Last Time:

- Energy Dissipation
- Gyrostats
- Superspin

Today:

- Dynamic Balance
- Attitude Determination
- Attitude Sensors

Superspin (Last time):

- Stable spin about non-major axis:

$$ {}^{p}h_2 = {}^{p}J_{22}\,{}^{p}\omega_2 + {}^{p}\rho_2 = \underbrace{\left(J_{22} + \frac{\rho_2}{\omega_2}\right)}_{J_{eff}} \omega_2 $$

- Choose ρ_2 so $J_{eff} > J_{33}$
- Good rule is $J_{eff} = 1.2\,J_{33}$

Dynamic Balance

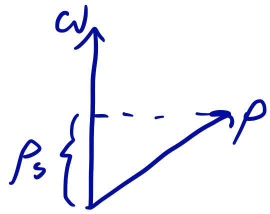
- What if we want to spin a satellite about a non-principle axis?
- Equilibrium spin conditions: ${}^{B}\dot{\omega} = \dot{\rho} = 0$

$$ J\cancel{\dot{\omega}} + \cancel{\dot{\rho}} + \omega \times (J\omega + \rho) = 0 $$

$$ \Rightarrow \omega \times (J\omega + \rho) = 0 $$

choose ρ to make this whole term zero

- We also need superspin condition for stability



$$\rho_s = \frac{p \cdot \omega}{\|\omega\|} \Rightarrow {}^B p^T\, {}^B\omega = \omega_s \rho_s$$

- only the component of p parallel to $\underline{\omega}$ contributes to J_{eff}:

$$J_{eff} = J_s + \frac{\rho_s}{\omega_s} \geq J_{33} \quad (= 1.2\, J_{33})$$

(solve for ρ_s)

- Stack no-wobble and superspin conditions:

$$\underbrace{\begin{bmatrix} {}^B\omega^T \\ {}^B\hat{\omega} \end{bmatrix}}_{4\times 3} \underbrace{{}^B p}_{3\times 1} = \underbrace{\begin{bmatrix} \omega_s \rho_s \\ -\omega \times J\omega \end{bmatrix}}_{4\times 1}$$

- Moore-Penrose Pseudoinverse:

$$\underbrace{A}_{m\times n} x = \underbrace{b}_{m\times 1}, \quad A \in \mathbb{R}^{m\times n}, \quad m > n$$

$$\underbrace{(A^T A)}_{n\times n} \underbrace{x}_{n\times 1} = \underbrace{A^T b}_{n\times 1} \Rightarrow x = \underbrace{(A^T A)^{-1} A^T}_{A^+ \text{ "pseudoinverse of } A\text{"}} b$$

- "least-squares" solution
- best solved with QR decomposition
- Just use backslash in MATLAB (x = A\b)

- Solve for p using pseudoinverse:

$$p = \left([\omega \;\vdots\; -\hat{\omega}] \begin{bmatrix} \omega^T \\ -\hat{\omega} \end{bmatrix} \right)^{-1} [\omega \;\vdots\; -\hat{\omega}] \begin{bmatrix} \omega_s \rho_s \\ -\omega \times J\omega \end{bmatrix}$$

Attitude Estimation:

- How to figure out attitude using onboard sensors
- How do we deal with attitude statistically?

Attitude Sensors:

* Gyroscopes
  - Measure ${}^B\omega$
  - Many kinds with wide variation in cost + performance

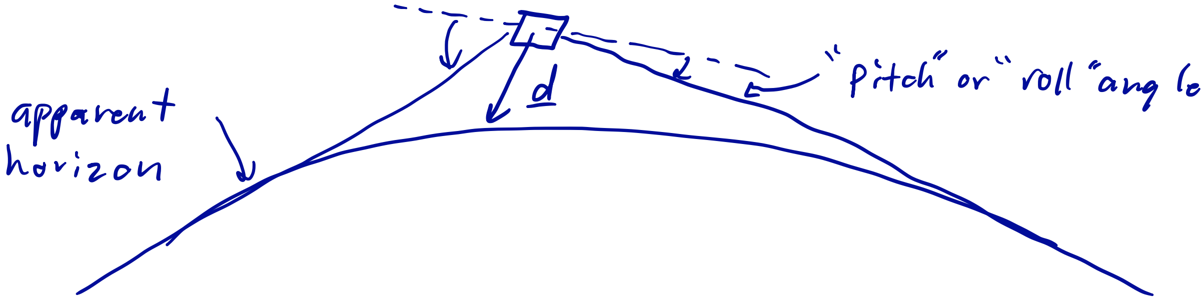
* Magnetometers
  - Measures Earth's magnetic field: ${}^B B$
  - Only useful in low-Earth orbits
  - Coarse: ~few degrees error
  - Issues with EM noise from other parts of the spacecraft
  - Simple + low cost

* Star Tracker
  - Camera looks at stars and compares to a database/map
  - Works anywhere but sensative to blinding by e.g. sun
  - Gives full attitude quaternion
  - High accuracy: ~1 arc sec (~.0003°) errors
  - expensive
  - Limited slew rates tollerated

* Sun Sensor
  - Measures vector to sun: ${}^B r_{sun}$
  - Coarse: ~few degrees error
  - Don't work in eclipse
  - Simple + low cost

* Earth Sensor

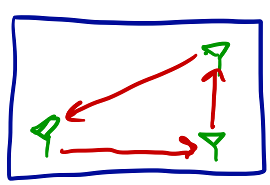
- Measures "pitch" and "roll" angles vs. horizon
- Gives "down" unit vector in body frame: ${}^B d$



- Moderate accuracy ~0.1° error
- Simple + robust
- Work well in lower orbits

* GPS

- Use 3 or more GPS antennas to get "baseline" vectors



- Moderate accuracy ~ 0.1°