DePaul University

Final Report

Win or Lose? The model to predict S&P 500

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CSC425: Time Series Analysis and Forecasting

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Non-Technical Report
(An executive summary)

Over the last 12 years, stock market has fluctuated from a market bubble in 2006, to a financial crisis in 2008, and finally to a period of stabilization and recovery in recent years. Stock market is influenced by various factors such as politics, monetary and fiscal policy, industry condition, and world economy. Technological advancements have changed business and opened the doors to an entirely different world of high frequency trading, making stock markets extremely volatile. Compared to the bond market, the stock market exhibits a higher level of uncertainty as the future payoffs are not guaranteed. However, the higher risks are associated with the possibility of higher returns. Thus, understanding risk-return tradeoff and how prices are determined is essential for investors in making investment decisions and managing their portfolios. Through the framework of a statistical modeling approach known as a technical analysis, this is a study of daily price movements of S&P 500 from January 2nd, 2013 to December 28th, 2017.

The S&P 500 dataset was obtained from Yahoo! Finance and used in this study because S&P 500 is one of the best representations of stock market in the world. It contains equity securities of 500 publicly traded companies, covering approximately 80% of total market capitalization in the United States. The findings of this study are useful for investors in making decisions on their investments and determining how risk and return of their portfolios compare to those of overall market.

The process was initiated by analyzing S&P 500 daily adjusted prices to examine the data distribution and trend. The analysis indicates that the S&P 500 prices have increased over the past five year except in 2016. There were major declines in S&P 500 prices because of big price drops in Chinese Stock Market and an unexpected result of Brexit. The relationship between the current price and its past values was analyzed at different time periods, and an average price or mean model was developed. The model diagnosis shows that the first model is adequate to explain the prices, but it assumes that the volatility of S&P 500 prices is constant. However the market prices are highly volatile in certain periods, implying market risks are not constant. Our study finds that negative circumstances such as an increase in interest rates that cause stock prices to go down have a stronger impact on the market volatility compared to positive circumstances. Additionally, the high volatility of prices does not decrease quickly, and stock market does not quickly recover after big price drops. Therefore, a different model was developed by taking into account the volatility and characteristics of price movements. As a result, the final statistical model captures both characteristics and effect of market volatility on price movements and yields a reliable forecast for risk and return of S&P 500.

The final model was used to forecast daily risk and return of S&P 500 in the first week of 2018. The predicted risks slightly increased, implying that stock market was more volatile in the first week of 2018. Similarly the prices in the week fluctuated from $2695 to $2747, indicating higher volatility of market prices. The model predicted an overall upward trend of risk and return, and a similar trend was observed in the actual series during the first week. The final model exhibits a high accuracy of prediction, in which the forecasted value is slightly different from the observed value.

There are some limitations in our study as the final model was developed by using the past market data, which did not reflect future expectations of investors on economy and market. However, stock prices usually increase if the investors have positive expectations on economy and company performance in the future. Therefore, it is recommended that economic factors including the industry and economy condition should be considered in future studies to improve the predictive performance of the model. Additionally, an advanced model that incorporates the effect of the economy growth on the changes in S&P 500 prices should be developed to capture the relationship between the economy and stock market.

Technical Report

Win or Lose? The model to predict S&P 500

Abstract

Understanding risk and return and how prices are determined is critical for investors in making investment decisions. This paper is a study in the application of time series analysis to model the risks and returns of S&P 500, which is one of the best representations of stock market in the world. Using technical analysis as a framework, the trend, volatility, and characteristics of price movements were considered in this study. The statistical models including ARMA, GARCH, EGARCH models were built based on historical data of S&P daily adjusted closing prices. All models had strong validation statistics and were compared in order to identify the final model that adequately explains the time series of S&P 500 and strongly exhibits predictive performance in forecasting risk and return of S&P 500. This study finds that negative circumstances have a stronger effect on the market volatility compared to positive circumstances, and the predicted volatility and returns show an upward trend. There are many macro and microeconomic forces that influence stock prices and fluctuate as market conditions change. Thus, it is recommended that future analyses include economic factors to improve the model.

Introduction

Risk-Return tradeoff is a key concept of investment, in which higher risks are associated with the possibility of higher returns, and low levels of risk are associated with potentially lower returns. Compared to the bond market, the stock market has a higher level of uncertainty as the stock market can be highly volatile in which prices can change dramatically over a short time period. For example, during the 2008 financial crisis, the S&P 500 lost approximately 38.5% of its value (Yahoo! Finance). Therefore, it is essential to consider risk-return tradeoff and find a balance of risk and return when managing portfolios and making investment decisions. In this paper, we employed principles of technical analysis to develop and compare different models for predicting daily return and risk of S&P 500. The main objective of this analysis is to determine a strong predictive model that helps investors forecast reasonable return and risk associated with their investments in stock market. Seven different models are developed and compared in order to select one final model that have good forecasting performance.

Methodology

The dataset consists of 1,260 observations of daily adjusted closing prices of S&P 500 from January 2013 to December 2017, retrieved from Yahoo! Finance: https://finance.yahoo.com/quote/%5EGSPC/. In technical analysis, historical market data including price movement and volume are examined and used in statistical analysis to forecast future price movements of market. Using the technical analysis as a framework, this analysis incorporated both trends as well as characteristic of price changes and seasonality. We used the following steps as a methodology for building the model.

- Explored trend and distribution of the observed prices of S&P 500 and identified outliers
- Investigated if there was a serial autocorrelation to check the stationary condition of process;
- Transformed original data by applying a first difference;
- Fitted ARMA model with drift and conducted residual analysis;
- Backtested the ARMA model with test set and forecasted the daily return;
- Examined the volatility of prices and conducted analysis of ARCH/GARCH effects;
- Fitted GARCH and EGRACH model and perform residual analysis;
- Conducted model selection and validated it by performing training/test set split;
- Predicted daily risk and return of S&P 500 based on final model

Data Exploration

Normal Distribution

We plotted graphs, computed basic statistics, conducted normality test to check the data distribution.

- Figure 1 displays a time plot of daily adjusted closing prices of S&P 500, in which the time series have an upward trend. The maximum price is $2,690, and the minimum price is $1,457. There are big drops in S&P prices. In January 2016, prices fell 3.9% as a result of big declines in Chinese Equity Market and in oil prices at 12-year low. In June 2016, the S&P prices significantly dropped again at 3.5% due to an unexpected result of Brexit.
- The histogram (Figure 2) shows a symmetric distribution of S&P prices, which the mean of $2035 is close to the median of $2050. The skewness of 0.116 is close to zero, and kurtosis of -0.369 is less than 3. Thus, the distribution of S&P daily prices is symmetric but light-tailed.
- The normal Q-Q plot (Appendix A-1) displays some outliers and minor patterns that several points do not lie on a straight line. The tsoutliers function identifies 10 potential outliers, but we decided to keep these outliers for the further analysis of risk in stock market. The Jarque-Bera normality test confirms that the distribution is not normal because the p-value of 0.0073 is less than 0.05, so the null hypothesis is rejected (Appendix A-2).

Stationary Condition

According to Figure 1, the mean of S&P prices increases over time, and the variance is not constant. Thus, the time series are not stationary.

- The ACF plot (Figure 3) displays that autocorrelations of prices are greater than zero and slowly decay, which indicate a non-stationary process. The results from Ljung-box test (Appendix A-2) show the p-value of < 2.2e-16, which is less than 0.05, so we rejected the null hypothesis and concluded that there are some serial correlations in the time series.
- The Augmented Dicky-Fuller tests were performed with time trend and with no time trend. The results in Figure 4 confirm that the process is not stationary because all p-values are greater than 0.05, so we accepted the null hypothesis and concluded that the first difference is required.

Figure 1: Time Plot of S&P Daily Prices

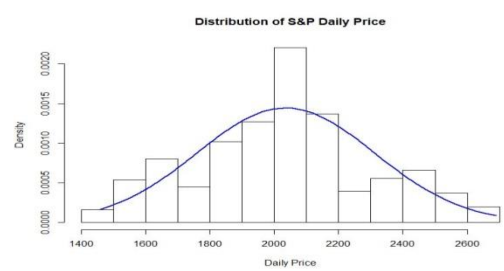


Figure 2: Histogram of S&P Daily Prices

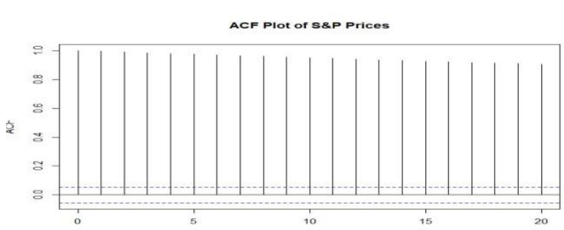


Figure3: ACF Plot of S&P Daily Prices

With time trend	With no time trend
Test Results: PARAMETER: Lag Order: 7 STATISTIC: Dickey-Fuller: -2.2005 P VALUE: 0.4934	Test Results: PARAMETER: Lag Order: 7 STATISTIC: Dickey-Fuller: -0.484 P VALUE: 0.8798

Figure 4: ADF Test of S&P Daily Prices

Data Transformation

<u>First Difference</u>

The time series of S&P prices are not stationary, so the first difference was applied to transform the original data. The first differenced value is the price at time t minus the price at time t-1.

The differenced time series: $Y_t=X_t-X_{t-1}$ or $Y_t=(1-B)X_t$, which is a stationary process.

- In Figure 5, the first differenced time series have a zero mean and constant variance.
- The Augmented Dicky-Fuller tests were performed with time trend and with no time trend. The results in Figure 6 confirm that the process after applying first difference is stationary because all p-values of 0.01 are less than 0.05, so we rejected the null hypothesis and concluded that the process of the differenced time series is stationary.
- Figure 7 displays that the autocorrelations of the differenced time series after lag 0 are close to zero, so there is no serial correlation. The PACF shows some seasonal trend. Thus, the seasonality was considered when fitting the model.

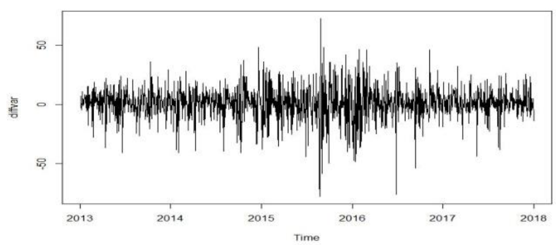


Figure 5: Time Plot of Differenced Prices

With time trend	With no time trend

```
Test Results:
  PARAMETER:
    Lag Order: 7
  STATISTIC:
    Dickey-Fuller: -13.1107
  P VALUE:
    0.01
```

```
Test Results:
  PARAMETER:
    Lag Order: 7
  STATISTIC:
    Dickey-Fuller: -13.1132
  P VALUE:
    0.01
```

Figure 6: ADF Test of Differenced Prices

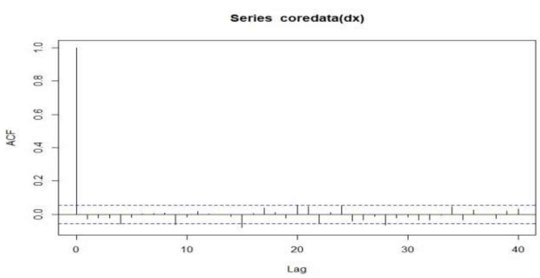


Figure 7: ACF Plot of of Differenced Prices

```
z test of coefficients:

        Estimate Std. Error  z value  Pr(>|z|)
ar1     0.937138   0.025685  36.4852 < 2.2e-16 ***
ma1    -0.966497   0.018537 -52.1382 < 2.2e-16 ***
drift   0.941217   0.223525   4.2108 2.545e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.'
```

Figure 8: Model M1 ARIMA(1,1,1) with drift

ARMA Model

<u>Model M1: ARIMA(1,1,1) with drift</u>

The ACF plot in figure 7 does not display autocorrelation at any lags, so we used AutoArima function to identify the orders. The BIC method suggests the ARIMA(0,1,0) or only the first difference. The AIC and Seasonal methods suggest the ARIMA(1,1,1) with drift, which has lower AIC and BIC than those of ARIMA(0,1,0). In figure 8 above, we fitted the S&P prices with the model, ARIMA(1,1,1) with drift because the S&P prices are unit-root non-stationary and have an upward trend.

The model equation is: $(1- 0.937B)(1-B)X_t = 0.941 + (1- 0.967B)a_t$

The model has a zero mean because we apply the first difference. Thus, the model is also stationary. It includes the order 1 of AR model, the first difference, drift, and the order 1 of MA model. All parameters in the model are significant because all p-values are less than 0.05.

Residual Analysis

- Normality of Residuals: The Q-Q plot in Figure 9 shows that residuals do not have a normal distribution. Many points on the left and right do not lie on a straight line.
- Constant variance assumption: The time plot in (Appendix A-3) displays that residuals have constant variance in this period except in 2016 in which the variance significantly fluctuated. Thus, the further analysis of volatility was performed and explained in next section of GARCH.
- Independence assumption: The ACF plot (Figure 10) identifies that there is no serial correlation. Ljung-Box tests were performed, and the results (Appendix A-3) show all p-values are greater than 0.05, so we accepted the null hypothesis and concluded that residuals are white noise.

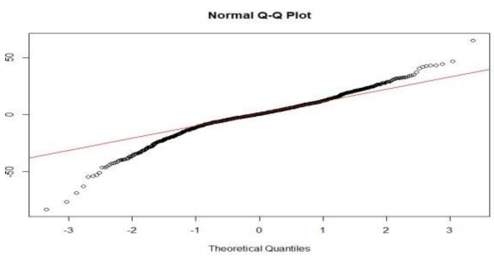


Figure 9: Q-Q Plot of Residuals

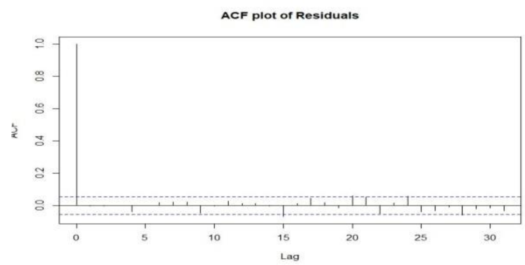


Figure 10: ACF plot of Residuals

Backtesting and Forecasting

As our main objective is to forecast the risk and return of S&P 500, the backtesting was performed using a validation set (20% of data) to assess the forecasting performance of model M1. The model was estimated using training set (80% of data). The results in Figure 11 indicates that the model M1 has a strong forecasting power as it has a low RMSE of 10.29 and a low mean absolute percentage error of 0.29%, which indicates a low percentage deviation of the forecasted value from actual value. In other words, one step ahead forecasted values from model M1 are 0.29% off from the observed values.

The model M1 is used to compute the predicted daily price of S&P 500 in the next five days. The plot (Figure 12) shows a slight upward trend of the forecasted prices. The predicted price of the first trading day in 2018 or (January 2, 2018) is $2673.50 with the 95% limit of ($2644.72, $2702.28) (Appendix A-4).

```
[1] "RMSE of out-of-sample forecasts"
[1] 10.29847
[1] "Mean absolute error of out-of-sample forecasts"
[1] 7.225655
[1] "Mean Absolute Percentage error"
[1] 0.002963082
[1] "Symmetric Mean Absolute Percentage error"
[1] 0.002964318
```

Figure 11: Backtesting Model M1: ARIMA(1,1,1) with Drift

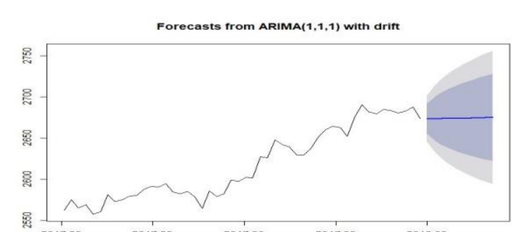


Figure 12: Forecasting Plot of Model M1

The model M1 Arima(1,1,1) with drift specifies the mean of process, which can be used to predict the S&P 500. However, it does not capture a significant GARCH effect that implies a non-constant volatility. Thus, we need to conduct a further analysis in the next section to find a better model that is appropriate to explain data with non-constant volatility and to predict both risk and return of S&P 500.

GARCH Model

<u>Converting price into simple returns</u>

From previous analysis, we used adjusted prices for our ARMA model. But for GARCH model, we converted the adjusted prices into simple returns using the following formula: Returns = $(P_t / P_{t-1}) / P_{t-1}$

<u>Analyzing Returns</u>

- Stationarity: The process of returns appears to be stationary as shown in plot Appendix B-1. The mean is equal to 0.000508, which is close to zero.
- Distribution of returns (Appendix B-2): The statistics indicate that the distribution is close to normal as mean and median are equal. The skewness is - 0.3 and kurtosis is 2.8 (thin tail). The histogram of returns also shows a normal distribution. However, from Jarque-Bera test, the null hypothesis of normal distribution is rejected as p-value is less than 0.05. Hence, the returns have distribution that is close to normal but not exactly normal.

<u>Volatility</u>

The first graph of returns (Figure 13) shows that there is variation in volatility. But to see it clearly, we plotted return^2 and absolute return. The volatility is more evident, varying among different time periods. Between 2015 and 2016, the volatility is quite high, whereas it is very low between 2017 and 2018. There are high spikes at some periods, in which volatility is significantly higher than others. Thus, the volatility is not constant, and it is various at different points and ranges of time.

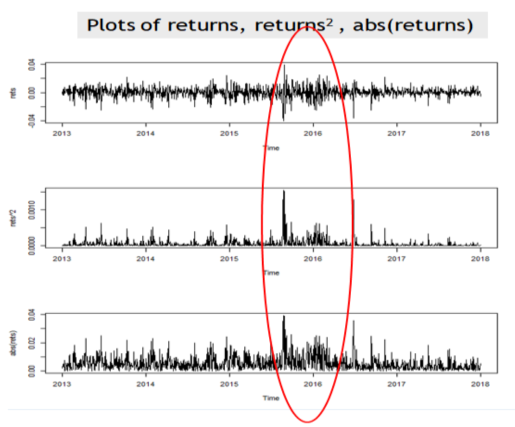


Figure 13: Plot of returns, returns^2 and abs(returns)

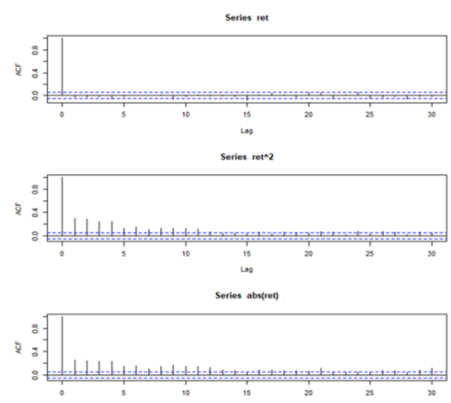


Figure 14: ACF Plot of returns, returns^2 and abs(returns)

<u>Analyzing the ARCH/GARCH effect</u>

Step 1: Check ACF plot of returns, returns^2, and absolute returns (displayed in Figure 14).
The ACF plot of returns shows a very weak serial correlation. But the plots of returns^2 and abs (returns) show a serial correlation, as autocorrelations at all lags are above the threshold or on the threshold. This shows that ARCH effect is significant, and a non-linear dependence is presented in the time series.
Step2: McLeod-Li test (Ljung Box test on Squared of returns, displayed in Appendix B-4)
The Ljung Box test hypothesis: H_0: All autocorrelations are zero. H_a: Some autocorrelation are not zero.
Since from the statistical test, all p-values are less the alpha value of 0.05, we reject the null hypothesis. The test suggests that the autocorrelation is not equal to zero and the series are serially correlated.

From both test results above, we conclude that there are significant ARCH effect, serial correlation, and non-linear dependence in the time series.

Leverage Effect

When we compare the time plot of prices with the volatility plot (Figure 15), we see that the highest volatility appears around 2016, when the S&P 500 index prices decline. The highest turbulence in the time series appeared at the same time when the prices dropped. This shows that negative shock has a stronger impact on the volatility compared to a positive shock of the same size. Referring to the graph of S&P 500 recent prices (Appendix B-5), market crashed in February 2018. Prices increase at a normal rate, but the downfall is very steep. This indicates that the series have an asymmetric behavior. To capture this asymmetric behavior, an EGARCH model would be more appropriate than a simple GARCH model.

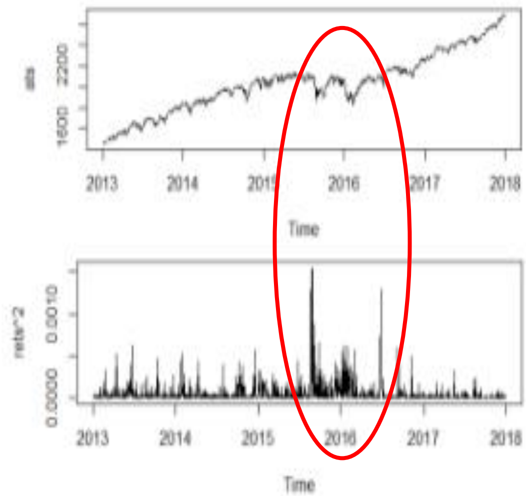


Figure 15: Price and Volatility

Model 1: ARMA(0,0)-EGARCH (1,1) with normal distribution

We fitted an EGARCH model with normal distribution (See Appendix B-6). The model expression is:

$r_t = 0.000278 + a_t$, $a_t = \sigma_t e_t$

$\ln(\sigma_t^2) = -0.674 + (-0.251\, e_{t-1} + 0.118\, (|e_{t-1}| - E(|e_{t-1}|)) + 0.9324 \ln(\sigma^2_{t-1})$

Parameters:

The parameter mu is not significant, as it has a p-value of more than the alpha value of 0.05. All other parameters appear to be significant. The parameter alpha1 of -0.251 is less than zero, indicating that volatility reacts more heavily to negative shocks.

Residual Analysis:

The residuals indicate that model 1 is not adequate (See Appendix B-6). The Ljung Box test of standardized residuals shows a serial correlation, as some p-values are less than 0.05. The ACF plot confirms that residuals are not white noises as ACF on many lags are above the threshold. The QQ plot also shows some variations from normality. The density curve does not display the best fit. The results from Goodness of fit test reject the null hypothesis of normal distribution. Hence, residuals are not white noise and not normally distributed. Thus, model 1 is inadequate, and advanced models are required.

Six different models were fitted and compared, including model m1: ARMA(0)-GARCH(1,1) with normal distribution, model m1.t: ARMA(0)-GARCH(1,1) with t-distribution, model em1:ARMA(0)-EGARCH(1,1) with normal distribution, model em1.t-ARMA(0)-EGARCH(1,1) with t distribution, model em2.t: ARMA(0,1)-EGARCH(1,1) with t distribution and model em3.t:- ARMA(1,0)-EGARCH(1,1) . The model em3.t: ARMA(1,0)-EGARCH(1,1) has the lowest BIC and AIC value(Fig. 16), so this model was chosen for a further analysis.

	m1	m1.t	em1	em1.t	em2.t	em3.t
Akaike	-7.157223	-7.218862	-7.230469	-7.277333	-7.282345	-7.282433
Bayes	-7.140888	-7.198444	-7.210050	-7.252831	-7.253759	-7.253847
Shibata	-7.157243	-7.218894	-7.230500	-7.277378	-7.282406	-7.282494
Hannan-Quinn	-7.151084	-7.211189	-7.222795	-7.268125	-7.271602	-7.271690

Figure 16: Comparing different models

Model 2: ARMA(1,0)-EGARCH(1,1) with t-distribution

The fitted ARMA(1,0)-EGARCH(1,1) with t-distribution can be written as

$r_t = 0.00051 - 0.08\, a_{t-1} + a_t$

$a_t = \sigma_t e_t \quad \ln(\sigma_t^2) = -0.549 - 0.243\, e_{t-1} + 0.153\, (|e_{t-1}| - E(e_{t-1})) + 0.946\, \sigma_{t-1}^2$

Where the error term e_t has t-distribution with 6 degrees of freedom.

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000510    0.000132   3.8655 0.000111
ar1    -0.081019    0.027798  -2.9146 0.003562
omega  -0.549286    0.011316 -48.5394 0.000000
alpha1 -0.242558    0.020657 -11.7424 0.000000
beta1   0.946033    0.001486 636.7963 0.000000
gamma1  0.152712    0.007539  20.2552 0.000000
shape   6.310544    1.089928   5.7899 0.000000
```

Parameters**:**

The p-values of all the parameters are less than the alpha of 0.05, so we reject the null hypothesis of no parameter being significant. Therefore, all parameters are significant for the model 2: ARMA(1,0) - EGARCH(1,1). The significant shape parameter indicates that t-distribution is a good choice. The parameter alpha1 (-0.242) is less than zero, which indicates that leverage effect is significant. The asymmetric behavior is also denoted by the gamma1 parameter, which is also significant. Therefore a negative stocks has a stronger impact on the volatility compared to a positive shock of the same size. In R, the leverage parameter is alpha1/gamma1 = -0.2426/0.1527 = -1.589.

Residual Analysis:

Step1: Checking serial correlation and ARCH/GARCH effect

The Ljung box test (Figure: 17) for serial correlation on standardized residuals shows that all p-values are more than the alpha value of 0.05. Similarly, the Ljung box test to check the ARCH/GARCH effect on standardized squared residuals shows that all p-values are more than 0.05. Therefore, we accept the null hypothesis of no serial correlation in the residuals. We can conclude that there is no evidence of serial correlation and ARCH/GARCH effect in the residuals. Thus, they are white noise.

Step2: Checking distribution

The QQ plot (Figure 18) and the density curve (Figure 19) indicate that model with t- distribution is an appropriate choice. Some departures can be seen in QQ plot near the right tail for extreme residuals, but majority of points lies on 45 degree line. The density curve shows that the overall fit of t-distribution is better than normal distribution. However, the Adjusted Pearson Goodness-of fit test in Figure 20 displays that p-values are less than the alpha value of 0.05, so we rejected the null hypothesis of error term having a t-distribution. The test indicates that t-distribution is not a good choice for error term.

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                    0.01092  0.9168
Lag[2*(p+q)+(p+q)-1][2]   0.03908  1.0000
Lag[4*(p+q)+(p+q)-1][5]   1.00153  0.9466
d.o.f=1
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                      0.345  0.5570
Lag[2*(p+q)+(p+q)-1][5]     1.036  0.8514
Lag[4*(p+q)+(p+q)-1][9]     2.552  0.8299
d.o.f=2
```

Figure 17: Ljung Box test on residuals

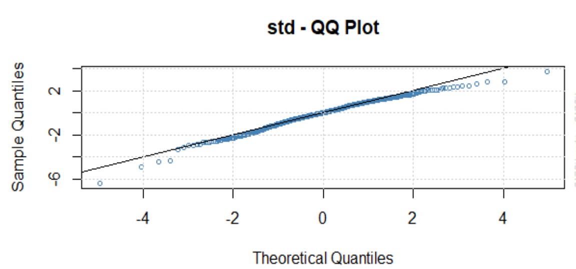


Figure 18: QQ plot of standardized residuals

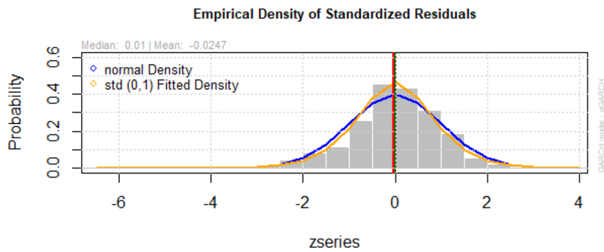


Figure 19: Empirical density curve of standardized residual

```
Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     42.00    0.0017724
2    30     51.94    0.0055409
3    40     56.34    0.0356559
4    50     89.54    0.0003625
```

Figure 20: Goodness of fit test

All analysis except the Goodness of fit test supports the choice of t-distribution. Additionally, the model 2 also has the lowest AIC and BIC value among other six models. Therefore, we selected this model 2: ARMA(1,0)-EGARCH(1,1) with t-distribution as our final model and use it for predictions.

Forecasting & Performance

The predicted conditional mean at time t+h is denoted by series in Figure 22, converges to unconditional mean (0.004757) of the series after four steps ahead. The predicted volatility at time t+h has an increasing trend. It converges to unconditional standard deviation of the time series which is 0.0000414. Since the sigma is increasing, the forecast of ARMA(1,0)-EGARCH(1,1) suggests that the volatility will have an increasing trend in the future. The Mean absolute error for the model is 0.00284, Mean squared error is 0.0000175 and Directional accuracy (DAC) is 0.65.

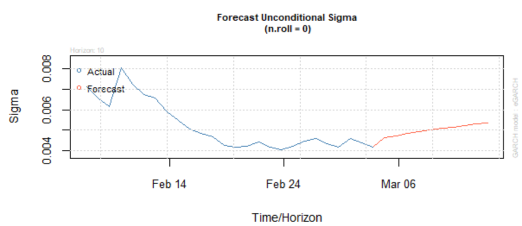


Figure 21: Forecasting sigma

```
0-roll forecast [T0=1973-03-03 18:00:00]:
       Series    Sigma
T+1 0.0006981 0.004644
T+2 0.0004586 0.004741
T+3 0.0004770 0.004834
T+4 0.0004756 0.004923
T+5 0.0004757 0.005007
```

Figure 22: Five step ahead forecast

Conclusion

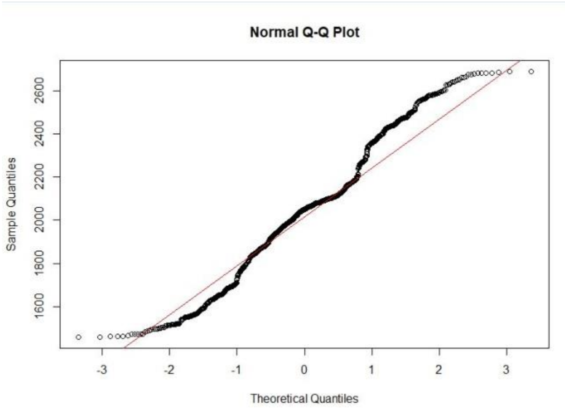
Our study used technical analysis as a framework and considered trend, volatility, and characteristics of price movements to understand how these factors influence prices and to develop a strong predictive model for forecasting the risk and return of S&P 500. According to our analysis and findings, the final model is AR(1)-EGARCH(1,1) with T distribution. Most of the analysis and statistical tests indicate that this model is adequate and it has the lowest AIC and BIC. It also exhibits a good forecasting performance as its MAE is as low as 0.0028. It forecasts a slight upward trend for the first week of 2018, which is similar to the actual trend observed. The model was used to compute one-step ahead forecast. The results imply that stock market was more volatile in the first week of 2018 than in the last week of 2017. The predicted daily volatility is 0.4644%, and the prices in the week fluctuated from $2695 to $2747, indicating higher volatility of market prices.

Limitations and Future Studies

We developed this model based on past market data. Thus, our analysis does not capture the future expectations of investors on the stock market and economy, which significantly drive the stock prices. To improve the predictive performance of our model, the macro and microeconomic factors including the market and industry condition should be considered in future studies. It is recommended that an advanced model is required to capture the effect of the economy growth on the S&P 500 prices.

Appendix A - S&P Prices Analysis and R Outputs

A-1: Q-Q plot and Basic Statistics of S&P 500 Prices




```
                    price
nobs         1.259000e+03
NAs          0.000000e+00
Minimum      1.457150e+03
Maximum      2.690160e+03
1. Quartile  1.863080e+03
3. Quartile  2.168760e+03
Mean         2.035666e+03
Median       2.050630e+03
Sum          2.562903e+06
SE Mean      7.776392e+00
LCL Mean     2.020410e+03
UCL Mean     2.050922e+03
Variance     7.613459e+04
Stdev        2.759250e+02
Skewness     1.162480e-01
Kurtosis    -3.691370e-01
```

A-2 Jarque-Bera and Ljung Box Test Results of S&P 500 Prices

```
Title:
 Jarque - Bera Normalality Test

Test Results:
  STATISTIC:
    X-squared: 9.8293
  P VALUE:
    Asymptotic p Value: 0.007338
```

```
> Box.test(ats,lag=3,type='Ljung-Box')

        Box-Ljung test

data:  ats
X-squared = 3709.6, df = 3, p-value < 2.2e-16

> Box.test(ats,lag=7,type='Ljung-Box')

        Box-Ljung test

data:  ats
X-squared = 8491.5, df = 7, p-value < 2.2e-16
```

A-3 Ljung Box Test Results of Residuals (Model M1)

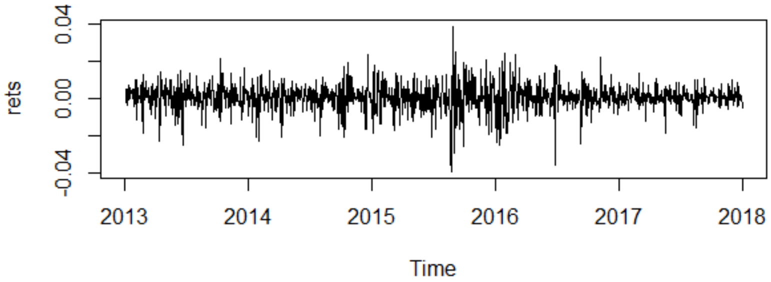
```
> Box.test(m1$residuals,lag=3,type='Ljung-Box', fitdf=2)

        Box-Ljung test

data:  m1$residuals
X-squared = 0.013103, df = 1, p-value = 0.9089

> Box.test(m1$residuals,lag=7,type='Ljung-Box', fitdf=2)

        Box-Ljung test

data:  m1$residuals
X-squared = 2.7697, df = 5, p-value = 0.7354
```

A-4 Forecasted Prices of S&P 500 from Model M1

	Point Forecast	Lo 80	Hi 80	Lo 95	Hi 95
2018.000	2673.503	2654.685	2692.321	2644.724	2702.282
2018.004	2673.461	2647.237	2699.686	2633.354	2713.569
2018.008	2673.482	2641.816	2705.147	2625.054	2721.910
2018.012	2673.560	2637.495	2709.625	2618.403	2728.717
2018.016	2673.693	2633.903	2713.482	2612.840	2734.545

Appendix B -GARCH Analysis and R Outputs

B-1 Plot of returns



B-2 Distribution of Simple Return

Basic Statistics

```
                    rets
nobs         1258.000000
NAs             0.000000
Minimum        -0.039414
Maximum         0.039034
1. Quartile    -0.002835
3. Quartile     0.004570
Mean            0.000508
Median          0.000531
Sum             0.638554
SE Mean         0.000210
LCL Mean        0.000095
UCL Mean        0.000920
Variance        0.000056
Stdev           0.007464
Skewness       -0.382317
Kurtosis        2.814137
```

Fig B-2.1 Basic statistics

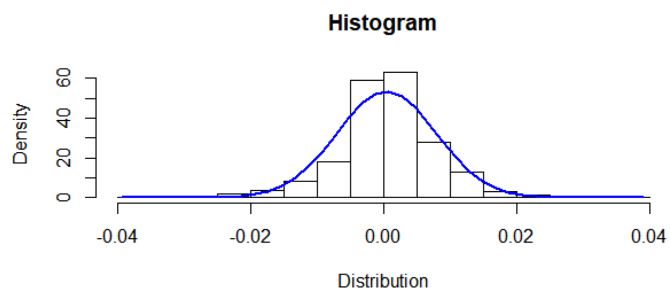
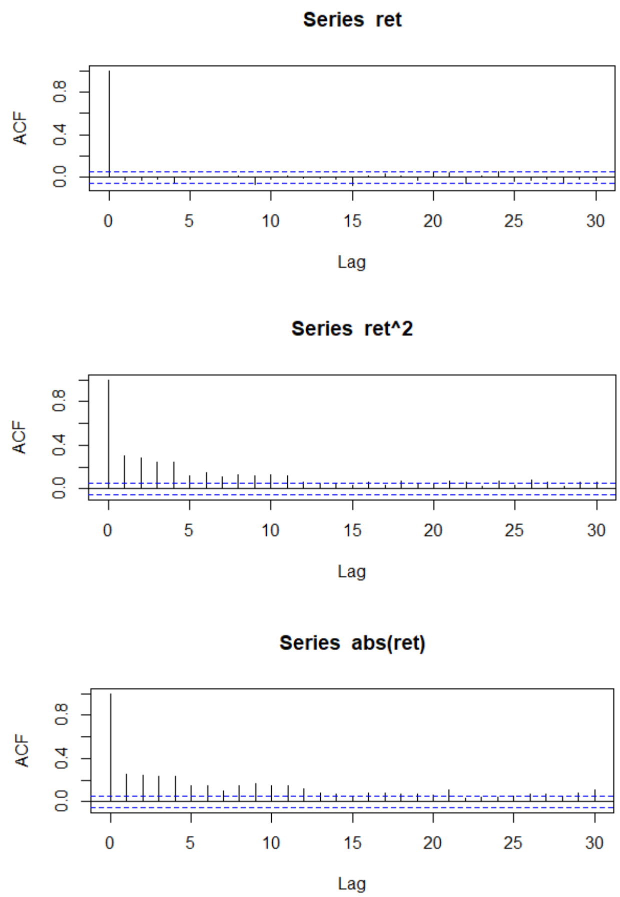


Fig B-2.2 Histogram

```
Title:
 Jarque - Bera Normalality Test

Test Results:
  STATISTIC:
    X-squared: 448.5608
  P VALUE:
    Asymptotic p Value: < 2.2e-16
```

Fig B-2.3 JB test output

B-3 ACF Plot of return, return^2 and abs(return)



B-4 McLeod-Li test

```
> Box.test(coredata(rets^2),lag=2,type='Ljung')

        Box-Ljung test

data:  coredata(rets^2)
X-squared = 216.35, df = 2, p-value < 2.2e-16

> Box.test(coredata(rets^2),lag=4,type='Ljung')

        Box-Ljung test

data:  coredata(rets^2)
X-squared = 370.95, df = 4, p-value < 2.2e-16

> Box.test(coredata(rets^2),lag=6,type='Ljung')

        Box-Ljung test

data:  coredata(rets^2)
X-squared = 418.92, df = 6, p-value < 2.2e-16
```

Fig. B- 4.1 Ljung Box Test of return^2

```
> Box.test(abs(coredata(rets)),lag=2,type='Ljung')

        Box-Ljung test

data:  abs(coredata(rets))
X-squared = 158.1, df = 2, p-value < 2.2e-16

> Box.test(abs(coredata(rets)),lag=4,type='Ljung')

        Box-Ljung test

data:  abs(coredata(rets))
X-squared = 293.4, df = 4, p-value < 2.2e-16

> Box.test(abs(coredata(rets)),lag=6,type='Ljung')

        Box-Ljung test

data:  abs(coredata(rets))
X-squared = 350.77, df = 6, p-value < 2.2e-16
```

Fig. B- 4.2 Ljung Box Test of abs(return)

B-5 Asymmetric Behavior

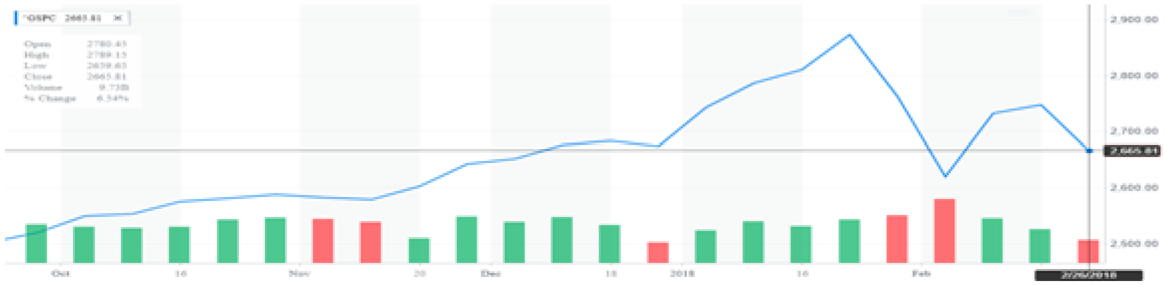


Figure B-5 S&P 500 graph from Oct 2017- Feb 2018

B-6 Model 1 ARMA (0)-EGARCH (1, 1)

```
Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.000278    0.000146    1.8958 0.057991
omega  -0.673531    0.007199  -93.5558 0.000000
alpha1 -0.251065    0.018638  -13.4704 0.000000
beta1   0.932406    0.000150 6235.8014 0.000000
gamma1  0.118328    0.009332   12.6798 0.000000
```

Fig B-6.1 Model output

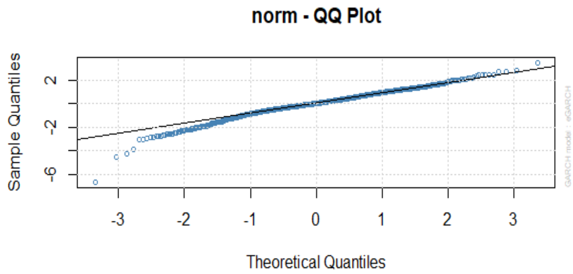
Residual Analysis

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      5.839 0.01568
Lag[2*(p+q)+(p+q)-1][2]     5.936 0.02287
Lag[4*(p+q)+(p+q)-1][5]     6.807 0.05777
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.9592  0.3274
Lag[2*(p+q)+(p+q)-1][5]    1.2166  0.8090
Lag[4*(p+q)+(p+q)-1][9]    2.1568  0.8855
d.o.f=2
```

Fig B-6.2 Ljung box test on residuals

```
Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     53.13    4.476e-05
2    30     73.11    1.124e-05
3    40     85.40    2.571e-05
4    50     92.48    1.728e-04
```

Fig B-6.3 Goodness of fit test on residuals



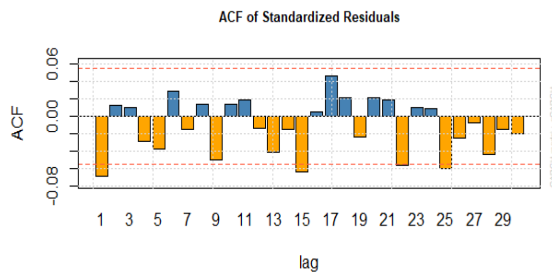


Fig B-6.3 QQ Plot of standardized residual

Fig B-6.3 ACF Plot of standardized residual

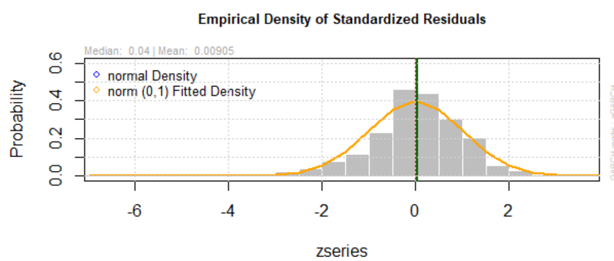


Fig B-6.3 Empirical density curve of standardized residual

B-7: Forecasting

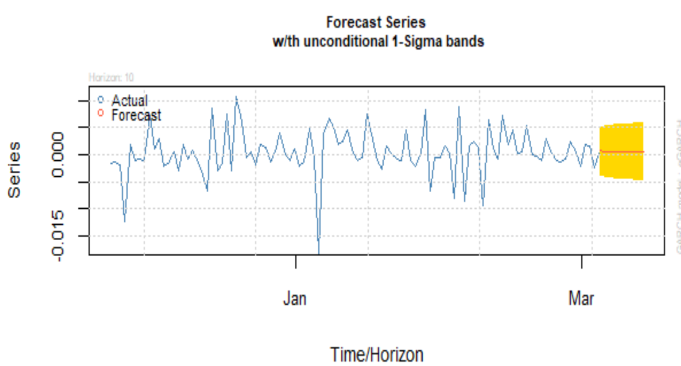


Fig B-7.1 Time series prediction

```
0-roll forecast [T0=1973-03-03 18:00:00]:
         Series    Sigma
T+1  0.0006981 0.004644
T+2  0.0004586 0.004741
T+3  0.0004770 0.004834
T+4  0.0004756 0.004923
T+5  0.0004757 0.005007
T+6  0.0004757 0.005087
T+7  0.0004757 0.005164
T+8  0.0004757 0.005236
T+9  0.0004757 0.005305
T+10 0.0004757 0.005371
```

Fig B-7.2 Ten step ahead forecast

Appendix C - R Codes

```
library(tseries)
library(fBasics)
library(zoo)
library(forecast)
library(lmtest)
library(fUnitRoots)
library(rugarch)

setwd("C:/Users/Desktop/csc425/New folder")
myd=read.table("S&P_5yrs.csv", header=T, sep=',')
head(myd)
price=myd[,6]
plot(price, main="Plot of Adjusted Price", type="l")
#create ts object
ats=ts(price, start=c(2013, 2), frequency = 252)
head(ats)
plot(ats)
plot(ats, main="Adjusted Closing Price", type="l")
#Checking outliers
tsoutliers(ats)
#removing them
atsclean=tsclean(ats)
plot(atsclean)
plot(ats, type='l')
#missing values
ats=na.approx(ats)
plot(ats)
#Analysis of distribution
basicStats(price)

# NORMALITY TESTS
# Perform Jarque-Bera normality test.
normalTest(ats,method=c("jb"))
par(mfcol=c(1,1))
hist(ats, xlab="Daily Price", prob=TRUE, main="Distribution of S&P Daily Price")
xfit<-seq(min(ats),max(ats),length=40)
yfit<-dnorm(xfit,mean=mean(ats),sd=sd(ats))
lines(xfit, yfit, col="blue", lwd=2)
qqnorm(price)
qqline(price, col = 2)
# ACF ANALYSIS
acf(coredata(ats),plot=T, lag=20, main="ACF Plot of S&P Prices")
#Pacf
pacf(coredata(ats), main="Pacf Plot")
#Ljung Box Test
Box.test(ats,lag=3,type='Ljung-Box')
Box.test(ats,lag=7,type='Ljung-Box')

# Dickey Fuller test
# tests for AR model with time trend
adfTest(ats, lags=3, type=c("ct"))
adfTest(ats, lags=7, type=c("ct"))
```

```
# tests for AR model with no time trend
adfTest(price, lags=3, type=c("c"))
adfTest(price, lags=7, type=c("c"))

# APPLYING DIFFERENCING TO DATA
dx=diff(ats)
acf(coredata(dx), plot=T, lag=40)
pacf(coredata(dx), plot=T, lag=40)
#check seasonal diferencing
sdx=diff(dx,6)
acf(coredata(sdx), plot=T, lag=40)
pacf(coredata(sdx), plot=T, lag=40)

# Dickey Fuller test #after first diff
adfTest(dx, lags=3, type=c("ct"))
adfTest(dx, lags=7, type=c("ct"))
adfTest(coredata(dx), lags=3, type=c("c"))
adfTest(coredata(dx), lags=7, type=c("c"))

#Unit-root tests on first difference
diffvar=diff(ats)
plot(diffvar)

#Finding model
auto.arima(ats, ic =c("bic"), trace=TRUE, stationary = F)
auto.arima(ats, ic =c("aic"), trace=TRUE, stationary = F)
auto.arima(ats, trace=T, seasonal=T)
#Fitting model
m1=Arima(ats, order=c(1,1,1), method='ML', include.drift=T)
coeftest(m1)

# RESIDUAL ANALYSIS
plot(m1$residuals)
acf(coredata(m1$residuals), main="ACF plot of Residuals")
Box.test(m1$residuals,lag=3,type='Ljung-Box', fitdf=2)
Box.test(m1$residuals,lag=7,type='Ljung-Box', fitdf=2)
qqnorm(m1$residuals)
qqline(m1$residuals, col = 2)

#FORECASTING
f=forecast(m1, h=5)
forecast(m1, h=5)
plot(f, include=50)
# BACKTESTING
source("backtest.R")
backtest(m1, ats, h=1, orig=length(price)*0.8)

############ GARCH MODEL###############
#simple return time series
rets =(ats-lag(ats,k=-1))/lag(ats,k=-1)
plot(rets)
ret=coredata(rets)
#compute statistics
basicStats(rets)
#histogram
```

```
hist(rets, xlab="Distribution", prob=TRUE, main="Histogram")
xfit<-seq(min(rets),max(rets),length=40)
yfit<-dnorm(xfit,mean=mean(rets),sd=sd(rets))
lines(xfit, yfit, col="blue", lwd=2)
# Perform Jarque-Bera normality test.
normalTest(rets,method=c("jb"))

par(mfrow=c(1,1))
# Plots ACF function of vector data
acf(ret)
# Plot ACF of squared returns to check for ARCH effect
acf(ret^2)
# Plot ACF of absolute returns to check for ARCH effect
acf(abs(ret))

#plot returns, square returns and abs(returns)
# Plots vector data
plot(rets, type='l')
# Plot squared returns to check for ARCH effect
plot(rets^2,type='l')
# Plot absolute returns to check for ARCH effect
plot(abs(rets),type='l')

# Computes Ljung-Box test on returns to test  independence
Box.test(coredata(rets),lag=2,type='Ljung')
Box.test(coredata(rets),lag=4,type='Ljung')
Box.test(coredata(rets),lag=6,type='Ljung')
# Computes Ljung-Box test on squared returns to test non-linear independence
Box.test(coredata(rets^2),lag=2,type='Ljung')
Box.test(coredata(rets^2),lag=4,type='Ljung')
Box.test(coredata(rets^2),lag=6,type='Ljung')
# Computes Ljung-Box test on absolute returns to test non-linear independence
Box.test(abs(coredata(rets)),lag=2,type='Ljung')
Box.test(abs(coredata(rets)),lag=4,type='Ljung')
Box.test(abs(coredata(rets)),lag=6,type='Ljung')

#checking for ARMA order
auto.arima(rets, max.p = 2, max.q = 2, stationary=TRUE, ic=c("aic"), stepwise=TRUE)

###### Model 1#########
#Fit ARMA(0,0)-GARCH(1,1) model-normal distribution
m1.spec=ugarchspec(variance.model = list(garchOrder = c(1,1)), mean.model =
list(armaOrder = c(0,0)))
m1.fit=ugarchfit(spec=m1.spec, data=rets)
m1.fit
coef(m1.fit)
#create selection list of plots for garch(1,1) fit
plot(m1.fit)
#conditional volatility plot
plot.ts(sigma(m1.fit), ylab="sigma(t)", col="blue")
infocriteria(m1.fit)

###### Model 2#########
#Fit ARMA(0,0)-GARCH(1,1) model with t-distribution
```

```
m1.t.spec=ugarchspec(variance.model=list(garchOrder=c(1,1)),
mean.model=list(armaOrder=c(0,0)), distribution.model = "std")
#estimate model
m1.t.fit=ugarchfit(spec=m1.t.spec, data=rets)
m1.t.fit
#plot of residuals
plot(m1.t.fit)
infocriteria(m1.t.fit)

###### Model 3#########
#Fit ARMA(0,0)-eGARCH(1,1) model with normal-distribution
em1.spec=ugarchspec(variance.model=list(model = "eGARCH", garchOrder=c(1,1)),
mean.model=list(armaOrder=c(0,0)))
#estimate model
em1.fit=ugarchfit(spec=em1.spec, data=rets)
em1.fit
plot(em1.fit)

###### Model 4#########
#Fit ARMA(0,0)-eGARCH(1,1) model with t-distribution
em1.t.spec=ugarchspec(variance.model=list(model = "eGARCH", garchOrder=c(1,1)),
mean.model=list(armaOrder=c(0,0)), distribution.model = "std")
#estimate model
em1.t.fit=ugarchfit(spec=em1.t.spec, data=rets)
em1.t.fit
plot(em1.t.fit)

###### Model 5#########
#Fit ARMA(0,1)-eGARCH(1,1) model with t-distribution
em2.t.spec=ugarchspec(variance.model=list(model = "eGARCH", garchOrder=c(1,1)),
mean.model=list(armaOrder=c(0,1)), distribution.model = "std")
#estimate model
em2.t.fit=ugarchfit(spec=em2.t.spec, data=rets)
em2.t.fit
plot(em2.t.fit)


###### Model 6#########
#Fit ARMA(1,0)-eGARCH(1,1) model with t-distribution
em3.t.spec=ugarchspec(variance.model=list(model = "eGARCH", garchOrder=c(1,1)),
mean.model=list(armaOrder=c(1,0)), distribution.model = "std")
#estimate model
em3.t.fit=ugarchfit(spec=em3.t.spec, data=rets)
em3.t.fit
plot(em3.t.fit)
#get unconditional mean and variance
uncmean(em3.t.fit)
uncvariance(em3.t.fit)

# MODEL COMPARISON # compare information criteria
model.list = list(m1 = m1.fit, m1.t = m1.t.fit, em1=em1.fit, em1.t = em1.t.fit, em2.t
= em2.t.fit, em3.t=em3.t.fit)
info.mat = sapply(model.list, infocriteria)
rownames(info.mat) = rownames(infocriteria(m1.fit))
info.mat
```

```
# RE-FIT MODELS LEAVING 100 OUT-OF-SAMPLE OBSERVATIONS FOR FORECAST
# EVALUATION STATISTICS
m1.fit = ugarchfit(spec=m1.spec, data=ret, out.sample=100)
m1.t.fit = ugarchfit(spec=m1.t.spec, data=ret, out.sample=100)
em1.fit = ugarchfit(em1.spec, data=ret, out.sample=100)
em1.t.fit = ugarchfit(em1.t.spec, data=ret, out.sample=100)
em2.t.fit = ugarchfit(em2.t.spec, data=ret, out.sample=100)
em3.t.fit = ugarchfit(em3.t.spec, data=ret, out.sample=100)

# COMPUTE 100 1-STEP AHEAD ROLLING FORECASTS W/O RE-ESTIMATING
m1.fcst = ugarchforecast(m1.fit, n.roll=100, n.ahead=1)
m1.t.fcst = ugarchforecast(m1.t.fit, n.roll=100, n.ahead=1)
em1.fcst = ugarchforecast(em1.fit, n.roll=100, n.ahead=1)
em1.t.fcst = ugarchforecast(em1.t.fit, n.roll=100, n.ahead=1)
em2.t.fcst = ugarchforecast(em2.t.fit, n.roll=100, n.ahead=1)
em3.t.fcst = ugarchforecast(em3.t.fit, n.roll=100, n.ahead=1)

# COMPUTE FORECAST EVALUATION STATISTICS USING FPM() FUNCTION
fcst.list = list(m1 = m1.fcst, m1.t = m1.t.fcst,em1=em1.fcst, em1.t = em1.t.fcst,
em2.t=em2.t.fcst,em3.t=em3.t.fcst)
fpm.mat = sapply(fcst.list, fpm)
fpm.mat

#FORECASTS for Model em3.t
#compute h-step ahead forecasts for h=1,2,...,10
em3.t.fit=ugarchfit(spec=em3.t.spec, data=rets, out.sample=100)
em3.fcst=ugarchforecast(em3.t.fit, n.ahead=10, n.roll=100)
em3.fcst
plot(em3.fcst)
```