ECE661 Fall 2024: Homework 3

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TASK 1

The goal is to remove projective and affine distortions in the given images. We do this using point-to-point correspondence, 2-step method, and 1-step method.

Point-to-Point Correspondence:

Here we manually find point-to-point correspondence between original undistorted scene (domain) and its photograph (range) which has projective and affine distortions. We start by finding the homography matrix (H) which relates the original scene(domain) to distorted image (range). To do we need to pick a set of 4 corresponding points and use them to find the elements of the homography matrix (H). While we can select the points on our distorted image, we use the given world coordinate frame measures to get the points for the undistorted image(original scene). For the given height & width, the points in the undistorted image are (0, 0), (0, width), (height, 0), and (height, width). Here, we keep the ratio of height and width in pixels same as that given in world coordinate frame.

We know that two points X and X' on domain and range plane respectively are related by the Homography matrix(H) with the equation as:

$$X = HX'$$

(Homography matrix is 3x3 non-singular & the points are in HC form rep. as 3D vectors)

We can also write it as:

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ v_1 & v_2 & v \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

In physical representation of point in domain plane, $x = x_1/x_3$ and $y = x_2/x_3$. We can set $x_3 = 1$ to get a direct relation of $x = x_1$ and $y = x_2$.

Similarly for the range plane point, $x' = x_1'/x_3'$ and $y' = x_2'/x_3'$.

We can also set $v = 1$ as ratios matter.

So we get,

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ v_1 & v_2 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix}$$

Expanding:

$$x_1' = a_{11}x_1 + a_{12}x_2 + t_x$$
$$x_2' = a_{21}x_1 + a_{22}x_2 + t_y$$
$$x_3' = v_1x_1 + v_2x_2 + 1$$

We know

$$x' = \frac{x_1'}{x_3'} = \frac{a_{11}x_1 + a_{12}x_2 + t_x}{v_1x_1 + v_2x_2 + 1} = \frac{a_{11}x + a_{12}y + t_x}{v_1x + v_2y + 1}$$

and

$$y' = \frac{x_2'}{x_3'} = \frac{a_{21}x_1 + a_{22}x_2 + t_y}{v_1x_1 + v_2x_2 + 1} = \frac{a_{21}x + a_{22}y + t_y}{v_1x + v_2y + 1}$$

Simplifying this we get,

$$x' = a_{11}x + a_{12}y + t_x - v_1xx' - v_2yx'$$
$$y' = a_{21}x + a_{22}y + t_y - v_1xy' - v_2yy'$$

We get 2 equations corresponding to a pair of points. Similarly, for 4 such pairs of points(denoted with subscript i=1,2,3,4), we will have 8 equations. Using this we can find the 8 unknowns in the Homography matrix H. Matrix representation of these equations is given below:

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1x_1' & -y_1x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -x_1y_1' & -y_1y_1' \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2x_2' & -y_2x_2' \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -x_2y_2' & -y_2y_2' \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3x_3' & -y_3x_3' \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -x_3y_3' & -y_3y_3' \\ x_4 & y_4 & 1 & 0 & 0 & 0 & -x_4x_4' & -y_4x_4' \\ 0 & 0 & 0 & x_4 & x_4 & 1 & -x_4y_4' & -y_4y_4' \end{bmatrix} \begin{pmatrix} a_{11} \\ a_{12} \\ t_x \\ a_{21} \\ a_{22} \\ t_y \\ v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \\ x_3' \\ y_3' \\ x_4' \\ y_4' \end{pmatrix}$$

This is of the form $Ax = b$. Vector containing unknown elements of H is given by $x = A^{-1}b$. Finally, we plug the solved values of unknowns in H to get our required homography matrix.

After obtaining H, we apply inverse homography to eliminate the distortions. We first map the vertices of the image to undistorted space and get the dimensions which act as a bounding box in this space. Next, we go through every pixel and map it accordingly using the homography matrix to get the undistorted image from a distorted one.

2-Step Method:

In this approach, we first remove the projective distortion from the image and then the affine distortion.

Remove Projective Distortion:

The projective distortion is removed by a homography which takes Vanishing Line (VL) back to l_∞. We start by picking a pair of lines which are parallel in the original scene. These lines can be found using the cross product of two points which lie on it. From a pair of such lines, we get a vanishing point which is the point of their intersection and is found using cross product. Similarly, we use other set of parallel lines and get another vanishing point. Using the cross product of these two vanishing points we obtain the vanishing line which can be given by $\begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix}$

Using this Vanishing line we get the homography that removes projective distortion and is given by: $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{bmatrix}$

Remove Affine Distortion:

We next eliminate affine distortion by the homography that restores the angle between two orthogonal lines in the original scene back to 90 deg.

The formula for $\cos(\theta)$ in dual degenerate conic is given as:

$$\cos(\theta) = \frac{l^T C_\infty^* m}{\sqrt{(l^T C_\infty^* l)(m^T C_\infty^* m)}} \text{ where } C_\infty^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This can be transformed by putting $l = HTl'$ and $m = HTm'$ plus setting $\cos(\theta)$ equal to 0.

$$cos(\theta) = l'^T H C_\infty^* H^T m' = 0$$

The above expression gives a constraint for H, where affine transformation matrix H is of the form: $H = \begin{bmatrix} A & t = 0 \\ 0^T & 1 \end{bmatrix}$

Expanding further,

$$(l_1' \; l_2' \; l_3') \begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0^T & 0 \end{bmatrix} \begin{bmatrix} A^T & 0 \\ t^T & 1 \end{bmatrix} \begin{pmatrix} m_1' \\ m_2' \\ m_3' \end{pmatrix} = 0$$

$$(l_1' \; l_2' \; l_3') \begin{bmatrix} AA^T & 0 \\ 0^T & 1 \end{bmatrix} \begin{pmatrix} m_1' \\ m_2' \\ m_3' \end{pmatrix} = 0$$

If $S = AA^T$ which is symmetric we get,

$$(l_1' \; l_2') \begin{bmatrix} s_{11} & s_{12} \\ s_{12} & s_{22} \end{bmatrix} \begin{pmatrix} m_1' \\ m_2' \end{pmatrix} = 0$$

We can put $s_{22} = 1$ and expand it to get the following constraint:

$$s_{11} l_1' m_1' + s_{12}(l_1' m_2' + l_2' m_1') = -l_2' m_2'$$

We need 2 set of such equations to solve for unknowns and thus 2 pairs of orthogonal correspondences from the original scene is required. This gives us:

$$\begin{bmatrix} l_1' m_1' & l_1' m_2' + l_2' m_1' \\ n_1' o_1' & n_1' o_2' + n_2' o_1' \end{bmatrix} \begin{pmatrix} s_{11} \\ s_{12} \end{pmatrix} = \begin{pmatrix} -l_2' m_2' \\ -n_2' o_2' \end{pmatrix}$$

Here $(l', m') and \; (n', o')$ are two sets of orthogonal lines from the image. The above equation is of the form $Px = q$ from which we solve for x to get the unknowns to form our S matrix.

After we have S, we can solve for A matrix using SVD.

$$S = AA^T = VDV^T VDV^T = V \begin{bmatrix} \lambda_1^2 & 0 \\ 0 & \lambda_2^2 \end{bmatrix} V^T$$

$$A = V \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} V^T$$

(eigen vectors remain same for A, but eigen values become sq. root)

Thus, we arrive at $H = \begin{bmatrix} A & 0 \\ 0^T & 1 \end{bmatrix}$

<u>1-Step Method:</u>

The projection of dual degenerate conic is given as,

$$C_\infty^{*\prime} = H C_\infty^* H^T$$

With this, $\cos(\theta) = l'^T C_\infty^{*\prime} m' = 0$

Let, $C_\infty^{*\prime} = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix}$ where we can put $f = 1$

For a pair of orthogonal lines l' & m' we can write:

$$(l_1' \; l_2' \; l_3') \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix} \begin{pmatrix} m_1' \\ m_2' \\ m_3' \end{pmatrix} = 0$$

This gives us,

$$\begin{pmatrix} l_1'm_1' & (l_1'm_2' + l_2'm_1')/2 & l_2'm_2' & (l_1'm_3' + l_3'm_1')/2 & (l_2'm_3' + l_3'm_2')/2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} = -l_3'm_3'$$

Similarly, for 5 sets of orthogonal pairs, we will get 5 such equations using which we find the 5 unknowns a,b,c,d,e, and form the $C_\infty^*{}'$ matrix.

After this, the task now is to derive the homography matrix H that satisfies:

$$C_\infty^*{}' = HC_\infty^*H^T$$

$$= \begin{bmatrix} A & 0 \\ v^T & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A^T & v \\ 0^T & 1 \end{bmatrix}$$

$$= \begin{bmatrix} AA^T & Av \\ v^TA^T & v^Tv \end{bmatrix}$$

Subsequently on comparing,

$AA^T = \begin{bmatrix} a & b/2 \\ b/2 & c \end{bmatrix}$ from which we can solve for A, and

$Av = \begin{bmatrix} d/2 \\ e/2 \end{bmatrix}$ will help us get v

With A & v we can form our homography matrix H.

1.1

Given Images:

(a) Img1: Tensor board

Points

On distorted image:

x1, y1 = 74,423
x2, y2 = 424,1783
x3, y3 = 1348,1946
x4, y4 = 1217,144

On undistorted scene:

x1, y1 = 0,0
x2, y2 = 1200,0
x3, y3 = 1200,800
x4, y4 = 0,800

(b) Img2: Corridor

Points

Distorted image:

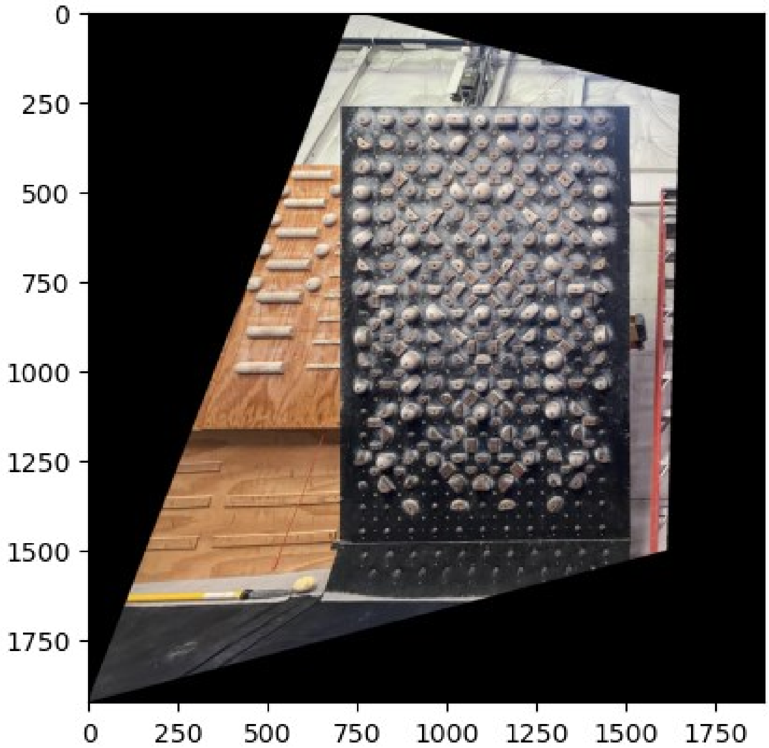
x1, y1 = 1078,521
x2, y2 = 1065,1221
x3, y3 = 1296,1349
x4, y4 = 1307,484

On undistorted scene:

x1, y1 = 0,0
x2, y2 = 600,0
x3, y3 = 600,300
x4, y4 = 0,300

Point-to-Point Correspondence outputs:

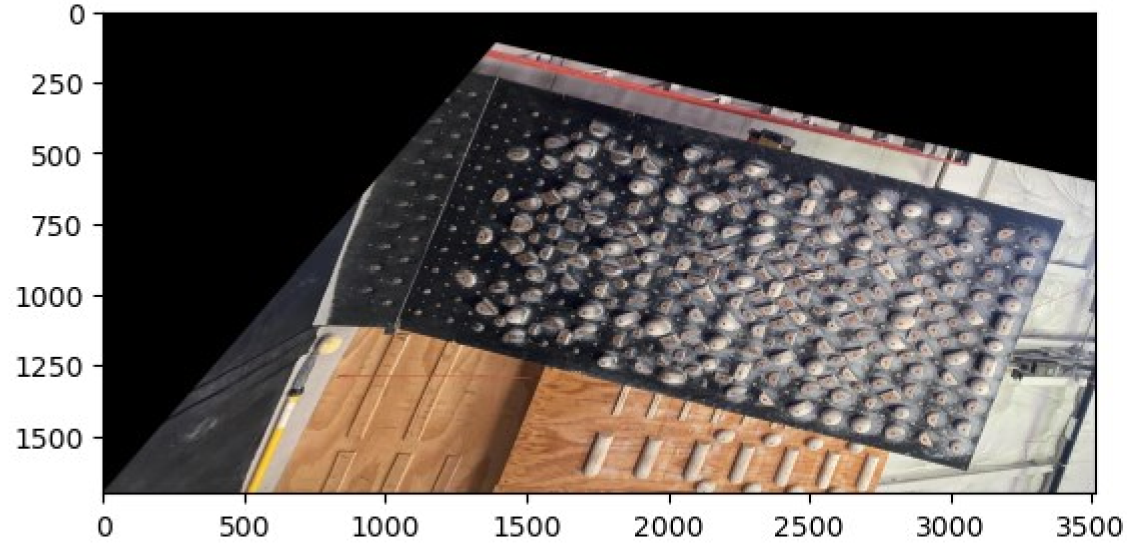
(a) Tensor Board



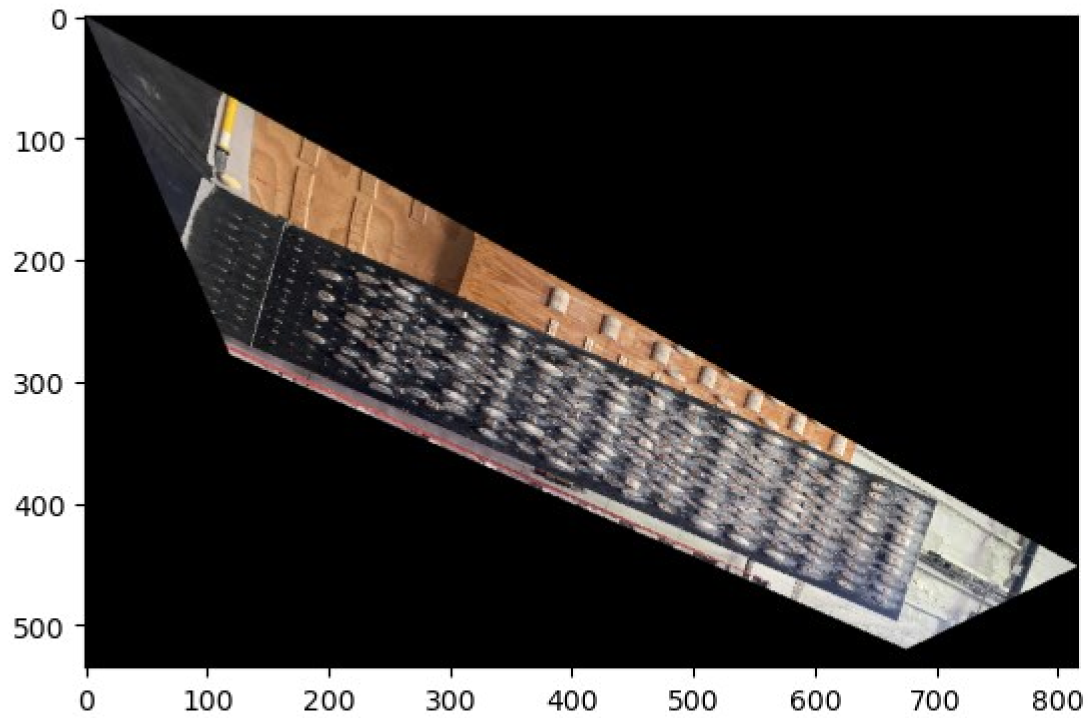
(b) Corridor

<u>1.2</u> 2-Step Method

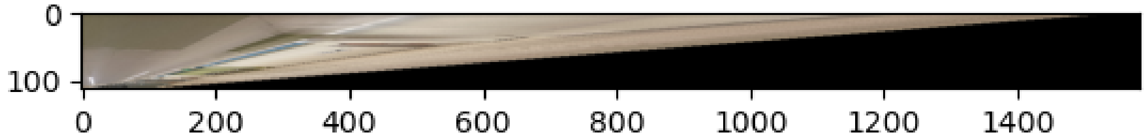
(a.1) Tensor Board after removing projective distortion



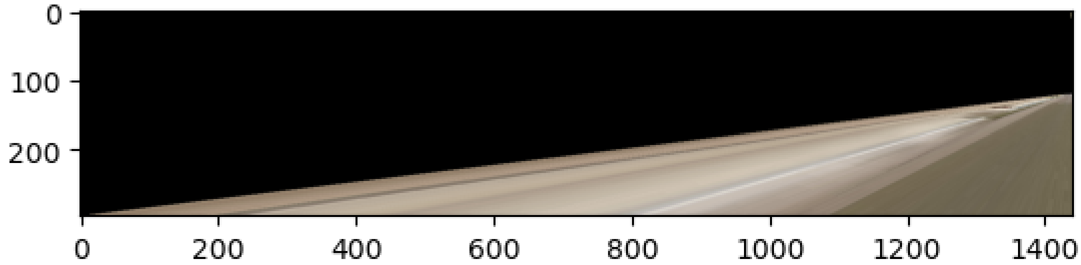
(a.2) Tensor board after removing projective and affine distortions



(b.1) Corridor after removing projective distortion:

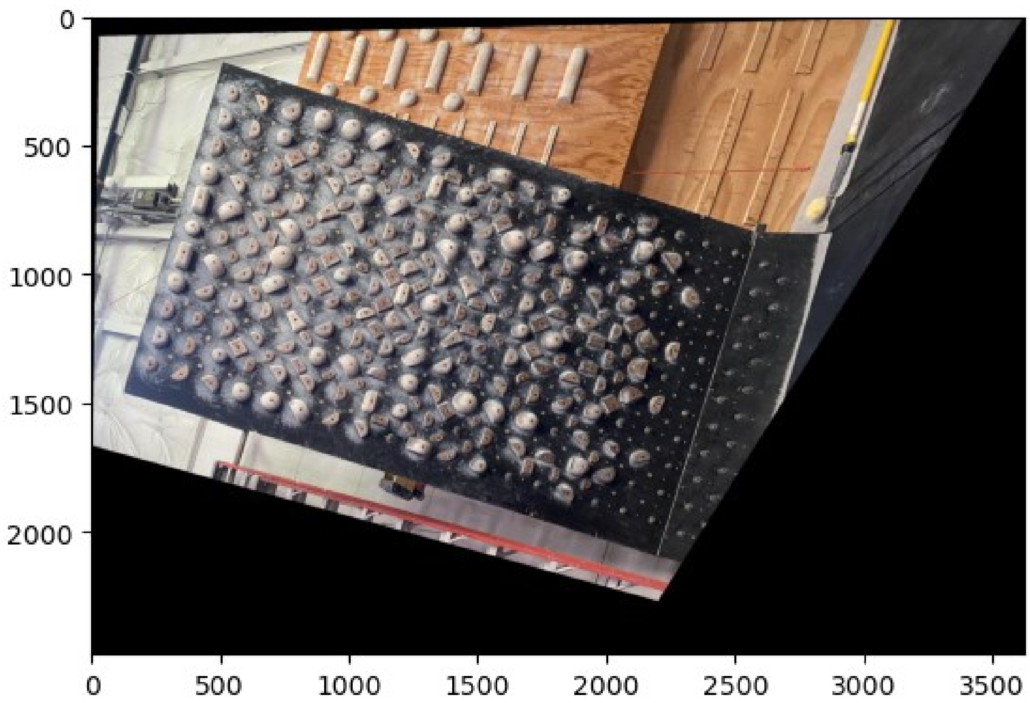


(b.2) Corridor after removing projective and affine distortions

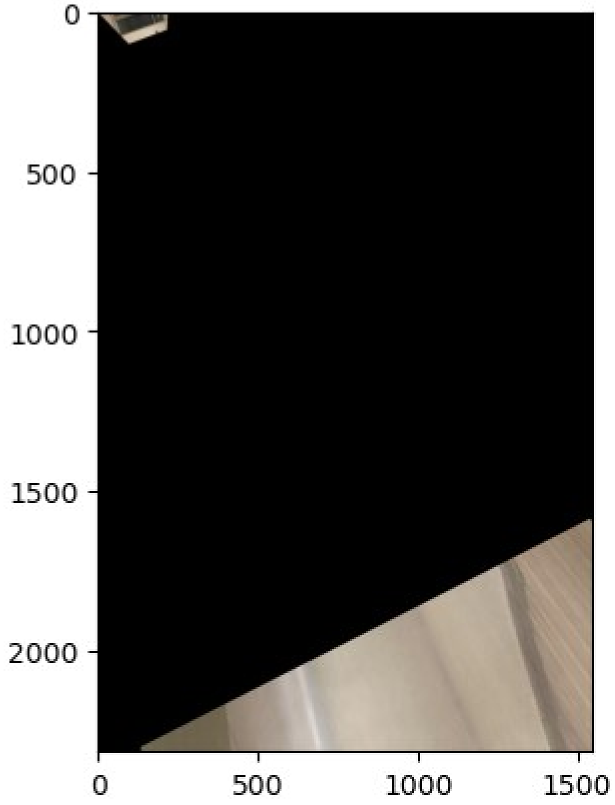


<u>1.3</u> 1 Step Method

(a) Tensor Board



(c) Corridor



2.

Other Images:

(c)Img3: Wall hanging
Dim: 1.5’x1.5’

Points:
On Distorted image:
x1, y1 = 511,875
x2, y2 = 116,2313
x3, y3 = 1658,2335
x4, y4 = 1921,430

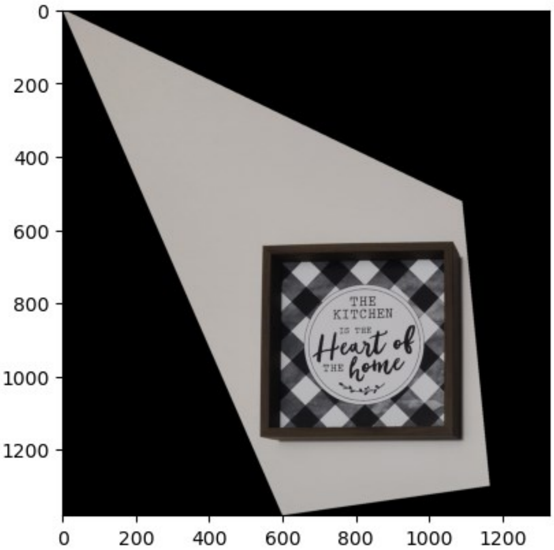
On Original scene:
x1, y1 = 0,0
x2, y2 = 500,0
x3, y3 = 500,500
x4, y4 = 0,500

(d)Img4: Amplifier
Dim:25cmX25cm

Points:
On Distorted image:
x1, y1 = 127,526
x2, y2 = 383,2130
x3, y3 = 1822,2120
x4, y4 = 2023,471

On Original scene:
x1, y1 = 0,0
x2, y2 = 600,0
x3, y3 = 600,600
x4, y4 = 0,600

point-point correspondence:

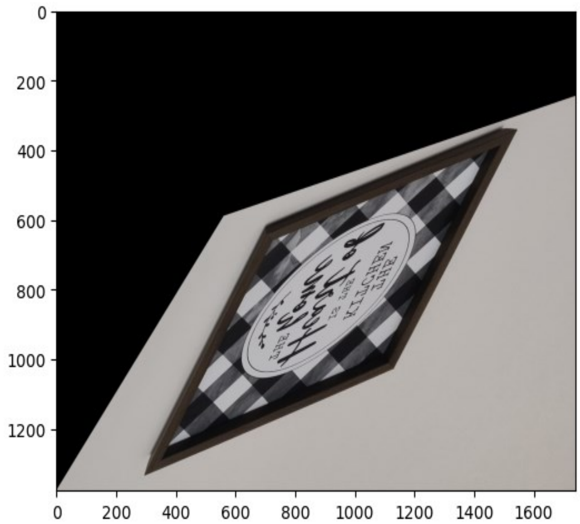


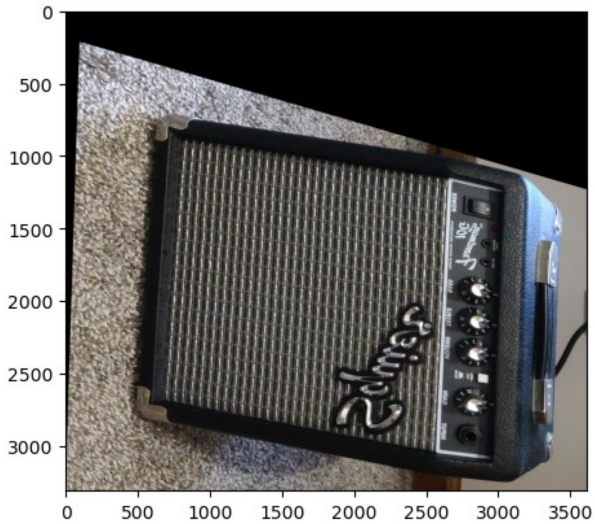
(c)Wall hanging

(d)Amplifier

2 Step Method:

After removing projective distortion

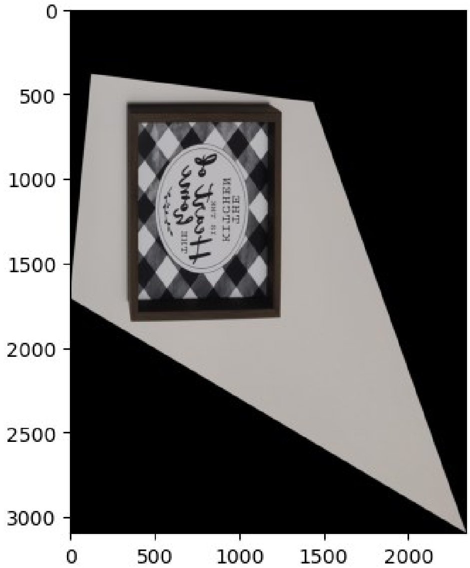




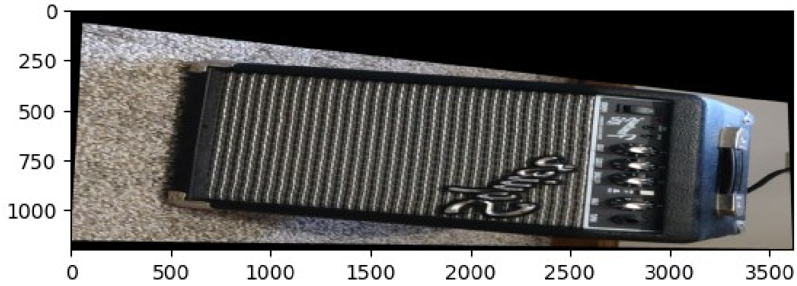
(c)Wall hanging

(d)Amplifier

After removing projective and affine distortions:

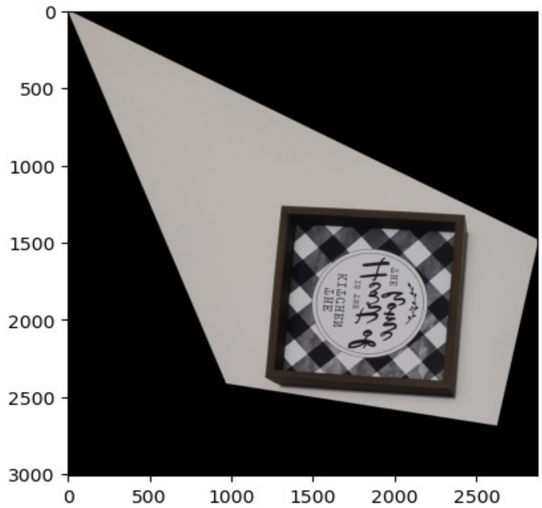


(c)Wall Hanging

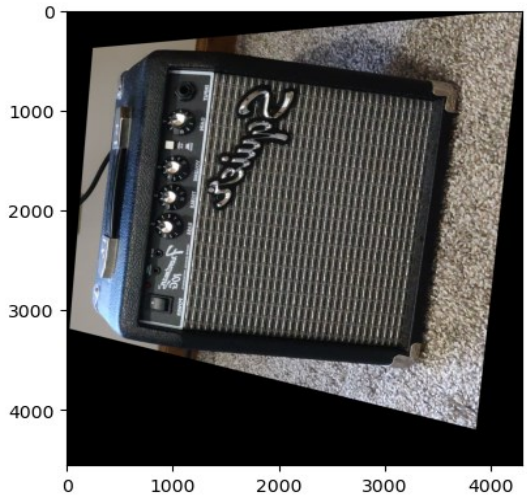


(d)Amplifier

<u>1-Step method:</u>



(c)Wall Hanging



(d)Amplifier

Observations:

Selection of points plays an important role in the quality of results and performace. Normalizing the HC representations is also important as it helps in numerical stability leading to faster and better results. The first approach of point-to-point correspondence resulted in very good outputs, almost identical to the original scene especially in the case of wall hanging and amplifier images. The performance of this method was usually faster than the other two approaches. In the second approach, we successively remove projective and affine distortions. This 2-step approach due to added complexity took more time than the previous approach but was usually faster than 1-step method. Its outputs were decent but somewhat inferior to the first approach. The 1-step method although slower than the second approach resulted in better outputs. Still, these were not as great as those from the first approach. While all three approaches were quite successful in removing projective and affine distortions from most images, corridor image proved to be very challenging. Difficulty in processing the corridor image could have been due to poor selection of points and fairly low distortions(far vanishing point).

Markings: