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Mathematical modelling and computer simulations in theory and practice

Documentation of laboratory task no 1

Title: **Complex roots**

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Field of studies: Informatics sem.V

1 Project Objective

The project objective is to develop a program that calculates the roots of complex number z of order n and then presents the result so that the roots of order n of number z are the vertices of a regular n-sided polygon inscribed in the circle of central point (0, 0) and radius $\sqrt[n]{|z|}$.

2 Description

The program consists of a single large function `complexRoots[z_, n_Integer]` composed of two smaller parts: computational part and visualization part.

2.1 Computational part

In the computational part, we first calculate the modulus, which gives the absolute value of z, and the argument, which gives the angle (phase) of the complex number z (denoted as Φ). Then, we compute the radius of the circle on which the roots should lie:

$$\text{radius} = \text{Power}[\text{modulus}, 1/n]. \tag{1}$$

Next, we calculate the n roots, where n is the number of sides of the polygon. We store the roots in a table as follows:

$$\text{roots} = \text{Table}[\text{radius} \cdot \text{Exp}[I \cdot (\text{argument} + 2\pi k)/n], \{k, 0, n-1\}]. \tag{2}$$

After computing the roots, we proceed to the visualization part of the function.

2.2 Visualization part

To prepare for plotting, we first convert the roots to points. The visualization part is divided into three sub-parts:

1. Creating Frames The frames are created progressively, with each frame showing an additional root point and connecting line. The code is as follows:

```
frames = Table[Graphics[
    {Blue, Circle[{0, 0}, radius],(*circle representing boundary*)
     Red, PointSize[Large],
     Point[rootPoints[[;; i]]],(*showing points progressively*)
     Line[rootPoints[[;; i]]] (*connecting points progressively*)},
    Axes -> True, AxesLabel -> {"Re", "Im"},
    PlotRange -> {{-radius - 1, radius + 1}, {-radius - 1,
      radius + 1}},
    PlotLabel ->
     "Roots of Order " <> ToString[n] <> " of " <> ToString[z],
    AspectRatio -> 1, GridLines -> Automatic,(*enabling grid lines*)
    Frame -> True,(*adding frame around the plot*)
    FrameLabel -> {"Re", "Im"} (*label for frame*)],
   {i, 1, n}];
```

2. Final Frame The final frame connects the last root point to the first, completing the polygon:

```
AppendTo[frames, Graphics[
   {Blue, Circle[{0, 0}, radius],
    Red, PointSize[Large], Point[rootPoints],(*showing all points*)
    Line[
     Join[rootPoints, {rootPoints[[1]]}]] (*connecting last to first*)},
   Axes -> True, AxesLabel -> {"Re", "Im"},
   PlotRange -> {{-radius - 1, radius + 1}, {-radius - 1,
     radius + 1}},
   PlotLabel ->
    "Roots of Order " <> ToString[n] <> " of " <> ToString[z],
   AspectRatio -> 1, GridLines -> Automatic,(*enabling grid lines*)
   Frame -> True,(*adding frame around the plot*)
   FrameLabel -> {"Re", "Im"} (*label for frame*)]];
```

3. Animating the Frames Finally, we animate the frames to visualize the construction of the polygon from the roots.

2.3 Result

To use this function, you can first declare the complex number z and the number of sides of the polygon n. Then, call the function as follows:

$$\text{complexRoots}[z, n] \tag{3}$$

Or you can call the function by directly substituting specific values for z and n. The result will be displayed as an animated plot showing the roots of z in the complex plane.
For example for $z = 23 + I$ and $n = 6$, the last frame would appear like shown below.

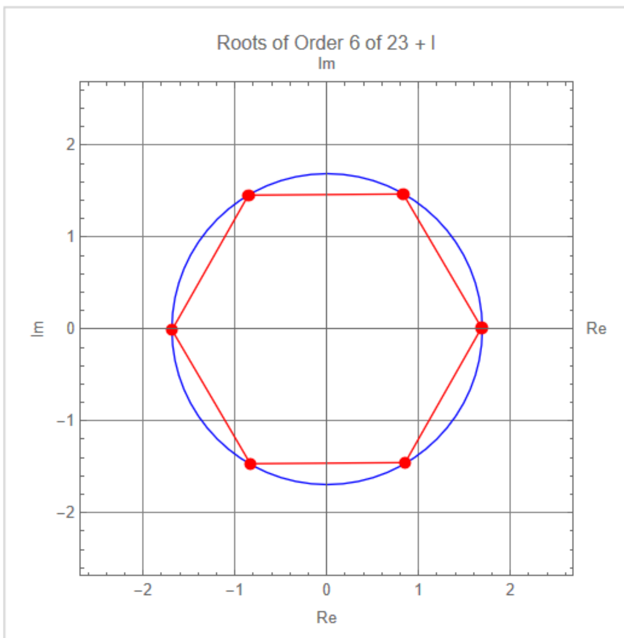

Figure 1: Roots of order 6 of 23 + I, last frame

Another example, for $z = -19 + 16I$ and $n = 10$, the sixth frame would appear like shown below.

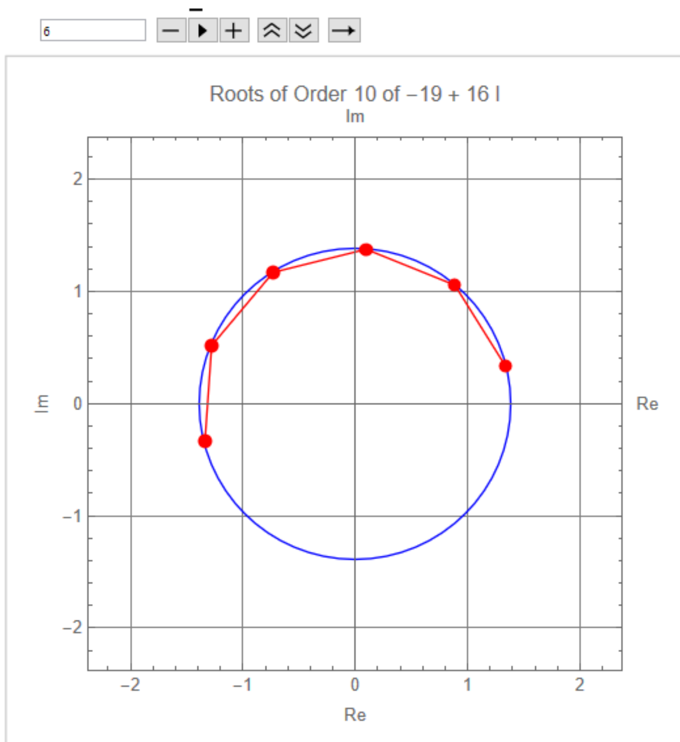

Figure 2: Roots of order 10 of -19 + 16 I, sixth frame

3 Enclosures

"STACHECKA Aleksandra Project 1.nb"