Dynamic pricing under competition in an e-commerce scenario: a demand learning and price optimization technique

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Outline

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3. Problem description
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4. Experiment design
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    - Value iteration;
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Goal and contributions

Goal

Predicting prices automatically by adapting to variations within the market;

Contributions

- The approximation of consumer demand and competitors' strategies;
- The continuous recalculation of the demand variables;
- The construction of a price optimization algorithm;

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Dynamic pricing

"It is the study of deciding ideal prices of products or services in a setting where prices can frequently be adjusted."
Van de Geer R. et al.[1]

Advantages

- Reducing prices updating costs;
- More accurate pricing decisions;
- Increase revenues;

Basic points

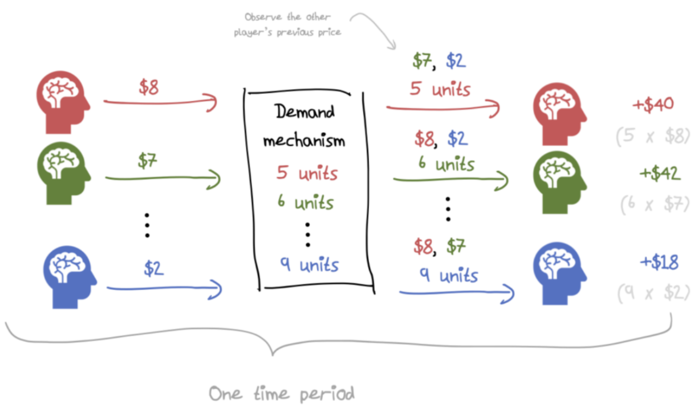
1. Treatment of historical data;
2. Processing a demand function or learning it;
3. Use of mathematical processes for price optimization;

[1] Dynamic pricing and learning with competition, Journal of Revenue Pricing Management 18, 185–203, 2019.

Dynamic pricing competition

- Dynamic Pricing Competition, Haensel AMS [a]

[a] https://dynamic-pricing-competition.com/



- Demand mechanism unknown: *demand uncertainty*;
- Heterogeneous customer behavior;
- Demand mechanism resembles a competitive e-commerce market;

Simulations settings

- Time period: $T = 1000$, indexed by $t = 0, 1, 2..T$;
- n competitors;
- Merchant could set price $p_{i,t} \in 0, 0.001, 0.002..100$ and no costs were involved.
- After each period, sales, $s_{i,t}$ and revenues, $p_{i,t}s_{i,t}$ are generated;
- The merchant could observe his sales, his price and competitors' prices from 0 to $t-1$;

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Demand learning

- *Exploration phase*: 150 periods
  - Random prices from a Uniform discrete distribution [1,100]
- With historical data: Regression model

Regression variables

1. x_0: intercept;
2. x_1: our price;
3. x_2: $min\ (p_{1t}, p_{2t}, p_{3t}..p_{nt})$;
4. x_3: $mean\ (p_{1t}, p_{2t}, p_{3t}..p_{nt})$;
5. x_4: $max\ (p_{1t}, p_{2t}, p_{3t}..p_{nt})$;
6. Interactions: $x_2 \cdot x_3$; $x_2 \cdot x_3$; $x_4 \cdot x_3$;

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Regression model

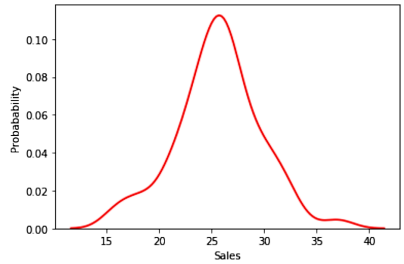
$$\hat{Y}_{sales}(a, \vec{p}, \vec{\beta}) = \hat{\beta}_0 + \hat{\beta}_1 x_1^3 + \hat{\beta}_2\, x_2^3 + \hat{\beta}_3\, x_3^3 + \hat{\beta}_4\, x_2 \cdot x_3 + \hat{\beta}_5\, x_2 \cdot x_4 + \hat{\beta}_6\, x_3 \cdot x_4$$

Why regression?

- Time requirements: 0.12 secs per iteration;
- Good compromise between accuracy and timing;

Why polynomial?

- Avoid linear relationship between prices and quantity: *heterogeneous customers*;



Sales probabilities

Schlosser R. and Boissier M.[a]

$$P(s, a, \vec{p}, \vec{\beta}) = e^{-\hat{Y}_{sales}(a,\vec{p},\vec{\beta})} \cdot \frac{\hat{Y}_{sales}(a,\vec{p},\vec{\beta})^{s}}{s!}$$

[a] Dynamic Pricing under Competition on Online Marketplaces, International Conference on Knowledge Discovery Data Mining, 2018.

- $\hat{Y}_{sales}(a, \vec{p}, \vec{\beta})$: mean sales per period;
- $s = [0, 100]$ is the number of maximum sales per period;

Effects

1. Customer buying behavior;
2. Customer arrivals;
3. Competitor pricing strategies;

Model formulation

Markov Decision Process

- State space: $\vec{S} = \{0, 1, 2, 3..100\}$;
- Actions: $A_t \in [1, 100]$;
- Sales probabilities: $P(s, a|\vec{p}, \vec{\beta})$;
- Reward: $R(a, \vec{p}, \vec{\beta}) = [a \cdot \hat{Y}_{sales}(a, \vec{p}, \vec{\beta})]$;
- Transition probability: $P_{s,s'}(s, s', a|\vec{p}, \vec{\beta})$;

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Value iteration algorithm

- The optimal value of each state s is find by performing the following update on all states:
$V(s) = \max_{a \in A} \sum_{s=0}^{100} [P_{s,s'}(s, s', a|\vec{p}, \vec{\beta}) + (a \cdot \hat{Y}_{sales}(a, \vec{p}, \vec{\beta})) + \gamma\ V(s')]$
- The optimal pricing decision for each state is find:
$a^*(s) = \arg\max_{a \in A} \sum_{s=0}^{100} [P_{s,s'}(s, s', a|\vec{p}, \vec{\beta}) + (a \cdot \hat{Y}_{sales}(a, \vec{p}, \vec{\beta})) + \gamma\ V(s')]$

Optimal policy

The greatest value function among all states will be chosen:
the corresponding action will be the optimal price.

Runtime efficiency

- Reduction in number of prices;
- Increase theta value;

repeat
  foreach $s \in S$: **do**
    $v \leftarrow V(s)$;
    $V(s) \leftarrow max_a \sum_{s',r} p(s', r|s, a) + [r + \gamma V(s')]$;
    $\triangle \leftarrow max(\triangle, |v - V(s)|)$;
until $\triangle < \theta$;

Value	Average runtime in ms
$\theta = 0.01$	21.94
$\theta = 0.05$	12.31
$\theta = 0.5$	11.87
$\theta = 1$	10.56

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Competition results

- Top half of participants;
- Competition results not fully available;

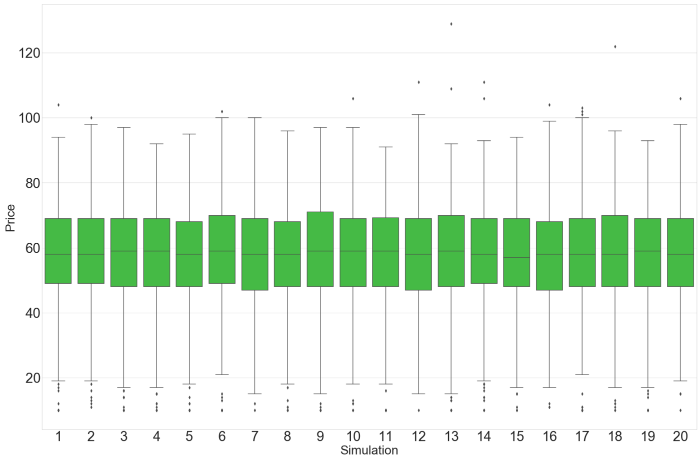

Figure 1: Merchant's average prices per simulation

Figure 2: Average price per competitor among all simulations

Competition results

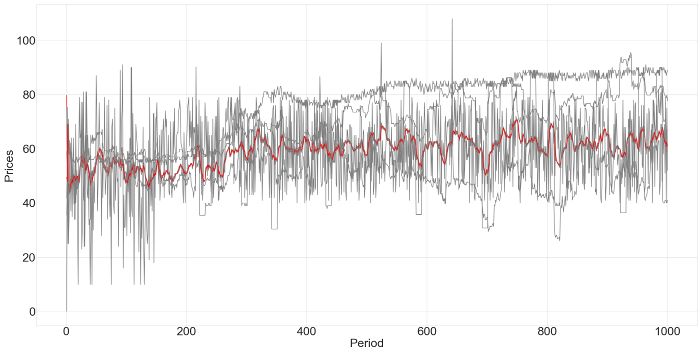

Figure 3: Prices trend in simulation 6

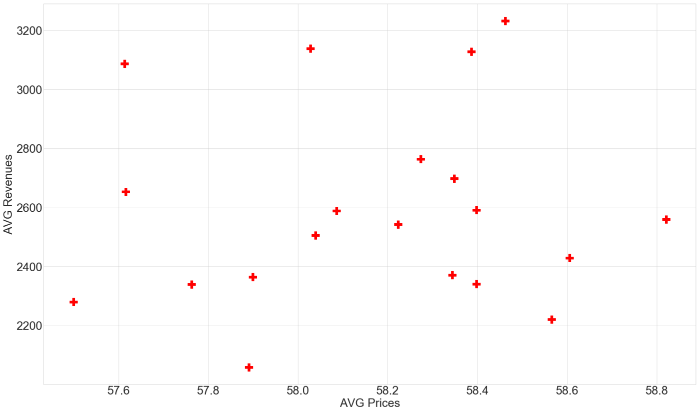

Figure 4: Average prices vs average revenues

Conclusions

Remarks

1. Ability to set prices and generate revenues;
2. Ability to compete in different situations and market contexts;
3. What seems reasonable in a simulated market may not be in a real one;

Future improvements

- Model customer behavior differently;
- Prediction of competitors' pricing strategies;
- Different transition probabilities;
- Lower θ value;
- Use all the available prices;

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Thank You
for your attention