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HybridSVD for feature selection

Introduction to Recommender Systems

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Outline

1. Introduction
2. Problem Statement
3. Why PureSVD is not relevant?
4. Hypothesis and experiments
5. Conclusions

PureSVD is equivalent to an eigenproblem for the scaled cosine similarity matrix

$$\mathrm{sim}(i,j) \sim \bar{a}_i^\top \bar{a}_j \qquad \mathrm{sim}(i,j) \sim \bar{a}_i^\top S\, \bar{a}_j$$

a_i is an i^{th} column of matrix A

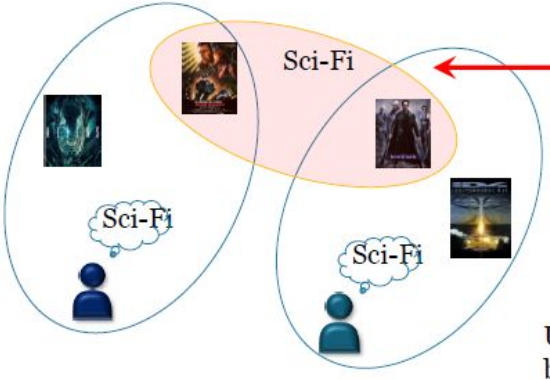
Off-diagonal elements of S encode a similarity between items based on their features.

Key idea: substitute scalar products with a bilinear form that takes side information into account.

“similarity” of users i and j depends on co occurrence of items in their preferences

$$C = A^{\top}A = V\Sigma^2V^{\top} \quad \leftrightarrow \quad c_{ij} = \bar{\boldsymbol{a}}_i^{\top}\bar{\boldsymbol{a}}_j$$

new formulation: $c_{ij} \sim \bar{\boldsymbol{a}}_i^{\top} S\, \bar{\boldsymbol{a}}_j$



“Virtual” connection based on item features.

User-based “side” similarity can be exploited in similar fashion.

Similarity matrix S

1			
	1	0.5	
	0.5	1	
			1

1. Build SPD similarity matrices K, S for users and items based on *side information*.
2. Solve a *new eigendecomposition* problem:

$$\begin{cases} ASA^\top = U\Sigma^2U^\top \\ A^\top KA = V\Sigma^2V^\top \end{cases}$$

Σ is a diagonal matrix of singular values.

$$\begin{cases} AA^\top = U\Sigma^2U^\top \\ A^\top A = V\Sigma^2V^\top \end{cases} \Rightarrow \begin{cases} ASA^\top = U\Sigma^2U^\top \\ A^\top KA = V\Sigma^2V^\top \end{cases}$$

Solution:

via SVD of an auxiliary matrix [Abdi 2007; Allen et al. 2014]: $L_K^\top A L_S = \hat{U}\Sigma\hat{V}^\top, \quad L_K L_K^\top = K, \; L_S L_S^\top = S$

link to the original latent space $L_K^{-\top}\hat{U} = U, \quad L_S^{-\top}\hat{V} = V$

Properties:

latent space structure: $U^\top K U = I, \quad V^\top S V = I$

"hybrid" folding-in: $\boldsymbol{p} = L_S^{-\top}\hat{V}\hat{V}^\top L_S^\top \boldsymbol{a}.$

HybridSVD parameters

$$\begin{cases} S = (1-\alpha)I + \alpha Z \\ K = (1-\beta)I + \beta W \end{cases}$$

- Z, W are real symmetric matrices

$$-1 \leq z_{ij}, w_{ij} \leq 1 \ \forall\, i, j$$

- $0 \leq \alpha, \beta \leq 1$ are free model parameters

control contribution of features into the model.

- Max 3 hyperparameters to tune: α, β and the rank of decomposition,
- Simplified procedure with rank truncation for any fixed pair of α and β.

Cold start with SVD

$$\begin{cases} S = (1-\alpha)I + \alpha Z \\ K = (1-\beta)I + \beta W \end{cases}$$

Z and W are users/items similarity in the features space.

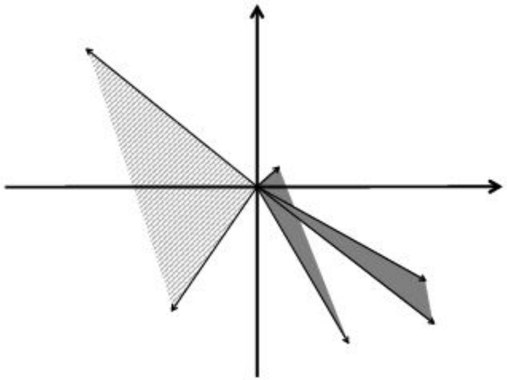
how to select the best features to include while calculating these matrices?
in other words, **introduce relevant ranking scheme for features**

Hypothesis

find a **compressed version of a feature matrix F** (which was used to calculate S) enriched with the correlations data computed based on ratings matrix A, then apply *maxvol algorithm* to select the most important features.

MAXVOL (Maximal-Volume Algorithm)

…algorithm, searches very efficiently for a submatrix of a factor matrix with the locally maximal determinant.



. An illustration of the intuition behind Maxvol for searching the seed

Related work: https://arxiv.org/pdf/1610.04850.pdf

Hypothesis

compressed version of a feature matrix F

Hypothesis 1

$$S_A = f(A^T A) = L_{S_A} L_{S_A}^T$$

$$L_{S_A}^T F_i = U \Sigma V^T$$

$$\text{feature ranking} = \text{maxvol}\,(V)$$

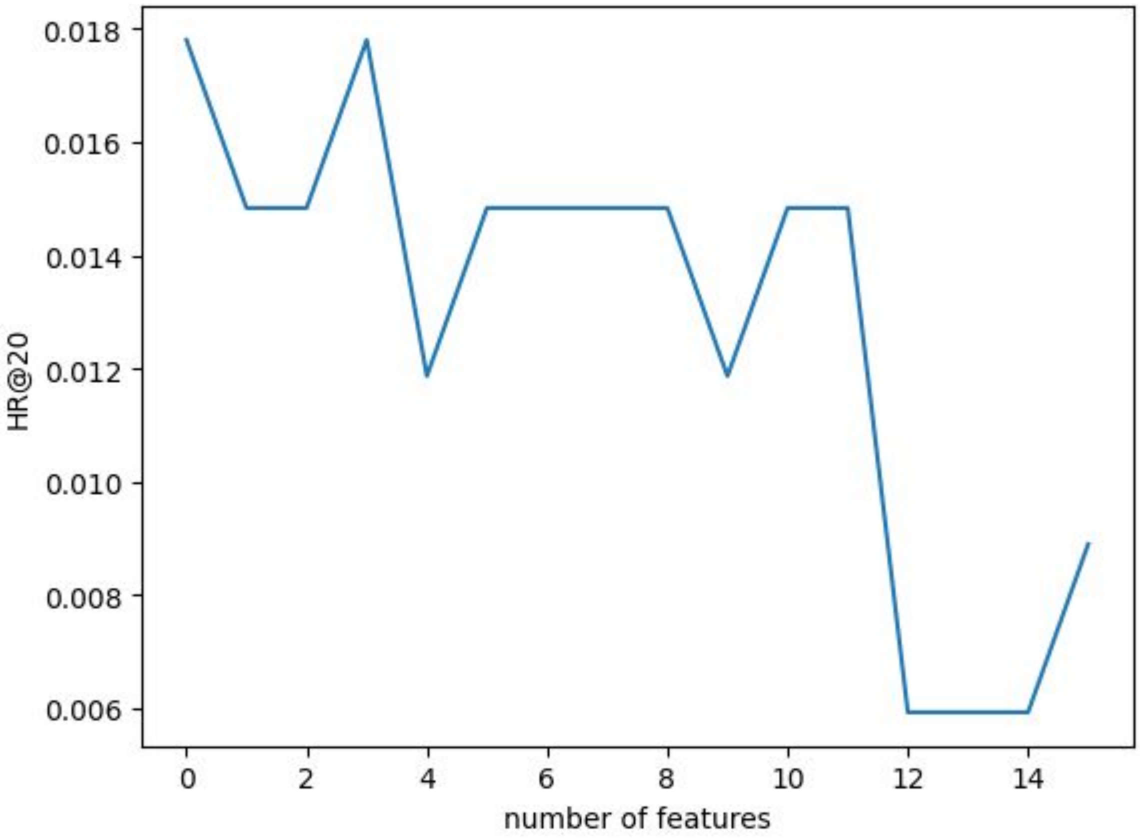
Hypothesis 2

$$B = AF_i$$

$$S_F = f(B^T B) = L_{S_F} L_{S_F}^T$$

$$F_i L_{S_F} = U \Sigma V^T$$

$$\text{feature ranking} = \text{maxvol}\,(V)$$


0.018
0.016
0.014
0.012
0.010
0.008
0.006
HR@20
0
2
4
6
8
10
12
14
number of features

Thank you for attention!