Classical v/s Quantum Optimization: Comparison of Runtime Scaling

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Motivation

Combinatorial Optimization

- central to logistics, network design, & theoretical computer science
- exponentially large discrete search spaces

Classical algorithms: CPLEX, Goemans-Williamson, MQLib
Quantum algorithms: LR-QAOA

- Normalized-Hamiltonian
- Extrapolation

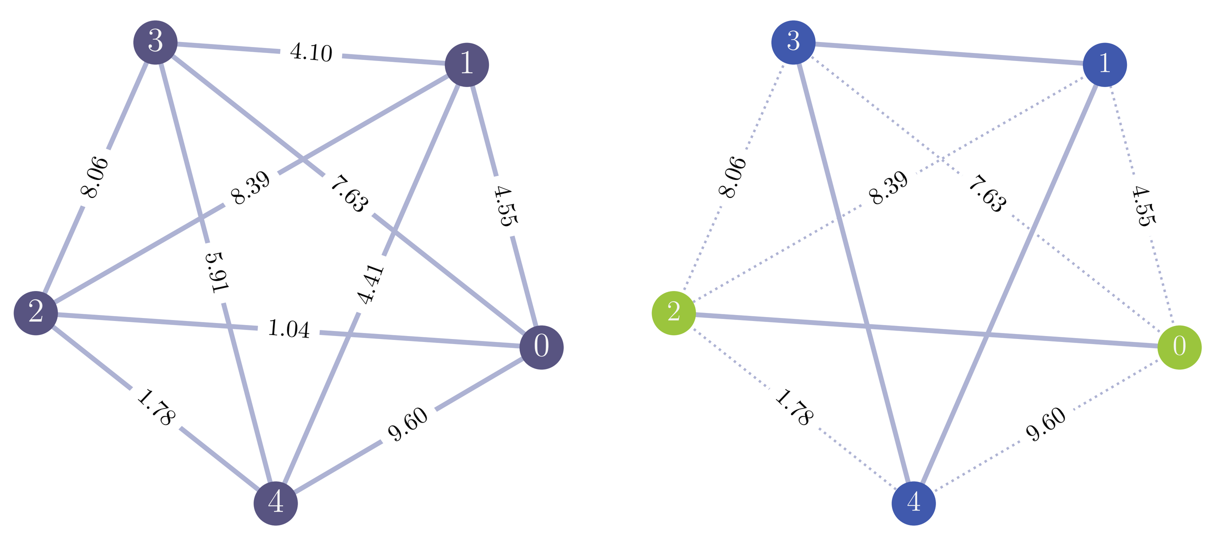
How does runtime scale with problem size?

QUBO Problems

Quadratic Unconstrained Binary Optimization

$$F(\mathbf{x}) = \mathbf{x}^T Q \mathbf{x}, \quad \mathbf{x} = \{x_1, x_2, \ldots, x_N\} \quad x_i \in \{0,1\}$$

P1. The Maximum Cut Problem (MaxCut)

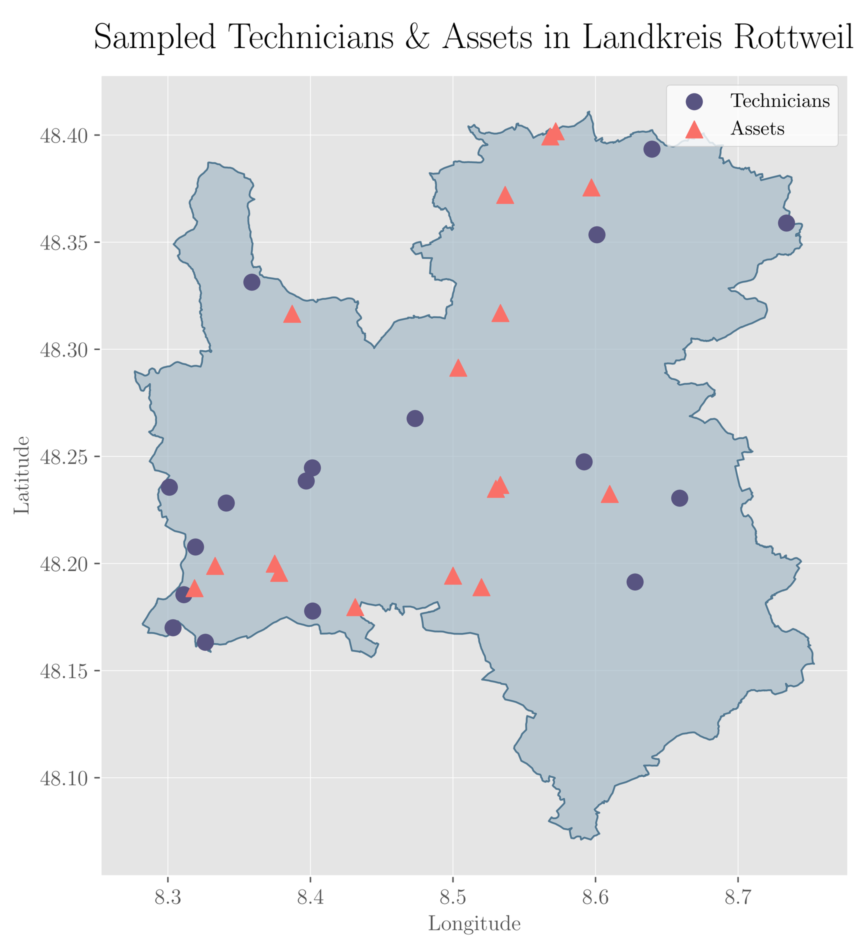


$$\text{maximize} \quad F(\mathbf{x}) = \sum_{(i,j)\in E} w_{ij}(x_i + x_j - 2x_i x_j) \qquad \text{(P1)}$$
$$x_i \in \{0,1\},\ i \in \{1,\ldots,V\}$$

P2. The Technician-Asset Allocation Problem (TA)

$T = \{t_1, t_2, \ldots, t_T\}$

$A = \{a_1, a_2, \ldots, a_A\}$



$$\tilde{F}(\mathbf{x}) = \sum_{i,j} W_{ij}^{(0)} (1 - x_{t,i})(1 - x_{a,j}) + \sum_{i,j} W_{ij}^{(1)} x_{t,i} x_{a,j}$$

$$\mathbf{x} = \{x_1, \ldots, x_N\} = \{x_{t_1}, \ldots, x_{t_T}, x_{a_1}, \ldots, x_{a_A}\}$$

$$W_{ij}^{(0)} = \begin{cases} 2D_{\text{lim}} & \text{if } D_{ij} > D_{\text{lim}} \\ \frac{D_{ij}}{(A-A_1)(T-T_1)} & \text{if } D_{ij} \le D_{\text{lim}} \end{cases}, \quad W_{ij}^{(1)} = \begin{cases} 2D_{\text{lim}} & \text{if } D_{ij} > D_{\text{lim}} \\ \frac{D_{ij}}{A_1 T_1} & \text{if } D_{ij} \le D_{\text{lim}} \end{cases}$$

$$P(\mathbf{x}) = \underbrace{2D_{\text{lim}} \left(T_1 - \sum_i x_{t,i} \right)^2}_{\text{penalty for } T} + \underbrace{2D_{\text{lim}} \left(A_1 - \sum_j x_{a,j} \right)^2}_{\text{penalty for } A}$$

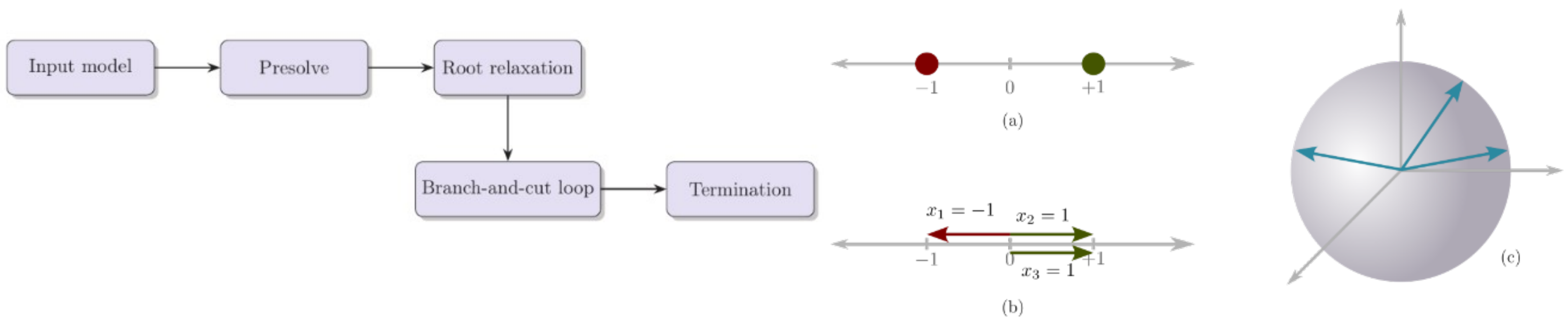
$$\text{minimize} \quad F(\mathbf{x}) = \tilde{F}(\mathbf{x}) + P(\mathbf{x}) \qquad \text{(P2)}$$

$$F \le 2D_{\text{lim}}$$

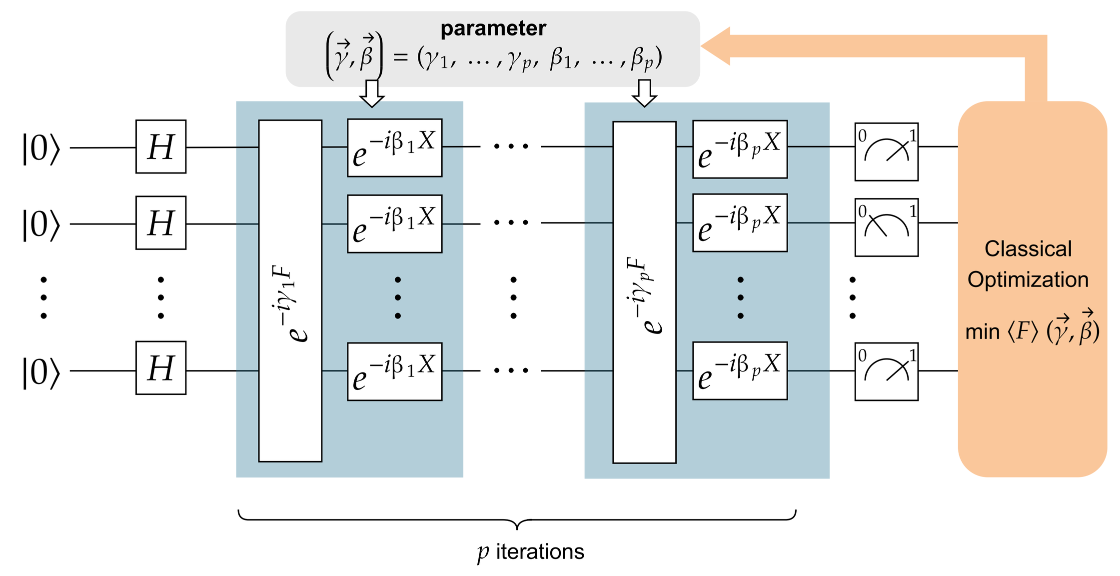
Feasibility criterion

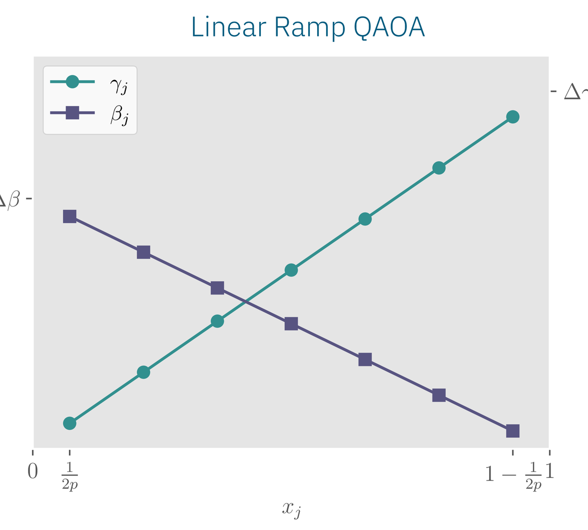
Classical Algorithms

1. Branch-and-cut-based optimization: CPLEX
2. Semi-definite relaxations: Goemans-Williamson
3. Heuristic: MQLib



Quantum Algorithms





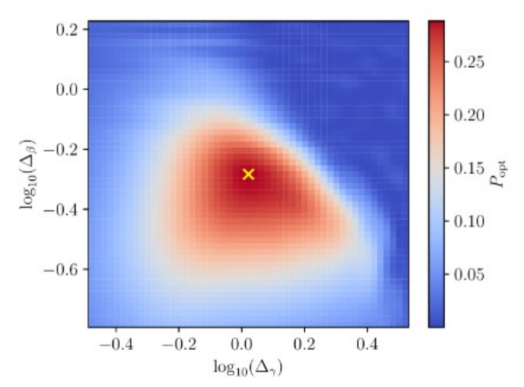
$$\gamma_j = \Delta_\gamma x_j$$
$$\beta_j = \Delta_\beta (1 - x_j)$$
$$x_j = \frac{j - \frac{1}{2}}{p}, \quad j = 1, 2, \ldots, p$$

1. Normalized Hamiltonian LR-QAOA

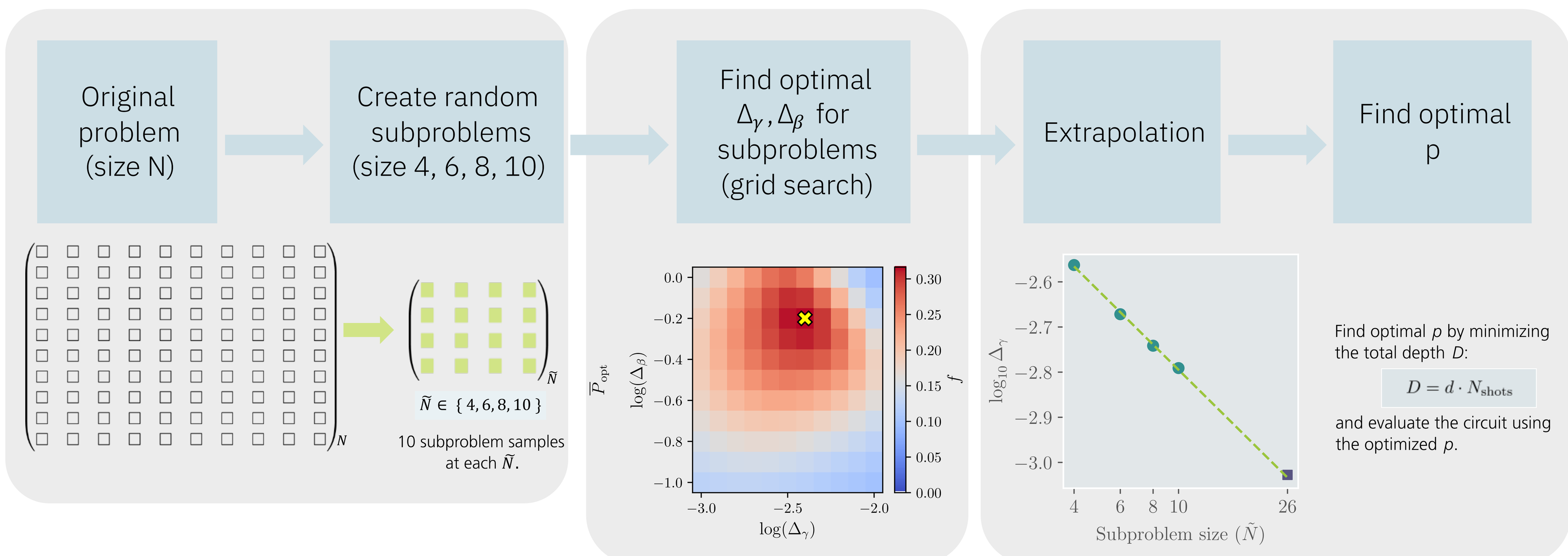
Goal: Use instance-agnostic $\Delta_\gamma, \Delta_\beta$ by identifying optimum for a small instance

Normalize the cost Hamiltonian of all problem QUBOs with the largest off-diagonal term

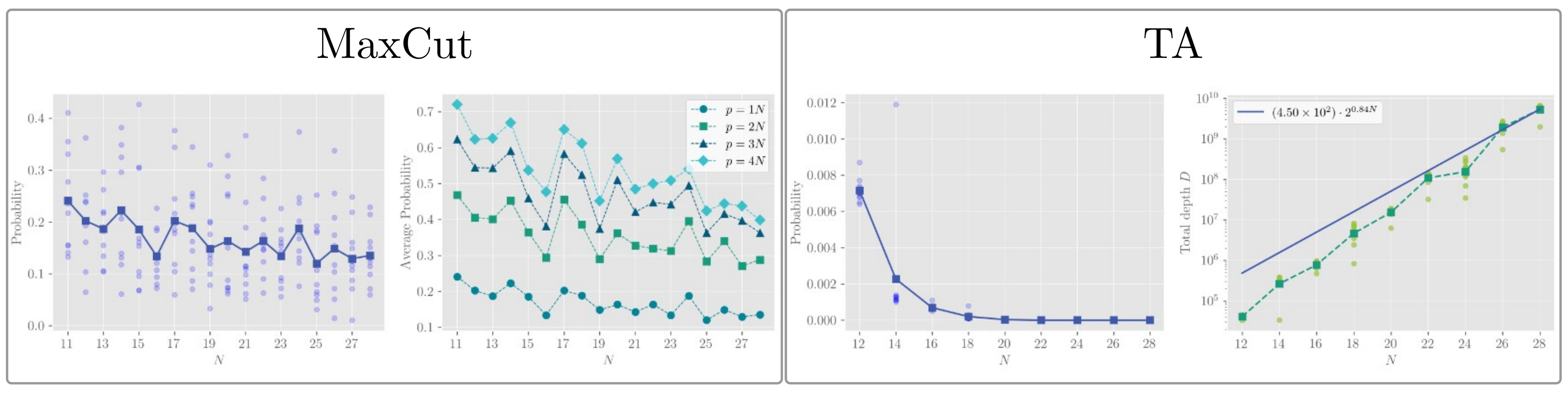
$$U_F(\gamma) = e^{-i\gamma \frac{H_F}{\lambda}} = e^{-i\frac{\gamma}{\lambda} H_F}, \quad \lambda = \max_{i<j} |H_{ij}|$$



2. Extrapolation LR-QAOA

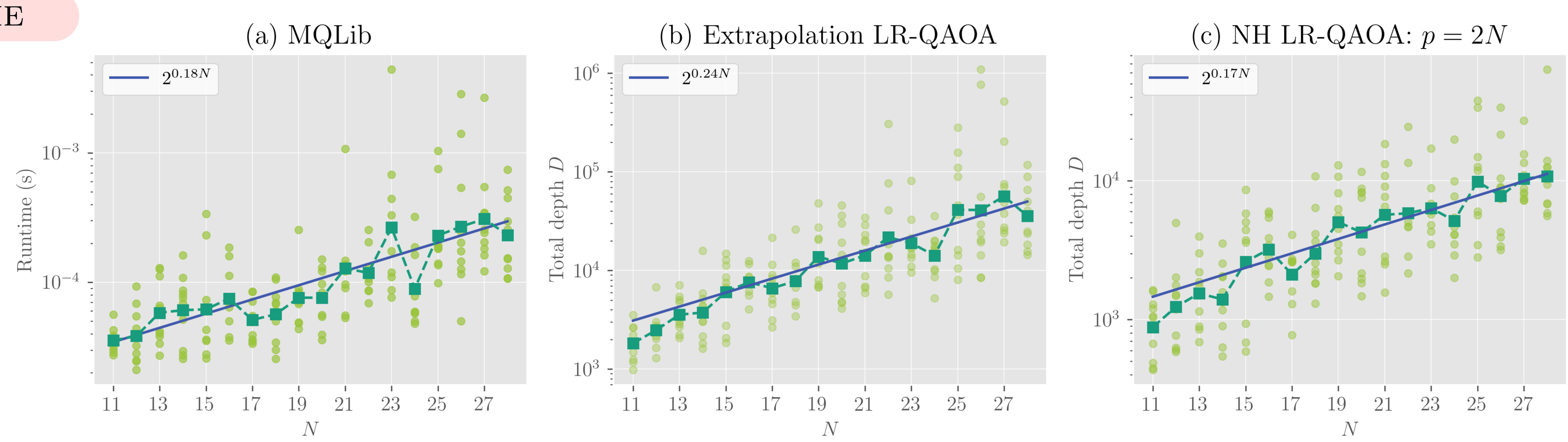


Results: Optimal Probabilities



Results: Scaling of Runtime

	Exponents	
	Weighted Complete	Weighted R3R
MQLib	0.182	**0.111**
Extrapolation LR-QAOA	0.236	
NH LR-QAOA		
$p = N$	0.176	0.211
$p = 2N$	**0.173**	0.181



I am seeking opportunities to further pursue this line of research. I would be happy to connect at qc@karthikjayd.com

Baden-Württemberg Ministerium für Wirtschaft, Arbeit und Tourismus