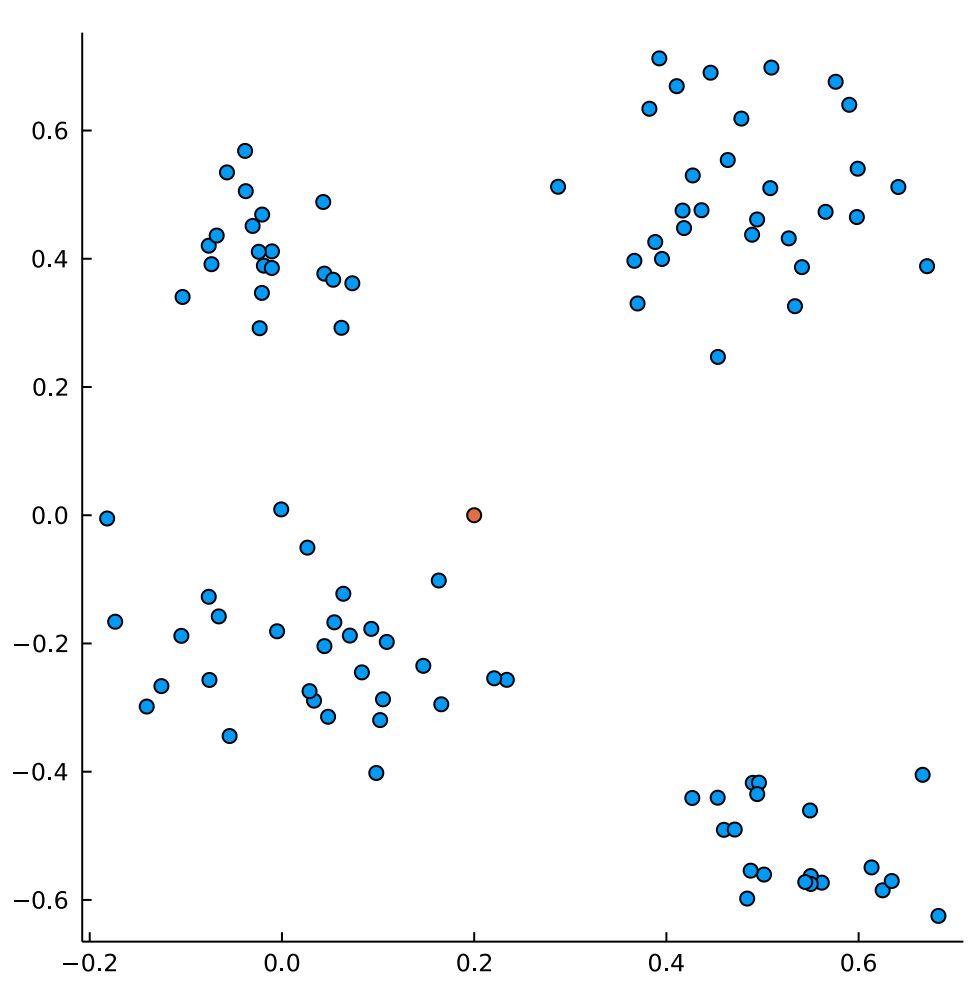
In [47]:
```julia
# 1

using LinearAlgebra
using Statistics
import Random
using Plots
using VMLS

Random.seed!(8)
X = vcat( [ [0, -0.2] + 0.1*randn(2) for i = 1:30 ],
 [ [0.5, 0.5] + 0.1*randn(2) for i = 1:30 ],
 [ [0.5, -0.5] + 0.1*randn(2) for i = 1:20 ],
 [ [0, 0.4] + 0.06* randn(2) for i = 1:20])

Che_res = [0.2, 0]


scatter([x[1] for x in X], [x[2] for x in X])
scatter!([Che_res[1]], [Che_res[2]])
plot!(legend = false, grid = false, size = (500,500))
```

Out[47]:



In [48]:
```julia
# 2 a,b

distances_1 = zeros(length(X))
for i in 1:length(X)
    distances_1[i] = norm(X[i] - Che_res)
end
println("These are all distances (in terms of 2-norm) between Chelsea's restaurant and all her customers:\n\n", dist
ances_1)

println("\nCustomer ", findmin(distances_1)[2], " is the closest customer to Chelsea's restaurant in terms of 2-nor
m.")
```

```
These are all distances (in terms of 2-norm) between Chelsea's restaurant and all her customers:

[0.2009677473503477, 0.25896260382301883, 0.42061324214195295, 0.25681145113350495, 0.18311296675364602, 0.3337325170
375205, 0.30235710582270836, 0.34911442689820105, 0.42816530554960197, 0.22796247114875975, 0.2735099500161391, 0.408
7802805445024, 0.3040375580970352, 0.33414103904197495, 0.18072514224273636, 0.25515907137194827, 0.2968302097625538,
0.45286928460732195, 0.3234524421184, 0.38190313311155316, 0.35804600583071267, 0.2214777808533561, 0.271509228858504
15, 0.37665689382076706, 0.41468352729322994, 0.24075740733699408, 0.30905153011315495, 0.21752699574550757, 0.207047
14569444144, 0.10825853896279175, 0.444915768740275, 0.6106488410454415, 0.5312820897959437, 0.6122261689301772, 0.59
79904266677406, 0.5766584137636611, 0.35393194713041715, 0.6718186266177286, 0.46656702131250816, 0.5196961812413883,
0.6136070015320558, 0.6759016621073926, 0.49828951069410593, 0.7497210949018761, 0.659654323821737, 0.465923012053291
8, 0.5244285277873351, 0.6781799833146084, 0.3714505462254292, 0.7015698508679072, 0.43039281510751, 0.73811138454117
1, 0.7736371010035272, 0.5157608183668018, 0.7636073781063303, 0.7327114255881644, 0.5223263796086925, 0.595970772746
6451, 0.5472016474256196, 0.5418461028791919, 0.7230630997432982, 0.6776574061779677, 0.5082080205663847, 0.636406147
0629753, 0.7898447526683505, 0.5081561804454912, 0.6627867064635058, 0.5780428715838136, 0.5116261644644154, 0.661919
2142769535, 0.7170718126318616, 0.525251441579659, 0.6733385934672023, 0.5552178155141244, 0.6177618325779338, 0.6874
809142523091, 0.6677648280019626, 0.5602004096553074, 0.4959624115790501, 0.6245714919588802, 0.3232963688534198, 0.4
465476652216085, 0.45597130863713103, 0.5182181548606503, 0.3833352254578807, 0.5066593191106892, 0.3672579758821926
7, 0.5585006119512392, 0.40777565446743036, 0.6161698869830765, 0.5131595037969192, 0.4111397415266218, 0.46217438505
98974, 0.39548945948269726, 0.4774786944055352, 0.5029716239002253, 0.5933484517569716, 0.4392977378986075, 0.4680315
189077585, 0.5119191981469117]

Customer 30 is the closest customer to Chelsea's restaurant in terms of 2-norm.
```

In [49]:
```julia
# 2 c

distances_2 = zeros(length(X))
for i in 1:length(X)
    distances_2[i] = abs(X[i][1] - Che_res[1]) + abs(X[i][2] - Che_res[2])
end
# println(distances_2)
println("Customer ", findmax(distances_2)[2], " is the farthest customer to Chelsea's restaurant in terms of 1-nor
m.")
```

```
Customer 65 is the farthest customer to Chelsea's restaurant in terms of 1-norm.
```

In [50]:
```julia
# 2 d 2-norm

distances_1 = zeros(length(X))
for i in 1:length(X)
    distances_1[i] = norm(X[i] - Che_res)
end

copy_distances_1 = copy(distances_1)

small_index_1 = []

for i in 1:5
    fake_index = findmin(copy_distances_1)[2]
    real_index = findall(x -> x==copy_distances_1[fake_index], distances_1)
    append!(small_index_1, real_index)
    deleteat!(copy_distances_1, findmin(copy_distances_1)[2])
end

println(small_index_1)
```

```
Any[30, 15, 5, 1, 29]
```

In [51]:
```julia
# 2 d 1-norm

distances_2 = zeros(length(X))
for i in 1:length(X)
    distances_2[i] = abs(X[i][1] - Che_res[1]) + abs(X[i][2] - Che_res[2])
end

copy_distances_2 = copy(distances_2)

small_index_2 = []

for i in 1:5
    fake_index = findmin(copy_distances_2)[2]
    real_index = findall(x -> x==copy_distances_2[fake_index], distances_2)
    append!(small_index_2, real_index)
    deleteat!(copy_distances_2, findmin(copy_distances_2)[2])
end

println(small_index_2)
```

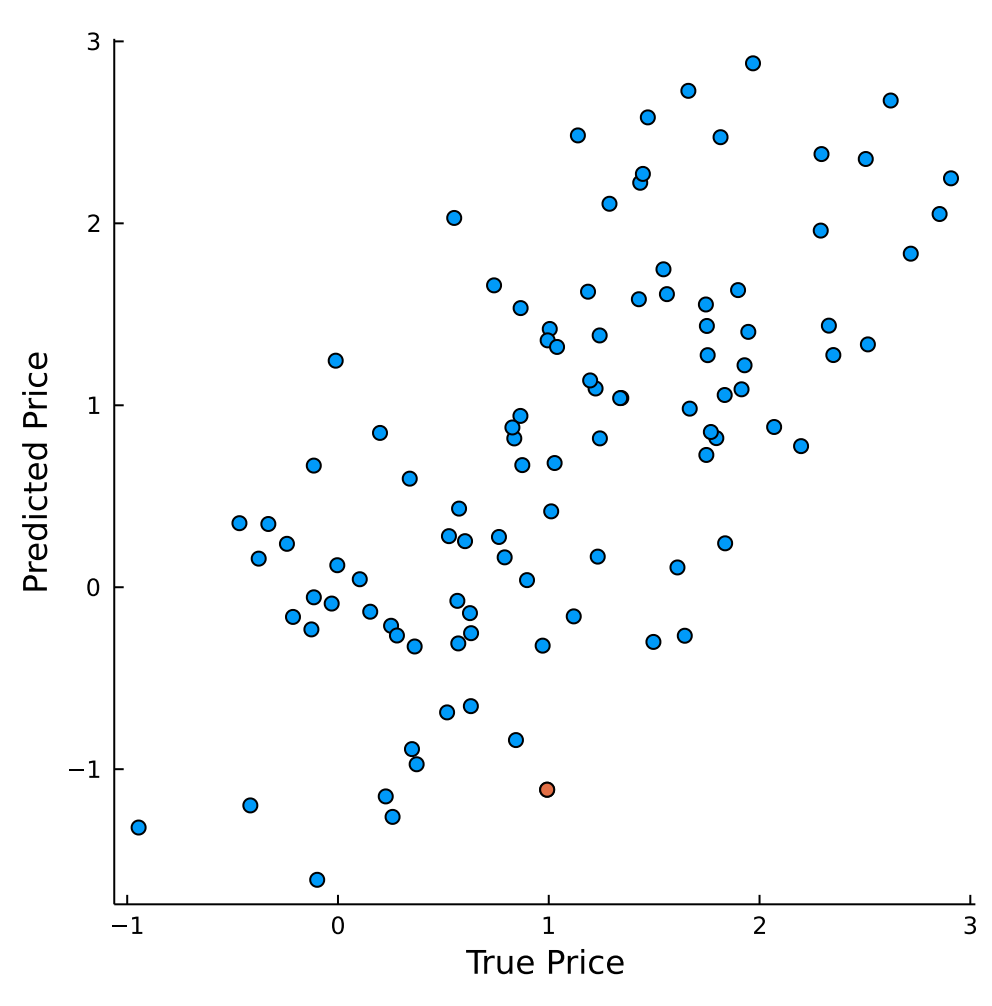
```
Any[30, 1, 15, 5, 16]
```

Answer 2d. : The 5 closest customers to Chelsea's restaurant calculated using 1-norm and 2-norm are "not" the same.

In [52]:
```julia
# 3

Random.seed!(8)
y = randn(100)
Random.seed!(8)
quality = y * 0.5 + rand(100) * 0.1
Random.seed!(10)
nutrition = y * 0.1 + rand(100) * 0.2
price = quality * 1 + nutrition * 0.1 + rand(100) * 2

predicted_price = 0.5*quality + 7.1*nutrition .+ 0.1

println("Data point ", findmax([norm(price[i] - predicted_price[i]) for i = 1:length(price)])[2], " is the farthest
 (in terms of 2-norm) to its prediction.")

scatter(price, predicted_price)
scatter!([price[97]], [predicted_price[97]])
plot!(legend = false, grid = false, size = (500,500), xaxis = "True Price", yaxis = "Predicted Price")
```

```
Data point 97 is the farthest (in terms of 2-norm) to its prediction.
```

Out[52]:



In [53]:
```julia
# 4 k-means algorithm
function kmeansfunction(x, k; maxiters = 100, tol = 1e-5)
N = length(x)     # number of points to be cluster
n = length(x[1])  # the dimension
distances = zeros(N)  # used to store the distance of each point to the nearest representative.
reps = [zeros(n) for i=1:k]  # used to store representatives.

# 'assignment' is an array of N integers between 1 and k.
# The initial assignment is chosen randomly.
assignment = [rand(1:k) for i=1:N]
Jprevious = Inf  # used in stopping condition

for iter = 1:maxiters
    # Cluster j representative is average of points in cluster j.
    for j = 1:k
        group = [i for i=1:N if assignment[i]==j]  # get which point index belongs to the group j
        reps[j] = sum(x[group]) / length(group);     # reps is the clustering center
    end;
    # For each x[i], find distance to the nearest representative
    # and its group index.
    for i = 1:N
        (distances[i], assignment[i]) = findmin([norm(x[i] - reps[j]) for j=1:k])
    end;

    # Compute clustering objective.
    J = norm(distances)^2 / N  # distance to the clustering center

    # Show progress and terminate if J stopped decreasing.
    println("Iteration ", iter, ": Jclust = ", J, ".")
    if (iter > 1) && (abs(J - Jprevious) < tol*J)
         return assignment, reps
    end
 Jprevious = J
 end

 end
```

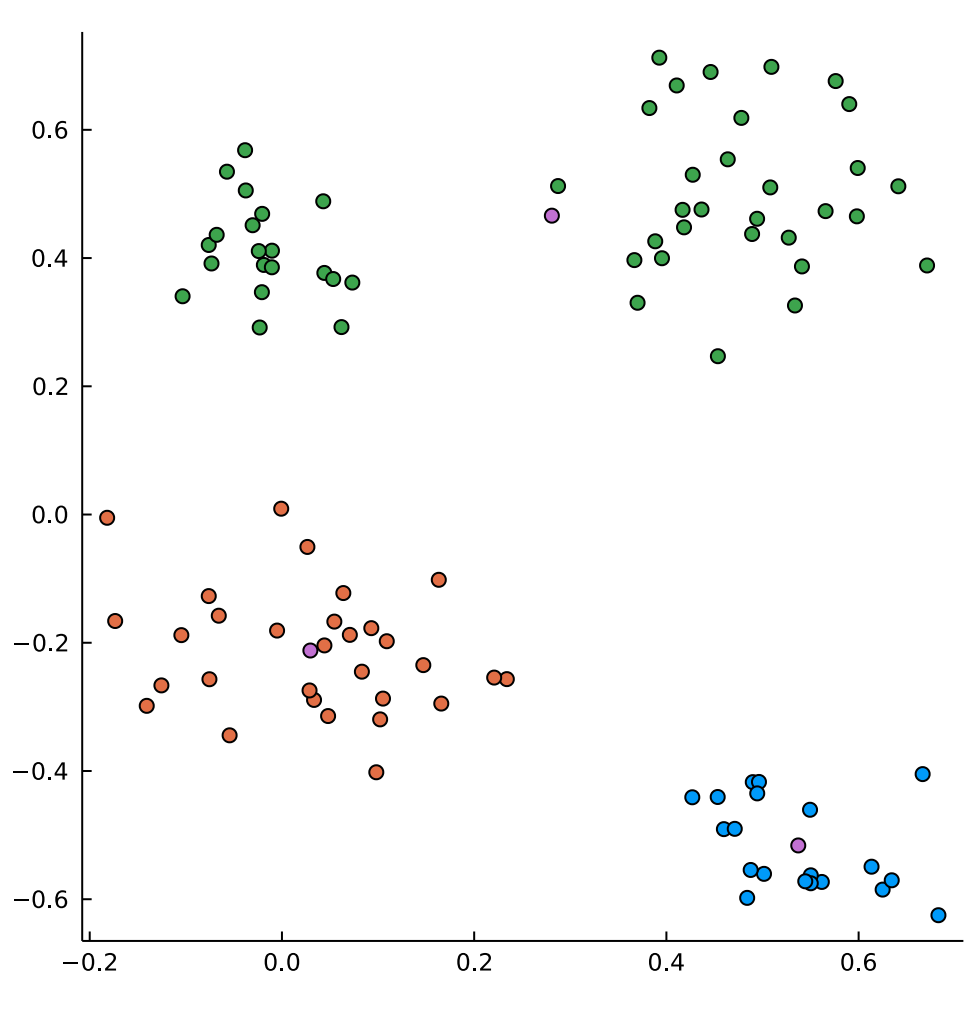
Out[53]: kmeansfunction (generic function with 1 method)

In [54]:
```julia
# 4 a,b
assignment, reps = kmeansfunction(X, 3)
println("\n\n", reps, " are the best locations to open these three restaurants for Chelsea's franchisee.")
```

```
Iteration 1: Jclust = 0.21805080743372227.
Iteration 2: Jclust = 0.054753907704250544.
Iteration 3: Jclust = 0.04740440133730133.
Iteration 4: Jclust = 0.04707074206229065.
Iteration 5: Jclust = 0.04707074206229065.


[[0.5371909960116741, -0.5161171797997566], [0.02958765596728597, -0.21210401348889638], [0.2808290177102955, 0.46611
675910208605]] are the best locations to open these three restaurants for Chelsea's franchisee.
```

In [55]:
```julia
k = 3   # Number of groups
N = length(X)  # Total number of points to be clustered
grps  = [[X[i] for i=1:N if assignment[i] == j] for j=1:k]
scatter([c[1] for c in grps[1]], [c[2] for c in grps[1]])
scatter!([c[1] for c in grps[2]], [c[2] for c in grps[2]])
scatter!([c[1] for c in grps[3]], [c[2] for c in grps[3]])
scatter!([c[1] for c in reps], [c[2] for c in reps])
plot!(legend = false, grid = false, size = (500,500))
```

Out[55]:



In [56]:
```julia
# 4 c
# 4 k-means algorithm with 1-norm
function kmeansfunction_2(x, k; maxiters = 100, tol = 1e-5)
N = length(x)     # number of points to be cluster
n = length(x[1])  # the dimension
distances = zeros(N)  # used to store the distance of each point to the nearest representative.
reps = [zeros(n) for i=1:k]  # used to store representatives.

# 'assignment' is an array of N integers between 1 and k.
# The initial assignment is chosen randomly.
assignment = [rand(1:k) for i=1:N]
Jprevious = Inf  # used in stopping condition

for iter = 1:maxiters
    # Cluster j representative is average of points in cluster j.
    for j = 1:k
        group = [i for i=1:N if assignment[i]==j]  # get which point index belongs to the group j
        reps[j] = sum(x[group]) / length(group);     # reps is the clustering center
    end;
    # For each x[i], find distance to the nearest representative
    # and its group index.
    for i = 1:N
        (distances[i], assignment[i]) = findmin([(abs(x[i][1] - reps[j][1]) + abs(x[i][2] - reps[j][2])) for j=1:k])
    end;

    # Compute clustering objective.
    J = norm(distances)^2 / N  # distance to the clustering center

    # Show progress and terminate if J stopped decreasing.
    println("Iteration ", iter, ": Jclust = ", J, ".")
    if (iter > 1) && (abs(J - Jprevious) < tol*J)
         return assignment, reps
    end
 Jprevious = J
 end

 end
```

Out[56]: kmeansfunction_2 (generic function with 1 method)

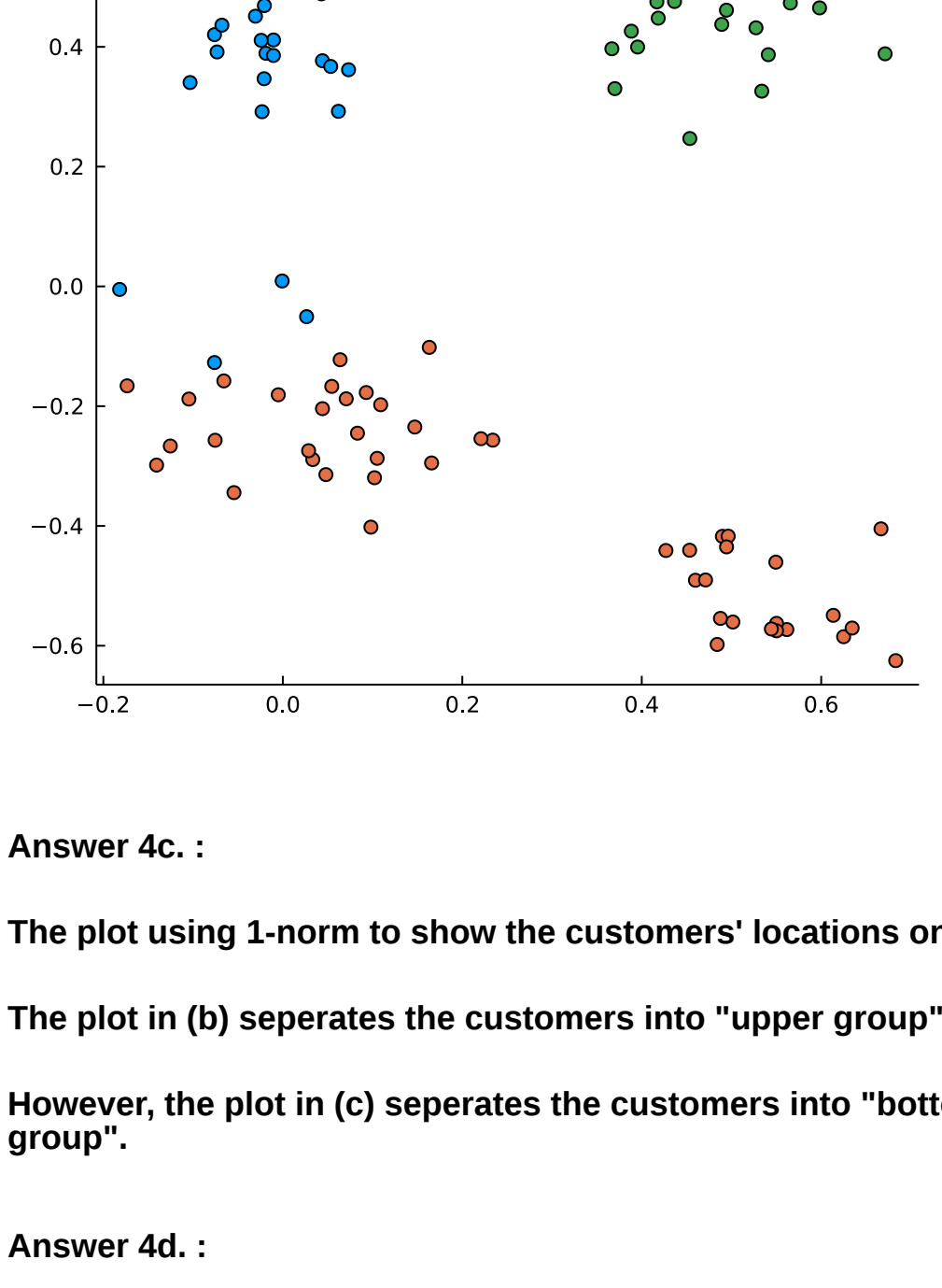
In [57]:
```julia
assignment, reps = kmeansfunction_2(X, 3)
```

```
Iteration 1: Jclust = 0.3403702703823343.
Iteration 2: Jclust = 0.09979403232802793.
Iteration 3: Jclust = 0.09823290083113136.
Iteration 4: Jclust = 0.09788499721599553.
Iteration 5: Jclust = 0.09773526641653874.
Iteration 6: Jclust = 0.09773526641653874.
```

Out[57]:
```
([1, 2, 2, 2, 2, 2, 2, 2, 2, 2  …  1, 1, 1, 1, 1, 1, 1, 1, 1, 1], [[-0.02373170768873549, 0.33606929730232826], [0.25
79074365705538, -0.35895024685859395], [0.4792906462350338, 0.5022140724893905]])
```

In [58]:
```julia
k = 3   # Number of groups
N = length(X)  # Total number of points to be clustered
grps  = [[X[i] for i=1:N if assignment[i] == j] for j=1:k]
scatter([c[1] for c in grps[1]], [c[2] for c in grps[1]])
scatter!([c[1] for c in grps[2]], [c[2] for c in grps[2]])
scatter!([c[1] for c in grps[3]], [c[2] for c in grps[3]])
plot!(legend = false, grid = false, size = (500,500))
```

Out[58]:



Answer 4c. :

The plot using 1-norm to show the customers' locations on the map is different from (b).

The plot in (b) seperates the customers into "upper group", "lower left group", and "lower right group".

However, the plot in (c) seperates the customers into "bottom group", "upper left group", and "upper right group".

Answer 4d. :

3 is not a good clustering number because apparently there are "4" seperated groups on the map.

4 would be a good number.

In [59]:
```julia
# 5

function Gram_Schmidt(a; tol = 1e-10)
    copy_a = copy(a)
    copy_a = convert(Vector{Vector{Float64}}, copy_a)

    for i = 1:length(copy_a)

        for j = 1:i-1
            copy_a[i] -= (copy_a[j]'*copy_a[i]) * copy_a[j]
        end

        if norm(copy_a[i]) < tol
            println("Vectors are linearly dependent.")
            return copy_a
        end

        copy_a[i] = copy_a[i]/norm(copy_a[i])
    end

    return copy_a
end
```

Out[59]: Gram_Schmidt (generic function with 1 method)