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(1) $(A, *)$ is a Semigroup. ⇒ two properties

a. closed

b. associative

$(a*b)*c = a*(b*c)$

$= a*(c*b)$

$= (a*c)*b$

$= (c*a)*b = c*(a*b)$ ✱

2. $(A, *)$ monoid ⇒ three properties a. closed

b. associative

c. Identity

abelian group ⇒ $ab = ba$.

$x*x = e$ suppose $y \in A$

$x*y = (y*y*(x*y)*(x*x)$

$= y*(y*x)*(y*x)*x$

$= y*x$ ⇒ match property

3. p: prime, group $(G, *)$ $|G| = p^2$

$g \in G$ a generator subgroup $(H, *)$

$H = \{ g, g^2 \cdots \quad g^{|H|} g \}$

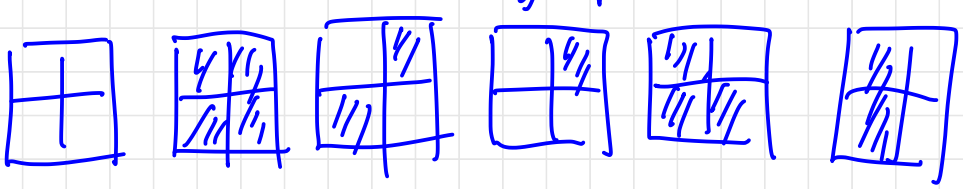
By Lagrange $1, p, p^2$

When $|H| = 1$ $g = e$ $|H| = p$, $H = G'$

$|H| = p^2 \Rightarrow G' = \{ e, e*g^p, e*g^{2p} \cdots \quad e*g^{(p-1)p} \}$

$\#\ = p$

4. (a)



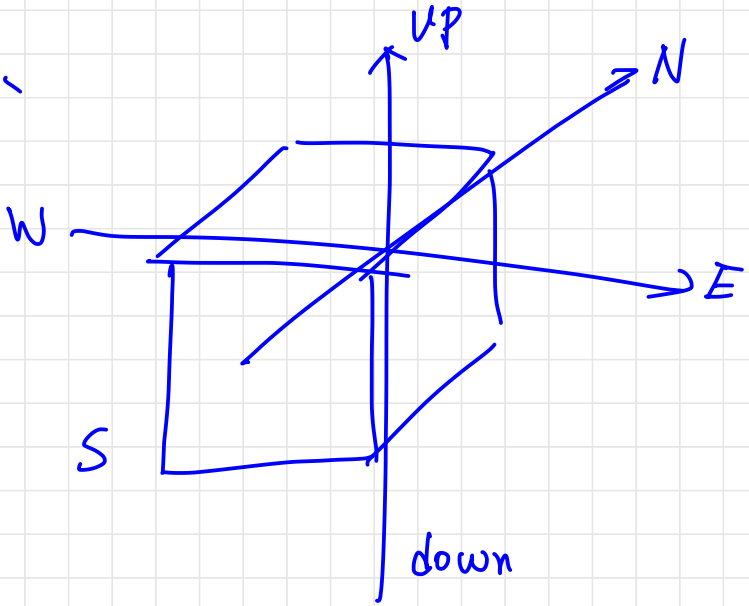
$\Rightarrow 6$ ✱

(b) 4 kinds of rotation, $0°$, $90°$ left, $90°$ right, $180°$

$0° \Rightarrow 2^{16}$ $\quad 90° \Rightarrow 2^4$ $\quad 180° \Rightarrow 2^8$

$$\frac{2^{16} + 2 \cdot 2^4 + 2^8}{4}$$ ✱

5.



no limit $\Rightarrow n^6$

rotate 2 face $\begin{cases} 180^\circ & 3n^4 \\ 90^\circ & 6n^3 \end{cases}$

rotate 2 edge 180° $6n^3$

rotate 2 vertices 120° $8n^2$

$\Rightarrow$ if we fix a side as down $\Rightarrow$ 6 choices
up side will be forced.
remaining 4 sides
fix one side as North, $\Rightarrow$ 4 choices
East, south, west will be forced
$6 \times 4 = 24$ fixed positons

$\Rightarrow \dfrac{n^6 + 3n^4 + 12n^3 + 8n^2}{24}$