What is Attitude?

- Relationship between 2 reference frames (typically S/C body and some inertial frame)
- A Lie group $SO(3)$

What is a Reference Frame?

- Set of mutually orthogonal basis vectors that form a right-handed coordinate system
- For our purposes "Reference Frame" and "Rigid body" go together
- We will primarily deal with 2 kinds:
  - "Inertial" or "Newtonian" reference frame ⇒ Newton's laws hold
  - "Body-fixed" or just "body" reference frame ⇒ attached to (and rotates with) a rigid body

Vectors and Reference Frames:

- Physical vectors exist independent of our choice of reference frame or coordinates

- When we want to perform calculations with numbers, we project $\underline{v}$ onto a set of basis vectors and write down its components:

$$\underline{v} = {}^N v_1 \underline{n}_1 + {}^N v_2 \underline{n}_2 + {}^N v_3 \underline{n}_3 = \begin{bmatrix} \underline{n}_1 \\ \underline{n}_2 \\ \underline{n}_3 \end{bmatrix}^T \begin{bmatrix} {}^N v_1 \\ {}^N v_2 \\ {}^N v_3 \end{bmatrix} = \underline{n}^T \, {}^N v$$

$$= {}^B v_1 \underline{b}_1 + {}^B v_2 \underline{b}_2 + {}^B v_3 \underline{b}_3 = \underbrace{\begin{bmatrix} \underline{b}_1 \\ \underline{b}_2 \\ \underline{b}_3 \end{bmatrix}^T}_{\text{"vectrix"}} \underbrace{\begin{bmatrix} {}^B v_1 \\ {}^B v_2 \\ {}^B v_3 \end{bmatrix}}_{\text{"column matrix"}} = \underline{b}^T \, {}^B v$$

- It is very important to distinguish a vector from its components in different reference frames!

How do we Parametrize Attitude?

- A convention usually followed in aerospace is to use the rotation from the vehicle body frame "B" to some chosen inertial frame "N"
- Unfortunately this is not universal and in physics the opposite convention is common
- Convenient for thinking about where sensors are pointed etc.

How do we Write Down Rotations?

- Euler Angles (roll pitch yaw):

  pro: Minimal (3 parameters), intuitive

  con: Singularities at 90°, trig functions in Kinematics

- Rotation Matrix:

  pro: Non-singular, easy to rotate vectors, linear Kinematics

  con: Redundant; 9 numbers for 3 DOF ⇒ 6 constraints

- Quaternions:

  pro: Non-singular, easy to compute dynamics

  con: Redundant; 4 numbers for 3 DOF ⇒ 1 constraint

- Gibbs Vector / Rodrigues Parameters

  pro: Minimal, polynomial Kinematics

  con: Singular at 180° (MRP at 360°)

- Rotation vector / Axis-Angle

  pro: Minimal, Intuitive

  con: Singularities in Kinematics at 180°