Last Time:

- Moment of Inertia
- Euler's Equation
- Equilibria + Stability

Today:

- Solutions to Euler's Equation
- Energy Dissipation
- Gyrostats

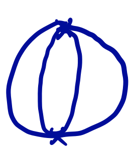
Solutions + Momentum Sphere:

- There is an analytic solution to Euler's equation but it is very ugly and not very useful in practice
- Usually numerical methods are used
- A lot of insight can still be gained from looking at the qualitative behavior

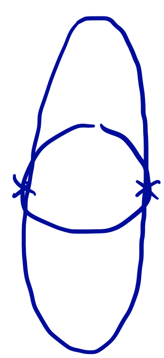
$$\|h\| = \text{constant} \Rightarrow \text{solutions lie on "momentum sphere"}$$

$$h^T J^{-1} h = \text{constant} \Rightarrow \text{solutions lie on "energy ellipsoid"}$$

- Trajectories are intersections of momentum sphere and energy ellipsoid

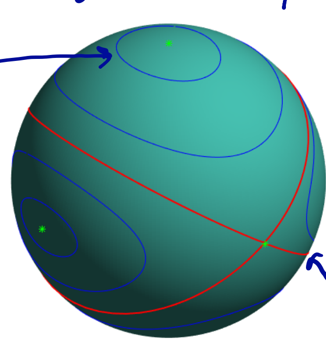


minimum energy (major-axis spin)



maximum energy (minor axis spin)

periodic motion is called "nutation"



"separatrix"

Energy Dissipation:

- Last time: minor + major axes are stable when there is no energy dissipation

$$\|h\| = \text{constant} \quad , \quad T = \frac{1}{2}\omega^T J \omega = \frac{1}{2} h^T J^{-1} h$$

* Maximum and minimum kinetic energy for a given momentum:

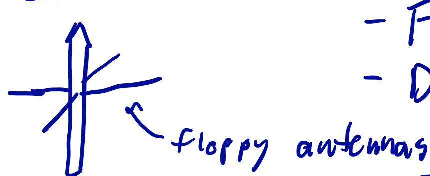
$$T_{max} = \frac{1}{2}\frac{1}{J_{11}}\|h\|^2 \quad , \quad T_{min} = \frac{1}{2}\frac{1}{J_{33}}\|h\|^2$$

$\Rightarrow$ **If there is any energy dissipation, only the maximum axis is stable since it is the minimum energy state**

* When does energy dissipation come from?

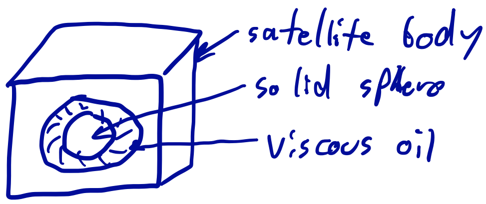
- Fluid slosh
- Damping associated with structural modes

* Explorer 1



- First US satellite
- Designed as a minor-axis spinner
- Entered a flat spin due to damping from flexible antennas

* A Simple Model of Internal Damping



$${}^B\tau_d = c_d({}^B\omega_d - {}^B\omega)$$

coupled system of ODEs

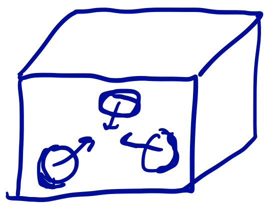
$$\begin{cases} {}^B J\, {}^B\dot{\omega} + {}^B\omega \times {}^B J\, {}^B\omega = c_d({}^B\omega_d - {}^B\omega) \\ J_d\, {}^B\dot{\omega}_d = c_d({}^B\omega - {}^B\omega_d) \end{cases}$$

- Known as a "Kane Damper"
- 2 parameters: J_d (scalar) and c_d (damping constant)
These can be fit to data to model e.g. fluid damping

Gyrostats:

- A system of rigid bodies whose relative motion does not change the total system moment of inertia

- Think of a box (spacecraft bus) with spinning rotors (momentum/reaction wheels) inside



- Spacecraft pointing and spin can be chosen arbitrarily

- Attitude disturbances can be handled with active control

* Gyrostat Dynamics

- Modify Euler's equation to include rotors

$$\underbrace{h}_{\text{total momentum}} = \underbrace{J\omega}_{\text{body momentum}} + \underbrace{\rho}_{\text{rotor momentum}}$$

$${}^N\dot{h} = {}^N\tau = {}^NQ^B\left({}^B\dot{h} + {}^B\omega \times {}^Bh\right)$$

$$\Rightarrow \boxed{{}^BJ\,{}^B\dot{\omega} + {}^B\dot{\rho} + {}^B\omega \times \left({}^BJ\,{}^B\omega + {}^B\rho\right) = {}^B\tau}$$ "gyrostat equation"

- We get to pick ρ and $\dot{\rho}$

Superspin:

- What if we want to spin a satellite about its minor or intermediate axis?

- Assume a nominal spin $^{\mathcal{P}}\omega = \begin{bmatrix} ^{\mathcal{P}}\omega_1 \\ 0 \\ 0 \end{bmatrix}$

$$^{\mathcal{P}}h_1 = {}^{\mathcal{P}}J_{11}\,{}^{\mathcal{P}}\omega_1 + {}^{\mathcal{P}}\rho_1 = \underbrace{\left(J_{11} + \frac{\rho_1}{\omega_1}\right)}_{\text{"Effective Inertia"}}\omega_1$$

- Given a desired ω_1, choose ρ so that $J_{eff} > J_{33}$

- A good engineering rule is $J_{eff} = 1.2 J_{33}$ (20% margin)