Last Time:

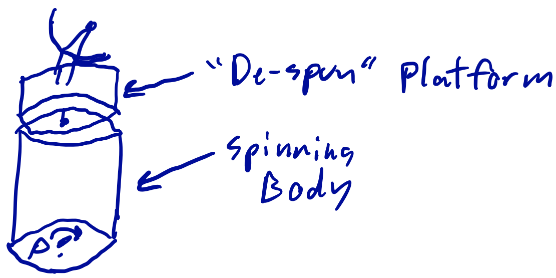
- Gravity Gradient Stabilization

Today:

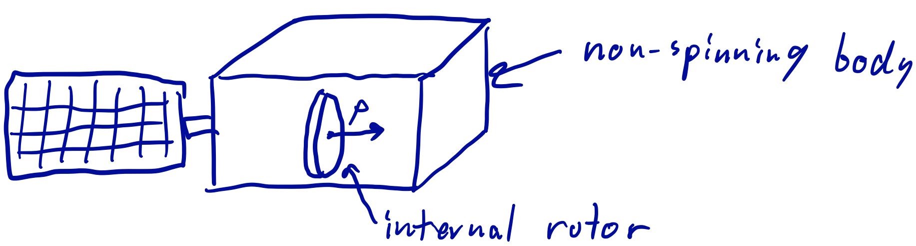
- Dual Spin + Momentum Bias
- Magnetic Torque Coils

Dual Spin / Momentum Bias:

- Dual Spin:



- Momentum Bias:



- Mathematically these are identical. They're both gyrostats:

$$ {}^B J\, {}^B\dot{\omega} + {}^B\dot{\rho} + {}^B\omega \times \left({}^B J\, {}^B\omega + {}^B\rho \right) = {}^B\tau $$

Gyrostat Stability:

- Assume rotor axis corresponds to $\underline{b}_3$
- Assume ${}^{B}J$ is diagonal:

$$ {}^{B}J = \begin{bmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_s \end{bmatrix} $$

- Assume a nominal spin ${}^{B}\omega = \begin{bmatrix} 0 \\ 0 \\ \omega_0 \end{bmatrix}$

$$ \begin{cases} J_x \dot{\omega}_x + (J_s - J_y)\omega_0 \omega_y + \rho \omega_y = 0 \\ J_y \dot{\omega}_y + (J_x - J_s)\omega_0 \omega_x - \rho \omega_x = 0 \\ J_s \dot{\omega}_z + \dot{\rho} = 0 \end{cases} $$

- Spin axis dynamics are decoupled and assumed stabilized by a motor controller to compensate for bearing friction.

- Transverse Dynamics:

$$ \frac{d}{dt}\begin{bmatrix} \omega_x \\ \omega_y \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1}{J_x}\left[(J_y - J_s)\omega_0 - \rho\right] \\ \frac{1}{J_y}\left[(J_s - J_x)\omega_0 + \rho\right] & 0 \end{bmatrix}}_{\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}} \begin{bmatrix} \omega_x \\ \omega_y \end{bmatrix} $$

* $\det(A - \lambda I) = 0 \Rightarrow \lambda = \pm\sqrt{ab} \Rightarrow$ we want $ab < 0$ for marginal stability

$$ \Rightarrow \boxed{\begin{array}{lcl} \rho > (J_x - J_s)\omega_0 & \text{and} & \rho > (J_y - J_s)\omega_0 \\ & \text{OR} & \\ \rho < (J_x - J_s)\omega_0 & \text{and} & \rho < (J_y - J_s)\omega_0 \end{array}} $$

Gyrostat Equilibria Revisited:

- Equilibrium $\Rightarrow \dot{\omega} = \dot{\rho} = 0$

  $$\Rightarrow {}^B\omega \times ({}^BJ\,{}^B\omega + {}^B\rho) = 0$$

- In words: ${}^B\omega$ must be parallel to ${}^BJ\,{}^B\omega + {}^B\rho = {}^Bh$

- Mathematically this can be written:

  $$J\omega + \rho = \lambda\omega$$

  scalar multiplier

  * $\lambda = \frac{\|h\|}{\|\omega\|}$ = "effective inertia"

- This is a "generalized eigenvalue problem" aka. "matrix pencil. It is surprisingly tricky to solve.

- Hughes solves this by turning it into a 6^{TH} order polynomial and finding its roots.

- Another solution is to solve the following eigenvalue problem:

  $$\det\left(\begin{bmatrix} 2\|h\|J^{-1} & J^{-2}(\rho\rho^T - h^T h I) \\ I & 0 \end{bmatrix} - \Omega I\right) = 0$$

  scalar magnitude $\|{}^B\omega\|$

- In general Ω_i will be complex. Only the real solutions correspond to physical equilibria.

- Unlike free rigid body, there can be 2, 4, or 6 equilibria.

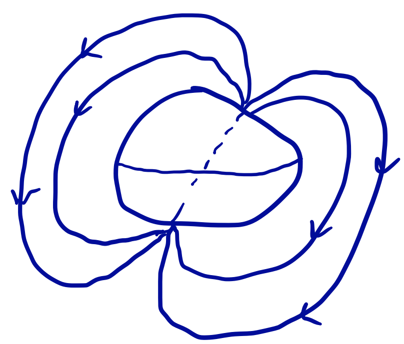
- Once you have Ω, solve for ${}^{B}\omega$:

$$ {}^{B}\omega = \left(\frac{\|h\|}{\Omega} I - J \right)^{-1} \rho $$

- For global stability, make sure effective inertia $\lambda = \lambda_{max}$

Magnetic Torque Coils:

- Earth's Magnetic field is a tilted dipole to 1st order:



*Tilt is $\approx 11°$

Earth radius

Scalar dipole strength

Earth dipole axis unit vector

$$ B(r) = \frac{-R_E^3 B_0}{\|r\|^3} \left(3 \frac{r r^T}{r^T r} - I \right) m_E $$

spacecraft position vector from center of Earth

- Note m_E is fixed in ECEF coordinates
- The torque on a coil is given by:

$$ \underline{\tau}_c = \underline{m}_c \times \underline{B} $$

magnetic field vector

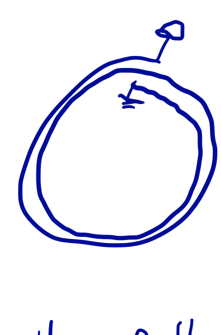
coil magnetic moment

$$ \|m_c\| = n I A $$

Enclosed area

current

number of turns



- Vector direction of $\underline{m}_c$ is right hand rule following current flow around coil (should be normal to coil)