Gliwice, 28.10.2024

Mathematical modelling and computer simulations in theory and practice

Documentation of laboratory task no 2

Title: **Falling body**

Author: Aleksandra Stachecka
Field of studies: Informatics sem.V

1 Project Objective

The project objective is to develop a program that illustrates the process of falling of the body, in dependence on: initial height, initial speed, mass of the body and air resistance coefficient. Two cases should be taken into consideration: simple case with ignoring the mass of the body and the air resistance and second case including the mass of the body and the air resistance.

2 Description

The program is divided into two main parts, with each part adressing a specific case of the problem. At the beginning we define the following parameters:

- $g = 9.81$ - gravitational acceleration [m/s^2]
- $h = 100$ - initial height [m]
- $v0 = 0$ - inital speed [m/s]
- $k = 0.9$ - air resistance coefficient [kg/s]
- $m = 1$ - mass of the body [kg]

These parameters provides the basis for the calculations in both parts of the program and can be adjusted to explore different scenarios.
As `DSolve` and `NDSolve` are not functioning as expected in this Mathematica version, all the calculations will be performed manually, step by step.

2.1 First case

In the first case - the simpler one, we will be ignoring the mass of the body and the air resistance. The free fall of the body can be described as:

$$y(0) = 0, \quad y'(0) = v_0, \quad y''(t) = g \tag{1}$$

In our calculations we will begin by integrating the equation $y''(t) = g$ to find the velocity $y'(t)$.

```
yPrime[t_] = Integrate[g, t] + v0;
```

After that, we will integrate $y'(t)$ to get the positon $y(t)$, which will be the result.

```
y[t_] = Integrate[yPrime[t], t];
```

Now we can solve the equation

$$h = y(t) \tag{2}$$

for the given height h to find the time T after which the body reaches the ground. We will get two solutions (one negative and one positive), so in Mathematica we will extract the positive one and assign it to variable T.

```
timeToGroundNoRes = Solve[y[t] == h, t, Reals]
T = timeToGroundNoRes[[2, 1, 2]];
```

The final step is to plot the relation

$$z(t) = h - y(t) \tag{3}$$

which represents distance to the ground over time. For the given parameters the resulting plot will look as following:

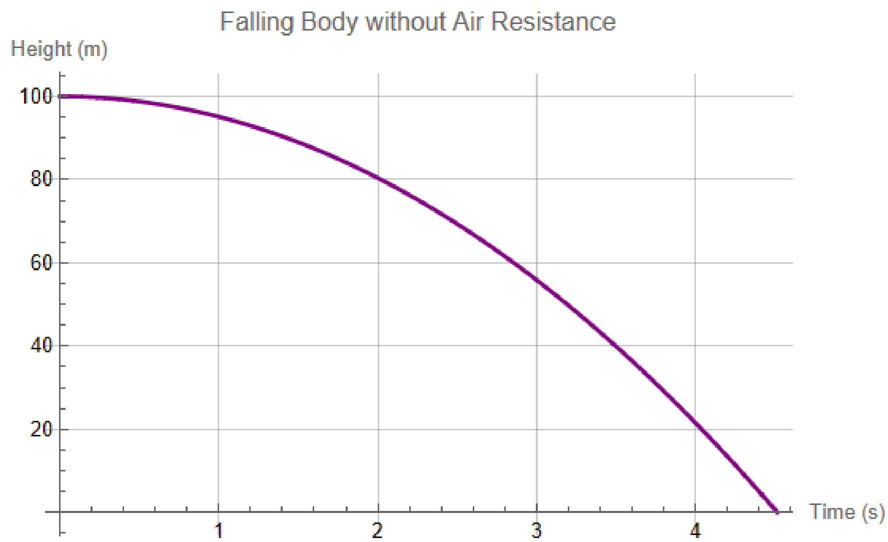

Figure 1: Falling Body without Air Resistance

2.2 Second case

In the second case we will take into consideration the mass of the falling body and the force of air resistance. The fall of the body can be described as:

$$y(0) = 0, \quad y'(0) = v0 \quad y''(t) + \frac{k}{m}y'(t) = g, \quad F = -kv(t) \tag{4}$$

where F is the force of air resistance and k is the air resistance coefficient. Second-order differential equation $y''(t) + \frac{k}{m}y'(t) = g$ can be transformed into a first-order differential equation for velocity by defining $v(t) = y'(t)$:

$$v(t) = \left(v0 - \frac{gm}{k}\right)\exp\left(-\frac{k}{m}t\right) + \frac{gm}{k} \tag{5}$$

which we will integrate to get the position function $y(t)$.

```
v[t_] = (v0 - (g*m)/k)*Exp[-(k/m)*t] + (g*m)/k;
y[t_] = Integrate[v[t], t] + c1;
```

We will solve the result of the integration for constant of integration $c1$ to find its value, we will use the initial condition $y(0) = 0$. After determining the value of $c1$ we will substitute it back into the equation for $y(t)$.

```
c1 = Solve[f[0] == 0, c1][[1, 1, 2]];
y[t_] = y[t] /. c1 -> c1;
```

Now we can solve the equation

$$h = y(t) \tag{6}$$

to find the time T after which the body reaches the ground. We will extract the positive solution.

```
T = Quiet[Solve[y[t] == h, t]][[2, 1, 2]]
```

The final step is to plot the relation

$$z(t) = h - y(t) \tag{7}$$

which represents distance to the ground over time. For the given parameters the resulting plot will look as following:

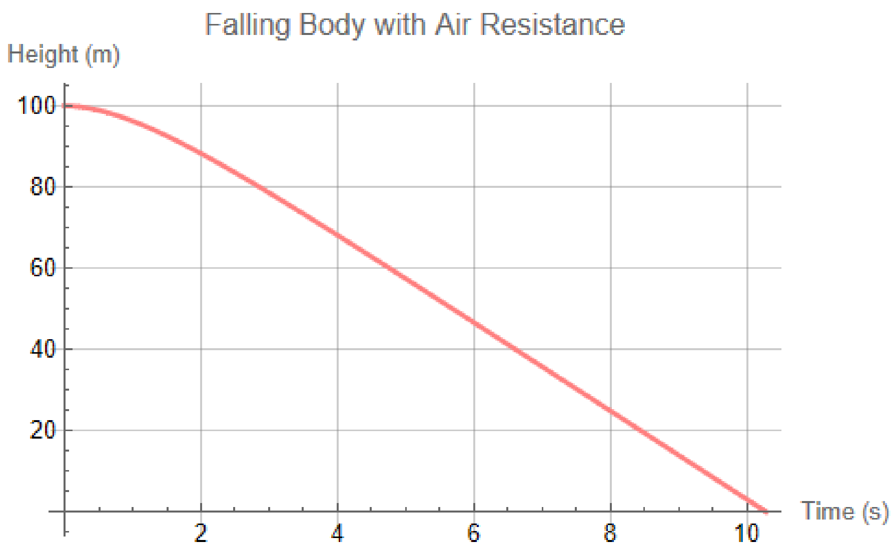

Figure 2: Falling Body with Air Resistance

3 Enclosures

"STACHECKA Aleksandra Project 2.nb"