

EE 602 (2018) End Semester Examination (Answers)

Q.1 Answer multi-choice questions

(1 X 6=6 Marks)

(i) - (b)

$$\text{PRP} = 1/\text{PRF} = 1/10000 = 100\mu\text{s}$$

$$\text{Average power} = (\text{PW}/\text{PRP}) \times P_t \rightarrow (8/100) (\mu\text{s}) \times 100 (\text{kW}) = 8\text{kW}.$$

(ii)-(c)

After the application of the Barker code, resolution pulse-width is $2\mu\text{s}$
 Resolution = $(c \times \text{XPW})/2 = (3 \times 10^8 \times 2 \times 10^{-6})/2 = 300\text{m}$

(iii)-(d)

$$\text{Noise Power} = 1.38 \times 10^{-23} \times 2900 \times 1000 = 4.002 \times 10^{-17}\text{W}$$

(iv)-(a)

$$F_{eq} = F_1 + \frac{F_2 - 1}{G_1} = 1.8 + \frac{2 - 1}{100} = 1.81$$

(v)-(b)

$$\text{Gain of Omni-antenna}=1. \text{ The power density at range } (R) = \frac{P_t G_t}{4\pi R^2} = \frac{\pi \times 10^3}{4\pi (5000)^2} = 10^{-5} \text{Wm}^{-2}$$

(vi)-(d)

$$10 \log (30\text{mW}/1\text{mW}) = 14.77\text{dBm}.$$

Q.2

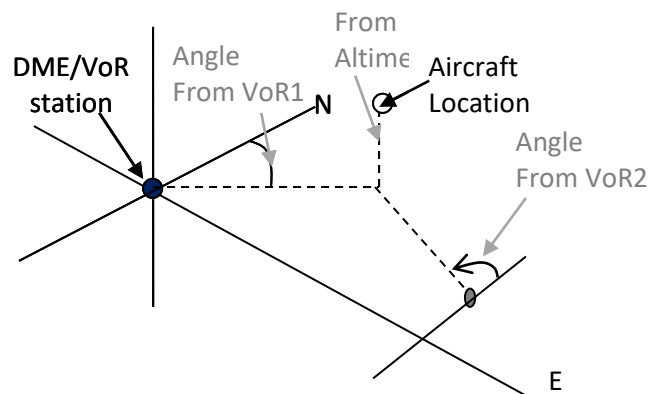
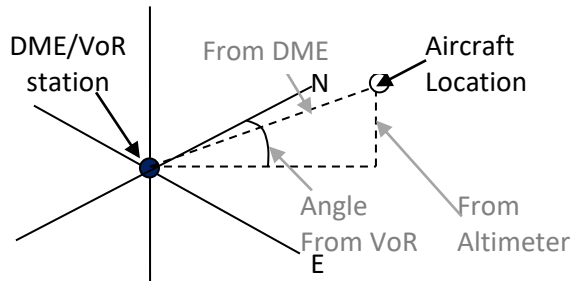
(1 X 6 = 6 Marks)

Provide short (one or two sentences) explanation for the following

(i) True. For Pulse radar, Range Resolution is $c\tau/2 = c/2\Delta F$; also for FMCW radars.

(ii) VoR, DME and altimeter Or 2 VoRs and altimeter Both answers are OK.

(Even one of the two answers without altimeter is OK; 2-D positioning is possible that way!)



- (iii) The particles (or electrons) get accelerated due to the voltage difference between cathode and Anode (or collector). In curvilinear devices, magnetic field adds to the particle (or electron) acceleration thereby giving higher velocity or energy/ power.
- (iv) The radar (under jamming attack) receives these re-transmitted pulses. It perceives these echoes as targets and estimates the target range based on the round-trip time. Thus the radar sees false targets with erroneous estimation of the range as the delays are not related to target(s).
- (v) A CW radar 'down-converts' the echo signals by mixing the transmitted waveform with it. Single frequency CW radar on direct down-conversion shall output DC signal proportional to the cosine of the phase difference between the transmitted and received signal. It is repetitive and will give identical output when the 'the round trip distance' increases by ' λ '. Hence the length of the range interval of unambiguous range estimation is ' $\lambda/2$ '. In other words, this radar can detect the range unambiguously between ' R ' and ' $R + \lambda/2$ ', for any R .
- (vi) Radar transmitters radiate high power (in kilo-Watts or Mega-Watts). For transmitters, the power efficiency is most important parameter as otherwise leads to unreasonably high (uneconomical) power consumption. On the other hand, the receiver is expected to be distortion less, extract the maximum information from the echo. This is only possible if it operates in 'linear mode'. Therefore, the amplifiers are operated in classes having desired properties. (any equivalent argument may be given credit)

Q3

(7 Marks)

(a) Find the unambiguous range and unambiguous velocity for each of the PRF?

Unambiguous range and Velocity $R_{un} = \frac{cT_p}{2} = \frac{c}{2f_p}$ $v_{un} = \frac{(PRF)c}{4f}$ Respectively

PRF of 10 KHz: $R_{un} = 15$ km. $V_{un} = \pm 50 \text{ms}^{-1}$

PRF of 15 KHz: $R_{un} = 10$ km. $V_{un} = \pm 75 \text{ms}^{-1}$

PRF of 20 KHz: $R_{un} = 7.5$ km. $V_{un} = \pm 100 \text{ms}^{-1}$

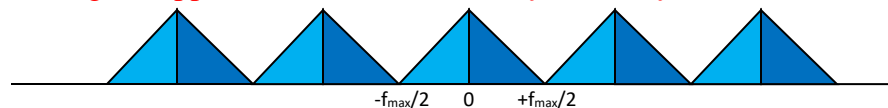
3 marks

(b) Find the correct range and velocity of each of the targets A, C.

Background Info:

Range Ambiguity leads to mapping of $R > R_{un}$ as $R \bmod R_{un}$

Velocity ambiguity can better be explained by following diagram. Frequencies in regions of the same colour get mapped fundamental band of $-f_{max}/2$ to $+f_{max}/2$.



Correct readings:

Target A: 9 km and -20ms^{-1}

{Different range (9 and 1.5 km) readings due to the range ambiguity}

Target C: 5km and -80ms^{-1} : {Different velocity readings of 20, 70 and -80 ms^{-1} are due to ambiguity. The correct value is more than unambiguous range for PRF1 and PRF2,

(One mark for each correct answer,

(1 X 4 = 4)

Q4

(7 Marks)

(a) Beam widths when pointed at Zenith

The antenna dimensions are $3.5\lambda \times 4.5\lambda$.

Hence the beam width in X-Z plane is $(70/3.5) = 20^\circ$

And beam width in Y-Z plane is $(70/4.5) = 15.556^\circ$ (1X2=2)

(b) -10 degree tilt.

Phase to the element at (0,0) is taken as reference = 0° .

Phase shift element at (1,0) = $(d \times \sin 10^\circ) / \lambda \times 360 = 1 \times 0.173648 / 2 \times 360 = 31.2566^\circ$

This phase will be lagging with respect to element at (0,0).

Phase shift element at (0,-2): This element is on Y axis and the beam tilt is in X-Z plane.

So will have same phase shift as (0,0) = 0° . (1X2=2)

(c) -20 degree tilt

Phase shift element at (-1,0) = $(d \times \sin 20^\circ) / \lambda \times 360 = 1 \times 0.342 / 2 \times 360 = 61.5636^\circ$

This phase will be leading with respect to element at (0,0).

Phase shift element at (2,2): = $(2d \times \sin 20^\circ) / \lambda \times 360 = 2 \times 0.342 / 2 \times 360 = 123.127^\circ$

This phase will be lagging with respect to element at (0,0).

Phase shift element at (-3,-2) = $(3d \times \sin 20^\circ) / \lambda \times 360 = 3 \times 0.342 / 2 \times 360 = 184.68^\circ$

This phase will be leading with respect to element at (0,0). (1X3=3)

Q5

(7 Marks)

(a) Matched filter is a 'Linear Time Invariant (LTI)' filter

Whose impulse response is the time reversed waveform of the transmit pulse.

OR Which correlates the received signal with replica of the transmit signal.

OR Mathematically, the impulse response is $h(t) = \frac{2K}{N_0} [s(t_0 - t)]^*$

(Where, K is any arbitrary constant and N_0 is the noise power)

OR the transfer function is

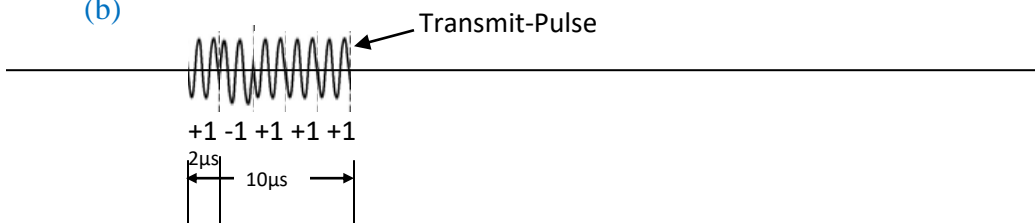
$$H(f) = \frac{2K}{N_0} S^*(f) e^{-j\omega t_0}$$

Any of the answers or equivalent is correct (give credit even if LTI is not mentioned) (1)

It maximizes the signal SNR

→ Thereby maximizing the 'probability of detection' / detectability (1)

(b)



(1)

(c) Range resolution will be 300 ($= c\tau/2 = 3 \times 10^8 \times 2 \times 10^{-6} / 2 = 300\text{m}$) (1)

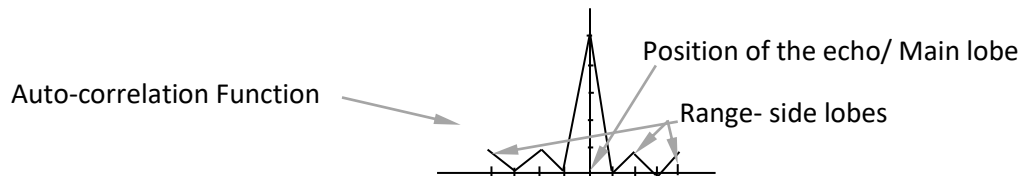
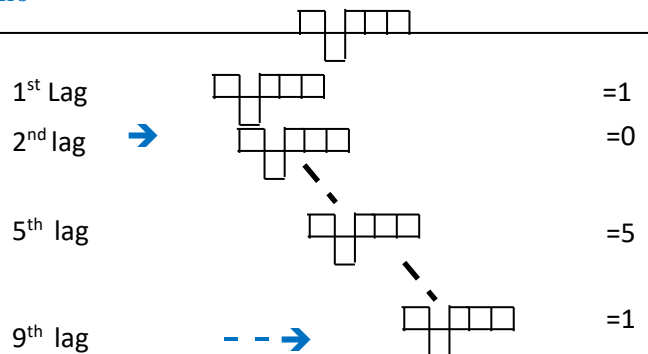
(d) This radar cannot process the echoes arriving before completing the transmit pulse.
Transmit pulse lasts for $10\mu\text{s}$.

$$(2 \times \text{range}/c = 10 \mu\text{s} \rightarrow \text{Range} = 10 \times 10^{-6} \times 3 \times 10^8 / 2 = 1500\text{m})$$

Hence, it corresponds to the range of 1500 m or 1.5 km. (1)

(e) Received Echo

Correlation template slides:



(1)

(f) Magnitude ratio 5: 1. (1)

Expressing it in dB, for magnitude, {use $20 \log(\text{ratio})$ }

$$20 \log 5 = 13.979 \text{ dB} \approx 14 \text{ dB}.$$

Q6

(7 Marks)

(a) What is the gain of the antenna? What will be the beam-width?

$$\text{Antenna gain} = 4 \pi A / \lambda^2 = 4 \pi (7 \times 7) / (0.5)^2 = 2463 \approx 33.91 \text{ dB} \quad (1)$$

$$\text{Beam width} = 70 D / \lambda = 5^\circ = 0.0873 \text{ radian}$$

$$\text{OR using expression for Gain} = 4 \pi / \Delta\theta \Delta\phi \rightarrow \Delta\theta = \Delta\phi = 0.0714 \text{ rad} = 4.0924^\circ \quad (1)$$

(Both answers may be given full credit)

(b) What will be the 'radar cross-section' of the 'air-mass' in the target at 5 km? (1)

The value of $c\tau/2 = 150\text{m}$

$$\text{The air: } \sigma = \eta \times \text{mass volume} = \eta \cdot \frac{c\tau}{2} \frac{\pi R \Delta\phi R \Delta\theta}{4}$$

(Volume of the cylinder, cylindrical shaped approximation, shown in diagram)

$$= 1 \times 10^{-17} \times 150 \times \pi (0.0714)^2 \times 25 \times 10^6 \times 0.25 = 1.5014 \times 10^{-10} \text{m}^2$$

$$\text{OR using other BW value} \quad = 1 \times 10^{-17} \times 150 \times \pi (0.0873)^2 \times 25 \times 10^6 \times 0.25 = 2.245 \times 10^{-10} \text{m}^2$$

$$\begin{aligned}
 \text{OR} \quad &= \eta \cdot \frac{c\tau}{2} R \Delta\phi R \Delta\theta \text{ (cuboids shaped approximation)} \\
 &= 1 \times 10^{-17} \times 150 \times (0.0714)^2 \times 25 \times 10^6 = 1.912 \times 10^{-10} \text{m}^2 \\
 &= 1 \times 10^{-17} \times 150 \times (0.0873)^2 \times 25 \times 10^6 = 2.858 \times 10^{-10} \text{m}^2
 \end{aligned}$$

(Any of the answers may be given full credit)

- (c) Calculate the power density incident on the radar antenna in terms of 'R'; (1)
And with target at 5 km. (1)

The total loss (L) = 2 X (antenna feed network loss) + (Duplexer & Blanking Switch loss)
= -3 dB (or a factor of 0.5; as we are putting in numerator)

The echo power density (EPD) on the receive antenna is given by....

$$EPD = \frac{P_t G_t}{4\pi R^2} \cdot \frac{c\tau}{2} \cdot \frac{\pi R^2 \Delta\theta \Delta\phi}{4} \eta \frac{L}{4\pi R^2} = \frac{P_t}{4} \cdot \frac{c\tau}{2} \cdot \frac{1}{4} \eta \frac{L}{R^2} = \frac{20 \times 10^3 \times 150 \times 10^{-17} \times 0.5}{16 \times R^2} = 9.375 \times 10^{-13}$$

Echo Power density (on receive antenna) = $9.375 \times 10^{-13} \text{ W.m}^{-2} \cdot \text{R}^{-2}$ (in terms of R)

(OR With cuboid assumption, This answer will be 1.194×10^{-12} .)

$$\begin{aligned}
 \text{Power density with target at 5km} &= 3.75 \times 10^{-20} \text{ W.m}^{-2} = -194.26 \text{ dBWm}^{-2} \\
 &= -164.26 \text{ dBWm}^{-2}.
 \end{aligned}$$

(OR With cuboid assumption, This answer will be $4.775 \times 10^{-20} = -193.21 \text{ dBWm}^{-2}$)

- (d) Find the received power received from target range-bin at 5km. (1)

Power received = Power density X Effective area of the antenna

$$\begin{aligned}
 &= 3.75 \times 10^{-20} \text{ W.m}^{-2} \times 49 \text{m}^2 = 1.8375 \times 10^{-18} \text{ W} \\
 &= -177.36 \text{ dBW} = -147.36 \text{ dBm}
 \end{aligned}$$

(OR With cuboid assumption, This answer will be $2.34 \times 10^{-18} = -176.3 \text{ dBW} = -146.3 \text{ dBm}$)

- (e) Knowing the sensitivity, compute the range of the radar (1)

Received power is inversely proportional to R^2 . We compute received power in terms of R.

Sensitivity of -150 dBm = 10^{-15} mW or 10^{-18} W

$$\text{Received power (in terms of R)} = 9.375 \times 10^{-13} \text{ W.m}^{-2} \cdot \text{R}^{-2} \times 49 \text{m}^2 = 4.59375 \times 10^{-11} \text{ W} \cdot \text{R}^{-2}$$

(OR With cuboid assumption, we get = $5.849 \times 10^{-11} \text{ WR}^{-2}$)

$$\begin{aligned}
 R_{\max}^2 &= 4.59375 \times 10^{-11} / 10^{-18} = \text{Received power (in terms of R) / sensitivity} \\
 &= 4.59375 \times 10^7
 \end{aligned}$$

$$R_{\max} = 6736 \text{ m} = 6.736 \text{ km}.$$

(OR With cuboid assumption, we get $R_{\max} = 7600.76 \text{m}$)