

CS5319 ADVANCED DISCRETE STRUCTURE

Homework 3

Due: November 22, 2021 (11:59pm)

Exam 2: December 07, 2021

1. A permutation of $\{1, 2, \dots, n\}$ is a *derangement* if integer k is not at the k th position for every k . Let D_n denote the number of derangements among the $n!$ permutations. So, $D_1 = 0$ and $D_2 = 1$.

Example: Suppose that $n = 3$. We see that $(2, 3, 1)$ and $(3, 1, 2)$ are the only two derangements. So, we have $D_3 = 2$.

(a) Show that $D_n = (n - 1)(D_{n-1} + D_{n-2})$ for any $n > 2$.

(b) Hence, or otherwise, show that $D_n = n D_{n-1} + (-1)^n$.

2. Solve the following recurrence relations:

(a) $a_r - 6a_{r-1} + 8a_{r-2} = 2^r$, where $a_0 = 2$ and $a_1 = 10$.

(b) $a_r - 5a_{r-1} + 3a_{r-2} + 9a_{r-3} = 0$, where $a_0 = 7$, $a_1 = 16$, and $a_2 = 65$.

3. Solve the following recurrence relations:

(a) $a_r^2 - 2a_{r-1}^2 = 1$, $a_0 = 2$ (*Hint:* Let $b_r = a_r^2$.)

(b) $a_r^2 - 2a_{r-1} = 0$, $a_0 = 4$ (*Hint:* Define an appropriate b_n as in part (a).)

4. For each of the following recurrence relations, find the generating function where a_r is the coefficient of x^r :

(a) $a_r - 5a_{r-1} + 6a_{r-2} = 0$, $a_0 = 6$ and $a_1 = 30$.

(b) $a_r - 2a_{r-1} - 3a_{r-2} = 4^r + 6$, $a_0 = 20$ and $a_1 = 60$.

5. Based on the definition from the lectures about pattern occurrences, answer the following questions.

(a) Let b_n denote the number of n -bit binary strings in which the pattern 01001 occurs at the n th bit. Find the generating function for (b_0, b_1, b_2, \dots) .

(b) Let c_n denote the number of n -bit binary strings in which the pattern 01001 first occurs at the n th bit. Find the generating function for (c_0, c_1, c_2, \dots) .