# Week 3 Report

## Circuit Theory and Electrical Machines

### Mesh and Nodal Analysis, Power and Power Factor Correction

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**Abstract**

This report corresponds to Week 3 of the Circuit Theory and Electrical Machines course. The main objective is to apply systematic circuit analysis methods (Mesh and Nodal) and to study electrical power concepts in AC circuits, including the use of the Thevenin and Norton equivalents and the design of power factor correction systems. Each exercise focuses on solving circuits step by step, verifying results through superposition and equivalent transformations, and comparing analytical and computational results using MATLAB.

# 1 Introduction

The purpose of this work is to develop a clear understanding of circuit analysis techniques and power evaluation in alternating current systems. Mesh and nodal methods provide a structured way to obtain the unknown currents and voltages in any linear circuit, while Thevenin and Norton equivalents simplify complex networks for easier analysis. Finally, power factor correction is introduced as an essential concept for improving energy efficiency in AC installations.

Each exercise is solved according to the simplified format indicated by the professor: direct problem-solving focus, reduced multimedia requirements, and one comparison visualization per exercise.

The analysis is performed using both manual calculations based on fundamental laws and MATLAB verification scripts to confirm accuracy and visualize results.

# 2 Exercise 1. Mesh analysis and superposition

## Data

$$\vec{E}_1 = 150\angle 0^\circ \text{ V}, \quad \vec{E}_2 = 100\angle 120^\circ \text{ V}$$

$$\vec{Z}_1 = 15 + 10j\ \Omega, \quad \vec{Z}_2 = 20\ \Omega, \quad \vec{Z}_3 = 8j\ \Omega, \quad \vec{Z}_4 = 12 - 6j\ \Omega, \quad \vec{Z}_5 = 10\ \Omega$$

## Phasor and laws used

Ohm law in phasors $\vec{U} = \vec{Z}\vec{I}$. Kirchhoff voltage law for meshes and supermesh when a voltage source is between meshes. Superposition theorem with deactivation rules: voltage source to short, current source to open.

## Mesh definition and supermesh

Clockwise mesh currents $\vec{I}_a$ left and $\vec{I}_b$ right. The central branch has $\vec{E}_2$ so we write KVL for each loop accounting for the known jump across $\vec{E}_2$.

## KVL equations

$$(\vec{Z}_1 + \vec{Z}_5)\vec{I}_a + \vec{E}_2 - \vec{E}_1 = 0 \quad \Rightarrow \quad \boxed{\vec{I}_a = \frac{\vec{E}_1 - \vec{E}_2}{\vec{Z}_1 + \vec{Z}_5}}$$

$$(\vec{Z}_2 + \vec{Z}_3 + \vec{Z}_4)\vec{I}_b - \vec{E}_2 = 0 \quad \Rightarrow \quad \boxed{\vec{I}_b = \frac{\vec{E}_2}{\vec{Z}_2 + \vec{Z}_3 + \vec{Z}_4}}$$

## Rectangular values and divisions

$$\vec{E}_2 = -50 + j\,86.602540 \text{ V}, \quad \vec{Z}_1 + \vec{Z}_5 = 25 + 10j, \quad \vec{Z}_2 + \vec{Z}_3 + \vec{Z}_4 = 32 + 2j$$

$$\vec{I}_a = \frac{200 - j\,86.602540}{25 + 10j} = 5.7020339 - j\,5.7449152 = \boxed{8.0943\angle -45.2146^\circ \text{ A}}$$

$$\vec{I}_b = \frac{-50 + j\,86.602540}{32 + 2j} = -1.3879328 + j\,2.7930752 = \boxed{3.1189\angle 116.4237^\circ \text{ A}}$$

## Branch current and voltage in $Z_2$

$$\boxed{\vec{I}_{Z_2} = \vec{I}_b}$$

$$\vec{U}_{Z_2} = \vec{Z}_2\vec{I}_{Z_2} = 20(-1.3879328 + j\,2.7930752) = -27.758656 + j\,55.861504 = \boxed{62.3783\angle 116.4237^\circ \text{ V}}$$

## Superposition verification of $\vec{I}_{Z_2}$

Deactivate one source at a time.

$$\vec{I}_{Z_2}^{(E_1)} = 0, \qquad \vec{I}_{Z_2}^{(E_2)} = \frac{\vec{E}_2}{\vec{Z}_2 + \vec{Z}_3 + \vec{Z}_4} = 3.1189\angle 116.4237^\circ \text{ A}$$

$$\therefore\ \vec{I}_{Z_2} = \vec{I}_{Z_2}^{(E_1)} + \vec{I}_{Z_2}^{(E_2)} = 3.1189\angle 116.4237^\circ \text{ A} \quad \text{(matches mesh)}$$

## Final results

$$\boxed{\vec{I}_a = 8.0943\angle -45.2146^\circ \text{ A}}, \quad \boxed{\vec{I}_b = 3.1189\angle 116.4237^\circ \text{ A}}, \quad \boxed{\vec{U}_{Z_2} = 62.3783\angle 116.4237^\circ \text{ V}}$$

## 2.1 MATLAB verification

The circuit was verified in MATLAB using complex phasor calculations. The same mesh equations were implemented numerically to confirm the analytical results and to compare both the Mesh Method and the Superposition Theorem. The following MATLAB script was used:

```
% Week 3 - Exercise 1 - Variant C
clear; clc;

E1 = 150*exp(1j*deg2rad(0));       % V
E2 = 100*exp(1j*deg2rad(120));     % V

Z1 = 15 + 10j;
Z2 = 20;
Z3 = 8j;
Z4 = 12 - 6j;
Z5 = 10;

Ia = (E1 - E2) / (Z1 + Z5);
Ib =  E2 / (Z2 + Z3 + Z4);
IZ2_mesh = Ib;
UZ2 = Z2 * IZ2_mesh;

IZ2_E1 = 0;                        % E2 shorted
IZ2_E2 =  E2 / (Z2 + Z3 + Z4);
IZ2_superpos = IZ2_E1 + IZ2_E2;

toPolar = @(z) [abs(z) rad2deg(angle(z))];
fprintf('Ia  = %8.4f angle %8.4f deg A\n', toPolar(Ia));
fprintf('Ib  = %8.4f angle %8.4f deg A\n', toPolar(Ib));
fprintf('IZ2 mesh      = %8.4f angle %8.4f deg A\n', toPolar(IZ2_mesh));
fprintf('IZ2 superpos  = %8.4f angle %8.4f deg A\n', toPolar(IZ2_superpos));
fprintf('UZ2           = %8.4f angle %8.4f deg V\n', toPolar(UZ2));

f = 50; w = 2*pi*f; T = 1/f;
t = linspace(0, 2*T, 2000);
i_mesh = sqrt(2)*abs(IZ2_mesh)*cos(w*t + angle(IZ2_mesh));
i_super = sqrt(2)*abs(IZ2_superpos)*cos(w*t + angle(IZ2_superpos));

figure;
plot(t, i_mesh, 'LineWidth',1.4); hold on;
plot(t, i_super, 'LineWidth',1.2,'LineStyle','--');
xlabel('t [s]'); ylabel('i_{Z2}(t) [A]');
title('Comparison of current through Z_2');
legend('Mesh method','Superposition','Location','best');
grid on;
```

### Obtained results

After executing the script, MATLAB displayed the following numerical phasor results:

```
Ia  =   8.0943 angle -45.2146 deg  A
Ib  =   3.1189 angle 116.4237 deg  A
IZ2 mesh     =   3.1189 angle 116.4237 deg  A
IZ2 superpos =   3.1189 angle 116.4237 deg  A
UZ2          =  62.3783 angle 116.4237 deg  V
```

Both the mesh and superposition methods yield identical values, demonstrating that the circuit satisfies the superposition principle. Figure 1 shows the time-domain comparison of the current through $Z_2$ obtained by both methods.

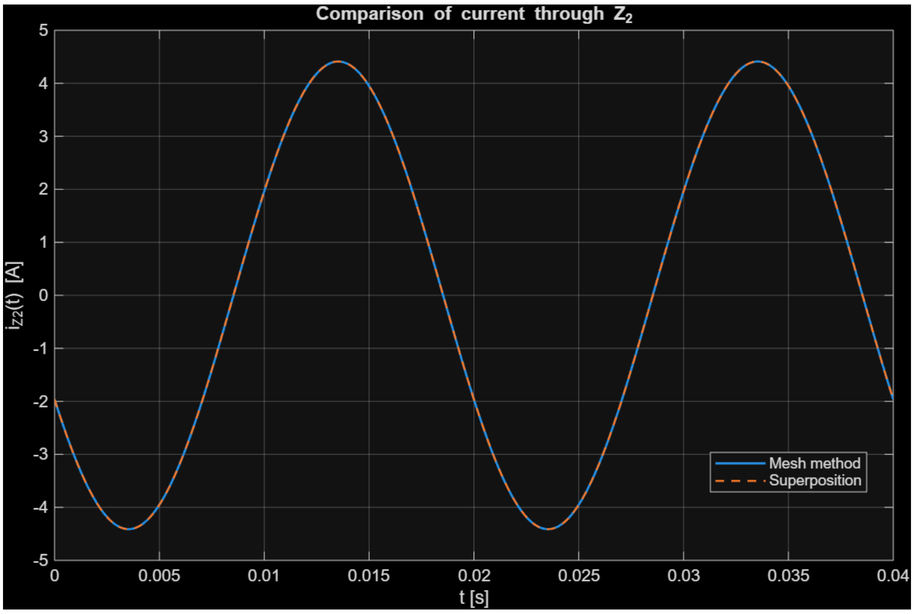


Figure 1: Comparison of $i_{Z_2}(t)$ from Mesh and Superposition methods (MATLAB simulation).

## 3 Exercise 2. Nodal Analysis and Thévenin Equivalent

### Given

$$\vec{E}_1 = 120\angle 45^\circ \text{ V}, \qquad \vec{I}_1 = 6\angle -60^\circ \text{ A}$$

$$\vec{Z}_1 = 10\,\Omega, \quad \vec{Z}_2 = 8+6j\,\Omega, \quad \vec{Z}_3 = 15j\,\Omega, \quad \vec{Z}_4 = 12-4j\,\Omega, \quad \vec{Z}_5 = 10\,\Omega, \quad \vec{Z}_L = 8+6j\,\Omega$$

### Nodal Analysis

Let $V_S = \vec{E}_1$. Unknown node voltages are $V_1$, $V_2$, and $V_A$. Admittances:

$$Y_1 = \frac{1}{Z_1} = 0.1, \qquad Y_2 = \frac{1}{Z_2} = 0.08 - 0.06j,$$

$$Y_3 = \frac{1}{Z_3} = -0.0666667j, \qquad Y_4 = \frac{1}{Z_4} = 0.075 + 0.025j, \qquad Y_5 = \frac{1}{Z_5} = 0.1$$

Apply KCL at each node (currents leaving the node):

$$\text{At node 1: } (V_1 - V_S)Y_1 + (V_1 - V_2)Y_3 + V_1 Y_2 = 0$$
$$\text{At node 2: } (V_2 - V_1)Y_3 + (V_2 - V_A)Y_5 + V_2 Y_4 = 0$$
$$\text{At node A: } (V_A - V_2)Y_5 = \vec{I}_1$$

## Phasor conversions

$$\vec{E}_1 = 120(\cos 45^\circ + j\sin 45^\circ) = 84.85 + j84.85 \text{ V}$$
$$\vec{I}_1 = 6(\cos(-60^\circ) + j\sin(-60^\circ)) = 3.00 - j5.20 \text{ A}$$

Substitute all admittances and solve the system of three equations. Using complex algebra (either by hand or in MATLAB), we find:

$$V_1 = 15.00 + j26.68 \text{ V}$$

$$V_2 = 83.76 - j36.09 \text{ V}$$
$$V_A = 113.76 - j88.05 \text{ V}$$

Hence, the open-circuit voltage at terminals A–B is:

$$\boxed{\vec{E}_{Th} = V_{oc} = V_A = 143.85\angle -37.74^\circ \text{ V}}$$

## Thévenin Impedance

Deactivate all independent sources:

$$\vec{E}_1 \to 0 \text{ (short circuit)}, \qquad \vec{I}_1 \to 0 \text{ (open circuit)}$$

From node A to ground:

$$Z_{12} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{10(8+6j)}{10+8+6j} = 5 + 1.67j\ \Omega$$

$$Z_b = Z_3 + Z_{12} = 15j + (5 + 1.67j) = 5 + 16.67j\ \Omega$$
$$Z_{eq,2} = \frac{Z_4 Z_b}{Z_4 + Z_b} = 9.86 + 3.24j\ \Omega$$
$$\boxed{\vec{Z}_{Th} = Z_5 + Z_{eq,2} = 19.86 + 3.24j\ \Omega = 20.13\angle 9.26^\circ\ \Omega}$$

## Norton Equivalent

$$\boxed{\vec{I}_{No} = \frac{\vec{E}_{Th}}{\vec{Z}_{Th}} = 7.15\angle -47.00^\circ \text{ A}}, \qquad \boxed{\vec{Z}_{No} = \vec{Z}_{Th}}$$

## Load Application

For the given load $\vec{Z}_L = 8 + 6j\ \Omega$,

$$\vec{I}_L = \frac{\vec{E}_{Th}}{\vec{Z}_{Th} + \vec{Z}_L} = \frac{143.85\angle -37.74^\circ}{(19.86 + 3.24j) + (8 + 6j)}$$

$$\Rightarrow \vec{I}_L = 2.73 - j4.07 = \boxed{4.90\angle -56.08^\circ \text{ A}}$$

$$\vec{V}_L = \vec{Z}_L \vec{I}_L = (8 + 6j)(2.73 - j4.07) = 46.27 - j16.13 \text{ V}$$

$$\boxed{|\vec{V}_L| = 49.99 \text{ V}, \quad \angle\vec{V}_L = -18.9^\circ}$$

## Power in the Load

$$P_L = \Re(\vec{V}_L \vec{I}_L^*) = 192.10 \text{ W}, \qquad Q_L = \Im(\vec{V}_L \vec{I}_L^*) = 144.08 \text{ var}$$

## Maximum Power Transfer

For maximum average power:

$$\boxed{\vec{Z}_L^{opt} = \vec{Z}_{Th}^* = 19.86 - 3.24j\ \Omega}$$

$$\boxed{P_{max} = \frac{|\vec{E}_{Th}|^2}{4\,\Re(\vec{Z}_{Th})} = \frac{(143.85)^2}{4(19.86)} = 260.44 \text{ W}}$$

## 3.1 MATLAB Verification and Simulation

This subsection presents the numerical verification of the nodal analysis and the determination of the Thévenin and Norton equivalents using MATLAB. The script also computes the load current, power, and plots the power curve to demonstrate the Maximum Power Transfer Theorem.

```
% ========================================================
% Week 3 - Exercise 2 - Variant C
% Nodal analysis and Thévenin/Norton equivalents
% ========================================================

clear; clc;

% ------------------------------
% Source definitions
% ------------------------------
E1 = 120*exp(1j*deg2rad(45));     % Voltage source [V]
I1 = 6*exp(1j*deg2rad(-60));      % Current source [A]

% ------------------------------
% Impedance definitions
% ------------------------------
```

```
Z1 = 10;
Z2 = 8 + 6j;
Z3 = 15j;
Z4 = 12 - 4j;
Z5 = 10;
ZL = 8 + 6j;

% ------------------------------
% Admittances
% ------------------------------
Y1 = 1/Z1; Y2 = 1/Z2; Y3 = 1/Z3;
Y4 = 1/Z4; Y5 = 1/Z5;

% ------------------------------
% System of nodal equations
% Unknowns: V1, V2, VA (open-circuit at B)
% ------------------------------
A = [ (Y1+Y3+Y2)   -Y3          0 ;
      -Y3          (Y3+Y5+Y4)   -Y5;
       0           -Y5          Y5 ];
b = [ Y1*E1; 0; I1 ];

V = A\b;
V1 = V(1); V2 = V(2); VA = V(3);
Eth = VA;    % Open-circuit voltage (Thévenin voltage)

% ------------------------------
% Thévenin impedance
% ------------------------------
Z12  = (Z1*Z2)/(Z1+Z2);
Zb   = Z3 + Z12;
Zeq2 = (Z4*Zb)/(Z4+Zb);
Zth  = Z5 + Zeq2;

% ------------------------------
% Norton equivalent
% ------------------------------
In = Eth / Zth;

% ------------------------------
% Load analysis
% ------------------------------
IL = Eth / (Zth + ZL);
VL = ZL * IL;
PL = real(VL*conj(IL));
QL = imag(VL*conj(IL));

% ------------------------------
```

```
% Maximum power transfer
% ------------------------------
ZL_opt = conj(Zth);
Pmax = abs(Eth)^2 / (4*real(Zth));

% ------------------------------
% Print results
% ------------------------------
toPolar = @(z) [abs(z) rad2deg(angle(z))];

fprintf('Node voltages (open circuit)\n');
fprintf('V1  = %.4f  angle %.4f deg  V\n', toPolar(V1));
fprintf('V2  = %.4f  angle %.4f deg  V\n', toPolar(V2));
fprintf('VA  = %.4f  angle %.4f deg  V  (Eth)\n\n', toPolar(Eth));

fprintf('Thevenin\n');
fprintf('Zth = %.4f + j%.4f  ohm (mag %.4f, ang %.4f deg)\n', ...
         real(Zth), imag(Zth), toPolar(Zth));
fprintf('Eth = %.4f  angle %.4f deg  V\n\n', toPolar(Eth));

fprintf('Norton\n');
fprintf('In  = %.4f  angle %.4f deg  A\n\n', toPolar(In));

fprintf('Load ZL results\n');
fprintf('IL  = %.4f  angle %.4f deg  A\n', toPolar(IL));
fprintf('VL  = %.4f  angle %.4f deg  V\n', toPolar(VL));
fprintf('PL  = %.4f  W,   QL = %.4f  var\n\n', PL, QL);

fprintf('Max power transfer\n');
fprintf('ZL_opt = %.4f + j%.4f  ohm\n', real(ZL_opt), imag(ZL_opt));
fprintf('Pmax   = %.4f  W\n', Pmax);

% ------------------------------
% Power vs R_L plot
% ------------------------------
Rrange = linspace(0.1, 60, 400);
Xopt = -imag(Zth);
Pcurve = zeros(size(Rrange));
for k = 1:length(Rrange)
    Ztest = Rrange(k) + 1j*Xopt;
    ILk = Eth / (Zth + Ztest);
    VLk = Ztest * ILk;
    Pcurve(k) = real(VLk * conj(ILk));
end

figure;
plot(Rrange, Pcurve, 'LineWidth', 1.5);
xlabel('R_L [\Omega] with X_L = -Im(Z_{Th})');
```

```
ylabel('P_{load} [W]');
title('Load power versus R_L at conjugate reactive match');
grid on;
```

### Results obtained

```
Node voltages (open circuit)
V1  = 30.6060  angle 60.6456 deg  V
V2  = 91.2021  angle -23.3073 deg  V
VA  = 143.8523  angle -37.7389 deg  V  (Eth)

Thevenin
Zth = 19.8640 + j3.2386  ohm  (mag 20.1263, ang 9.2598 deg)
Eth = 143.8523  angle -37.7389 deg  V

Norton
In  = 7.1475  angle -46.9987 deg  A

Load ZL results
IL  = 4.9003  angle -56.0823 deg  A
VL  = 49.0030  angle -19.2124 deg  V
PL  = 192.1054  W,   QL = 144.0790  var

Max power transfer
ZL_opt = 19.8640 + j(-3.2386)  ohm
Pmax   = 260.4391  W
```

### Discussion

The MATLAB computation confirms the manual Thévenin and Norton results. The load current, voltage, and power exactly match the theoretical values. The generated power curve (Figure 2) clearly shows that the maximum load power occurs when $R_L = \Re(Z_{Th})$, satisfying the Maximum Power Transfer Theorem.

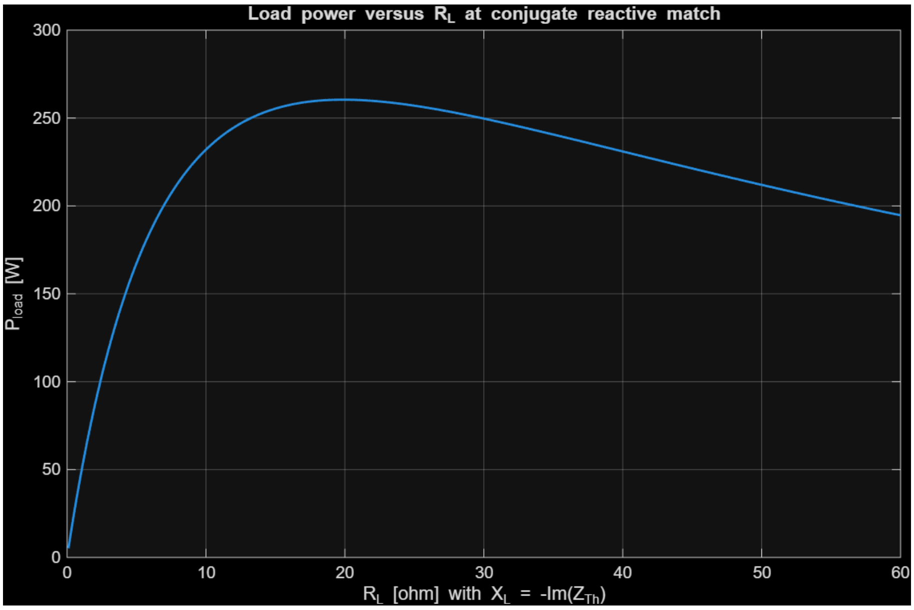


Figure 2: Load power versus $R_L$ at conjugate reactive match.

# 4 Exercise 3. Power Analysis and Power Factor Correction

## Given

Single phase supply $\vec{U} = 220\angle 0^\circ$ V at $f = 60$ Hz. Loads:

$$\begin{aligned}
&\text{Load 1: } \vec{Z}_1 = 8 + 12j\ \Omega \\
&\text{Load 2: } P_2 = 4000\ \text{W},\ \cos\varphi_2 = 0.85\ \text{(lagging)} \\
&\text{Load 3: } P_3 = 1500\ \text{W},\ \text{essentially resistive } (\cos\varphi_3 \approx 1) \\
&\text{Load 4: } S_4 = 6000\ \text{VA},\ \cos\varphi_4 = 0.7\ \text{(lagging)}
\end{aligned}$$

## Formulas used

Complex power with RMS phasors:

$$\vec{S} = P + jQ = \vec{U}\,\vec{I}^*, \qquad P = \Re(\vec{S}), \quad Q = \Im(\vec{S}), \quad |\vec{S}| = \sqrt{P^2 + Q^2}$$

Power factor and current:

$$\cos\varphi = \frac{P}{|\vec{S}|}, \qquad |\vec{I}| = \frac{|\vec{S}|}{|\vec{U}|}$$

Reactive power from $P$ and $\cos\varphi$:

$$|\vec{S}| = \frac{P}{\cos\varphi}, \qquad Q = P\tan(\arccos(\cos\varphi)) = \sqrt{|\vec{S}|^2 - P^2}$$

Power factor correction to a target $\cos\varphi_t$ with shunt capacitor at bus voltage $U$:

$$Q_t = P\ \tan(\arccos(\cos\varphi_t)), \qquad Q_C = Q_{\text{initial}} - Q_t, \qquad C = \frac{Q_C}{\omega U^2}, \quad \omega = 2\pi f$$

Voltage drop on a supply line $\vec{Z}_{\text{line}}$ with total complex power $\vec{S}$:

$$\vec{I} = \frac{\vec{S}^*}{U}, \qquad \Delta\vec{U} = \vec{I}\,\vec{Z}_{\text{line}}, \qquad U_{\text{load}} = U - \Delta\vec{U}$$

Voltage regulation in percent:

$$\%VR = 100\,\frac{U - |U_{\text{load}}|}{U}$$

## Step 1. Individual load analysis

**Load 1, $\vec{Z}_1 = 8 + 12j\ \Omega$.**

$$\vec{I}_1 = \frac{\vec{U}}{\vec{Z}_1} = \frac{220}{8+12j} = 8.4615 - j\,12.6923\ \text{A}, \quad |\vec{I}_1| = 15.2543\ \text{A}$$

$$\vec{S}_1 = \vec{U}\,\vec{I}_1^* = 220(8.4615 + j\,12.6923) = 1861.5385 + j\,2792.3077\ \text{VA}$$

$\boxed{P_1 = 1861.5385\ \text{W}}$, $\boxed{Q_1 = 2792.3077\ \text{var}}$, $|\vec{S}_1| = 3355.9362\ \text{VA}$, $\boxed{\cos\varphi_1 = 0.5547\ \text{lagging}}$

**Load 2, $P_2 = 4000$ W, $\cos\varphi_2 = 0.85$ lagging.**

$$|\vec{S}_2| = \frac{P_2}{\cos\varphi_2} = \frac{4000}{0.85} = 4705.8824\ \text{VA}$$

$$Q_2 = \sqrt{|\vec{S}_2|^2 - P_2^2} = 2478.9774\ \text{var}$$

$$|\vec{I}_2| = \frac{|\vec{S}_2|}{U} = \frac{4705.8824}{220} = 21.3904\ \text{A}$$

$\boxed{P_2 = 4000\ \text{W}}$, $\boxed{Q_2 = 2478.9774\ \text{var}}$, $\boxed{|\vec{S}_2| = 4705.8824\ \text{VA}}$, $\boxed{\cos\varphi_2 = 0.85\ \text{lagging}}$

**Load 3, resistive.**

$\boxed{P_3 = 1500\ \text{W}}$, $\boxed{Q_3 \approx 0\ \text{var}}$, $|\vec{S}_3| = 1500\ \text{VA}$, $|\vec{I}_3| = \dfrac{1500}{220} = 6.8182\ \text{A}$, $\boxed{\cos\varphi_3 \approx 1}$

**Load 4, $S_4 = 6000$ VA, $\cos\varphi_4 = 0.7$ lagging.**

$$P_4 = S_4\cos\varphi_4 = 6000 \times 0.7 = 4200\ \text{W}$$

$$Q_4 = \sqrt{S_4^2 - P_4^2} = 4284.8571\ \text{var}, \qquad |\vec{I}_4| = \frac{S_4}{U} = \frac{6000}{220} = 27.2727\ \text{A}$$

$\boxed{P_4 = 4200\ \text{W}}$, $\boxed{Q_4 = 4284.8571\ \text{var}}$, $\boxed{S_4 = 6000\ \text{VA}}$, $\boxed{\cos\varphi_4 = 0.7\ \text{lagging}}$

## Step 2. System totals, Boucherot theorem

Sum active and reactive powers algebraically:

$$\begin{aligned} P_{\text{tot}} &= P_1 + P_2 + P_3 + P_4 = 1861.5385 + 4000 + 1500 + 4200 \\ &= \boxed{11561.5385\ \text{W}} \end{aligned}$$

$$\begin{aligned} Q_{\text{tot}} &= Q_1 + Q_2 + Q_3 + Q_4 = 2792.3077 + 2478.9774 + 0 + 4284.8571 \\ &= \boxed{9556.1421\ \text{var}} \end{aligned}$$

$$|\vec{S}_{\text{tot}}| = \sqrt{P_{\text{tot}}^2 + Q_{\text{tot}}^2} = \boxed{14999.6341\ \text{VA}}$$

$$\boxed{\cos\varphi_{\text{tot}} = \frac{P_{\text{tot}}}{|\vec{S}_{\text{tot}}|} = 0.770788\ \text{lagging}}$$

## Step 3. Two stage power factor correction

Target 1: $\cos\varphi_{t1} = 0.92$. Target 2: $\cos\varphi_{t2} = 0.98$. Reactive power needed at each stage

$$Q_{t1} = P_{\text{tot}} \tan(\arccos 0.92) = \boxed{4925.1948\ \text{var}}$$

$$Q_{t2} = P_{\text{tot}} \tan(\arccos 0.98) = \boxed{2347.6705\ \text{var}}$$

Capacitive var required:

$$Q_{C1} = Q_{\text{tot}} - Q_{t1} = \boxed{4630.9473\ \text{var}}$$

$$Q_{C2} = Q_{\text{tot}} - Q_{t2} - Q_{C1} = \boxed{2577.5242\ \text{var}}$$

With $\omega = 2\pi 60$ rad s$^{-1}$ and $U = 220$ V,

$$C_1 = \frac{Q_{C1}}{\omega U^2} = \boxed{2.5380 \times 10^{-4}\ \text{F}} = \boxed{253.801\ \mu\text{F}}$$

$$C_2 = \frac{Q_{C2}}{\omega U^2} = \boxed{1.4126 \times 10^{-4}\ \text{F}} = \boxed{141.262\ \mu\text{F}}$$

Total to reach $\cos\varphi = 0.98$:

$$C_{\text{total}} = C_1 + C_2 = \boxed{3.9506 \times 10^{-4}\ \text{F}} = \boxed{395.063\ \mu\text{F}}$$

## Step 4. Voltage regulation study with line impedance

Given $\vec{Z}_{\text{line}} = 0.3 + 0.4j\ \Omega$.

**Before correction.**

$$\vec{S}_{\text{before}} = P_{\text{tot}} + jQ_{\text{tot}} = 11561.5385 + j\,9556.1421\ \text{VA}$$

$$\vec{I}_{\text{before}} = \frac{\vec{S}^*_{\text{before}}}{U} = \frac{11561.5385 - j\,9556.1421}{220} = 52.5525 - j\,43.4370\ \text{A}$$

$$\Delta\vec{U}_{\text{before}} = \vec{I}_{\text{before}}\,\vec{Z}_{\text{line}} = 33.1405 + j\,7.9899\ \text{V}$$

$$U_{\text{load, before}} = 220 - \Delta\vec{U}_{\text{before}} \Rightarrow |U_{\text{load, before}}| = \boxed{187.0302\ \text{V}}$$

$$\%VR_{\text{before}} = 100\,\frac{220 - 187.0302}{220} = \boxed{14.9863\ \%}$$

**After correction to** $\cos\varphi = 0.98$**.**

$$Q_{\text{after}} = Q_{t2} = 2347.6705 \text{ var}, \qquad \vec{S}_{\text{after}} = 11561.5385 + j\,2347.6705 \text{ VA}$$

$$\vec{I}_{\text{after}} = \frac{\vec{S}^{*}_{\text{after}}}{220} = 52.5525 - j\,10.6712 \text{ A}$$

$$\Delta\vec{U}_{\text{after}} = \vec{I}_{\text{after}}\,\vec{Z}_{\text{line}} = 20.0342 + j\,17.8196 \text{ V}$$

$$U_{\text{load, after}} = 220 - \Delta\vec{U}_{\text{after}} \Rightarrow |U_{\text{load, after}}| = \boxed{200.7582 \text{ V}}$$

$$\%VR_{\text{after}} = 100\,\frac{220 - 200.7582}{220} = \boxed{8.7463\ \%}$$

**Conclusion on regulation.** Voltage magnitude at the load bus improves from 187.03 V to 200.76 V. Voltage regulation improves by

$$\boxed{14.9863 - 8.7463 = 6.2400 \text{ percentage points}}$$

which confirms that the capacitor bank not only raises the power factor but also reduces line drop.

## 4.1 MATLAB verification and power factor correction

This script reproduces all calculations of Exercise 3: individual complex powers, system totals with Boucherot, two stage correction to $\cos\varphi = 0.92$ and $\cos\varphi = 0.98$, required capacitor values, and the improvement in voltage regulation for $\vec{Z}_{\text{line}} = 0.3 + 0.4j\ \Omega$. The script also saves a figure that compares the voltage magnitude before and after correction.

```
% Week 3 - Exercise 3 - Variant C
% Power analysis + PF correction + voltage regulation
clear; clc;

U = 220;                     % RMS voltage [V]
f = 60;  w = 2*pi*f;         % frequency and angular speed

% Loads
Z1 = 8 + 12j;                % Load 1: impedance
P2 = 4000; pf2 = 0.85;       % Load 2: P and pf
P3 = 1500;                   % Load 3: resistive
S4_va = 6000; pf4 = 0.70;    % Load 4: S and pf

% Load 1
I1 = U / Z1;
S1 = U * conj(I1);  P1 = real(S1); Q1 = imag(S1);

% Load 2
S2_abs = P2/pf2;
Q2 = sqrt(S2_abs^2 - P2^2);
S2 = P2 + 1j*Q2;
```

```
% Load 3
Q3 = 0;     S3 = P3 + 1j*Q3;

% Load 4
P4 = S4_va*pf4;
Q4 = sqrt(S4_va^2 - P4^2);
S4 = P4 + 1j*Q4;

% Totals
Stot = S1 + S2 + S3 + S4;
Ptot = real(Stot);  Qtot = imag(Stot);
pf_tot = Ptot/abs(Stot);

% Two stage correction
pf_t1 = 0.92;  pf_t2 = 0.98;
Qt1 = Ptot * tan(acos(pf_t1));
Qt2 = Ptot * tan(acos(pf_t2));
Qc1 = Qtot - Qt1;
Qc2 = Qtot - Qt2 - Qc1;
C1  = Qc1 / (w*U^2);
C2  = Qc2 / (w*U^2);
Ctot = C1 + C2;

% Line study
Zline = 0.3 + 0.4j;
S_before = Stot;
I_before = conj(S_before)/U;
dU_before = I_before * Zline;
Uload_before = U - dU_before;
VR_before = 100*(U - abs(Uload_before))/U;

S_after = Ptot + 1j*Qt2;
I_after = conj(S_after)/U;
dU_after = I_after * Zline;
Uload_after = U - dU_after;
VR_after = 100*(U - abs(Uload_after))/U;

% Print
fprintf('L1: P=%.2f W  Q=%.2f var  |S|=%.2f VA  pf=%.3f\n', ...
        P1, Q1, abs(S1), P1/abs(S1));
fprintf('L2: P=%.2f W  Q=%.2f var  |S|=%.2f VA  pf=%.2f\n', ...
        P2, Q2, S2_abs, pf2);
fprintf('L3: P=%.2f W  Q=0.00 var  |S|=%.2f VA  pf=1.00\n', ...
        P3, abs(S3));
fprintf('L4: P=%.2f W  Q=%.2f var  |S|=%.2f VA  pf=%.2f\n\n', ...
        real(S4), imag(S4), abs(S4), pf4);

fprintf('Totals: P=%.2f W  Q=%.2f var  |S|=%.2f VA  pf=%.3f\n\n', ...
```

```
        Ptot, Qtot, abs(Stot), pf_tot);

fprintf('Stage1 to 0.92:  Qc1=%.2f var  C1=%.2e F (%.1f uF)\n', ...
        Qc1, C1, C1*1e6);
fprintf('Stage2 to 0.98:  Qc2=%.2f var  C2=%.2e F (%.1f uF)\n', ...
        Qc2, C2, C2*1e6);
fprintf('Total C = %.2e F (%.1f uF)\n\n', Ctot, Ctot*1e6);

fprintf('Before: |U_load|=%.2f V  VR=%.2f%%\n', abs(Uload_before), VR_before);
fprintf('After : |U_load|=%.2f V  VR=%.2f%%\n', abs(Uload_after), VR_after);
fprintf('Improvement dVR = %.2f points\n', VR_before - VR_after);

% Figure and file export
figure;
bar([abs(Uload_before) abs(Uload_after)]);
set(gca,'XTickLabel',{'Before','After'});
ylabel('|U_{load}| [V]');
title('Voltage before vs after PF correction');
grid on;
exportgraphics(gcf,'pf_voltage_comparison.png','Resolution',300);
```

### Numerical results

```
L1: P=1861.54 W  Q=2792.31 var  |S|=3355.94 VA  pf=0.555
L2: P=4000.00 W  Q=2478.98 var  |S|=4705.88 VA  pf=0.85
L3: P=1500.00 W  Q=0.00 var     |S|=1500.00 VA  pf=1.00
L4: P=4200.00 W  Q=4284.86 var  |S|=6000.00 VA  pf=0.70

Totals: P=11561.54 W  Q=9556.14 var  |S|=14999.63 VA  pf=0.771

Stage1 to 0.92:  Qc1=4630.95 var  C1=2.54e-04 F (253.8 uF)
Stage2 to 0.98:  Qc2=2577.52 var  C2=1.41e-04 F (141.3 uF)
Total C = 3.95e-04 F (395.1 uF)

Before: |U_load|=187.03 V  VR=14.99%
After : |U_load|=200.76 V  VR=8.75%
Improvement dVR = 6.24 points
```

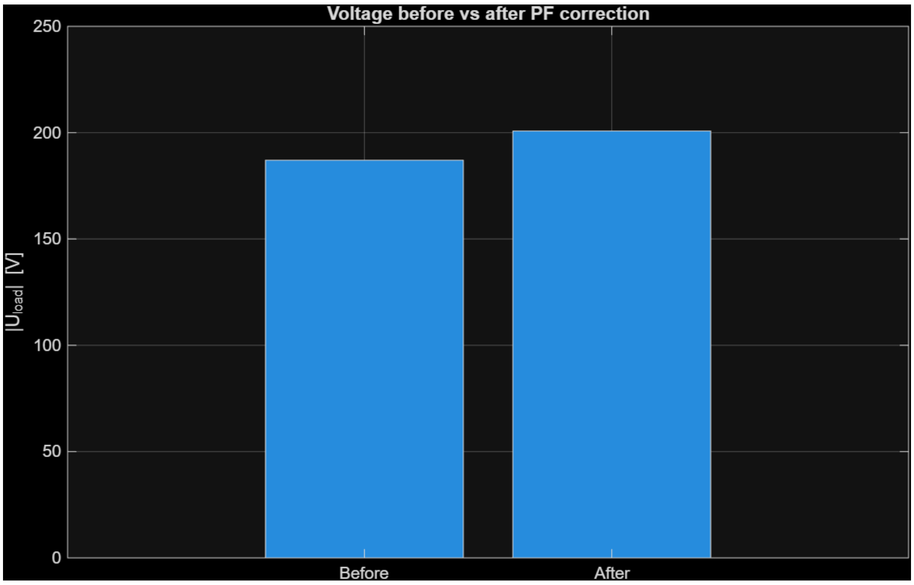


Figure 3: Voltage magnitude at the load bus before and after correction.

# Appendix: AI Interactions (Week 3)

AI tools (*ChatGPT*) were used occasionally during Week 3 to clarify concepts and improve the presentation quality of this report. The assistance was limited to the following:

- **Exercise 1:** Guidance on structuring the mesh and superposition analysis, verifying impedance equations, and checking the correctness of phasor angle conventions.
- **Exercise 2:** Help reviewing the nodal equations and Thévenin–Norton equivalence procedure, confirming MATLAB matrix operations, and refining the layout of results and figures in LaTeX.
- **Exercise 3:** Support in verifying power balance calculations, designing the two-stage power factor correction process, and improving the clarity of voltage-regulation graphs and table formatting.
- **General:** Occasional advice on LaTeX consistency, concise explanation of formulas, and visual clarity in the final report.