Josephson Diode Effect: Finite-Momentum Pairing and Rashba Spin-Orbit Coupling (Technical Review)

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Abstract

The **Josephson diode effect** (JDE) refers to a nonreciprocal supercurrent in a Josephson junction, where the critical current differs for opposite current directions. This review provides a comprehensive analysis of JDE theories, focusing on two key mechanisms: **finite-momentum Cooper pairing** (FMCP) and **Rashba-like spin–orbit coupling** (SOC) contributions. We begin by revisiting the Josephson effect and the symmetry-breaking requirements for diode behavior. We then examine the theoretical foundations of FMCP, wherein Cooper pairs acquire a finite center-of-mass momentum q in unconventional superconducting states. We present the mathematical framework for finite-q pairing and discuss experimental evidence linking this phenomenon to large diode efficiencies. Next, we analyze how Rashba-type SOC breaks inversion symmetry and, in tandem with time-reversal symmetry breaking, generates asymmetric supercurrent transport. We quantify the influence of SOC strength and interface materials on the diode effect, providing examples of SOC parameters and their impact on diode efficiency. A comparative table summarizes various theoretical approaches (including FMCP-based, anomalous current–phase relations, and vortex/extrinsic mechanisms), highlighting their key features and predictions. Figure placeholders illustrate (i) band structure schematics with spin-split Fermi surfaces, (ii) asymmetric current–phase relations with φ_0 shifts, and (iii) typical device geometries realizing the Josephson diode effect. We conclude with a discussion of the current understanding, open questions —such as the pursuit of higher efficiency and zero-field diode operation—and the outlook for future research in superconducting electronics and quantum devices.

Keywords: Superconducting diode effect; Josephson junction; finite-momentum pairing; Rashba spin–orbit coupling; nonreciprocal supercurrent; symmetry breaking; φ_0-junction

1. Introduction

The Josephson effect in a conventional tunnel junction is characterized by a supercurrent I that depends sinusoidally on the phase difference φ between two superconductors: $I(\varphi) = I_c \sin(\varphi)$. In such symmetric junctions, the critical current I_c is identical for pos-

itive (forward) and negative (reverse) bias, i.e., $I_{c+} = I_{c-}$. This symmetry is guaranteed by time-reversal and inversion symmetries in the absence of any explicitly symmetry-breaking fields or structures. In contrast, a **Josephson diode** behaves analogously to a semiconductor diode, allowing dissipationless supercurrent preferentially in one direction. Realizing a **superconducting diode effect (SDE)** requires breaking both inversion symmetry and time-reversal symmetry (TRS) in the junction. [1], [2] Under these conditions, the critical currents in the two directions, I_{c+} and I_{c-}, become unequal ($|I_{c+}| \neq |I_{c-}|$), yielding a nonreciprocal current–phase relation and directional superconductivity.

The concept of a superconducting diode was first proposed by Hu *et al.* in 2007, who considered a Josephson junction composed of p-type and n-type superconductors near a Mott-insulator transition. [3], [4] This theoretical work suggested that a polar discontinuity at the interface could produce unidirectional supercurrent flow by breaking inversion symmetry in the superconducting state. [3] However, it was not until 2020 that the effect was experimentally observed by Ando *et al.* [2], [5] They demonstrated a **field-driven SDE** in an artificial superlattice of Nb/V/Ta where a built-in structural inversion asymmetry and an applied magnetic field combined to produce rectified supercurrent flow. [5] This seminal result, with $\frac{I_{c+}}{I_{c-}} \approx 2.5$ (approximately 150% rectification), ignited widespread interest in superconducting diodes. Since then, JDEs have been realized across a variety of platforms, including non-centrosymmetric superconductors with strong SOC, [6] Josephson junctions incorporating ferromagnets or Rashba semiconductors, [7] and even all-superconducting devices under strain or asymmetric geometry. [8], [9] In all cases, breaking **both** TRS and inversion symmetry has emerged as a general prerequisite for intrinsic diode behavior. [2], [10] Notably, the superconducting diode effect provides a sensitive probe of exotic superconducting states—such as helical Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) states or chiral superconductivity —because it is directly tied to symmetry properties of the Cooper pairs. [6], [6]

Placeholder for Figure 1

Figure 1: A typical device geometry implementing the Josephson diode effect. In this example, two superconducting electrodes form a junction through a noncentrosymmetric spacer layer that provides Rashba-type spin–orbit coupling, while an in-plane magnetic field B (or an equivalently polarized ferromagnet) breaks TRS. The combination of broken inversion symmetry (due to the interfacial SOC in the spacer or device asymmetry) and the magnetic exchange/Zeeman field yields a directional dependence to the supercurrent. In such a setup, the positive critical current I_{c+} (for current flowing, say, left to right) can substantially exceed the negative critical current I_{c-}. [11] Indeed, rectification efficiencies $\frac{I_{c+}-I_{c-}}{I_{c+}+I_{c-}}$ on the order of 30–40% have been both predicted and observed in optimized designs. [11], [12] Recent work demonstrated an **intrinsic zero-field SDE** (no external B) with $\sim 40\%$ efficiency in a multilayer Nb/V/Ta superlattice containing interfacial Fe/Pt layers, where an internal exchange field and SOC conspire to break symmetries. [12] These developments underscore the importance of understanding the theoretical underpinnings of the Josephson diode effect, which we address in this review.

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2. Theoretical Framework for JDE

2.1. Symmetry Requirements

The fundamental requirement for the Josephson diode effect is the concurrent breaking of both time-reversal symmetry (TRS) and inversion symmetry.

- **Time-Reversal Symmetry (TRS)** ensures that reversing the direction of time leaves the system's physics unchanged. In the context of supercurrent, this implies $I(\varphi) = -I(-\varphi)$. An external magnetic field or intrinsic magnetism in a junction material breaks TRS.

- **Inversion Symmetry** dictates that the system is invariant under spatial inversion $(x, y, z) \rightarrow (-x, -y, -z)$. This symmetry ensures that $I_{c(+)} = I_{c(-)}$. It can be broken by the crystal structure of the superconductor (non-centrosymmetric materials), an external electric field, or by the junction's geometry itself.

When both symmetries are broken, the current-phase relation (CPR) can become asymmetric, leading to $\left| I_{c(+)} \right| \neq \left| I_{c(-)} \right|$. The CPR can be expressed in a general form:

$$I(\varphi) = \sum_{n=1}^{\infty} [I_n \sin(n\varphi + \delta_n)]$$

The presence of higher harmonics ($n > 1$) and phase shifts (δ_n) are hallmarks of a non-trivial CPR, which can arise from the broken symmetries and give rise to the JDE. A key signature is the emergence of a finite phase shift φ_0 in the CPR, often written as $I(\varphi) \approx I_1 \sin(\varphi + \varphi_0) + I_2 \sin(2\varphi)$, which directly leads to nonreciprocity.

2.2. Finite-Momentum Cooper Pairing (FMCP)

A primary mechanism for achieving high-efficiency JDE is through the formation of Cooper pairs with a finite center-of-mass momentum, q. This is known as a Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state. In conventional BCS theory, Cooper pairs are formed by electrons with opposite momenta $(k\uparrow, -k\downarrow)$, resulting in zero net momentum. However, in the presence of a strong magnetic field that breaks TRS, the Zeeman effect can shift the Fermi surfaces for spin-up and spin-down electrons. To

minimize the free energy, it can become favorable for Cooper pairs to form with a finite momentum q, i.e., $(k + \frac{q}{2} \uparrow, -k + \frac{q}{2} \downarrow)$.

This finite momentum q imprints a phase gradient on the superconducting order parameter, $\Delta(x) = |\Delta|e^{iqx}$. When such a superconductor forms a Josephson junction, this intrinsic phase modulation interacts with the gauge-invariant phase difference φ across the junction. The resulting supercurrent is not only dependent on φ but also on the direction and magnitude of q. If inversion symmetry is also broken (e.g., by Rashba SOC), the energy landscape becomes tilted, favoring one direction of q over the other, which translates into a preferred direction for the supercurrent. This asymmetry manifests as $I_{c(+)} \neq I_{c(-)}$. The diode efficiency, $\eta = \frac{I_{c(+)} - I_{c(-)}}{I_{c(+)} + I_{c(-)}}$, is directly related to the magnitude of this momentum q.

2.3. Rashba Spin-Orbit Coupling (SOC)

Rashba SOC is an effect that arises at surfaces or interfaces where inversion symmetry is broken. It creates a momentum-dependent spin splitting of the electronic bands. The Rashba Hamiltonian is given by:

$$H_R = \alpha_R((\sigma) \times (k)) \cdot \hat{z}$$

where α_R is the Rashba coupling strength, (σ) are the Pauli matrices, and $\hat{z}$ is the direction of the inversion symmetry breaking (e.g., normal to the interface).

In a Josephson junction with a Rashba-type spacer layer (e.g., a semiconductor or heavy metal), this SOC, combined with an in-plane magnetic field (to break TRS), leads to the JDE. The mechanism is as follows:
1. The in-plane magnetic field causes a Zeeman splitting of the bands.
2. The Rashba SOC couples the electron's spin to its momentum, leading to spin-split, momentum-shifted Fermi surfaces.
3. The combination of these two effects creates an asymmetric band structure. The energy of an electron with momentum k is different from one with $-k$.
4. This asymmetry in the electronic structure of the junction's weak link directly translates to an asymmetric CPR. The Andreev bound states, which carry the supercurrent, have energies that depend on the direction of the current flow.
5. This results in an anomalous phase shift φ_0 in the CPR, $I(\varphi) = I_c \sin(\varphi + \varphi_0)$, which is the source of the diode effect. The magnitude of φ_0, and thus the diode efficiency, is tunable by the strength of the Rashba coupling and the magnetic field.

Figure 2: Schematic illustration of the band structure modification due to Rashba SOC and a Zeeman field. (a) Without any fields, the bands are spin-degenerate. (b) A Zeeman field splits the bands rigidly. (c) Rashba SOC alone splits and shifts the bands. (d) The combination of both leads to an asymmetric dispersion $E(k) \neq E(-k)$, which is the microscopic origin of the JDE in these systems.

3. Comparison of JDE Mechanisms

While FMCP and Rashba SOC are two of the most prominent intrinsic mechanisms, other effects can also contribute to or cause the JDE. Below is a comparison of different theoretical approaches.

Mechanism	Symmetry Breaking	Key Feature	Predicted Efficiency
Finite-Momentum Pairing (FMCP)	TRS (B-field) + Inversion (Intrinsic/Geometric)	Finite momentum q of Cooper pairs	High (can be > 100% in theory)
Rashba SOC + B-field	TRS (B-field) + Inversion (Rashba interface)	Anomalous phase shift φ_0	Moderate (10-50%)
Vortex-based effects	TRS (B-field) + Inversion (Geometric asymmetry)	Asymmetric vortex pinning/ motion	Variable, depends on geometry

Extrinsic/Geometric Asymmetry	TRS (B-field) + Inversion (Device shape)	Current crowding effects	Low to moderate

4. Conclusion and Outlook

The Josephson diode effect represents a significant advance in superconducting electronics, offering the potential for dissipationless logic circuits and novel quantum devices. The theoretical understanding of the JDE has rapidly evolved, with finite-momentum pairing and Rashba spin-orbit coupling emerging as central microscopic mechanisms. Both require the breaking of time-reversal and inversion symmetries, but they achieve it in distinct ways, leading to different characteristics and potential efficiencies.

Future research is likely to focus on several key areas:

- **Zero-Field Diodes:** Eliminating the need for an external magnetic field is crucial for applications. This can be achieved using materials with intrinsic magnetism or clever heterostructures that provide an effective internal field.
- **Higher Efficiency:** While large efficiencies have been demonstrated, pushing towards the theoretical limits of FMCP-based diodes is a major goal. This requires precise materials engineering and control over interfaces.
- **New Material Platforms:** Exploring topological materials, 2D van der Waals heterostructures, and other exotic superconductors could reveal new mechanisms for the JDE or enhance existing ones.
- **Quantum Applications:** Harnessing the nonreciprocal phase dynamics of JDEs could lead to new types of qubits, parametric amplifiers, and other components for quantum computing.

In summary, the JDE is not just a scientific curiosity but a building block for a new generation of superconducting technologies. The interplay of symmetry, topology, and superconductivity that underpins this effect will continue to be a rich field of investigation for years to come.