Last Time:

- Dual Spin + Momentum Bias
- Magnetic Torque Coils

Today:

M+C Ch. 7

- Detumbling With Torque Coils
- Momentum Dumping
- Actuator Jacobians

Detumbling:

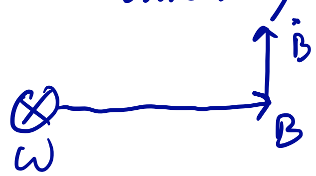
- Very common, especially on small satellites (e.g. CubeSats)
- Even if the spacecraft is using some other method for attitude stabilization, we need to get rid of initial angular momentum
- A very simple + effective control law is "B-dot" ($\dot{B}$)
- Only required hardware: Magnetometer + torque coils
- Derivative of B is proportional to ${}^B\omega$:

$$ {}^B\dot{B} \approx -{}^B\omega \times {}^BB $$

* Assuming ${}^NB$ changes slowly as the spacecraft moves around the Earth

* Typically there will be a residual spin roughly at the orbit frequency due to ${}^N\dot{B}$

- $\dot{B}$ is also conveniently perpendicular to B:



- Torque from coils is given by:

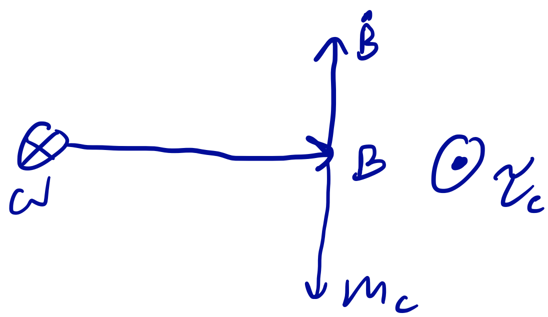
$$\underline{\tau}_c = \underline{m}_c \times \underline{B} \quad , \quad m_c = nIA$$

- A reasonable control law is:

$$^{B}m_c = -K\,^{B}\dot{B}$$

K ← scalar gain

* Can also do "bang-bang" control $m_c = -m_{max}\,\text{sign}(\dot{B})$ but this can lead to chattering due to noise in $\dot{B}$ measurements (especially if using finite differencing).



- Since ^{N}B varies over the orbit, we'll eventually be able to zero out all 3 components of $\underline{h}$. Typically, this takes a few orbits.

Momentum Dumping:

- Widely used on e.g. geostationary communication satellites.

- Reaction wheels build up angular momentum over time and eventually saturate.

- An external torque is needed to get rid of this momentum.

- Thrusters can be used, but that requires valuable fuel

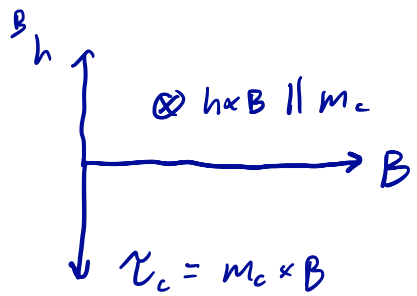
- Typical approach: Keep reaction wheel attitude controller running to maintain pointing while pulsing torque coils.

- Assuming spacecraft is inertially pointing, ${}^{B}h = \rho$ and we know this exactly from wheel speeds.

- Coil moment command:

$$ {}^{B}m_c = K({}^{B}h \times {}^{B}B) $$

scalar gain



$$ \tau_c \parallel -{}^{B}h $$

- Averaged over an orbit, we can zero out all components of ${}^{B}h$.

Actuator Jacobians:

- So far we have written our dynamics in terms of ρ and τ:

$$ {}^{B}J\,{}^{B}\dot{\omega} + {}^{B}\dot{\rho} + {}^{B}\omega \times ({}^{B}J\,{}^{B}\omega + {}^{B}\rho) = {}^{B}\tau $$

- We need mappings between actuator commands (reaction wheel torques, thruster forces) and ρ and τ

- It turns out this mapping is always linear

- For thrusters:

$${}^B\tau_T = \sum_{i=1}^{N} {}^Br_i \times {}^BF_i = \sum_{i=1}^{N} {}^Br_i \times {}^Ba_i \, u_i$$

Thruster force command

Thruster position from COM

Thruster axis unit vector

$$\Rightarrow {}^B\tau_T = \underbrace{\left[{}^Br_1 \times {}^Ba_1 \;\vdots\; {}^Br_2 \times {}^Ba_2 \;\vdots\; \cdots \;\vdots\; {}^Br_N \times {}^Ba_N \right]}_{B_T} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}$$

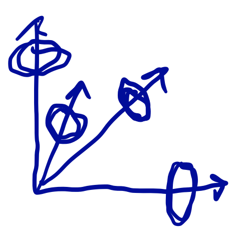
- Reaction Wheels

  - While 3 reaction wheels are needed for full control authority, most spacecraft have 4 or more for redundancy.

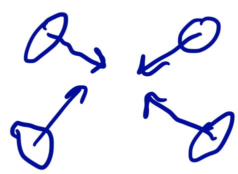
  - Four wheels can be arranged in various ways to provide higher performance + single-fault tollerance.

  - Some common configurations:

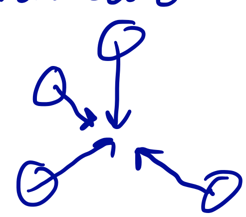
JPL:



Pyramid:



Tetrahedron:



- Wheel Jacobian:

wheel axis unit vector

$${}^B\rho = \underbrace{\left[{}^Ba_1 \;\vdots\; {}^Ba_2 \;\vdots\; \cdots \;\vdots\; {}^Ba_N \right]}_{B_w} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$$

$$\begin{cases} \tau_w = B_w u_w \\ \dot{w} = u_w \end{cases}$$

wheel momenta

- To Produce a Desired Torque:

$$\tau = Bu \Rightarrow u = \underbrace{(B^T B)^{-1} B^T}_{\text{Pseudo inverse of } B} \tau$$

* Pseudoinverse gives minimum 2-norm solution (usually corresponds to minimum overall power or fuel consumption). Other choices (e.g. 1-norm) are possible.