# CSCE 420 Homework #2 (2022 Fall)

# Game Playing ; Logic and Theorem Proving

## Total 100 points

See Canvas for submission details.

## 1 Game Playing

### 1.1 Minmax Search

**Question 1 (15 pts):** Using the following figure 1, use minmax search to assign utility values for each internal node (i.e., non-leaf node) and indicate which path is the optimal solution for the MAX node at the root of the tree. Assume you explore the successors from left to right.

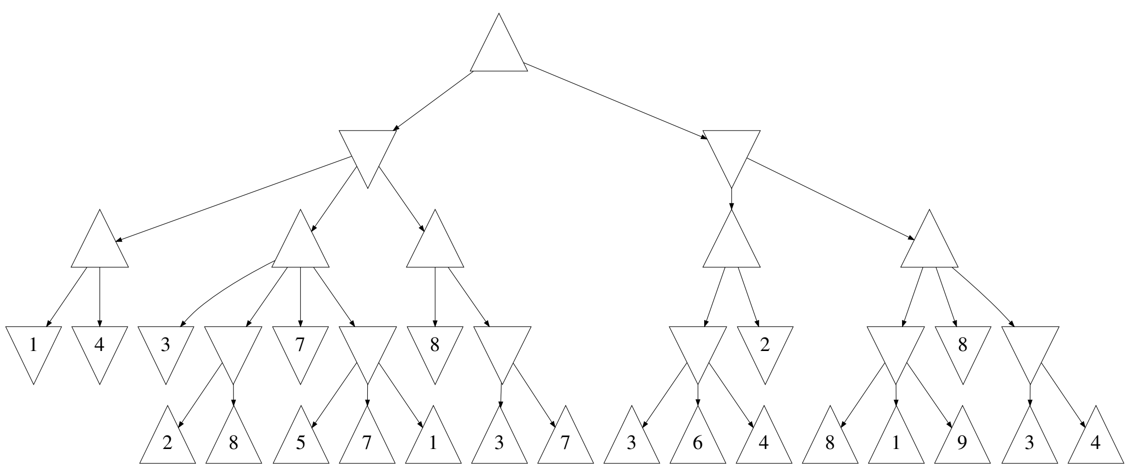



Figure 1: **Game Tree.** Solve using minmax search.

### 1.2 $\alpha - \beta$ pruning

**Question 2 (20 pts):** Using the following figure 2, use $\alpha - \beta$ pruning to (1) assign utility values for each internal node (i.e., non-leaf node) and indicate which path is the optimal solution for the MAX node at the root of the tree. (2) For each node, indicate the final $\alpha$ and $\beta$ values. (Note that initial values at the root are $\alpha = -\infty, \beta = \infty$.) (3) For each cut that happens, draw a line to cross out that subtree.

**Hint:** There are 5 places that need to be cut.

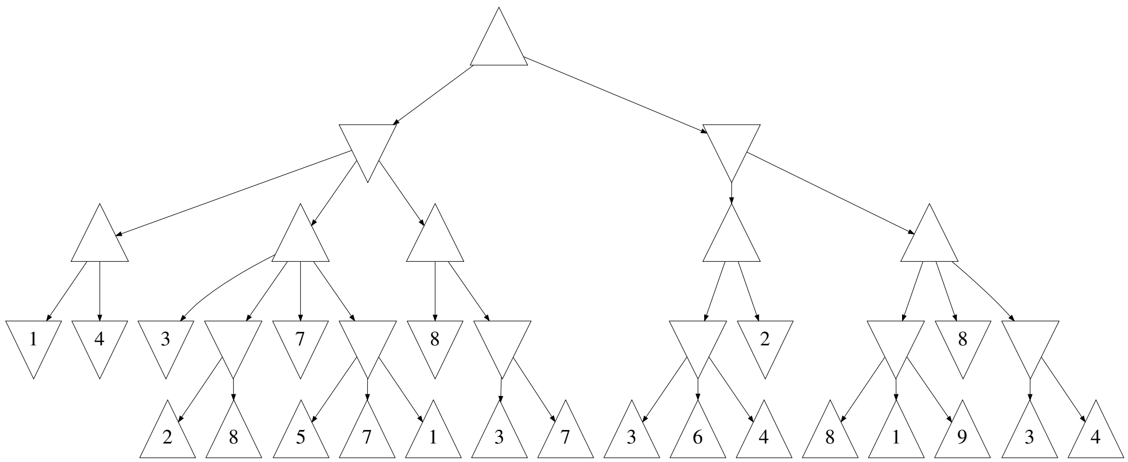



Figure 2: **Game Tree.** Solve using $\alpha - \beta$ pruning. This tree is the same as figure 1.

**Question 3 (10 pts):** In Minmax search, we used a depth-first exploration through the use of recursion. We know that Minmax gives an optimal solution, however, we also know that depth-first search is suboptimal. Explain why Minmax gives an optimal solution even when it is using a depth-first exploration.

# 2 Propositional Logic

In this section, assume $P, Q, R, S, T, U, V$ are atoms (propositions).

### 2.1 Inference rule

**Question 4 (20 pts):** Using a truth table, show that the resolution inference rule is valid (if the premises are true, the conclusion is also true, or, $((P \vee R) \wedge (Q \vee \neg R)) \rightarrow (P \vee Q)$ is valid). Note: valid means "true under all interpretations".

$$\frac{P \vee R, \;\; Q \vee \neg R}{P \vee Q}$$

| $P$ | $Q$ | $R$ | $(P \vee R)$ | $(Q \vee \neg R)$ | $(P \vee R) \wedge (Q \vee \neg R)$ | $(P \vee Q)$ | $((P \vee R) \wedge (Q \vee \neg R)) \rightarrow (P \vee Q)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | | | | | |
| T | T | F | | | | | |
| T | F | T | | | | | |
| T | F | F | | | | | |
| F | T | T | | | | | |
| F | T | F | | | | | |
| F | F | T | | | | | |
| F | F | F | | | | | |

### 2.2 Normal forms

In all of the problems in this section, show each step of the derivation and indicate which law (or other rules) you used: For example, *distributive law, by definition, etc.*

**Question 5 (15 pts):**

(1) Convert $P \vee (Q \wedge \neg R)$ into conjunctive normal form.

(2) Convert $P \vee (Q \wedge \neg R) \vee (Q \wedge S)$ into conjunctive normal form.

(3) Convert $(P \rightarrow Q) \rightarrow (R \vee S)$ into disjunctive normal form.

### 2.3 Theorem proving

Using resolution, show that $P \vee Q$ is a logical consequence of the following premises:

1. $R \rightarrow (Q \vee S)$
2. $(U \rightarrow V) \wedge \neg V$
3. $S \rightarrow (P \vee V)$
4. $U \vee R$

**Question 6 (10 pts):** Transform the above problem into a set of clauses (premises and the conclusion), suitable for resolution-based theorem proving.

- Turn each axiom in the list of premises above into conjunctive normal form.
  - One premise may result in multiple clauses.
  - For example, one premise $\neg((P \wedge \neg R) \vee S)$ will convert to CNF as $(\neg P \vee R) \wedge \neg S$, which results in two clauses:
    Clause 1: $\neg P \vee R$
    Clause 2: $\neg S$

- Don't forget to negate the conclusion ($P \vee Q$), before adding to the clause list. Multiple clauses may (or may not) result from the negated conclusion.

C1:

C2:

C3:

C4:

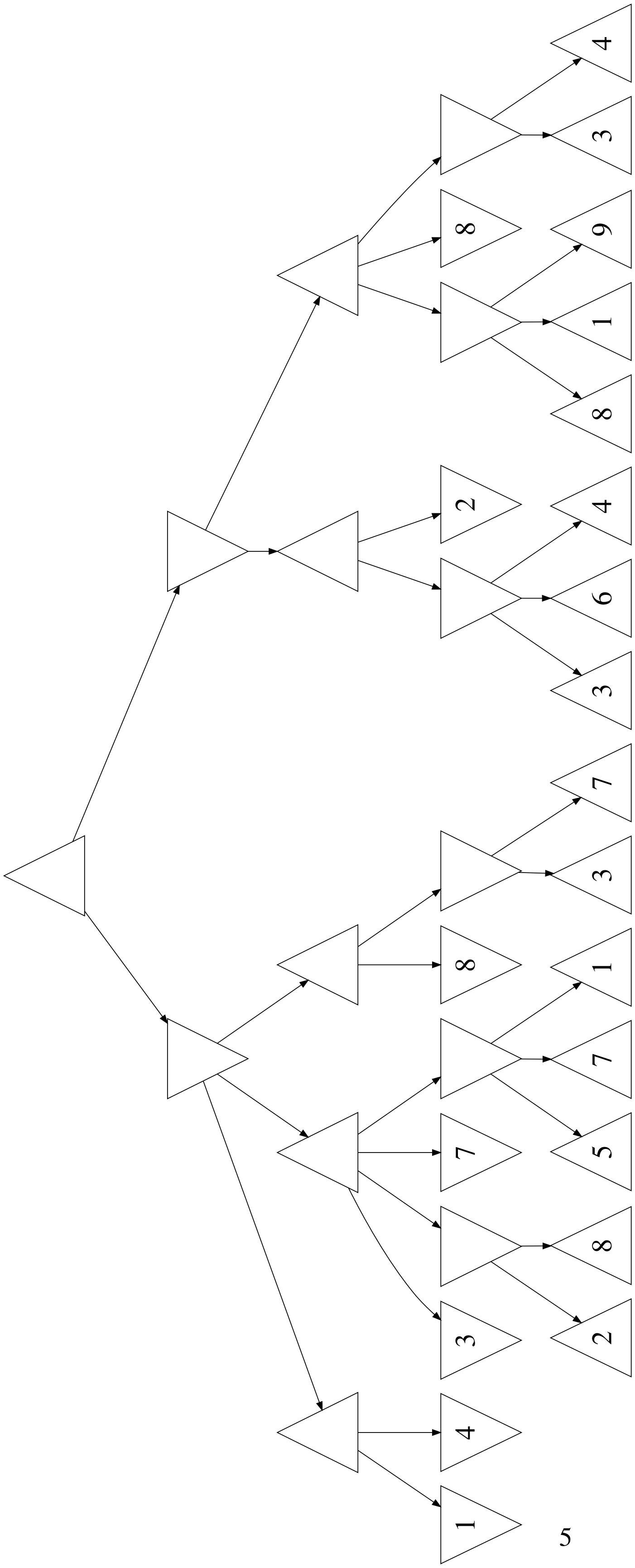
C5:

C6:

...

**Question 7 (10 pts):** Use resolution to derive **False**. Show every step. DO NOT USE any other infernce rule.

**Full-size print of the game tree, for practice, etc. (copy 1)**



## Full-size print of the game tree, for practice, etc. (copy 2)