1. (a) we turn 100 into triad

$100_d = 81 + 9 + 9 + 1 = (10201)_{triad}$

since each digit of 3^n can be picked twice, it meets the property of a triad.
and you can express any number from 0~100 with a five digit triad.

(b) we pick weighs

(1, 1) (3, 3) (9, 9) (27, 27) (81, 81)

we can express any number from 0 ~ 2 with two ones.

denote 0, 1, 2, 3 .... n as Range (0, n)

Range (0, 8) with (3,3) and (1,1)

Range (0, 26) with (9,9), (3,3) and (1,1)

Range (0, 80) with (27,27), (9,9) (3,3) and (1,1)

81 + 19 = 100

19 can be presented as 9 + 9 + 1.

2. (a) $\frac{13!}{13} = 12!$

$\frac{12!}{3!\,10!} = 22.$

(b) If they are not in a circle

There are 103 objects

permutation will be

$\frac{103!}{3!\,100!}$

They are in a circle. according to the lecture, there are $\frac{103!}{103}$ ways since 103 objects could be the start.

$\Rightarrow \frac{103!}{3!\,100!\,103} = \frac{102!}{3!\,100!}$ ✗

3.

$$\frac{x+1}{(x-2)(1+5x)} = \frac{\alpha}{x-2} + \frac{\beta}{1+5x}$$

$$= \frac{(5\alpha+\beta)x + (\alpha-2\beta)}{(x-2)(1+5x)}$$

We have to let

$5\alpha+\beta=1 \qquad \alpha-2\beta=1 \qquad \alpha=\frac{3}{11}$

to match $\frac{x+1}{(x-2)(1+5x)} \qquad \beta=\frac{-4}{11}$

$$\frac{3}{11(x-2)} + \frac{-4}{11(1+5x)}$$

$$= \frac{3}{11}\times\left(\frac{-1}{2}\right)\times\frac{1}{\left(1-\frac{x}{2}\right)} + \frac{-4}{11}\times\frac{1}{1-(-5x)}$$

$$= \frac{3}{11}\left(-\frac{1}{2}\sum_{j\geq 0}\left(\frac{1}{2}x\right)^j\right) - \frac{4}{11}\left(\sum_{j\geq 0}(-5x)^j\right)$$

$$\text{Coeff} = \frac{3}{-22}\left(\frac{1}{2}\right)^n - \frac{4}{11}(-5)^n$$

4. This is similar to the problem the proffesor asked when not all n objects have to be used

$$EGF_1(x) = e^x - 1$$

$$EGF_2(x) = x + \frac{2!}{2!}x^2 + \frac{3!}{3!}x^3 \dots = \frac{1}{1-x} - 1$$

$$EGF_3(x) = EGF_2(x) + 1$$

since case 3 cannot be empty

$$EGF_{1,2,3}(x) = (e^x - 1)\left(\frac{1}{1-x} - 1\right)\left(\frac{1}{1-x}\right)$$

$$= (e^x - 1)\frac{x}{(1-x)^2}$$

(b)

$a_6 \Rightarrow$ cal coeff $x^6 \cdot 6!$

$$\left(x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots\right)(x + x^1 + x^2 + \dots)((1 + x + x^2 + \dots)$$

$$\Rightarrow 5 + 2 + \frac{1}{2} + \frac{1}{120} + \frac{1}{12}$$

$$= 7\frac{71}{120}$$

$$= \left(\frac{7 \times 120 + 71}{120}\right) \times 6!$$

$$= 5040 + 426$$

$$= 5466$$

$$\begin{array}{r} 42 \\ \times\ 12 \\ \hline 84 \\ 42 \\ \hline 5040 \end{array} +$$

5. express the notation with a graph.

suppose $n=6$.

# of balls in row

is the value after partition

swap the row and column

since the total number of balls is unchanged, it still meets the requirement of total # $= 6$

a_n is with at least 3 elements

$\Downarrow$

b_n max at least 3 elements

we can always find another way to interpret a_n that matches b_n

we find a one-to one correlation between a_n and b_n

$a_n = b_n$

6. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots$

$$e^x - 1 - x = \frac{x^2}{2!} + \frac{x^3}{3!} \cdots$$

$$\frac{e^x - 1 - x}{x} = \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} \cdots$$

$$\left(\frac{e^x - 1 - x}{x}\right)' = \frac{1}{2!} + \frac{2}{3!}x + \frac{3}{4!}x^3 + \cdots$$

$$= F(x)$$

$$\left(\frac{e^x - 1 - x}{x}\right)' \qquad \left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

$$\parallel$$

$$\frac{(e^x - 1)x - (e^x - 1 - x)}{x^2}$$

$$F(x) = \frac{(e^x - 1)x - (e^x - 1 - x)}{x^2}$$

$$F(1) = \frac{e - 1 - (e - 1 - 1)}{1} = 1_{\text{※}}$$