CS 2112 | Assignment 4 Written Problem

Parsing and Fault Injection

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Proving Asymptotic Complexity

For each of the following functions, show by giving a witness that it is $O(n^2)$, or else show that it isn't $O(n^2)$ by arguing that no such witness can exist.

1. $3n^2 + 10n + 1$

   We choose a witness $k = 5$ and $n_0 = 9$. $g(n) = n^2$.
   For all $n \geq n_0$, $f(n) \leq k * g(n)$:
   $f(n) = 3n^2 + 10n + 1 \leq 5 * n^2$ for all $n \geq 9$.

2. 2^n

   2^n is NOT $O(n)$ because the function 2^n is raised to an exponent n that increases linearly, whereas function n^2 is raised to a constant exponent 2. No matter what k and n_0 we choose, 2^n will always approach ∞ faster than n^2.

   By contradiction: Say that there is a k and n_0 such that $n \geq n_0$, $f(n) \leq k * g(n)$. Then:
   $2^n \leq k * n^2$
   $n \leq \log_2(k * n^2)$
   k varies according to a non-constant, so k can never be constant.

3. $n \lg n$

   We choose a witness $k = 1$ and $n_0 = 0$. $g(n) = n^2$.
   For all $n \geq n_0$, $f(n) \leq k * g(n)$:
   $f(n) = n \lg n \leq 1 * n^2$ for all $n \geq 0$.

4. $n^3/\lg n$

   $n^3/\lg\ n$ is NOT $O(n)$ because lg n will always be less than n, so $n^3/\lg\ n$ will always be greater than n^3/n, which is equal to n^2.

   By contradiction: Say that there is a k and n_0 such that $n \geq n_0$, $f(n) \leq k * g(n)$.
   Then:
   $n^3/\lg\ n \leq k * n^2$
   $n/\lg\ n \leq k$
   The left hand side of the equation will ultimately approach ∞ as n increases, so k is forced to vary according to this non-constant function, and k can therefore not be a constant.

5. $f(n) + h(n)$, where each of $f(n$ and $h(n)$ are $O(n^2)$.

   By definition of $O(n^2)$, we know that there is a k_1 such that:
   $f(n) \leq k_1 * g(n)$
   and a k_2 such that:
   $h(n) \leq k_2 * g(n)$

   Adding these two equations will produce:
   $f(n) + h(n) \leq k_1 * g(n) + k_2 * g(n)$
   $f(n) + h(n) \leq (k_1 + k_2)g(n)$

   By the definition of $O(n^2)$, this means that $f(n) + h(n)$ must also be $O(n^2)$.