monte Carlo Localization Package
- Localization for a robot moving in a 2D space
- Localizes against a pre-existing map

AMCL Parameters

`min_particles`

Default: 100

The minimum number of particles to be used for calculating correlation

`max_particles`

Default: 500

The maximum number of particles to be used for calculating correlation

AMCL Parameters

update_min_d

Default: 0.2m

The minimum translation movement required by the vehicle before an pose update is published

update_min_a

Default: $\pi/6$ radians

The minimum angular movement required by the vehicle before an pose update is published

AMCL Parameters

initial_pose_x Default: 0

initial_pose_y Default: 0

initial_pose_a Default: 0

The initial mean position of the particles to initialize the particle filter

AMCL Parameters

initial_cov_xx

Default: 0

initial_cov_yy

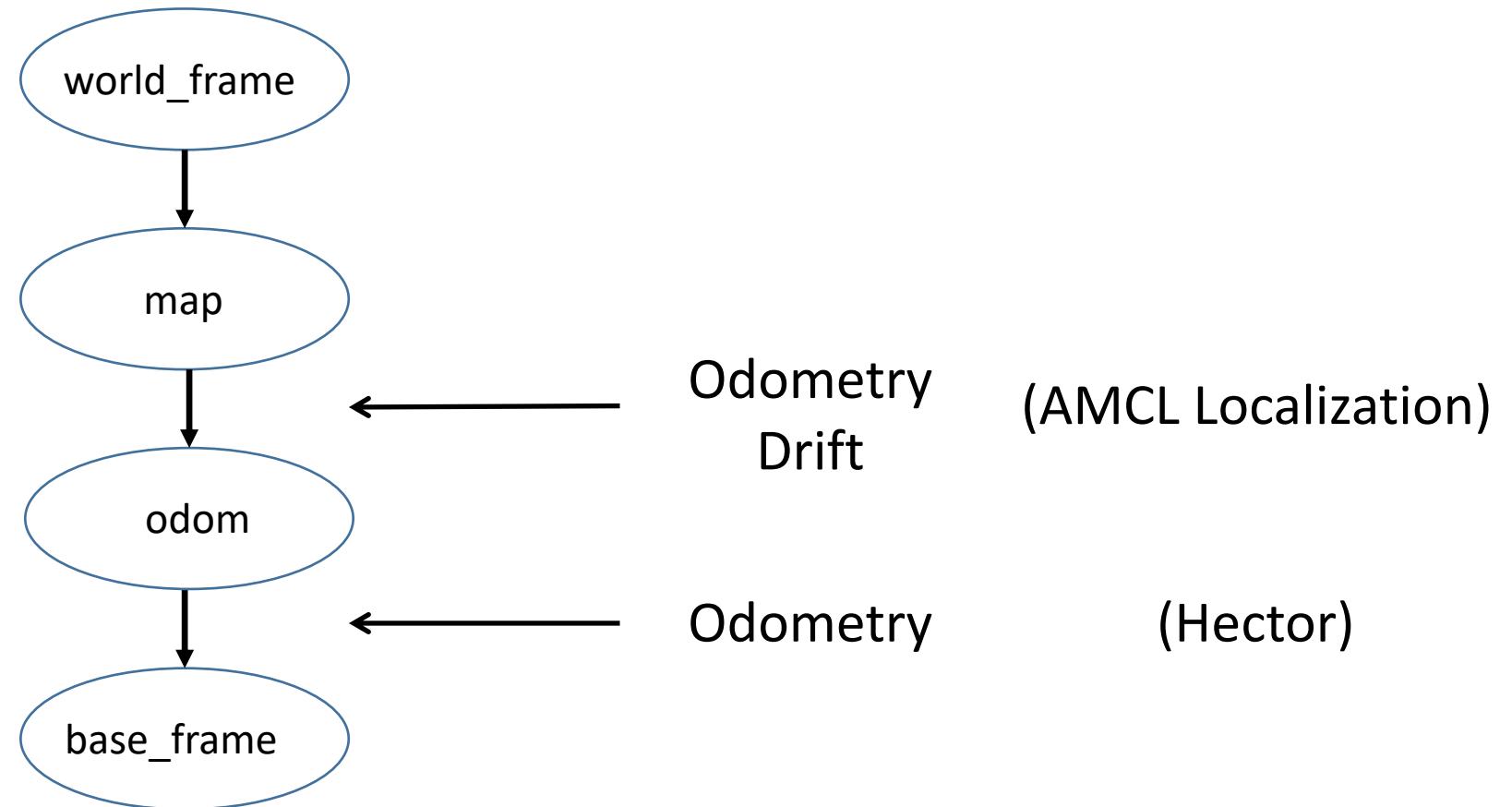
Default: 0

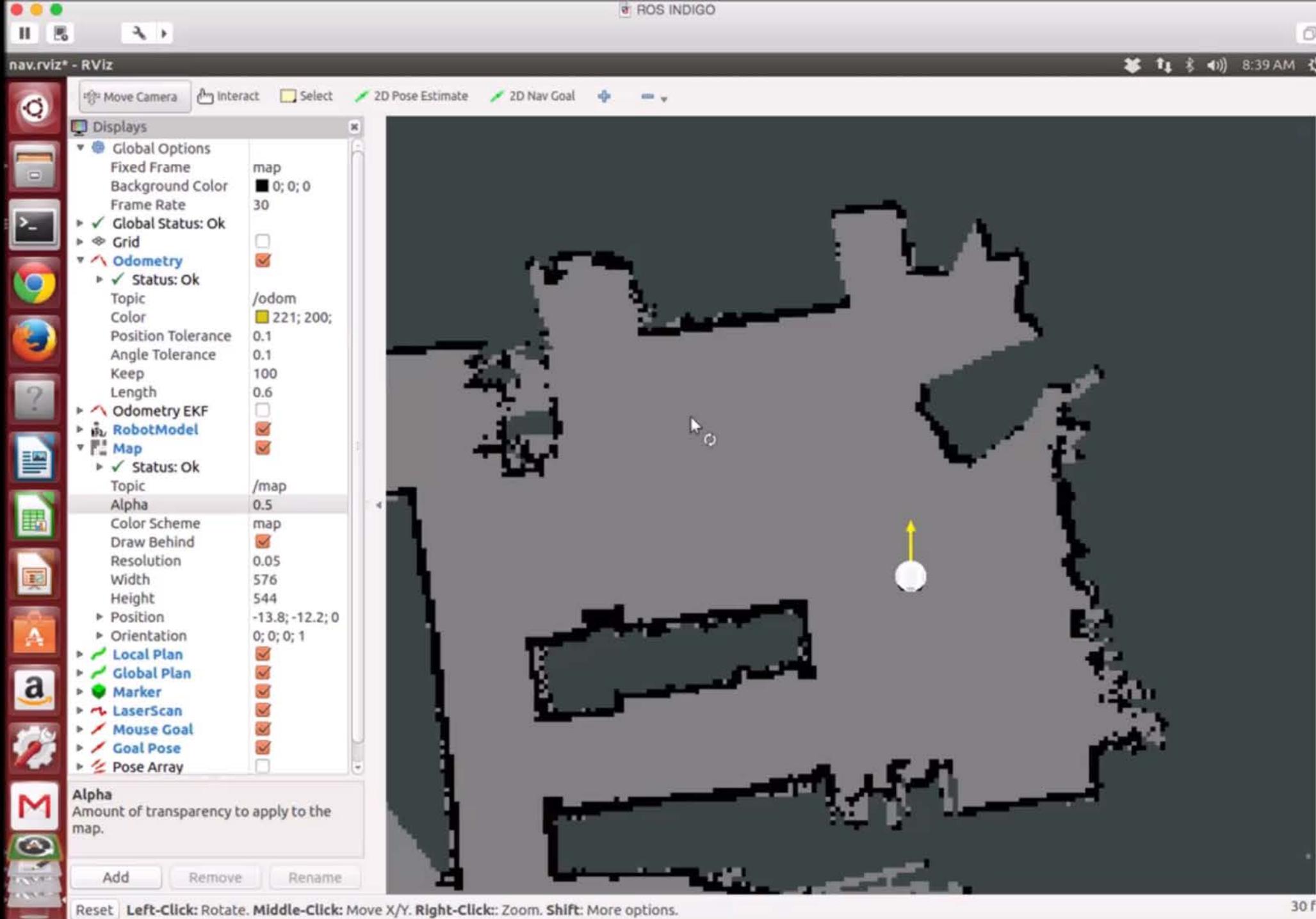
initial_cov_aa

Default: 0

The covariance of particles distributed around the mean

Tf tree – Where does AMCL fit in





Alpha

Amount of transparency to apply to the map.

Add

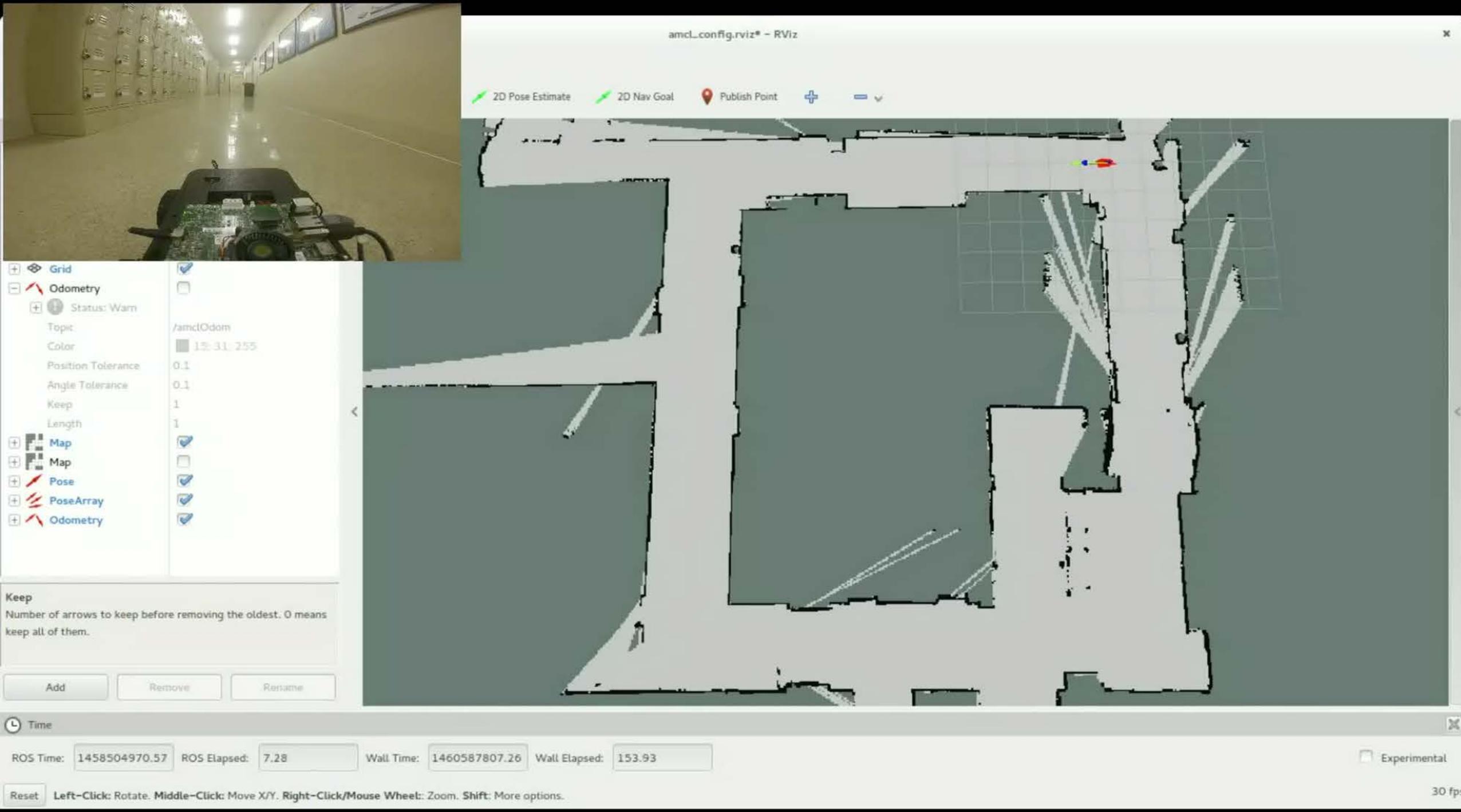
Remove

Rename

Reset

Left-Click: Rotate. Middle-Click: Move X/Y. Right-Click: Zoom. Shift: More options.

30 fps



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