Physics Informed Neural Networks (PINNs)

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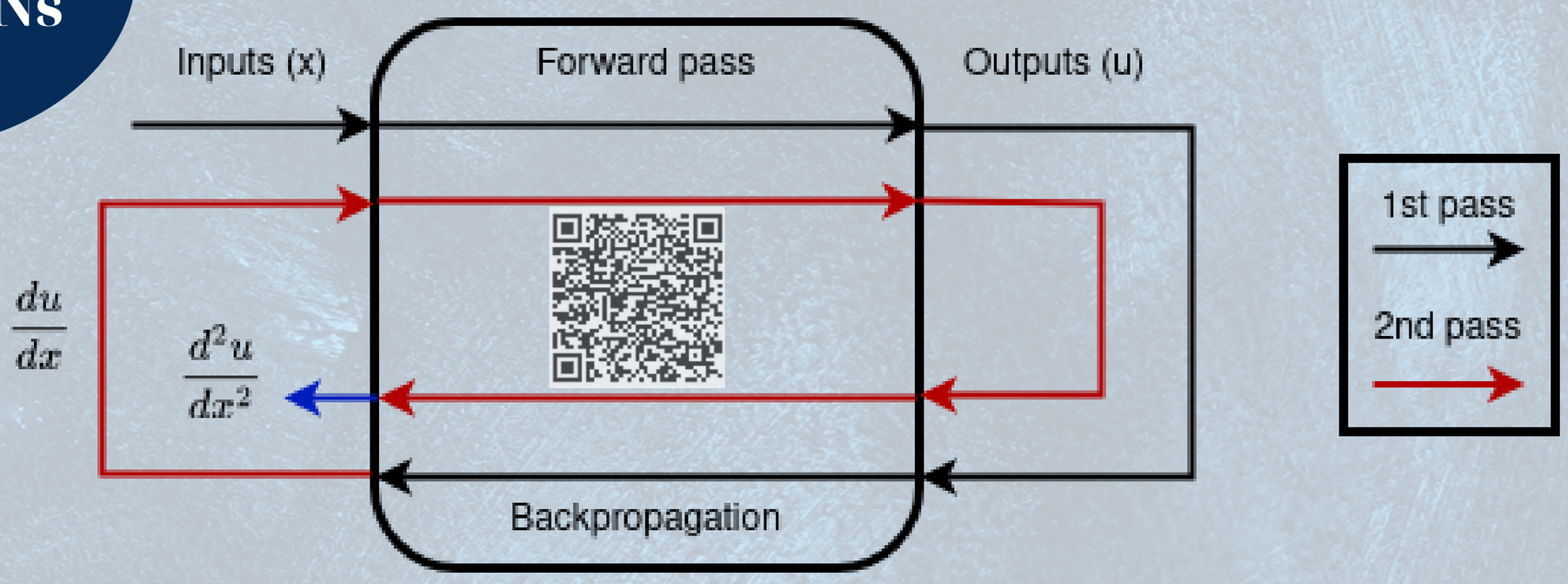
Introduction

Physics-Informed Neural Networks (PINNs) are a type of machine learning approach that combines the power of neural networks with the principles of physics. PINNs were proposed to solve Ordinary Differential Equations (ODEs) and Partial Differential Equations (PDEs), taking advantage of automatic differentiation.

Motivation

PINNs are a distinguished paradigm in modeling and computation that enriches deep learning with mathematical physics. Usage of differential equations is widespread, especially when it comes to mechanics, fluid dynamics or even in population dynamics, epidimiology and economics problems. This wide field that can be used and the fact that are not so common in the bibliography motivate us to study them and their solutions using PINNs.

The main idea behind PINNs



Diffusion Equation

Solving the Diffusion equation:

$u_t - u_{xx} + e^{-t}(sin(\pi x) - \pi^2 sin(\pi x)) = 0$

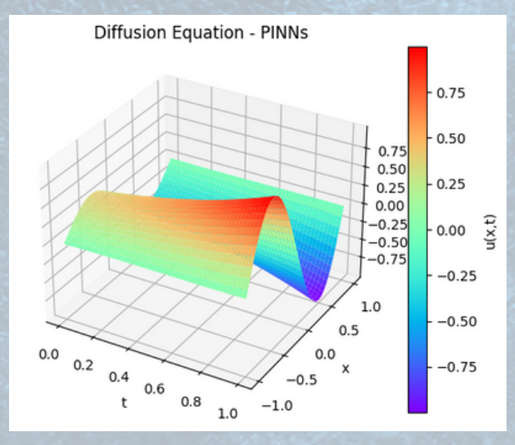
- $x \in [-1, 1]$
- $t \in [0, 1]$

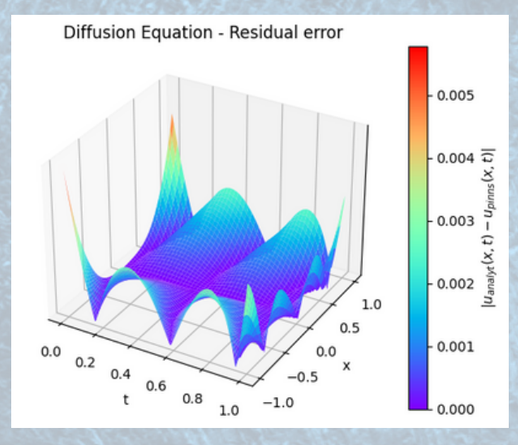
1. Initial condtion:

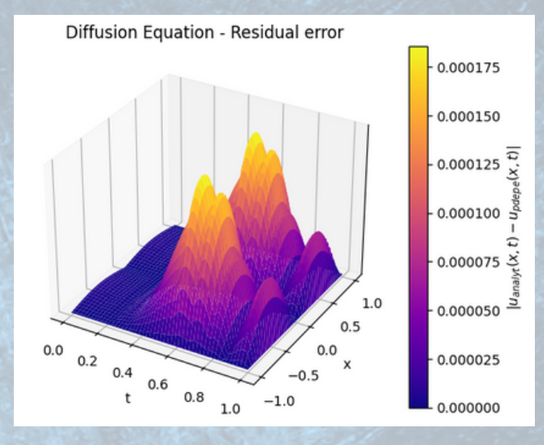
$u(x, 0) = sin(\pi x)$

2. Boundary conditions:

- $u(-1, t) = 0$
- $u(1, t) = 0$







Burgers' equation

Solving the Burger's equation:

$u_t + uu_x - \frac{0.01}{\pi} u_{xx} = 0$

- $x \in [-1, 1]$
- $t \in [0, 1]$

1. Initial condtion:

$u(x, 0) = -sin(\pi x)$

2. Boundary conditions:

- $u(-1, t) = 0$
- $u(1, t) = 0$