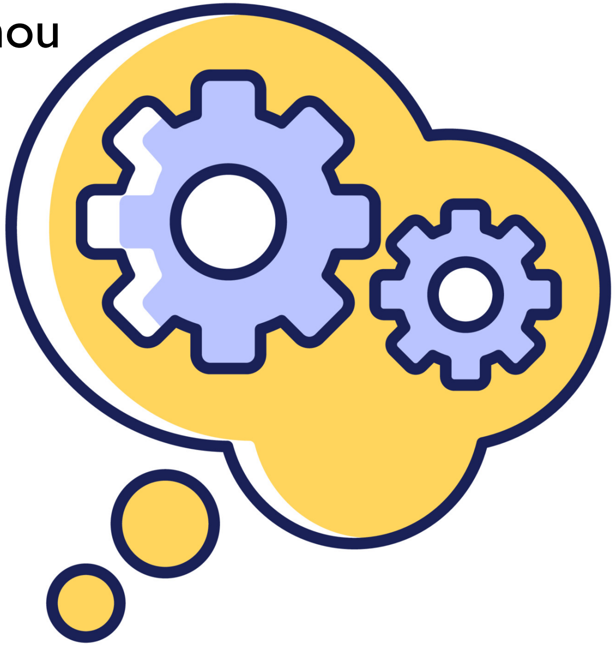
Bridging Machine Learning and Logical Reasoning by Abductive Learning

Wang-Zhou Dai, Qiuling Xu, Yang Yu, Zhi-Hua Zhou



Pisa

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Machine Learning

- Very powerful and good in tasks such as:
  - Image recognition
  - Speech recognition
  - ..

⇒ Map sensory information to a concept

- Not so good in:
  - Visual question answering
  - Learning simple relations
  - Do maths in natural language
  - …

⇒ Reason over context and input data

<u>Tip: Not treat reasoning as perception!</u>

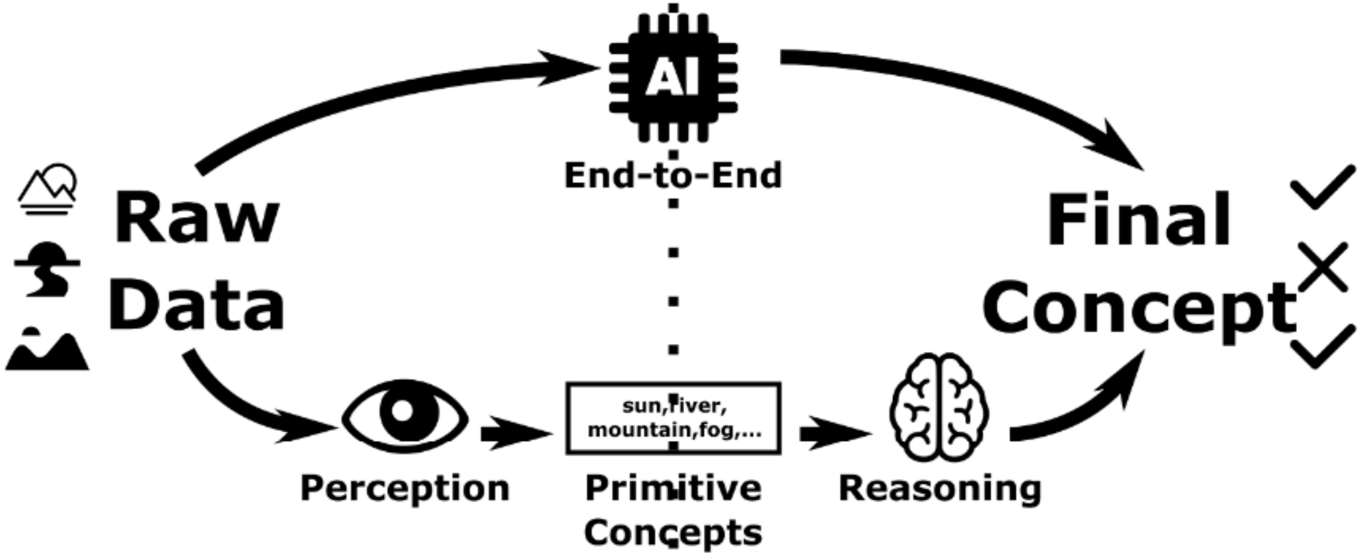
Machine reasoning

- Very first task in AI

- Boolean satisfiability
- Expert systems
- Logic programming
- Inductive Logic Programming
- Probabilistic Logic Programming
- Constrain Logic Programming
- Answer Set Programming
- …

Bringing machine learning and reasoning

- Perception: ML model
- Reasoning: abductive reasoning



Abductive reasoning:

Explain (specific) **observations** based on (general) **background knowledge**.

KB ∪ **H** ⊨ **O**

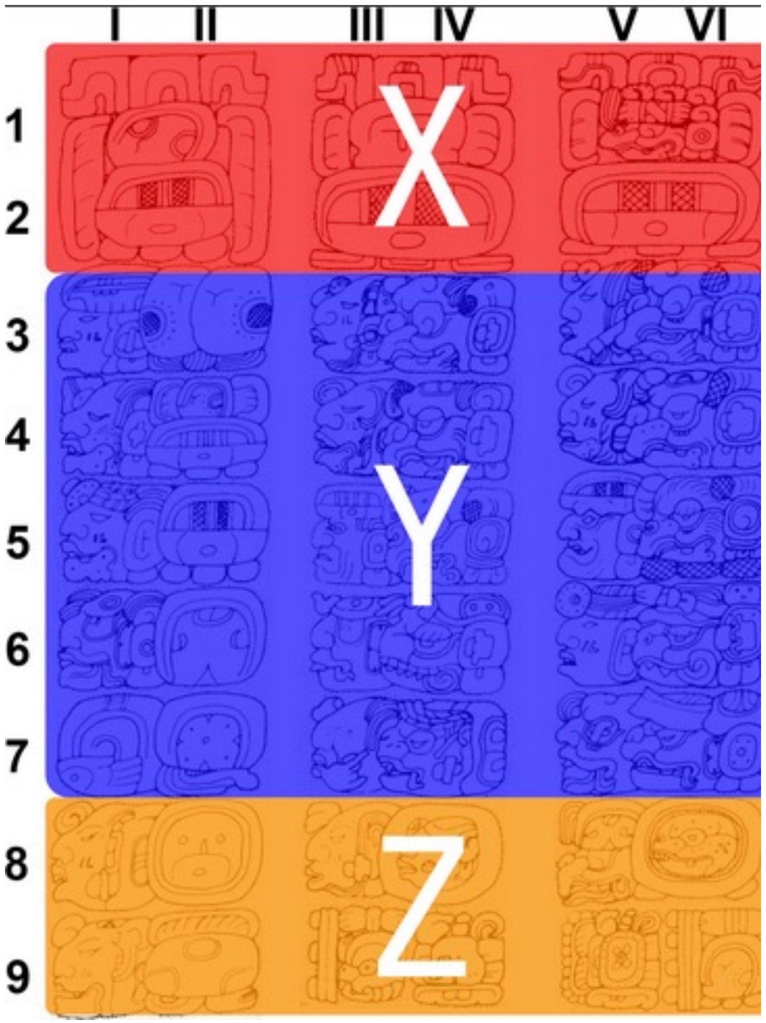
Human Abductive Problem-Solving

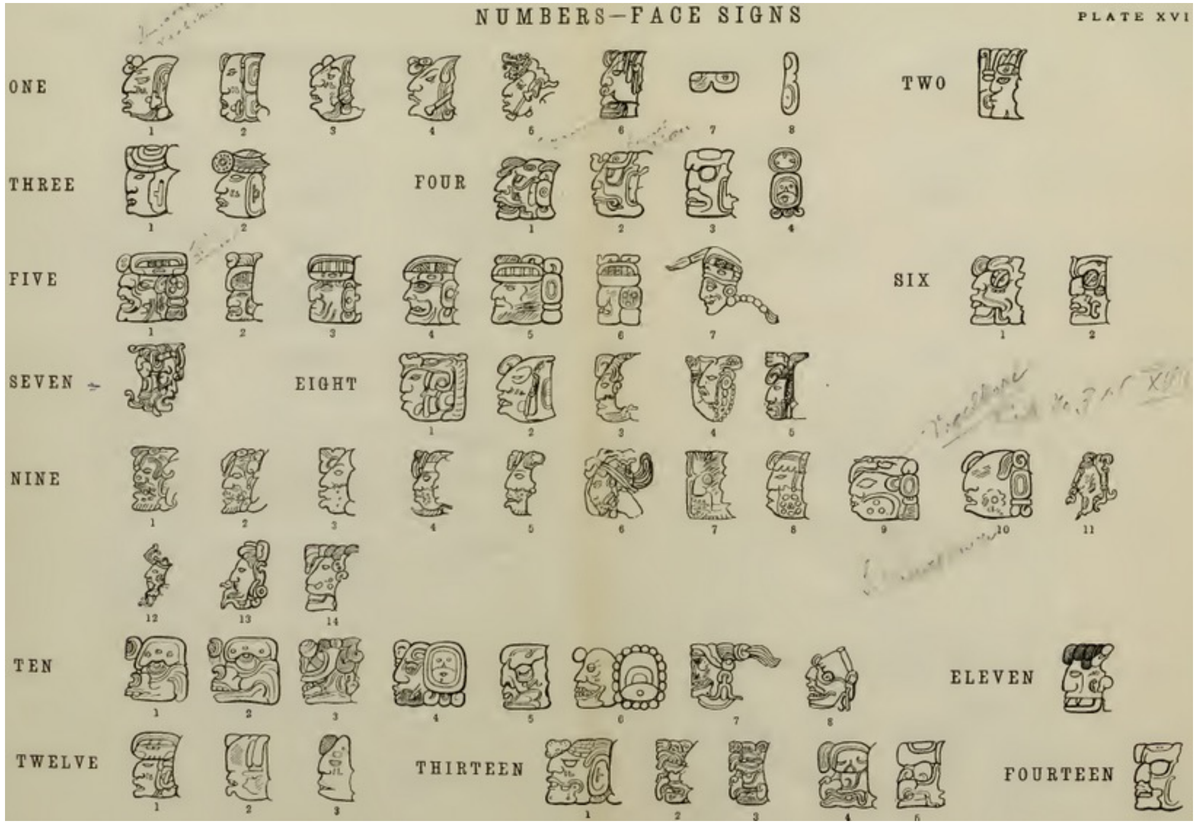
Structure

- Row 1-2: X
- Row 3-7: Y
- Row 8-9: Z

Calculation

$X \oplus Y = Z$





Human Abductive Problem-Solving

Column 1.		Column 2.
9.18.5.0.0.,	9 Ahau 13 Mac.	4 Ahau 13 Ceh (18)
9.18.5.1.0.,	9 Ahau 13 Mac.	11 Ahau 13 Mac (18)
9.18.5.2.0.,	9 Ahau 13 Mac.	5 Ahau 13 Kankin (18)
9.18.5.3.0.,	9 Ahau 13 Mac.	12 Ahau 13 Muan (18)
9.18.5.4.0.,	9 Ahau 13 Mac.	6 Ahau 13 Pax (18)
9.18.5.5.0.,	9 Ahau 13 Mac.	13 Ahau 13 Kayab (18)
8.18.5.8.0.,	8 Ahau 13 Mac	9 Ahau 3 Zac (40)
8.18.5.9.0.,	8 Ahau 13 Mac	3 Ahau 3 Ceh (40)
8.18.5.10.0.,	8 Ahau 13 Mac	10 Ahau 3 Mac (40)
8.18.5.11.0.,	8 Ahau 13 Mac	4 Ahau 3 Kankin (40)
1.18.5.2.0.,	1 Ahau 13 Mac	13 Ahau 13 Zac (34)
1.18.5.3.0.,	1 Ahau 13 Mac	7 Ahau 13 Ceh (34)
1.18.5.4.0.,	1 Ahau 13 Mac	1 Ahau 13 Mac (34)
1.18.5.5.0.,	1 Ahau 13 Mac	8 Ahau 13 Kankin (34)
1.18.5.6.0.,	1 Ahau 13 Mac	2 Ahau 13 Muan (34)
1.18.5.7.0.,	1 Ahau 13 Mac	9 Ahau 13 Pax (34)

[Bowditch, 1901]

Perception

Glyphs (image) -> Numbers (symbol)

Abductive reasoning

Observation: equations on table are correct

KB:

Structure -> $X \oplus Y = Z$,

Calculation rules -> 20-based

Trial-and-errors:

Until perception and reasoning are consistent

Framework

- **Input:**
  - $D = \{\langle\, \mathbf{x}_1, y_1 \,\rangle, \ldots, \langle\, \mathbf{x}_m, y_m \,\rangle\}$;
  - KB is defined by FOL clauses:
    - Primitive symbols: $\mathcal{P} = \{p_1\ , p_2\ , ..\ , p_r\ \}$;

- **Target:** $H = p \cup \Delta_c$
  - Perception model (maps input data to primitive symbols): $p : \mathcal{X} \mapsto \mathcal{P}$;
  - Knowledge model, Δ_c

- $\mathbf{KB} \cup \mathbf{H} \vDash \mathbf{O}$ -> $KB \cup \Delta_c \cup p(\mathbf{x}_i) \vDash y_i$.

(1) $\forall\, (x, y) \in D\ (KB \cup \Delta_c \cup p(\mathbf{x}_i) \vDash y_i)$.

Framework

- **Intuition:**
  - Maximise the number of instances in D that are **consistent** with H:

$$\max_{H=p\cup\Delta_c} \mathrm{Con}(H \cup D)$$

where $\mathrm{Con}(H \cup D)$ is the size of subset $\widehat{D}_c \in D$ consistent with H:

$$\widehat{D}_c = \arg\max_{D_c \in D} |D_c|$$

$$\text{s.t. } \forall\, (\mathbf{x}_i, y_i) \in D_c\ (KB \cup \Delta_c \cup p(\mathbf{x}_i) \vDash y_i).$$

Framework

- **Challenge:**
  - Labels for training are unknown: need to be inferred by logical reasoning
  - Reasoner require perceived symbols as input

- **Assumption:**
  - When the perception model is undertrained, $p : \mathcal{X} \mapsto \mathcal{P}$ is possibly incorrect and (1) is inconsistent.

Framework

- Mark up the "possibly wrong" pseudo-labels:

$$\delta(p^t(X)) \subseteq p^t(X),$$

where δ is an heuristic function to **guess which perceived symbols are wrong.**

Framework

- Change optimization objective:

$$\max_{\delta} \mathrm{Con}(H_\delta \cup D)$$

$$\text{s.t.} \mid \delta[p^t(X)] \mid \leq M$$

where M is the maximum revision size of δ

- **KB** $\cup$ **H** $\vDash$ **O** -> $KB \cup \Delta_c \cup p^t(X) - \delta[p^t(X)] \cup \Delta_\delta[p^t(X)] \vDash Y$

$$H_\delta = [p^t(X) - \delta[p^t(X)] \cup \Delta_\delta[p^t(X)] \cup \Delta_c.$$

- Abduce the revised pseudo-labels $r_\delta(X)$ and reasoning model Δ_c based on δ,

  where: $r_\delta(X) = [p^t(X) - \delta[p^t(X)] \cup \Delta_\delta[p^t(X)]$

Framework

- Use revised pseudo-label $\mathrm{r}_{\delta}(\mathrm{X})$ to train perception model p^{t+1}.

$$p^{t+1} = \arg\min_{p} \sum_{i=1}^{m} Loss(p(\mathbf{x}_i), r(\mathbf{x}_i))$$

In short:

(a) Given the training data and the KB,
(b) Use ML model to obtain the pseudo-labels,
(c) Treat pseudo-labels as groundings of the primitive concepts,
(d) Provide pseudo-labels to the reasoner to abduce ΔC.
(e) If the abduction terminated due to inconsistency, revise the pseudo-labels,
(f) Retrain the ML model with new abduced pseudo-label.

Conclusion

- Abductive reasoning connects **high-level reasoning** and **low-level perception**;
- Abduction is neither sound or complete, humans/machines need **trial-and-errors**.
- The dividing line between **high-level** and **low-level** is **unclear**, how to combine symbolic and sub-symbolic AI more efficiently is still an open question.