

# Report

1. In Experiment 1, estimate the probability of winning \$80 within 1000 sequential bets. Explain your reasoning.

**Answer:** According to my observation , probability of winning 80\$ within 1000 sequential bets will be approximately 1.

**Reasoning:** In extremely rare scenario ,it may not reach 80. Which is very unlikely. Since to win 80\$ in 1000 sequential bets , player has to get win 80 times(Each win is recovering all previous losses with 1\$ net gain), which is very likely in 1000 sequential bets considering the probability of single win is 0.4736  
Also the figure 1 shows all 10 simulations eventually hit the 80\$ mark  
If we run it for 1000 simulations then also all the simulations are converging at the 80\$ mark eventually.

2. In Experiment 1, what is the expected value of our winnings after 1000 sequential bets?

**Answer:** 80\$

**Reasoning:**

**Case 1: Considering the upper cap of 80\$ as per experiment 1**

According to Answer 1 , the probability of reaching 80\$ is Approx. 1  
which means If we run the simulation 1000 times where (1 simulation = 1000 bets)  
 $X=80$  ,  $P(x)=1$  ,  $N=1000$

Expected Value = Mean of expected values of 1000 simulations

$$= \sum_{i=1}^N P(x) * x / N$$

$$= (1 * 80 * 1000) / 1000$$

$$= 80$$

**Case 2: Removing the upper cap condition of 80\$**

I calculated expected value of winnings after 1000 sequential bets using frequentist approach ,programmatically.

1 simulation =1000 bets

Ran the simulation 1000 times and calculated the mean of episode\_winning values resulting after 1000 spin of each simulation

X1 = episode winning after 1000 spin (simulation 1)

X2 = episode winning after 1000 spin (simulation 2)

X3 = episode winning after 1000 spin(simulation 3)

X4 = episode winning after 1000 spin (simulation 4)

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X1000 = episode winning after 1000 spin (simulation 1000)

$$\text{Expected Value} = (X1+X2+X3+X4.....X1000)/1000$$
$$= 469.685 \text{ (Approx.)}$$

This can also be explained by looking at the Figure1 and Figure2 graphs that episode winning shows overall linear growth trend (with sudden drops due to some consecutive losses encountered , these exponential drops are due to bet amount getting doubled after every consecutive losses).

We see that between 160 to 200 spins all the simulations have reached the winning goal of 80\$. We can conclude that after 1000 spins , the winning amount will reach somewhere between 400 to 500 \$ (based on overall linear growth trend), which proves the value 469.685 is valid expected value.

3. In Experiment 1, does the standard deviation reach a maximum value then converge or stabilize as the number of sequential bets increases? Explain why it does (or does not).

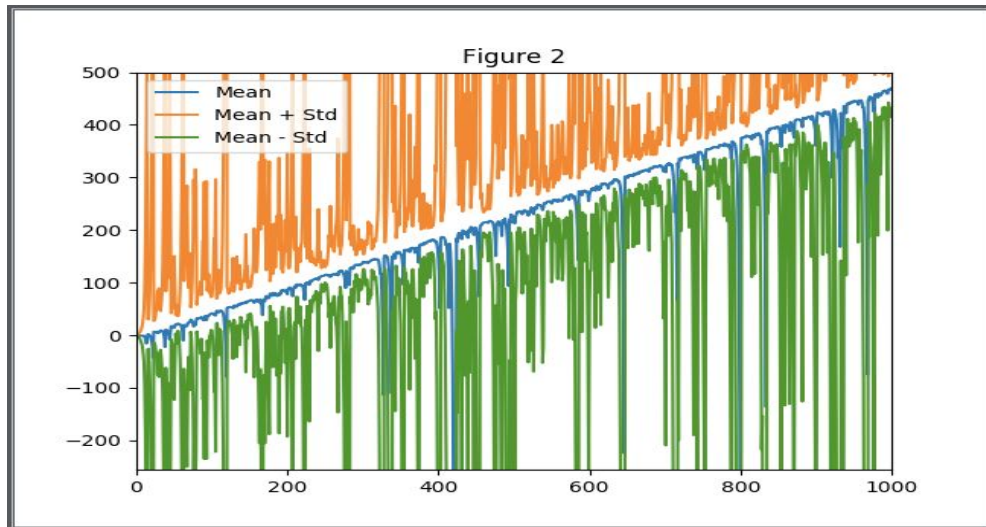
**Answer: It Does Not Converge after reaching the maximum value**

In Experiment 1, we are considering that player has unlimited bank roll. Even after few consecutive losses eventually a single win is recovering all previous losses with net gain of 1\$. That means the episode\_winning amount will continue to show the overall linear growth for large number of bets.

As per figure 2 , standard deviation is converging at 80, because we are stopping to spin after reaching the winning goal at 80 and filling the remaining winnings with value 80.

However , if we do not stop at any winning goal the standard deviation **DOES NOT** converge as shown in below graph and continues to maintain the upward trend(with sudden and frequent shots)

Below is the figure 2 (When we do not stop spinning after reaching the goal of 80\$).



4. In Experiment 2, estimate the probability of winning \$80 within 1000 sequential bets. Explain your reasoning.

**Answer: Probability = 0.65**

**Reasoning :** I have calculated the probability again using frequentist approach here.

Programmatically , ran 1000 simulations out of which 656 resulted in reaching the goal of 80\$. In rest 344 simulations gambler ran out of money.

Ran the experiment 2 again with 10000 simulation. 6495 simulations hit the goal of 80\$  
3505 resulted in -256\$ as shown in table below:

No. of Simulations	Simulations hit 80\$ Mark	Simulations hit -256\$ mark
1000	656	344
10000	6495	3505

**Probability is calculated here by applying the law of large numbers.”As the number of experiments increases, the actual ratio of outcomes will converge on the theoretical, or expected, ratio of outcomes”**

5. In Experiment 2, what is the expected value of our winnings after 1000 sequential bets? Explain your reasoning.

**Answer: -37.77**

**Case 1: Considering upper cap of 80\$ limit**

**According to law of large numbers and data shown in table for Q4, i am considering for 10000 simulation run**

$$\begin{aligned}
 \text{Expected Value} &= (\text{Sum of } P(x) \cdot x \text{ for } N \text{ bets})/N \\
 &= (6495 \text{ bets} \cdot 80) + (-256 \cdot 3505)/10000 \\
 &= (519600 - 897280)/10000 \\
 &= -37.77
 \end{aligned}$$

**Case2: When there is no upper cap limit**

Ran the simulation 1000 times after removing the condition for 80\$ goal in experiment 2 and calculated the mean of episode\_winning values resulting after 1000 spin of each simulation

X1 = episode winning after 1000 spin (simulation 1)

X2 = episode winning after 1000 spin (simulation 2)

X3 = episode winning after 1000 spin(simulation 3)

X4 = episode winning after 1000 spin (simulation 4)

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X1000 = episode winning after 1000 spin (simulation 1000)

$$\begin{aligned}
 \text{Expected Value} &= (X1+X2+X3+X4.....X1000)/1000 \\
 &= (444+474-256-256.....+472)/1000 \\
 &= -124845/1000 \\
 &= -124.85
 \end{aligned}$$

**Note: On running simulation for 10000 times also it is coming approx. -123**

6. In Experiment 2, does the standard deviation reach a maximum value then converge or stabilize as the number of sequential bets increases? Explain why it does (or does not).

**In experiment 2 , yes the standard deviation converges/stabilizes after reaching the maximum value.** Reason being that this time gambler has limited payroll of 256\$ ,After approx. 8 or 9 consecutive losses( $2^8 = 256$  ). Since bet amount is increasing by the factor of 2 at each loss). Gambler is getting out of game.

Since the win has linear growth but on every loss bet amount is shooting exponentially.  
After Approx. 9 consecutive losses gambler is quitting the game at winning of -256.

Probability of losing the 8 consecutive bets =  $(20/38)^8 = .00588$

So in 1000 consecutive bets possibility of hitting the -256\$ mark is 5.88 times.

As the number of sequential bets increases probability of hitting the -256\$ mark goes high and the standard deviation graph will converge eventually at -256.

**Note: the calculation given above is not accurate , consider it as approx.**

**calculation for large number of bets.**

## 7. Include figures 1 through 5.

Figure 1:

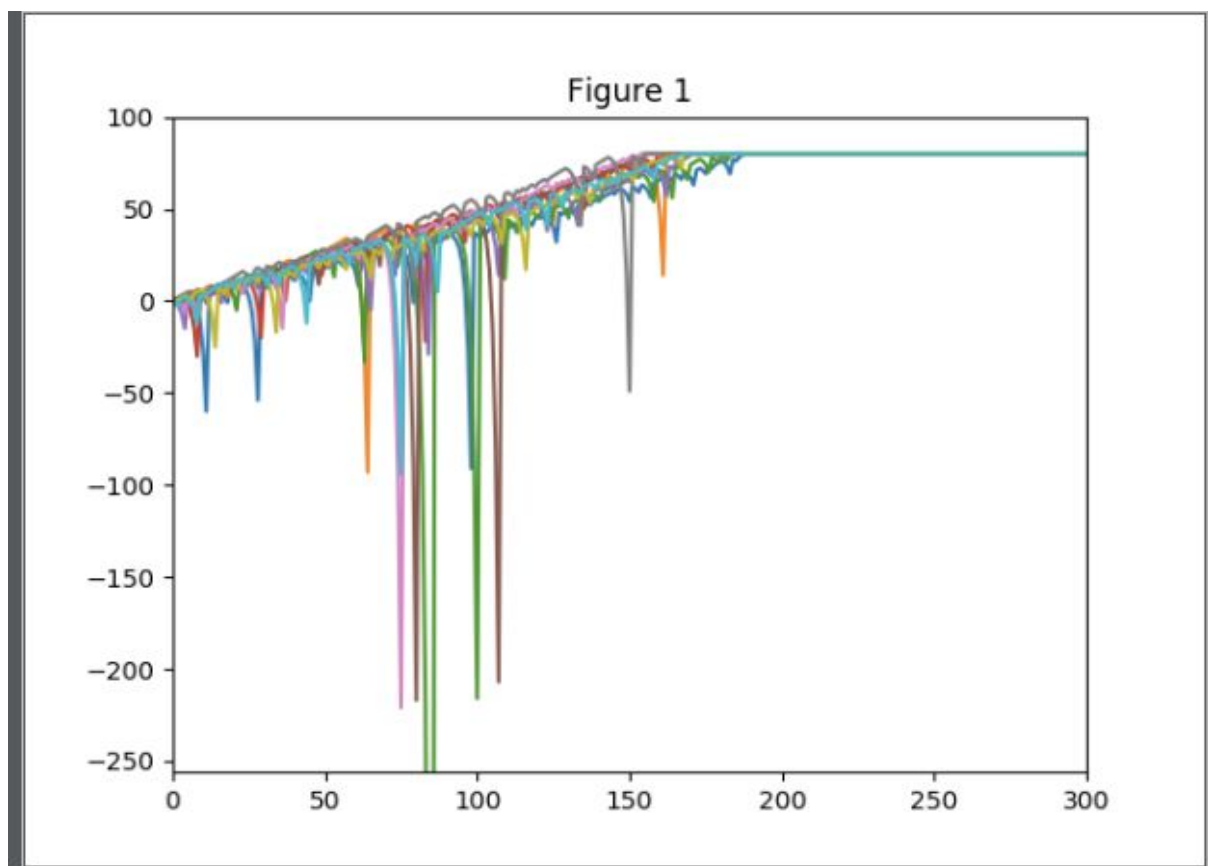


Figure2:

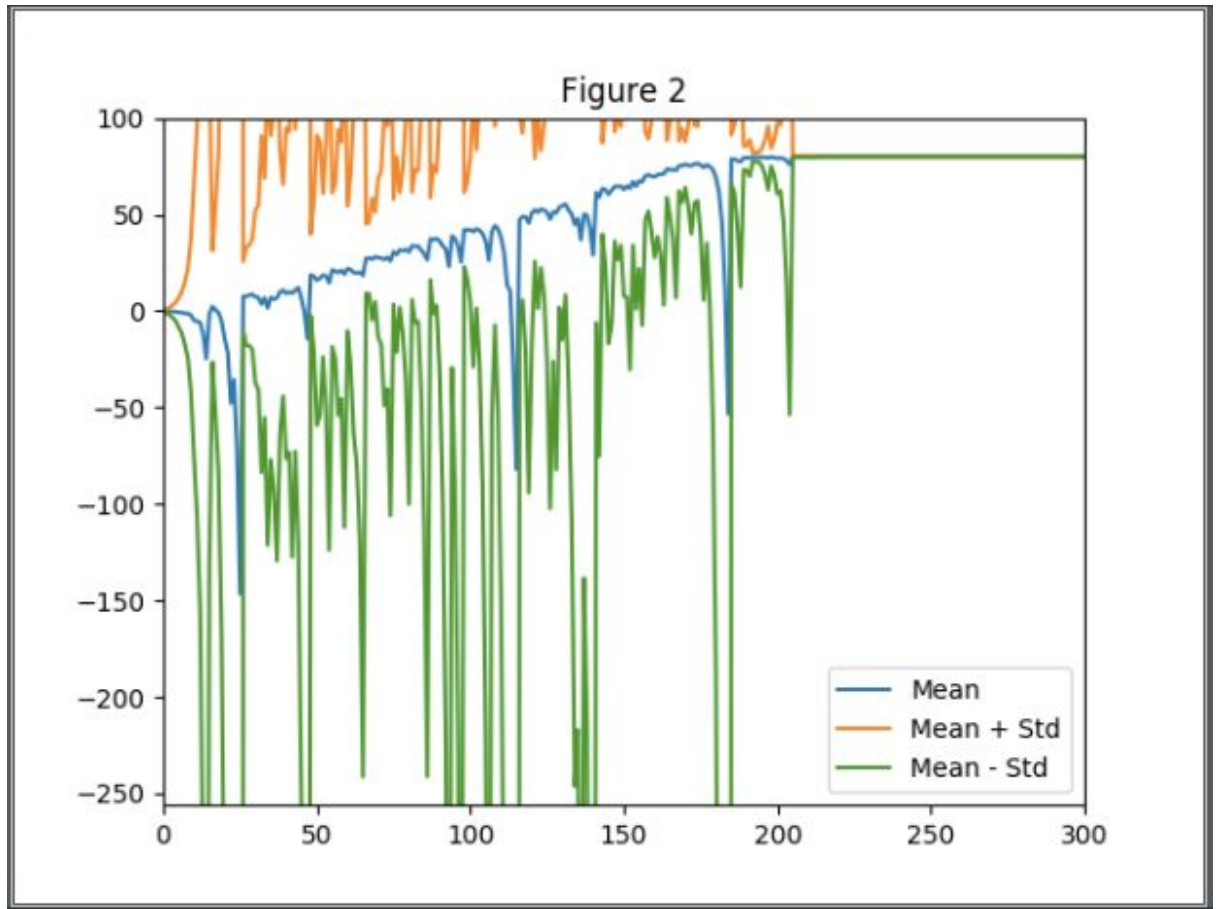


Figure3:

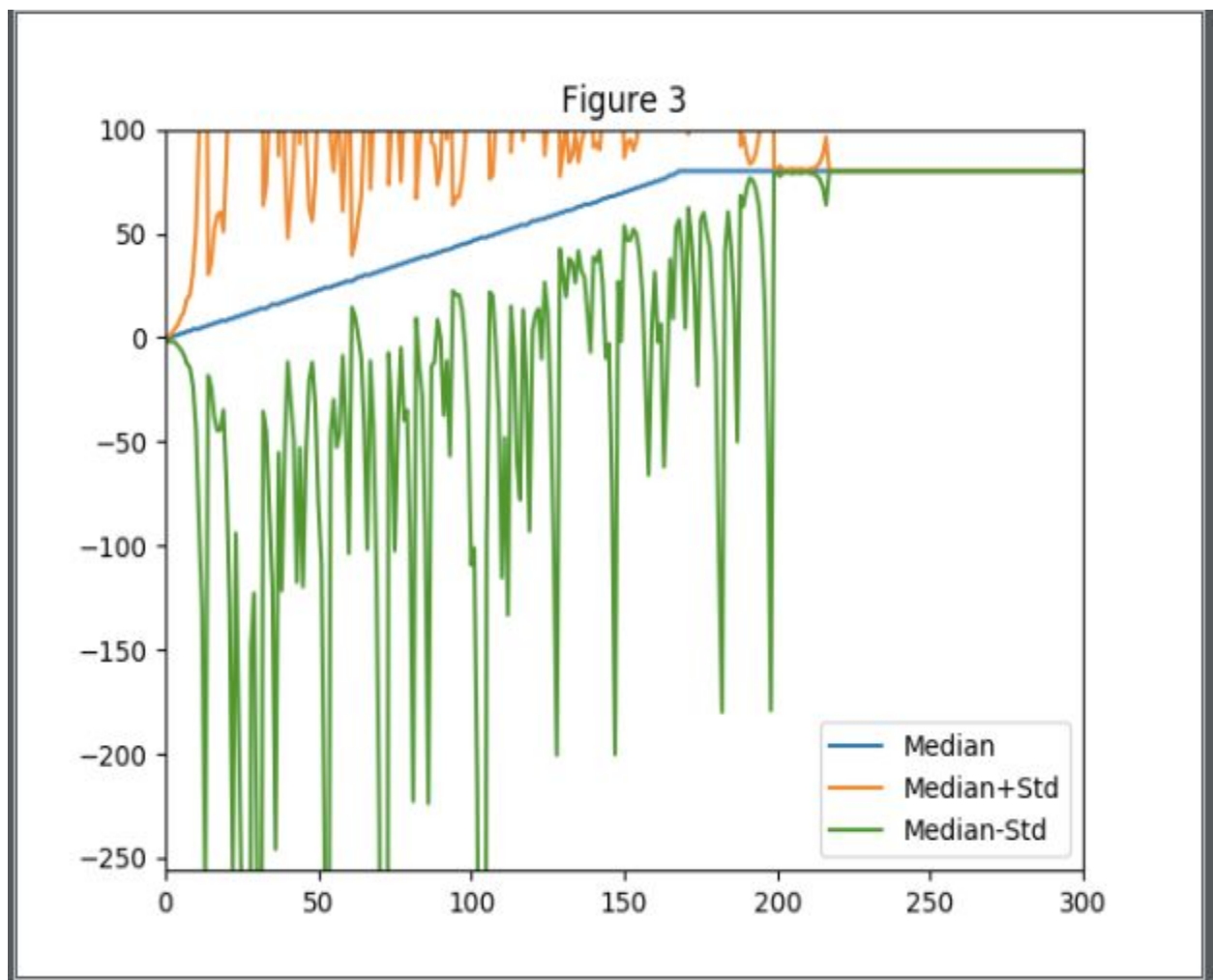


Figure4:

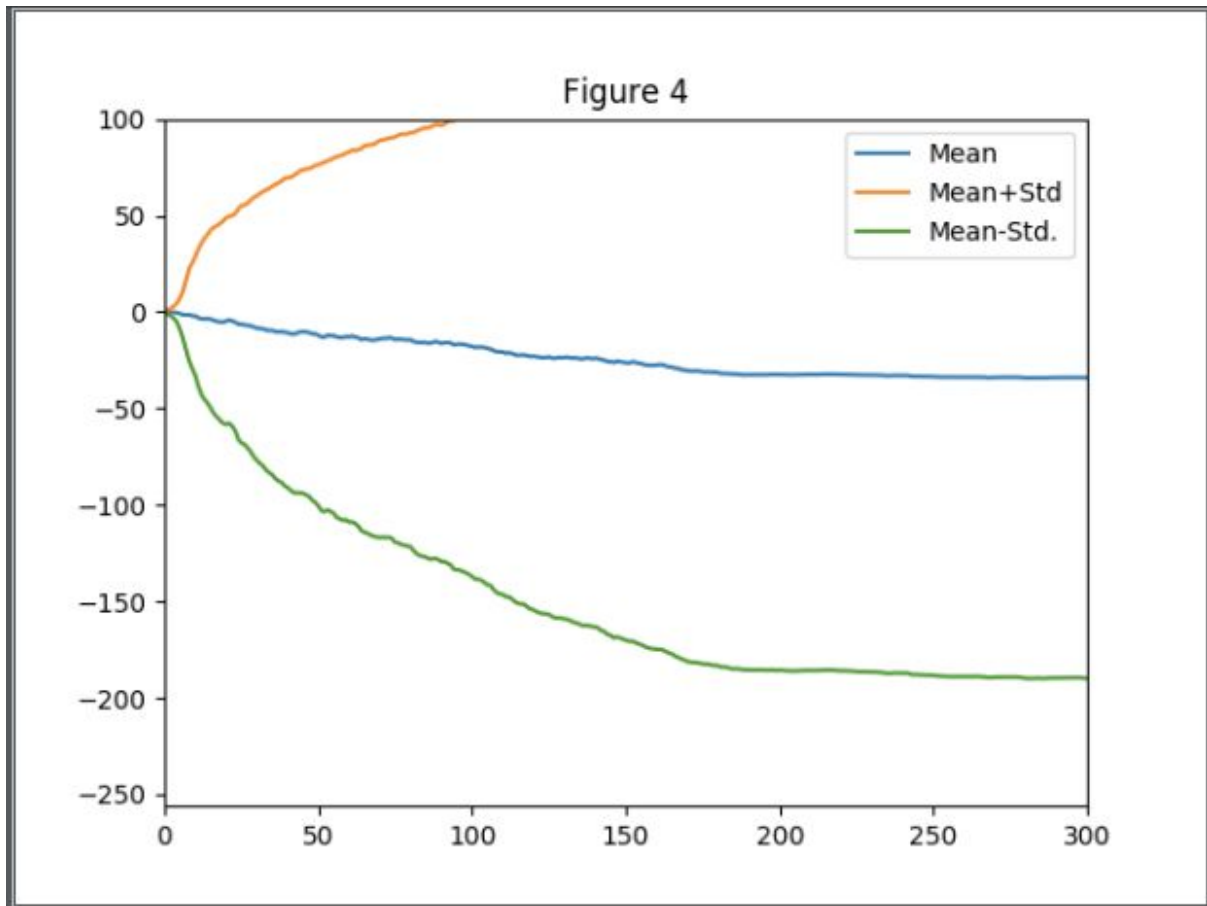




Figure5:

