Last Time:
- B-dot Control
- Actuator Jacobians

Today:

- Flat Spin Recovery
- Attitude Regulation

Flat Spin Recovery:

- Spacecraft can end up in an undesired major-axis spin (flat spin) due to malfunctions.

- We need a control law to recover the spacecraft's nominal minor-axis spin (especially applicable to dual-spin spacecraft).

- Basic Idea: Minor-axis spin is a maximum energy state for the system, so just pump kinetic energy into the system with the controller.

$$T = \frac{1}{2}\omega^T J \omega \Rightarrow \dot{T} = \omega^T J \dot{\omega}$$

$$J\dot{\omega} + \dot{\rho} + \omega \times (J\omega + \rho) = \tau$$

$$\Rightarrow \dot{\omega} = -J^{-1}(\dot{\rho} + \omega \times (J\omega + \rho) - \tau)$$

$$\Rightarrow \dot{T} = -\omega^T \cancelto{I}{J J^{-1}}(\dot{\rho} + \cancelto{0}{\omega \times (J\omega + \rho)} - \tau)$$

$$= -\omega^T \dot{\rho} + \omega^T \tau$$

* We want $\dot{T} \geq 0$, so a reasonable control law is:

$$\dot{\rho} = -K\omega$$

$\uparrow$ gain

- Can also implement with thrusters, but this is less common because it uses limited fuel.

- This same control law also works as a nutation damper. Often implementing damping with a wheel offers better performance for less weight than a mechanical/fluid damper.

- Only one wheel is required to implement this controller in most cases (including a dual spinner since the rotor axis is perpendicular to ω in a flat spin).

- This controller cannot guarantee the final sign of ${}^B\omega$, since there are two mino-axis equilibria with the same energy.

Attitude Regulation:

- Holding a desired attitude is pretty straight forward.

- Let's assume we want to maintain a desired attitude $q = q_0$ and $\omega = 0$

- The full dynamics of the spacecraft are:

$$\dot{q} = \frac{1}{2} q \begin{bmatrix} \omega \\ 0 \end{bmatrix} \quad (\text{or } \dot{Q} = Q\hat{\omega})$$

$$J\dot{\omega} + \dot{\rho} + \omega \times (J\omega + \rho) = \tau$$

$$\dot{\rho} = Bu$$

- Lets linearize about $q = q_0$, $\rho = 0$, $\omega = 0$:

$$\dot{q}_e = \frac{1}{2} q_0^+ q \begin{bmatrix} \omega \\ 0 \end{bmatrix} \leftarrow \text{error quaternion}$$

$$J\dot{\omega} + \dot{\rho} = \tau$$