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Mathematical modelling and computer simulations in theory and practice

Documentation of laboratory task no 3

Title: **Shadow of a Figure**

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Field of studies: Informatics sem.V

1 Project Objective

The goal of the project is to compute and visualize the shadow (2D projection) of a 3D polygon on a flat ground ($z = 0$) when lit by a point light source.

2 Description

The program consists of a function `shadow[vertices, lightSource]` which takes 2 inputs: *vertices* (a list of coordinates of polygon vertices) and *lightSource* (the coordinates of a point light source). Within the function, five key sections are implemented, each responsible for generating and displaying a specific part of the graphical visualization. The smallest of them being `lightPoint` which displays a light source in yellow.

```
    lightPoint =
  Graphics3D[{PointSize[0.05], Yellow, Point[lightSource]}];
```

2.1 Figure and axis graphics

The second section which is `figureGraphics` is responsible for displaying the polygon defined by the input vertices. We make sure to connect the last point to the first one so the shape is closed.

```
    figureGraphics =
  Graphics3D[{RGBColor[0, 0, 1], PointSize[0.03], Point[
      vertices],
    Line[Append[vertices, vertices[[1]]]]}];
```

Then we have the third section which displays axes as the arrows. The axes are scaled to ensure that the light source is positioned within the visible range of the axes.

```
  axisGraphics =
Graphics3D[{Arrow[{{0, 0, 0}, {lightSource[[3]] + 1, 0,
   0}}],
  Arrow[{{0, 0, 0}, {0, lightSource[[3]] + 1, 0}}],
  Arrow[{{0, 0, 0}, {0, 0, lightSource[[3]] + 1}}]}];
```

2.2 Shadow calculation

We use the *projections* variable to store the shadow points of the input vertices after being projected onto the $z = 0$ plane. These points define the shape of the shadow.

```
  projections = Table[Module[{t, parametricLine},
  parametricLine = lightSource + (vertex - lightSource) t;
  t = Solve[parametricLine[[3]] == 0, t][[1, 1, 2]];
  lightSource + t (vertex - lightSource)], {vertex,
    vertices}];
```

Using a table enables us to iterate over each vertex in vertices, performing the same calculations for all vertices of the polygon. To prevent possible conflicts with other parts of the program we will do the calculations in `Module`.
Now we define the parametric equation of the line passing through the light source and a vertex.

```
  parametricLine = lightSource + (vertex - lightSource) t;
```

t is the scalar parameter.
The equation describes all points along this line as

$$P(t) = L + t * (V - L) \tag{1}$$

where L is the light source, and V is the vertex.

Then we can solve for t.

```
  t = Solve[parametricLine[[3]] == 0, t][[1, 1, 2]];
```

At the end we compute the shadow point. The resulting shadow point is $P = (x, y, 0$.

```
  lightSource + t (vertex - lightSource)
```

After computing the shadow points we can display the shape of the shadow. We make sure to make the shape closed.

```
  shadowGraphics =
Graphics3D[{RGBColor[1, 0, 0], PointSize[0.03], Point[
   projections],
  Line[Append[projections, projections[[1]]]]}];
```

At the end of the `shadow` function we make sure to show all the graphics.

```
    Show[{axisGraphics, lightPoint, figureGraphics,
        shadowGraphics},
    Boxed -> False, Axes -> False, PlotRange -> All]];
```

2.3 Result

We will see the result on two different polygons. We will define the light source as following:

```
    lightSource = {0, 0, 6};
```

and we will use the same light source for both polygons. Firstly, we will test the `shadow` function of triangle.

```
    triangleVertices = {{1, 1, 1}, {3, 3, 4}, {1, 5, 3}};
    shadow[triangleVertices, lightSource]
```

This will be our result:

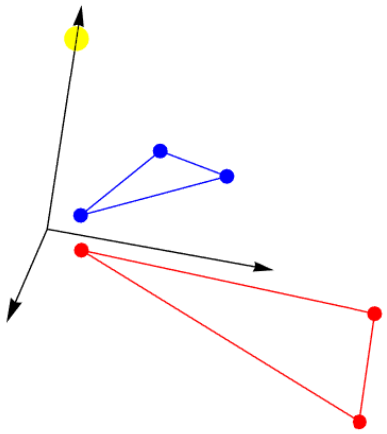

Figure 1: The shadow of the triangle

Then we will use the function to create a shadow of the rectangle.

```
rectangleVertices = {{1, 1, 1}, {4, 1, 2}, {4, 5, 3},
    {1, 5, 4}};
shadow[rectangleVertices, lightSource]
```

This will be our result:

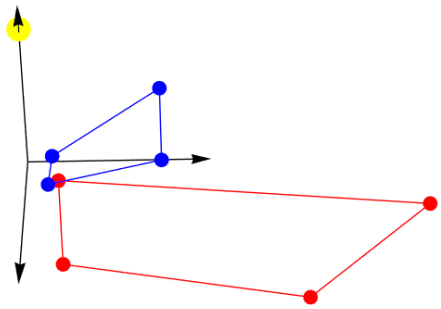

Figure 2: The shadow of the rectangle

3 Enclosures

"STACHECKA Aleksandra Project 4.nb"