1. (a) $F_n = 2^{2^n} + 1$

$$F_n - 2 = 2^{2^n} - 1$$

$$= \underbrace{\left(2^{\frac{2^n}{2}} - 1\right)}\ \underbrace{\left(2^{\frac{2^n}{2}} + 1\right)}_{F_{n-1}}$$

$$= \left(2^{\frac{2^n}{4}} - 1\right) \underbrace{\left(2^{\frac{2^n}{4}} + 1\right)}_{F_{n-2}} F_{n-1}$$

$$\vdots$$

$$= \left(2^{\frac{2^n}{2^{n-1}}} - 1\right) F_{n-1} F_{n-2} \cdots F_1$$

$$= (2^2 - 1) F_{n-1} F_{n-2} \cdots F_1$$

$$= 3 \cdot F_1 \cdot F_2 \cdot F_3 \cdot F_4 \cdots F_{n-1}$$

$$= F_0 \cdot F_1 \cdot F_2 \cdot F_3 \cdot \cdots \cdot F_{n-1} \ \#$$

(b) The remain of F_n divide $F_0, F_1, F_2 \ldots F_{n-1}$ is all 2.

Suppose $0 \le k \le n-1$

$$\gcd(F_n, F_k) = \gcd(F_n, 2) = \gcd(2^{2^n} + 1, 2) = 1$$

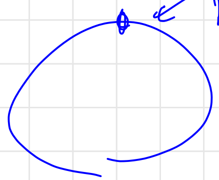
They are pairwise prime.

(c) If they are not distinct, The gcd will be the common factor, this ($\Rightarrow\Leftarrow$) with the proof in (b)

(d) Since their are infinite fermat numbers, and we know $F_0 = 3$, 3 is a prime number, we could discover infinite primes since fermat numbers are pairwise relatively prime, and by C, the prime factors for F_n will be distinct, we can obtain ∞ primes.

2. for each column, since we could decide to color each point Red/white, we have $2\times2\times2=8$ choices, or 8 different ways to color the 3 points, however in this situation, we have 9 rows. By pigeon hole principle, there must exist 2 rows that are colored the same. Causing a rectangle whose four corners have the same color

3. Base $n=2$

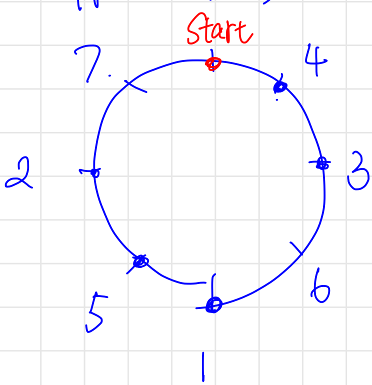


pick here then move to opposite side.

opponet decide left, right does not matter

For $n>2$ since n is a power of 2 we adopt the strategy similar as above, we try to clean opposite sides consecutively, then move left/right $\frac{n}{2}$ steps and continuosly clean up the cakes.

suppose $P(8) \to$ move 4 $\to$ move 2 (left, right is the same)



→ clean opposite side (move 4)

→ move 1 (left right same)

→ clean opposite side

→ move 2

→ clean opposite

→ finish

Since n is power of 2, we guarantee $\frac{n}{2}$ is divisible and this table can always be cleaned up.

4 we assign it as the following way

V_1	V_2	V_3	V_4	V_5	V_6	$\cdots$	V_n
1	$n-1$	2	$n-2$	3	$n-3$		

$V_{12} = n-2$

$V_{23} = n-3$

$V_{34} = n-4$ $\cdots$

$|f(x) - f(x-1)| = n - x + 1 \quad 1 \le x \le n$

no two will be the same

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