CS5319 Advanced Discrete Structure

Homework 2
Due: October 25, 2021 (11:59pm)
Tutorial: October 26, 2021
Exam 1: November 02, 2021

1. Find the generating function of the sequence $(a_0, a_1, a_2, \ldots)$ where a_r is the number of ways in which the sum r will show when two distinct dice are rolled, with the first one showing even and the second one showing odd.

2. How many different ways are there to color n distinct objects $(n \geq 3)$ using 3 colors if every color must be used at least once?

3. Find the coefficient of x^n in the following expansion

$$\frac{x+1}{x^2 - x - 6}$$

   Hint: Review partial fraction.

4. Express the following sum in closed form:

$$\sum_{n=0}^{\infty} \sum_{m=0}^{n} \binom{n}{m} w^m \frac{x^n}{n!}$$

5. Find the exponential generating function of the sequence

$$(1,\ 1 \times 5,\ 1 \times 5 \times 9,\ 1 \times 5 \times 9 \times 13,\ \ldots,\ 1 \times 5 \times \cdots \times (4r+1),\ \ldots)$$

   Hint: Consider $(1 - ax)^b$, for some integer a and some related fraction b.

6. (No marks) Show that the number of partitions of the integer $2r + k$ into exactly $r + k$ parts is the same for any nonnegative integer k.

   Hint: Study Ferrers diagram.