

# Formatting Instructions for RLC Submissions

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## Abstract

Hello

## 1 Introduction

In reinforcement learning, ...

## 2 Problem Setting

Reinforcement learning is often modeled as a Markov decision process (MDP) given by the tuple  $M = (\mathcal{S}, \mathcal{A}, P, R)$ . In this model, an agent and environment interact over a sequence of time steps  $t$ . At each time step, the agent receives a state  $S_t \in \mathcal{S}$  from the environment, where  $\mathcal{S}$  denotes the set of all possible states. The agent uses the information given by the state to select an action  $A_t$  from the set of possible actions  $\mathcal{A}$ . Based on the state of the environment and the agent's behavior, i.e. action, the agent receives a scalar reward  $R_t = R(S_t, A_t)$  and transitions to the next state  $S_{t+1} \in \mathcal{S}$  according to the state-transition probability  $P(s'|s, a) = P(S_{t+1} = s' | S_t = s, A_t = a)$  for each  $s, s' \in \mathcal{S}$  and  $a \in \mathcal{A}$ .

The behavior of an agent is given by a policy  $\pi(a|s)$ , which is a probability distribution over actions given a state. The agent's goal is to learn the optimal policy,  $\pi^*$ , which is the policy that maximizes the expected discounted return

$$G_t = R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \dots = \sum_{k=1}^{T-t} \gamma^{k-1} R_{t+k-1}$$

either for a discounted factor  $\gamma \in [0, 1)$  when the task is continuing,  $T = \infty$ , or  $\gamma \in [0, 1]$  and  $T < \infty$  in episodic task.

Through the process of policy iteration, Monte-Carlo algorithms strive to maximize the expected return by computing value-functions that estimate the expected future returns. The state-value function quantifies the agent's expected return starting in state  $s$  and following policy  $\pi$ , i.e.  $v^\pi(s) = \mathbb{E}_\pi[G_t | S_t = s]$ . When learning to control in RL, we often want to estimate the action-value function, which is the agent's expected return starting in state  $s$ , taking action  $a$  and then following policy  $\pi$ , i.e.

$$q_\pi(s, a) = \mathbb{E}_\pi[G_t | S_t = s, A_t = a].$$

Often in RL, it is useful to use function approximation to learn the action-values.

## 3 Algorithm

**Proposition 3.1.** *Given tolerance  $\tau \geq 0$  and a real-valued list  $xs$  of size  $N$ , Algorithm 1 returns an approximate sum  $Q$  that satisfies  $|Q - \text{sum}(xs)| \leq \tau$ .*

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**Algorithm 1** ADAPTIVE-QUADRATURE

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Input: A list  $xs$  and tolerance  $\tau \geq 0$ 
 $N \leftarrow \text{length}(xs)$ .
if  $N > 2$  then
     $Q \leftarrow N \cdot (xs[0] + xs[-1])/2$ .
else
    return  $\text{sum}(xs)$ .
end if
 $\varepsilon \leftarrow |Q - \text{sum}(xs)|$ 
if  $\varepsilon \geq \tau$  then
     $c \leftarrow \lfloor N/2 \rfloor$ .
     $Q \leftarrow \text{ADAPTIVE-QUADRATURE}(xs[:c], \tau/2) + \text{ADAPTIVE-QUADRATURE}(xs[c:], \tau/2)$ 
end if
return  $Q$ .

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*Proof.* This proof will follow by induction. Assume that  $N < 3$ . Then Algorithm 1 returns  $Q = \text{sum}(xs)$ . Therefore  $|Q - \text{sum}(xs)| = |\text{sum}(xs) - \text{sum}(xs)| = 0 \leq \tau$  for all  $\tau \geq 0$ .

Now assume that for a fixed  $N > 2$  and  $\tau \geq 0$  it holds that  $|Q - \text{sum}(xs)| \leq \tau$ . □

## A Experimental Details