5. $C_n = \frac{1}{n+1}\binom{2n}{n}$

$C_p = \frac{1}{p+1}\binom{2p}{p}$

$= \frac{1}{\cancel{p+1}}\left(\frac{(2p)(2p-1)\cdots(p+2)\cancel{(p+1)}}{p\cdot(p-1)\cdots\cdots 1}\right)$

do mod operation

$(C_p \bmod p)$

$= 2\cdot\frac{(p-1)(p-2)(p-3)\cdots\cancel{2}}{1\cdot(\cancel{p}-1)(\cancel{p}-2)\cdots\cancel{2}\cdot 1}$

$= 2$

2、 $53^{723} = (53^{720})\cdot(53)^3$

$\equiv (53^2)^{360}\cdot(53)^3 \pmod{100}$

$\equiv (9)^{360}\cdot 53^3 \pmod{100}$

$\equiv (81)^{180}\cdot 53^3 \pmod{100}$

$\equiv (61)^{90}\cdot 53^3 \pmod{100}$

$\equiv (21)^{45}\cdot 53^3 \pmod{100}$

$= (21)^{44}\cdot 21\cdot 53^3$

$\equiv (41)^{22}\cdot 21\cdot 53^3 \pmod{100}$

$\equiv (81)^{11}\cdot 21\cdot 53^3 \pmod{100}$

$\equiv (61)^5\cdot 81\cdot 21\cdot 53^3 \pmod{100}$

$\equiv \underline{(21)^2\cdot 61\cdot 21}\cdot 53^3\cdot \underline{81} \pmod{100}$

$= \underline{xx}\,\underline{01}\times 53^3$

$= \underline{xxx}\ \underline{77}$

1、 Closure property : positive rational number × positive rational number

= positive rational number

$$x = \frac{q}{p} \quad y = \frac{\beta}{\alpha} \quad p, q, \alpha, \beta > 0.$$

$$x \cdot y = \frac{q\beta}{p\alpha} > 0$$

Reserve : $\forall\ p/q \in Q^+$ there exist an unique inverse

$$\frac{q}{p} \Rightarrow p/q \cdot \frac{q}{p} = 1 = \text{identity } e$$

Identity = 1 is the identity

Associative = $x = \frac{q}{p} \quad y = \frac{b}{a} \quad z = \frac{\beta}{\alpha}$

$$\left(\frac{q}{p} \times \frac{b}{a}\right)\frac{\beta}{\alpha} = \frac{qb}{pa} \times \frac{\beta}{\alpha} = \frac{qb\beta}{pa\alpha}$$

$$\frac{q}{p}\left(\frac{b}{a} \times \frac{\beta}{\alpha}\right) = \frac{q}{p} \times \frac{b\beta}{a\alpha} = \frac{qb\beta}{pa\alpha}$$

the same

$\Rightarrow$ 4 properties all match $\Rightarrow$ it is a group.

3.

$779 = 19 \times 41$

$\phi(779) = 18 \times 40 = 720$

$101 \cdot d = 22321 \Rightarrow d = 221$

calculate with 輾轉相除法

$n = 779$

$\phi = 720$

$d = 221$

$e = 101$

$299^{221} \equiv 29 \pmod{779}$

$656^{221} \equiv 41 \pmod{779}$

$280^{221} \equiv 48 \pmod{779}$

$47^{221} \equiv 35 \pmod{779}$

$216^{221} \equiv 30 \pmod{779}$

29 - 41 - 48 - 35 - 30

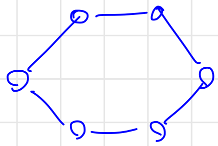
$a = 27$

29 - 41 - 48 - 35 - 30

2 . 14 . 21 . 8 . 3

c - o - v - $\bar{i}$ - d

4.



don't rotate or

rotate clock wise anticlockwise

60° 60°

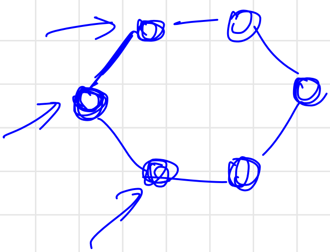
120° 120°

180° ⇒ same ⇐ 180°

⇒ total ⇒ 6 ways

(1) rotate 0° $2^6 \cdot 3^6$

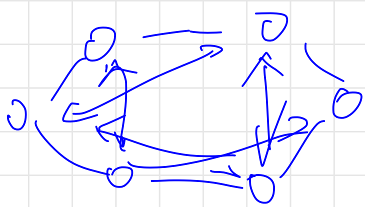
(2) rotate 60 (clock, anticlock same)



adjacent edge / vertex have to be same color

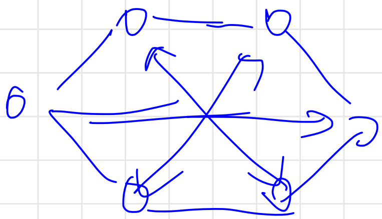
2×3

(3) rotate 120



$2^2 \cdot 3^2$

4. rotate $180°$



$2^3 \cdot 3^3$.

$$\frac{2^6 \cdot 3^6 + 6 \times 2 + 2^2 \cdot 3^2 \times 2 + 2^3 \cdot 3^3}{6}$$

$= 1826$.