Last Time:

- Flat Spin Recovery

Today:

- Attitude Regulation
- Large Angle Maneuvers

Attitude Regulation:

- Problem: Hold a fixed attitude with a feedback controller using reaction wheels or thrusters.

* Full Gyrostat Dynamics:

$$\dot{q} = \frac{1}{2} q \begin{bmatrix}\omega\\0\end{bmatrix} \quad (\text{or } \dot{Q} = Q\hat{\omega})$$

$$\dot{\omega} = -J^{-1}\left(\omega \times (J\omega + \rho) + \dot{\rho} - \tau\right)$$

* State is $x = \begin{bmatrix}q\\ \omega\\ \rho\end{bmatrix}$

- Approach: Lineariz about a desired attitude q_0 and $\omega = 0$

$$\Rightarrow \quad \dot{\omega} \approx -J^{-1}\dot{\rho} + J^{-1}\tau$$

* Linearize Kinematics with an axis-angle vector:

$$q_e = q_0^+ q \approx \begin{bmatrix}\frac{1}{2}\phi\\ 1\end{bmatrix}$$

$$\dot{q}_e = q_0^+\dot{q} = \frac{1}{2} q_0^+ q \begin{bmatrix}\omega\\0\end{bmatrix} = \frac{1}{2} q_e \begin{bmatrix}\omega\\0\end{bmatrix}$$

2nd order

$$\Rightarrow \dot{q}_e \approx \begin{bmatrix}\frac{1}{2}\dot{\phi}\\ 0\end{bmatrix} \approx \frac{1}{2}\left(\begin{bmatrix}0\\1\end{bmatrix} + \begin{bmatrix}\phi\\0\end{bmatrix}\right)\begin{bmatrix}\omega\\0\end{bmatrix} = \frac{1}{2}\left(\begin{bmatrix}\omega\\0\end{bmatrix} + \cancel{\begin{bmatrix}\phi\times\omega\\ -\phi^T\omega\end{bmatrix}}\right)$$

$$\Rightarrow \boxed{\dot{\phi} \approx \omega}$$

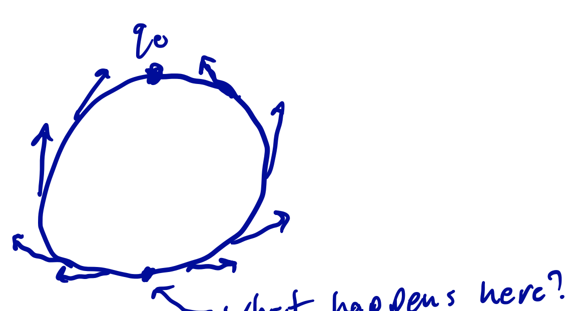
- Since ρ doesn't show up in our linearized dynamics, we'll ignore it for the purposes of this controller.
- Putting everything together:

$$\underbrace{\begin{bmatrix} \dot{\phi} \\ \dot{\omega} \end{bmatrix}}_{\dot{x}} \approx \underbrace{\begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \phi \\ \omega \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 & 0 \\ -J^{-1} & J^{-1} \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \dot{\rho} \\ \tau \end{bmatrix}}_{u}$$

- To 1^{st} order the satellite is a double integrator plant.
- Any linear control technique can be used (e.g. pole placement or LQR).
- Similar to the MEKF, we have a step outside the linear controller to convert quaternions to axis-angle vectors:

$$\phi = \check{\log}(Q_0^T Q) \approx 2\,\text{vec}(q_0^+ q)$$

- Note that because of linearization the controller will have a limited basin of attraction (it won't necessarily converge from very large errors).
- There is a general result that no time-invariant feedback law can be globally stabilizing when the configuration manifold is a sphere:



- This is related to the "Hairy Ball Theorem"

Large-Angle Maneuvers:

- For large-angle maneuvers it's not a good idea to simply command the new q_0 in the regulator controller.

- We really want a "motion plan" - a nominal trajectory that includes $q(t)$, $\omega(t)$, $\rho(t)$, $\dot{\rho}(t)$, $\tau(t)$.

- Given a motion plan, we can track it with a time-varying linear controller.

- There are many ways to generate one of these trajectories, from closed-form analytic solutions to numerical trajectory optimization.

* Eigen-Axis Slew:

- The rotation from our current attitude Q to our desired attitude Q_0 is given by:

$$^{B_0}Q^{B} = {}^{B_0}Q^{N}\,{}^{N}Q^{B} = Q_0^T Q$$

- If we want to spin about a fixed axis to get from Q to Q_0, that axis can be found using eigendecomposition of $^{B_0}Q^{B}$.

- The axis of rotation corresponds to the only real eigenvector of $^{B_0}Q^{B}$

- While this is the classic method in the litterature, we can also use the matrix log:

$$\phi = \overset{\vee}{\log}({}^{B_0}Q^{B})$$

- We can use this axis-angle vector to create a trajectory:

$$\phi(t) = \theta(t)\, r$$

unit vector axis of rotation

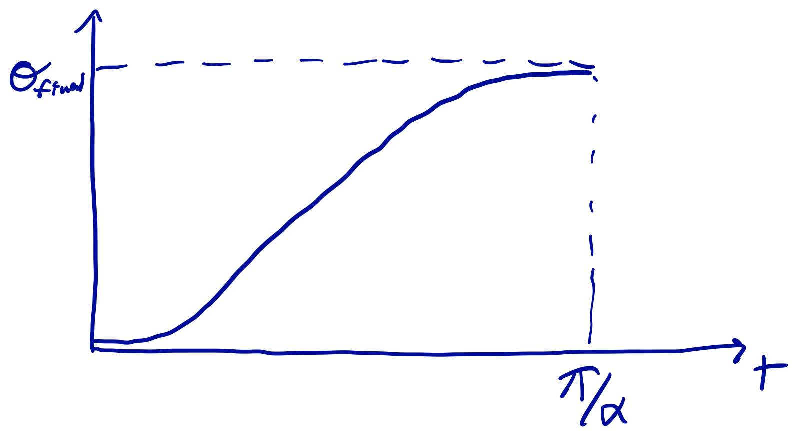
angle is a function of time

$$\Rightarrow \quad Q(t) = e^{\hat{\phi}(t)}, \quad \omega(t) = \dot{\theta}(t)\, r$$

- A common choice for $\theta(t)$ is the "versine" function:

$$\theta(t) = \|\theta_{final}\| \tfrac{1}{2}(1 - \cos(\alpha t))$$

time constant sets how long the maneuver takes



- ρ, $\dot{\rho}$, and/or τ can be found by plugging ω and $\dot{\omega}$ into the gyrostat equation.

Inverse Dynamics:

- Assume we have a smooth trajectory $q(t)$ that we can differentiate to get $\omega(t)$ and $\dot{\omega}(t)$

$$J\dot{\omega} + \dot{\rho} + \omega \times (J\omega + \rho) = 0$$

$$\Rightarrow \quad \dot{\rho} = -J\dot{\omega} - \omega \times (J\omega + \rho)$$

- Start with $\rho(0)$, solve for $\dot{\rho}(t)$ and integrate to get $\rho(t)$.

- Even easier with thrusters:

$$\tau = J\dot{\omega} + \omega \times J\omega$$

- For a generic mechanical system:

$$\underbrace{M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)u}_{\text{"Manipulator equation"}}$$

$$\Rightarrow \quad u = B^{-1}\left[M\ddot{q} + C\dot{q} + G\right]$$

* Note that this is only possible if B is full rank. This property is known as being "fully actuated". If B is not full rank, the system is said to be "underactuated".

- We can still solve the inverse dynamics problem when the system is underactuated, but it's a lot harder.