Quadratic Voting Meets Groups

Mechanism Design

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Overview

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Motivation for QV

"Perhaps the basic problem underlying existing collective decision procedures is that they rely on the **principle of rationing** (viz., every individual is rationed a single vote on each political contest or issue) rather than on the market **principle of trade** (viz., individuals can exchange influence on issues less important to them for influence on those more important to them)." [Posner and Weyl, 2017]

Equality or Intensity

"As Hirschman (1982, p. 104) put it,

> *"the 'one man one vote' rule gives everyone a minimum share in public decision-making, but it also sets ... a maximum or ceiling; for example, it does not permit citizens to register the different intensities with which they hold their respective political convictions and opinions."*

QV, by allowing individuals truthfully to express this intensity, expands the freedom of individuals to participate in the political process. Even if this added freedom may be outweighed by the lesser equality allowed by QV, it seems to us a basic error to exclude it from the balance" [Posner and Weyl, 2017]

Lalley and Weyl's Original Model

- All voters are voting on single proposal. They can either vote for or against said proposal.
- Without loss of generality, assume that a vote for the proposal is positive, and a vote against the proposal is negative.
- Each voter i chooses $v_i \in \mathbb{R}$ number of votes to buy, paying v_i^2 dollars for these votes. Let u_i be the utility voter i receives if the vote is in her favor and 0 if the vote does not go in her favor. Thus the payoff for voter i is:

$$\Psi(V)u_i, \text{ where } V = \sum_{i=1}^{N} v_i \tag{1}$$

- V is the vote total. They specify a highly specific form for $\Psi(V)$, and describe it "as the **payoff function**, because it determines the quantity by which values u_i are multiplied to obtain the allocative component of each individuals utility."[Lalley and Weyl, 2014]

Lalley and Weyl's Original Model

Thus, in a *type-symmetric Bayes-Nash equilibrium*, a voter will be maximizing

$$E[u\Psi(V_{-1}+v)] - v^2 \tag{2}$$

where $V_{-1} := \sum_{i \neq 1} v_i$ is termed the *one-out vote total*, or all the votes except for the votes of one individual.

Core Rationale of QV

Differentiating on v we get the following first order condition:

$$uE[\psi(V_{-1}+v)] = 2v \Rightarrow v(u) = \underbrace{\frac{E[\psi(V_{-1}+v)]}{2}}_{\text{marginal pivotality}} u \tag{3}$$

The marginal benefit of an additional vote is thus the vote's marginal pivotality, or the amount her vote increases the probability of an alternative being adopted, multiplied by her value.

Core Rational of QV

- As N grows large, most voters will realize that their vote will have an almost inconsequential impact on the vote total
- Assuming that there are a limited number of extreme voters, a large number of voters can reach a consensus marginal pivotality p that is constant throughout the population.

Thus the number of votes a voter buys given a utility should satisfy the proportionality rule, where:

$$v(u) = pu \tag{4}$$

Moreover, they argue that this implies QV is efficient, as then the vote total would be $V = p \sum_i u_i$. *Theoretically, since the vote total is proportional to the sum of values, the vote would always result in an efficient outcome.*

Assume Groups, Not Individuals

A key assumption of the QV mechanism is an independent symmetric private values environment.

I posit a few key empirical observations:

- People tend to vote in groups
- In divisive collective decisions, voters in each group are largely aligned in interest
- There is information sharing and coordination within groups, an issue that Weyl notes for future research[1]

[1] *"...Third, none of my analyses allowed for endogenous information acquisition or pre-election communication among voters. Such factors are crucial to how real-world elections operate." [Weyl, 2017]*

Full Cooperation Model: Assumptions

- Assume that there are only two groups in the company, groups $i \in \{a, b\}$.
- Assume full cooperation within each group, or that each group can collectively agree on a total number of votes to be purchased, and that each group can ensure each members purchases the same number of votes.
- Assume that each member in group i receives the same u_i utility if their preferred outcome results.
- Let v_i denote the number of votes an individual in group i purchases, and let $V_i = \sum_{n_i} v_{i_j}$ be the vote total for group i.
- Assume that you minimum vote you can cast as a group is 1.

Individual Voter Payoff

Let the vote total of group $h \in \{a, b\}, h \neq i$. Define function $\Psi(V_i, V_h)$ such that:

$$\Psi_i(V_i, V_h) = \begin{cases} 1, & \text{if } V_i \geq V_h + \delta \\ \frac{V_i - V_h + \delta}{2\delta}, & \text{if } V_h - \delta \leq V_i \leq V_h + \delta \\ 0, & \text{if } V_i \leq V_h - \delta \end{cases}$$

Thus, the payoff for a member in group i is:

$$\Psi_i(V_i, V_h)u_i - \left(\frac{V_i}{n_i}\right)^2 \tag{5}$$

Tyrannical Equilibrium

One group wins the vote with probability 1 and the other group has probability 0 of winning the election. Without loss of generality assume group a wins with probability 1.

To show that δ is a best response for group a, we need to show that

$$u_a - \left(\frac{\delta}{n_a}\right)^2 \geq \Psi(\widetilde{V_a}, 0)u_a - \left(\frac{\widetilde{V_a}}{n_a}\right)^2 \tag{6}$$

To show that 0 is a best response for group b, we need to show that:

$$0 \geq \Psi(\widetilde{V_b}, 0)u_b - \left(\frac{\widetilde{V_b}}{n_b}\right)^2 \tag{7}$$

Conditions for Tryannical Equilbirium

Thus, we derive the following conditions for a tyrannical equilibrium:

$$\frac{n_b^2 u_b}{2} \leq \delta \leq n_a \sqrt{u_a} \tag{8}$$

$$\frac{n_a^2 u_a}{2} \leq \delta \leq n_b \sqrt{u_b} \tag{9}$$

Which thus implies that if both groups are voting:

$$n_a \sqrt{u_a} < \delta < \frac{n_b^2 u_b}{2} \tag{10}$$

$$n_b \sqrt{u_b} < \delta < \frac{n_a^2 u_a}{2} \tag{11}$$

Probabilistic Equilibrium

To find a probabilistic equilibrium we consider when both groups are voting. Solving this out from the perspective of group a we find that:

$$\max_{V_a} \Psi(V_a, V_b)u_a - \left(\frac{V_a}{n_a}\right)^2 \tag{12}$$

$$= \left(\frac{V_a - V_b + \delta}{2\delta}\right)u_a - \left(\frac{V_a}{n_a}\right)^2 \tag{13}$$

Taking the FOC, we find that the optimal total vote count is:

$$\frac{u_a}{2\delta} - \frac{2V_a}{n_a^2} = 0 \tag{14}$$

$$V_a = \frac{n_a^2 u_a}{4\delta} \tag{15}$$

Probabilistic Equilibrium

Thus, wlog of generality, group a has a greater probability of winning if

$$V_a > V_b \Rightarrow n_a^2 u_a > n_b^2 u_b \tag{16}$$

However, the efficient outcome would be where group a wins only if

$$n_a u_a > n_b u_b \tag{17}$$

Numerical Example

Assume $u_a = 5$, $n_a = 2$ and $u_b = 1.6$, $n_b = 5$. Let $\delta = 5$. It would be efficient for a to win:

$$5 \times 2 > 1.6 \times 5 \tag{18}$$

$$V_a = \frac{2^2 \times 5}{4 \times 5} = 1 \tag{19}$$

$$V_b = \frac{5^2 \times 1.6}{4 \times 5} = 2 \tag{20}$$

Thus since $2 > 1$ group b would win.

Implications for QV

Weyl argues that QV is robust to collusion because:

> *While collusion and fraud typically benefit their perpetrators, incentives for unilateral deviation and reactions from other participants force collusive groups to be quite large or fraud to be extremely ambitious in order to be efficacious if the broader population is large. [Weyl, 2017]*

Weyl's Comparative Robustness

Lalley and Weyl not only argue that QV is more efficient than 1p1v mechanisms, but also that it is simpler and more robust than other social choice mechanisms:

> *"in the same environment [large population, iid values] LW consider,* **many of these mechanisms are efficient even in small populations or converge to efficiency more quickly than QV does.** *LW argue that the principal advantage of QV over these alternatives is that it is is* **simpler than these other mechanisms and that its structure suggests that it may also be more robust to a variety of deviations from the baseline model.**"*...While VCG has a number of potential weaknesses, it is widely believed that its* **greatest limitation for collective decision-making is its sensitivity to collusion and fraud (Ausubel and Milgrom 2005).** *[Weyl, 2017]*

Corporate Quadratic Voting: Poison Pill

- A poison pill deters takeovers typically by giving existing shareholders (including management) the right to buy and vote additional shares at reduced price when a takeover is initiated. It was invented in the 1980s in reaction to a wave of corporate takeovers.
- Managers disliked takeovers because they frequently lost their jobs in the resulting reorganization, and argued that takeovers disrupted a firms operations, caused layoffs, and destroyed shareholder value.
- Shareholder activists replied that managers used poison pills to entrench their positions at shareholders expense.
- *How to Make Poison Pills Palatable*, New York Times, [Posner and Weyl, 2013]

Corporate Quadratic Voting

- Quadratic vote buying, or Q.V.B., aggregates shareholders independent judgments as to the advisability of the merger, taking into account the level of intensity with which they care about the outcome. As a result, the takeover can take place only if it maximizes shareholder value.
- By contrast, if a takeover is put to a regular vote, outsiders can exploit the passivity of most shareholders to obtain a controlling stake and thereby to transfer resources from remaining shareholders to themselves.
- For example, if one shareholder buys four votes in favor of a takeover at cost of \$16, and another buys five votes against the takeover at cost of \$25, the takeover would be voted down (5-4). The funds would be paid into the corporate treasury, and hence ultimately distributed back to the shareholders albeit on a per-share basis, so if the shareholders each own one share, they receive \$20.50 each.
- *How to Make Poison Pills Palatable*, New York Times, [Posner and Weyl, 2013]

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