EE 602: Mid Semester Examination

Date: 27th February 2018 **Total marks: 25**

Q. 1 (1 X 5 = 5 marks)

(a) Find the power density at a target situated at a distance of 50 km from a radar radiating a power of 100 MW from a lossless isotropic antenna.

(b) If this radar now employs a lossless antenna with a gain of 5000 and the target has a radar cross-section of 1.2 m^2, then what is the power density of the echo signal at the receiver?

(c) If the minimum detectable signal of the radar is 10^{-8} MW and the wavelength of the transmitted energy is 0.02 m, then what is the maximum range at which the radar can detect targets of the kind mentioned in (b)?

(d) What is the effective area of the receiving antenna?

(e) Suppose, due to some modifications made in the radar system components, the antenna gain is doubled while keeping the antenna effective aperture constant. Find the new radar range.

Q. 2 (2 X 3 = 6 marks)

(a) A target is closing on a radial of a radar site (travelling radially towards the radar) with a relative velocity of 200 knots. The radar transmits continuous wave energy at a wavelength of 5 cm. What will the Doppler shift of the target be? What will the Doppler shift be if the target alters its course by 45^o? (1 Knot = 1 nautical mile·hr^{-1} and 1 nautical mile= 1852 m)

(b) If the receiver has a receiver sensitivity of -109 dBm, what is the value of the minimum discernible signal (S_{min}) in watts? If the noise factor of 2. Thermal noise is the only 'non-signal' input to the receiver. Assume that the signal with SNR of 5 dB gets barely detected. What is the bandwidth of the receiver? (Boltzman's constant= 1.38 X 10^{-23})

(c) Increasing the transmitter power of a radar by a factor of 5 will increase the maximum range by what percent? What will be percent decrease in range if receiver input cable loss increases by 6 dB?

Q. 3 (5 marks)

Ku band scanning radar operates at 15 GHz. It scans a solid angle Ω = 2 sr (Steradian) by covering the scan volume by sequential spots of radar beam. The radar acquires the echoes from three pulse for each beam spot; total number of beam spots being $\Omega/d\theta d\phi$, where $d\theta$ and $d\phi$ being the widths of radar beam in orthogonal directions. Total time required for all beam spots, scan time is T_{SC} = 3 seconds

Compute the power aperture product (Average transmitted power $\mathbf{P_{average}}\times$ effective antenna area, **Ae**) for following parameters: signal-to-noise ratio SNR = 10 dB; losses L = 9 dB; effective noise temperature T_e = 1000 degree Kelvin; noise figure F = 4 dB. Assume target cross section of 11 dBm^2 and range R = 150 km.

Also, compute the peak transmitted power corresponding to 10% duty factor, if the effective antenna area is 5 dBm^2.

Q. 4 (7 marks)

FMCW radar altimeter is used to measure the height of a flying platform. The antennas of the radar is pointed towards the ground. The operating frequency range is 2.925-3.075 GHz. This system measures the heights from 10 m to 100 m.

(a) Design the linear chirp transmit wave and the processing scheme so that the beat frequency is in the range of LF frequency band (30 kHz to 300 kHz). Draw an appropriate diagram illustrating the design. Write the expression to estimate the height. What will be height resolution?

(b) What will be the sweep time?

(c) If the distance estimation is performed by adjusting sweep rate to get 50 ± 0.5 kHz. What is the minimum and maximum sweep time?

(d) What will be the advantage of the of the beat frequency detection method by changing the sweep rate? What will be the disadvantage?

(e) Give approximate quantification of these advantage and disadvantage. (T he signal advantages in ratio of sensitivity/ signal power and the time advantage should be quoted in the approximate ratio of processing time. Make convenient assumptions)

Q. 5 (1 X 2 = 2 marks)

A Pulsed Doppler radar is operating at 3 GHz.

(a) What will be maximum unambiguous range if the radar is operating at PRF of 1 KHz. ?

(b) What will be the velocity resolution of this radar if the echoes from 1000 pulses are analyzed for frequency estimation?

Answers (and guidelines for the evaluation)

(a) Find the power density at a target situated at a distance of 50km from a radar radiating a power of 100 MW from a lossless isotropic antenna.

(b) If this radar now employs a lossless antenna with a gain of 5000 and the target has a radar cross-section of 1.2 m2, then what is the power density of the echo signal at the receiver?

(c) If the minimum detectable signal of the radar is 10^{-8}mW and the wavelength of the transmitted energy is 0.02 m, then what is the maximum range at which the radar can detect targets of the kind mentioned in (b)?

(d) What is the effective area of the receiving antenna?

(e) Suppose, due to some modifications made in the radar system components (design, frequency etc), the antenna gain is doubled while keeping the antenna effective aperture constant. Find the new radar range.

1 (1X 5= 5 marks)

(a) $PowerDensity = \frac{P_t}{4\pi R^2} = \frac{100\times 10^6}{4\pi \times (50\times 10^3)^2} = 0.3183\times 10^{-2}Wm^{-2}$

(b) $PowerDensity\,\mathrm{Re}\,ceiver = \frac{P_t G}{4\pi R^2}\cdot\frac{\sigma}{4\pi R^2} = \left(\frac{100\times 10^6}{4\pi \times (50\times 10^3)^2}\right)\frac{5000\times 1.2}{4\pi\times(50\times 10^3)^2} = 6.079\times 10^{-10}Wm^{-2}$

(c) $R_{\max} = \left[\frac{P_t G^2\lambda^2\sigma}{(4\pi)^3 S_{\min}}\right]^{0.25} = \left[\frac{100\times 10^6\times(5000)^2\times(0.02)^2\times 1.2}{(4\pi)^3\times 10^{-11}}\right]^{0.25} = 88.1836km$

The approximate calculation in dB would lead to ➔ (80+74-34+0.8-33+110)/4= 49.45

dB(meter) =88.104

(d) $A_e = \frac{G\lambda^2}{4\pi} = \frac{5000\times(0.02)^2}{4\pi} = 0.159m^2$

Doing approximate dB calculations, we get ➔: 37-34-11=-8 dBm^2.= 0.1584m^2

(e) Doubling gain keeping effective antenna aperture same is possible only by changing the frequency. As the gain is proportional to λ^{-2}, the wavelength will be reduced by the factor $2^{0.5}$. Substituting the new gain and wavelength values, we get Range increase by $2^{0.25}$. => R_{max}= 104.8 km

By normal approach, the students would write expression ias in (c), and substitute new values. However, intelligent approach would be as follows:

(Full credit may be given for these following approaches also)

Gain is doubled (+3dB) if λ is reduced by a factor of squre root 2 (1.4142) Hence the expression of the gain in (c) would get multiplied by 2. Hence the range gets multiplied by $2^{0.25}$.

88.1836 X 1.1892=104.868

Approximate dB calculations, R_{maxold}(in dB(meter))+(6 (for gain)-3)/4=50.3dB(meter)=107.151

2 (a) 2X 3= 6 Marks

1 knot = 1852m/hr= 0.5144 m/s,

Therefore, 200 kts = 102.8888 m/s or 10288.88cm/s (conversion 0.5)

$f_{Doppler}$ for 200 kts = 2 X 10288.88cm / 5cm = **4115.552 Hz** (correct computation 0.5)

For a course change of 45^o, the velocity component in radial direction is
10288.88 cos(45^o) =7275.3368cm/s (0.5)
Frequency shift = 2(7275.3368cm/s)/(5 cm) = **2910.13 Hz** (0.5)
(Direct division by sqrt2 to Doppler shift also gets 1)

(b) S_{min} (watts) = $10^{(-109/10)}$ mW = 1.2589 X 10^{-11} mW (0.5)
= **1.2589 X 10^{-14} Watts** = 12.589 femto Watts. (0.5)
Thermal Noise power (dB) = S_{min} (dB) - SNR min
= **-114 dBm** = 3.98107 X 10^{-12}mW (0.5)
3.98107 X 10^{-12}mW = kTB= 1.38 X 10^{-23} X 290 X Bandwidth
=> Bandwidth ≈ **1 MHz**. (0.5)
(The noise factor/figure was given to mislead, however, people who have taken 'Output SNR' and computed the following may be given full credit)
Thermal Noise power (dB) = S_{min} (dB) - SNR min- Noise figure (in dB)/ factor
= **-117 dBm** = 1.99526 X 10^{-12}mW (0.5)
1.99526 X 10^{-12}mW = kTB= 1.38 X 10^{-23} X 290 X Bandwidth
=> Bandwidth≈ **500 kHz**.

(c) Range is proportional to the 4^{th} root of P_t. If Pt is increased by a factor of 5 the range will be increased by a factor of ($5^{0.25}$) = 1.4953. Approximate increase by **49 %** (1)
6dB loss would decrease to ($10^{-0.15}$)= 0.7079 approximately **30%** (29.21%) decrease. (1)
The students who have calculated second part including both the effects should also be given full credit.
i.e., 1.4953 X 0.7079 =1.058522 (increase by **5.8 %**)

3. We need to derive the equation for the relation between SNR and power aperture product

$$\left(\frac{P_r}{Noise}\right) = SNR = \frac{P_t G_t \sigma A_e}{(4\pi)^2 R^4 kT_e BLF} = \frac{T_{PRP} P_{av} G_t \sigma A_e}{\tau(4\pi)^2 R^4 kT_e BLF}$$

We have

T_{sc}= nT_{PRF} X *No. of beam spots*= (Ω/dθdϕ) Where dθ and dϕ are the beam-widths in orthogonal direct ions. We also know that Gain (G=4π/ dθdϕ) and B=(1/τ) and *n* is the number of pulses per spot. The expression becomes…

$$SNR = \frac{T_{PRP} P_{av} G \sigma A_e}{\tau(4\pi)^2 R^4 kT_e BLF} = \frac{T_{sc} P_{av} \sigma A_e}{n4\pi R^4 kT_e LF\Omega}$$

Re-arranging and substituting, we have

$$P_{av} A_e = \frac{n4\pi R^4 kT_e LF\Omega \times SNR}{T_{sc}\sigma}$$

(2 marks)

Computing in dB terms

Power Aperture Product= 5+11+207-198.62+9+4+3+10-5-11=34.38 dBWm^2=2741.57 Wm^2.

(1mark)

The calculation with actual values (not in dB) to get answer may also be given full credit.

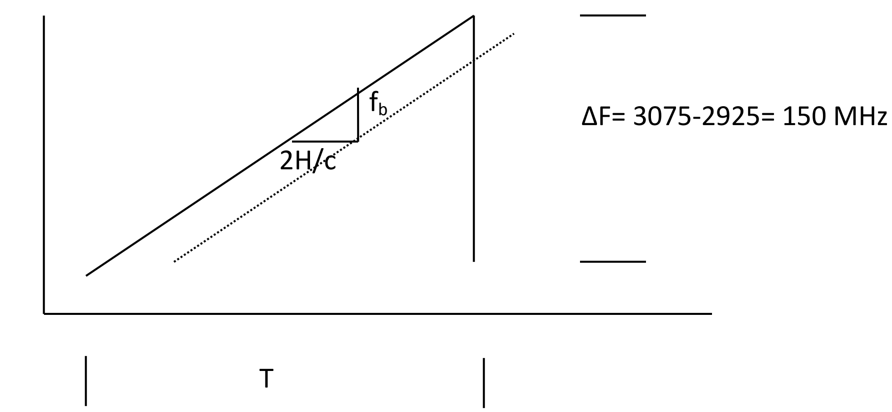
Now, computing Peak Transmitted power

Effective area is 5 dbm^2= 3 m^2, Average power P_{av}=2741.57/3 = 913.86

With 10% duty, we have P_t=913.86/0.1= 9138.6 W= 9.138 kW

(2 Marks)

4.(a)



$$H\left(\frac{2\Delta F}{cT}\right) = H\left(\frac{2\times 1.5\times 10^8}{3\times 10^8 \times T}\right) = f_b$$

Diagram and expression … (1 Mark)

For 10 m, f_b will be minimum= 30 kHz. For that condition, **T= 0.3334 ms. Or 333.4 µs.**

For 100m the f_b will be 300 kHz. (1mark)

----- Any equivalent expression of understanding may be given credit.

Hence the signal processing will estimate height by the expression. **H= f_b / 3000**

As the signal is observed for 1/3 ms. The **frequency resolution will be 3 KHz.**

Hence the height **resolution will be 1m.**

(Check whether resolution was specifically asked?)

(a) The sweep time is, **T= 0.3334 ms**

(1marks)

c) In order to get a fixed $\mathbf{f_b}$ of 50 kHz,

The sweep time corresponding to 10 m will be **200 µs** (Minimum)

And that corresponding to 100m will be **2 ms** (Maximum). (1 mark)

d) Advantage less bandwidth less noise

Disadvantage more processing time (1 mark)

e) Method of Changing sweep time requires only 1 KHs of bandwidth.

Whereas the classical scheme requires 270 KHz

(Bandwidth / noise power **advantage by a factor of 270**) (0.5 mark)

In order to cover entire sweep range, the radar must transmit different frequency sweeps one after another.

Considering the capture bandwidth is ±1 %. Therefore a specific frequency sweep would cover height slice of ±2 % (Total of 10 %). Therefore, subsequent sweep time must have incremented by 2%. The frequency sweep times will form a geometric series of ratio 1.02 (reciprocal≈0.98). Approximately 25 sweeps would cover complete height range (10times).

$(1.02)^y = 10$ ➔ y=117

The total time is calculated by the sum of geometric series formula.

Approximate time for all the sweeps, the total time = 2 {(1-0.1)/(1-0.98)}= **90 ms**

But depending on the height, different number of sweeps are used.

Additionally, for the adjustment time will be for one-two sweeps.

The processing time for conventional is 334µs (computation time is negligible/ neglected)

The processing time for variable sweep method is 90,000 µs

90,000/334 ≈270

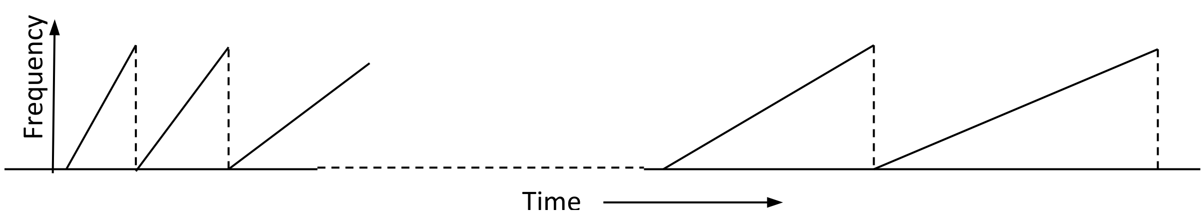
Capture bandwidth reduce by 270 and calculation time increases by 270.

This or any thinking on this line (1.5 mark)

Some background info is given below(Blue font) for reference:

The range estimation is using linear chirp signal can also be done by varying the frequency sweep rate as shown below. In this method, a narrow band receiver is used for the processing the beat frequency signal which can process narrow band beat frequency signals (say Δf_b). Due to this the echoes from a small range slice (ΔH, corresponding to Δf_b) can be processed from one sweep rate. To cover the desired height range the FM chirp signals at different chirp rates are transmitted one after another. The process stops when beat frequency in the range Δf_b is received. The sweep time is further adjusted so that the exact center frequency is obtained. Thus the sweep time is indicative of the range.

$$H\left(\frac{2\Delta F}{cT}\right) = f_b \quad \Longrightarrow \quad \Delta H\left(\frac{2\Delta F}{cT}\right) = \Delta f_b$$



5.

(a)Max unambiguous range is c X PRP/2= **150 km.** (1 mark)

(b) The observation time of 1 ms X 1000= 1 s. Hence the frequency estimation will be with a resolution of 1 Hz.

Doppler frequency resolution = 2 ΔV/0.1=1 Hz.

Corresponding frequency resolution is = **0.05 ms^{-1}.** (1 mark)